

# **EECE6086 - HW 3**

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## 1 Objective

The objective of this lab is to implement an algorithm based on the unate recursive paradigm (URP) which uses a heuristic called `BINATE_SELECT` to choose a variable in the recursive Shannon expansion. Therefore, given a cover `F`, we must determine if the cover is a `tautology` or not. If cover `F` is found to not be a `tautology` then we must perform an algorithm (the complement) to find the missing covers in order to make the cover a `tautology`.

## 2 Implementation Details

### 2.1 Tautology Checking

We implemented two main algorithms to perform the tautology checking: the standard heuristic unate reduction method, and a brute force enumeration method which sets a flag for every unique cube that it comes across in the input file. These algorithms are referred to in this project as `Heur` (Heuristic) and `Flags` respectively.

We run these two algorithms in parallel using two separate threads. The main thread then waits for one of the two algorithms to complete after which it terminates the losing algorithms thread. As we will see in the results section the flags algorithm is usually faster in determining if the cover is a `tautology` whereas the heuristic algorithm is faster at determining that the cover is **not** a `tautology`.

#### 2.1.1 Unate Reduction (Heur)

The algorithm starts off by reading the whole input cover file into a matrix, which is implemented as an array of char arrays. The whole matrix is then passed into the recursive `check_tautology(matrix)` function.

We then check for the following three cases

- If there are no rows in the matrix its **not** a `tautology`
- If there is a row of all dashes in the matrix it is a `tautology`.
- If there is only one row left and it is not all dashes its **not** a `tautology`

If our cube list does not fit any of the special cases as described above we then perform unate reduction. During this process we rearrange the cover so that the input matrix is in this form:

$$F = \begin{bmatrix} U & F1 \\ D & F2 \end{bmatrix}$$

Where `U` are the unate columns and `D` is a matrix of all don't cares. In reality we simply mark each column and row that fits into the `F2` quadrant and then allocate a new matrix that just has those values in.

Once we have reduced all the unate columns (if there were any) we again check to see if there are any cubes with all don't cares which would indicate a `tautology`.

We then try to find the most binate column which will tell us the variable that is most dependent, meaning that all other variables depend heavily on this variable. If it turns out there are no binate columns this means this matrix is not a `tautology` so we can return immediately.

After finding the most binate variable, we are now ready to cofactor (Shannon expansion). We do this by following this simple boolean equation:  $F = x * F_x + \bar{x} * \bar{F}_x$ . The two new matrixes are then sent back into `check_tautology(matrix)` and the results checked to see if they are both `tautologies`. If they are both `tautologies` we can say that the original input matrix is also a `tautology`.

All matrixes are stored in a single structure type called `matrix_t` which contains a dynamically allocated array of arrays to hold the variables of each cube. We also store the number of rows and columns so that we can loop through the matrix with known bounds.

### 2.1.2 Cube Enumeration (Flags)

Each cube in the cover file is converted into an integer value which is used to index into an array of flags. For instance, the input cube 1011 would set flag number 11. If the cube has don't cares we recursively create the other cubes. For instance, the input cube 1--0 would first generate 10-0 and 11-0 and then it would recurse sending in those new cubes. Once there are no don't cares in the cube it computes the flag index by treating the cube as a binary number.

We also check for the special case of cubes with don't cares only at the end. For instance, with the cube 10--- we know this covers all flags between 10000 and 10111 so we can skip the recursion and just set that range of flags immediately.

If we find that we have set  $2^n$  new flags, where  $n$  is the number of variables in the cube, we know we have a tautology and we can exit immediately. If we have checked all cubes in the cover file and the number of flags set is less than  $2^n$  we know that we do not have a tautology.

The flags list is represented by a `char` array where each bit represents a flag. This allows for a constant initial allocation of  $(2^n)/8$  bytes, where  $n$  is the number of variables in the cube. While this allocation size scales exponentially, and will not support increasingly large number of variables, for our covers, which have at most 30 variables (resulting in a 134MB array) it is acceptable.

## 2.2 Complement

Like the tautology checker, we use two competing algorithms in order to more quickly find the complement of a given cover file, the standard heuristic method and the enumerated cubes method.

### 2.2.1 Heuristic

The heuristic method is pretty similar to the heuristic method in the tautology checker and starts by reading the whole input cover file into a matrix. It then sends the matrix to the recursive `check_complement(matrix)` function which performs the following initial checks:

- If there is a row of all dashes we return an empty matrix with zero rows
- If there are no rows in the matrix we return a matrix with a single row of all don't cares.
- If there is only one row left we create a new matrix that has the complement.

The complement is performed by assigning each value (other than don't care) to its own row and with the opposite polarity.

```

                                0-----
                                -1-----
10-11-0    =>    ---0---
                                ----0--
                                -----1

```

If none of the above three conditions are satisfied we continue by trying to find the most binate column. If it turns out there are no binate columns we simply choose a unate column. We then positively and negatively `co_factor` our matrix and send it back into `check_complement(matrix)`. Once `check_complement(matrix)` returns we have two new positive and negative matrices. We then need to restore the binate or unate column we chose before. If we had chosen a positive unate column we set all variables in that column of the negative matrix to 0. If we had chosen a negative unate column we set all the variables in that column of the positive matrix to 1. If we had chosen a binate column we do both.

We then concatenate the positive and negative matrices together and return this new matrix.

### 2.2.2 Flags

The flags algorithm, described in the tautology checking section, has the ability to immediately provide us with the missing cubes as soon as it finishes processing all the lines in the cover file. As we will see in the results section, the flags algorithm is always faster than the heuristic algorithm at providing the full list of missing cubes. The main disadvantage, however, is that it does not coalesce adjacent cubes to form don't care variables. For instance the heuristic algorithm might provide two missing cubes as  $0-10$  where as the flags algorithm will provide each cube separately,  $0010$  and  $0100$ . Choosing which algorithm to use is a matter of time space trade off. The flags algorithm will complete faster but return more cubes while the heuristic algorithm will be slower but return much less cubes. If you have the disk space to store the un coalesced cubes the flags algorithm seems to be the way to go.

### 3 Usage

Building is accomplished via a Makefile which generates two separate executables `tc` and `cc`. Additional build options are shown below.

---

```

1 $ make           # build both tautology checker (tc) and complement checker (cc)
2 $ make tc        # only builds tc
3 $ make cc        # only builds cc
4 $ make clean     # removes all binaries, object files, and benchmark results
5 $ make benchmarks # runs all the benchmarks in the 'benchmark' directory
6 $ make pngs      # generate memory and recursion depth plots from the benchmark results

```

---

Both `tc` and `cc` accept 3 optional flags and as well as the input file name. