

# Energy hub model documentation

Course: Energy Innovation & Management

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This document contains the details of the energy hub model given as part of the 2nd assignment in the “Energy Innovation & Management” class at ETH Zurich.

## Model requirements

In order to run the energy hub optimization model on your computer (Windows/macOS), you will need to follow the following instructions:

1. Install Python on your computer. You can follow the beginner guide here: <https://www.python.org/>.
2. Install CVXPY on your computer. You can follow the instructions here: <https://www.cvxpy.org/install/index.html>.
3. If you want to use the Mosek solver in CVXPY (optional):
  - (a) Request an academic license and follow the installation instructions here: <https://www.mosek.com/>.
  - (b) Add `import mosek` to your code.

## Model documentation

### Introduction

This document presents the main parts of a simple optimization model based on the energy hub concept [1]. The complete model formulation is given in the file `EnergyHub.py`. The model is written in Python with CVXPY as the optimization environment.

The model considers a series of candidate energy conversion and storage technologies and performs the following tasks:

- Identifies the optimal design of the multi-energy system (MES), *i.e.* selects which technologies need to be installed and what their capacities should be
- Calculates the optimal operation of each technology over a defined horizon
- Minimizes the total system cost or the total CO<sub>2</sub> emissions for the envisioned energy system

The set of candidate technologies considered in this version of the energy hub model is schematically shown as a superstructure in Fig. 1.

In this version, the modeled horizon is one full year composed of 8760 hourly time steps. It is assumed that this year is representative of all years in the lifetime of the MES. Hence, the model calculates the optimal operation of each component of the MES during this representative year.

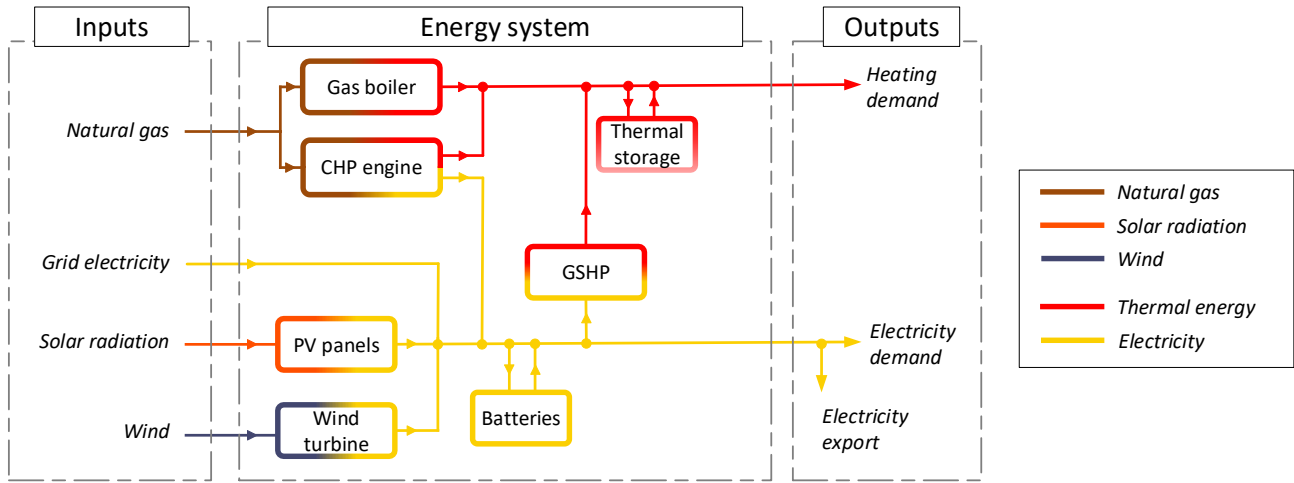


Figure 1: Superstructure representation of a multi-energy system (MES) showing the candidate energy conversion and storage technologies

### Model assumptions

- The costs of the energy networks (thermal, electrical etc.) needed to connect the MES with the end-users are not considered in the cost objective.
- Investments are to happen in one stage at the beginning of the project and no further reinvestment is possible.
- The performance of the energy technologies during part-load operation is not considered. Additionally, no minimum part-load limit during operation is imposed.

### Model parameters

In this section, the input parameters of the model are discussed. These need to be defined by the modeler in order to be used by the model for the optimal design of the energy system.

#### Optimization horizon

- **Horizon** = 8760: Used to define the hourly time steps  $t$  for the representative year

#### Energy demand parameters

- **heat\_demand( $t$ )** (kWh): Heat demand by end-users in time step  $t$
- **elec\_demand( $t$ )** (kWh): Electricity demand by end-users in time step  $t$

#### Renewable energy availability

- **solar( $t$ )** (kWh/m<sup>2</sup>): Incoming solar radiation in time step  $t$
- **wind( $t$ )** (m/s): Wind speed in time step  $t$

### Technical parameters

- **eff\_gb (%)**: The thermal conversion efficiency of the gas boiler
- **eff\_heat\_chp (%)**: The thermal conversion efficiency of the combined heat and power engine
- **eff\_elec\_chp (%)**: The electrical conversion efficiency of the combined heat and power engine
- **eff\_gshp (%)**: The thermal conversion efficiency of the ground-source heat pump
- **eff\_pv (%)**: The electrical conversion efficiency of the photovoltaic panels
- **cut\_out\_wind\_speed (m/s)**: The cut-out wind speed i.e. the speed above which the wind turbine is put to a standstill and does not generate energy for safety reasons
- **cut\_in\_wind\_speed (m/s)**: The cut-in wind speed i.e. the speed at which the turbine first starts to rotate and generate power
- **rated\_wind\_speed (m/s)**: The rated output wind speed i.e. the speed above which the wind turbine output is equal to the rated capacity
- **ch\_eff\_bat (%)**: The charging efficiency of the battery
- **dis\_eff\_bat (%)**: The discharging efficiency of the battery
- **self\_dis\_bat (%)**: The self-discharging losses of the battery
- **ch\_eff\_ts (%)**: The charging efficiency of the thermal storage tank
- **dis\_eff\_ts (%)**: The discharging efficiency of the thermal storage tank
- **self\_dis\_ts (%)**: The self-discharging losses of the thermal storage tank
- **max\_ch\_ts (%)**: Maximum rate of charge of the thermal storage tank (given as a percentage of the tank's capacity)
- **max\_dis\_ts (%)**: Maximum rate of discharge of the thermal storage tank (given as a percentage of the tank's capacity)
- **max\_ch\_bat (%)**: Maximum rate of charge of the battery (given as a percentage of the battery's capacity)
- **max\_dis\_bat (%)**: Maximum rate of discharge of the battery (given as a percentage of the battery's capacity)

### Economic parameters

- **cost\_gb (CHF/kW)**: The investment cost of the gas boiler
- **cost\_chp (CHF/kW<sub>e</sub>)**: The investment cost of the combined heat and power engine
- **cost\_gshp (CHF/kW)**: The investment cost of the ground-source heat pump
- **cost\_pv (CHF/m<sup>2</sup>)**: The investment cost of the photovoltaic panels
- **cost\_wind (CHF/kW)**: The investment cost of wind turbines
- **cost\_bat (CHF/kWh)**: The investment cost of the battery
- **cost\_ts (CHF/kWh)**: The investment cost of the thermal storage tank
- **d (%)**: Discount rate used to discount future operating costs and revenues
- **price\_gas (CHF/kWh)**: The price per imported kWh of natural gas from the natural gas grid

- **price\_elec** (CHF/kWh): The price per imported kWh of electricity from the electricity grid
- **exp\_price\_elec** (CHF/kWh): The compensation that the MES receives for every kWh of electricity that is exported to the electricity grid
- **esc\_gas** (%): The escalation factor of the natural gas price per year
- **esc\_elec** (%): The escalation factor of the electricity price per year
- **esc\_elec\_exp** (%): The escalation factor of the compensation rate for exported electricity per year

### Environmental parameters

- **co2\_gas** (kgCO<sub>2</sub>/kWh): The carbon emission factor per consumed kWh of natural gas
- **co2\_elec** (kgCO<sub>2</sub>/kWh): The carbon emission intensity per imported kWh of electricity from the electrical grid

### Other parameters

- **max\_solar\_area** (m<sup>2</sup>): The maximum available area to accommodate photovoltaic panels
- **max\_wind\_cap** (kW): The maximum possible capacity of wind turbines that can be accommodated

### Model variables

The decision variables of the model and a brief description of each one are given as follows:

#### Design variables

- **Cap\_gb** (kW): The capacity of the gas boiler (gb)
- **Cap\_chp** (kW<sub>e</sub>): The electrical capacity of the combined heat and power (chp) engine
- **Cap\_gshp** (kW): The capacity of the ground-source heat pump (gshp)
- **Cap\_pv** (m<sup>2</sup>): The capacity of the PV (pv) panels in terms of installed panel area
- **Cap\_wind** (kW): The capacity of the wind turbines (wind)
- **Cap\_bat** (kWh): The storage capacity of the battery (bat)
- **Cap\_ts** (kWh): The storage capacity of the thermal storage (ts) tank

#### Operating variables

- **Imp\_gas(t)** (kWh): The imported natural gas in the energy system in time step **t**
- **Imp\_elec(t)** (kWh): The imported grid electricity in the energy system in time step **t**
- **Exp\_elec(t)** (kWh): The exported grid electricity from the system in time step **t**
- **P\_in\_gb(t)** (kWh): The input natural gas stream to a gas boiler (gb) in each time step **t**
- **P\_in\_chp(t)** (kWh): The input natural gas stream to a combined heat and power (chp) engine in each time step **t**
- **P\_in\_gshp(t)** (kWh): The input electricity stream to a ground-source heat pump (gshp) in each time step **t**
- **P\_out\_gb(t)** (kWh): The output thermal energy stream from a gas boiler (gb) in each time step **t**

- $P_{out\_elec\_chp}(t)$  (kWh): The output electrical energy stream from a combined heat and power (chp) engine in each time step  $t$
- $P_{out\_heat\_chp}(t)$  (kWh): The output thermal energy stream from a combined heat and power (chp) engine in each time step  $t$
- $P_{out\_gshp}(t)$  (kWh): The output thermal energy stream from a ground-source heat pump (gshp) in each time step  $t$
- $P_{out\_pv}(t)$  (kWh): The output electrical energy stream from photovoltaic panels (pv) in each time step  $t$
- $P_{out\_wind}(t)$  (kWh): The output electrical energy stream from wind turbines (wind) in each time step  $t$
- $Q_{in\_bat}(t)$  (kWh): The charging energy stream into the battery in time step  $t$
- $Q_{out\_bat}(t)$  (kWh): The discharging energy stream out of the battery in time step  $t$
- $E_{bat}(t)$  (kWh): The amount of energy stored in the battery in time step  $t$
- $Q_{in\_ts}(t)$  (kWh): The charging energy stream into the thermal storage tank in time step  $t$
- $Q_{out\_ts}(t)$  (kWh): The discharging energy stream out of the thermal storage tank in time step  $t$
- $E_{ts}(t)$  (kWh): The amount of energy stored in the thermal storage tank in time step  $t$

## Model constraints

Model constraints define the relationships between the different parameters and variables of the model, as well as the feasible values that each variable can take.

### Input-output relationships

These constraints describe the relationships between the input and the output energy streams in each conversion device of the MES taking into account conversion efficiencies.

- $P_{out\_gb}(t) = P_{in\_gb}(t) * eff\_gb$
- $P_{out\_elec\_chp}(t) = P_{in\_chp}(t) * eff\_elec\_chp$
- $P_{out\_heat\_chp}(t) = P_{in\_chp}(t) * eff\_heat\_chp$
- $P_{out\_gshp}(t) = P_{in\_gshp}(t) * eff\_gshp$
- $P_{out\_pv}(t) = solar(t) * Cap\_pv * eff\_pv$
- $P_{out\_wind}(t) = \begin{cases} Cap\_wind \frac{wind(t) - cut\_in\_wind\_speed}{rated\_wind\_speed - cut\_in\_wind\_speed}, & \begin{matrix} wind(t) > cut\_in\_wind\_speed \text{ and} \\ wind(t) < rated\_wind\_speed \end{matrix} \\ Cap\_wind, & \begin{matrix} wind(t) \geq rated\_wind\_speed \text{ and} \\ wind(t) < cut\_out\_wind\_speed \end{matrix} \\ 0, & \begin{matrix} wind(t) \leq cut\_in\_wind\_speed \text{ or} \\ wind(t) \geq cut\_out\_wind\_speed \end{matrix} \end{cases}$

### Energy storage balances

These constraints describe the relationship between the amount of energy stored in an energy storage technology and the charging and discharging energy flows taking into account self-discharging losses and charging/discharging efficiencies.

- $E_{bat}[1] = 0$
- $E_{bat}(t) = (1 - self\_dis\_bat) * E_{bat}[t-1] + ch\_eff\_bat * Q_{in\_bat}(t) - (1/dis\_eff\_bat) * Q_{out\_bat}(t), \forall t \geq 2$
- $E_{ts}[1] = 0$
- $E_{ts}(t) = (1 - self\_dis\_ts) * E_{ts}[t-1] + ch\_eff\_ts * Q_{in\_ts}(t) - (1/dis\_eff\_ts) * Q_{out\_ts}(t), \forall t \geq 2$

### Non-violation of capacities

These constraints prevent the violation of the conversion and storage capacities during the operation of the energy technologies of the MES.

- $P_{out\_gb}(t) \leq Cap\_gb$
- $P_{out\_elec\_chp}(t) \leq Cap\_chp$
- $P_{out\_gshp}(t) \leq Cap\_gshp$
- $E_{bat}(t) \leq Cap\_bat$
- $E_{ts}(t) \leq Cap\_ts$

### Physical constraints

Constraints are needed for the photovoltaic panels and the wind turbines to define the maximum possible installed area due to roof area availability and maximum turbine capacity, respectively.

- $Cap\_pv \leq max\_solar\_area$
- $Cap\_wind \leq max\_wind\_cap$

### Charging rate constraints

These constraints are needed to ensure that the maximum charging and discharging limits of energy storage technologies are obeyed during the energy system's operation.

- $Q_{in\_ts}(t) \leq max\_ch\_ts * Cap\_ts$
- $Q_{out\_ts}(t) \leq max\_dis\_ts * Cap\_ts$
- $Q_{in\_bat}(t) \leq max\_ch\_bat * Cap\_bat$
- $Q_{out\_bat}(t) \leq max\_dis\_bat * Cap\_bat$

### Energy balances

The first two constraints are perhaps the most important constraints of the model as they ensure that the system can satisfy the heat and electricity demands of the end-users. The third constraint represents the balance of the natural gas flows in the system.

- $P_{\text{out\_gb}}(t) + P_{\text{out\_heat\_chp}}(t) + P_{\text{out\_gshp}}(t) + Q_{\text{out\_ts}}(t) - Q_{\text{in\_ts}}(t) = \text{heat\_demand}(t)$
- $\text{Imp\_elec}(t) + P_{\text{out\_elec\_chp}}(t) + P_{\text{out\_pv}}(t) + P_{\text{out\_wind}}(t) - P_{\text{in\_gshp}} + Q_{\text{out\_bat}}(t) - Q_{\text{in\_bat}}(t) = \text{elec\_demand}(t) + \text{Exp\_elec}(t)$
- $\text{Imp\_gas}(t) - P_{\text{in\_gb}}(t) - P_{\text{in\_chp}}(t) = 0$

### Non-negativity constraints

All defined variables of the model are continuous and positive.

- $\text{Cap\_gb} \geq 0$
- $\text{Cap\_chp} \geq 0$
- $\text{Cap\_gshp} \geq 0$
- $\text{Cap\_pv} \geq 0$
- $\text{Cap\_wind} \geq 0$
- $\text{Cap\_bat} \geq 0$
- $\text{Cap\_ts} \geq 0$
- $\text{Imp\_gas}(t) \geq 0$
- $\text{Imp\_elec}(t) \geq 0$
- $\text{Exp\_elec}(t) \geq 0$
- $P_{\text{in\_gb}}(t) \geq 0$
- $P_{\text{in\_chp}}(t) \geq 0$
- $P_{\text{in\_gshp}}(t) \geq 0$
- $P_{\text{out\_gb}}(t) \geq 0$
- $P_{\text{out\_elec\_chp}}(t) \geq 0$
- $P_{\text{out\_heat\_chp}}(t) \geq 0$
- $P_{\text{out\_gshp}}(t) \geq 0$
- $P_{\text{out\_pv}}(t) \geq 0$
- $P_{\text{out\_wind}}(t) \geq 0$
- $Q_{\text{in\_bat}}(t) \geq 0$
- $Q_{\text{out\_bat}}(t) \geq 0$
- $E_{\text{bat}}(t) \geq 0$
- $Q_{\text{in\_ts}}(t) \geq 0$
- $Q_{\text{out\_ts}}(t) \geq 0$
- $E_{\text{ts}}(t) \geq 0$

## Objective functions

- Total system cost:  $\text{Cost} = \text{Inv} + \sum_{y=1}^{25} \frac{\text{Op}_y}{(1+d)^y}$ 
  - $\text{Inv} = \text{Cap\_gb} * \text{cost\_gb} + \text{Cap\_chp} * \text{cost\_chp} + \text{Cap\_gshp} * \text{cost\_gshp} + \text{Cap\_pv} * \text{cost\_pv} + \text{Cap\_wind} * \text{cost\_wind} + \text{Cap\_bat} * \text{cost\_bat} + \text{Cap\_ts} * \text{cost\_ts}$
  - $\text{Op}_y =$ 
    - +  $\sum_{t=1}^{8760} \left\{ \text{Imp\_gas}(t) * [\text{price\_gas} * (1+\text{esc\_gas}^{(y-1)})] \right\}$
    - +  $\sum_{t=1}^{8760} \left\{ \text{Imp\_elec}(t) * [\text{price\_elec} * (1+\text{esc\_elec}^{(y-1)})] \right\}$
    - $\sum_{t=1}^{8760} \left\{ \text{Exp\_elec}(t) * [\text{exp\_price\_elec} * (1+\text{esc\_elec\_exp}^{(y-1)})] \right\}$
- Total system CO<sub>2</sub> emissions:  $\text{CO2} = 25 * \sum_{t=1}^{8760} \left\{ \text{Imp\_gas}(t) * \text{co2\_gas} + \text{Imp\_elec}(t) * \text{co2\_elec} \right\}$

## Extracting model results

Although extracting the results of the model is more a matter related to Python and CVXPY syntax, a brief note is provided here nonetheless. For the model's decision variables (*e.g.* `Cap_gb`) and objectives (*e.g.* `Cost`), one must use the `value` property of the CVXPY variable. For instance, to display the total cost of the optimal solution, we would need to call `Cost.value`. More info can be found in <https://www.cvxpy.org/examples/index.html#basic-examples>.

## References

- [1] Martin Geidl, Gaudenz Koeppel, Patrick Favre-Perrod, et al. "Energy hubs for the future". In: *IEEE Power and Energy Magazine* 5.1 (Jan. 2007), pp. 24–30. ISSN: 1558-4216. DOI: 10.1109/MPAE.2007.264850.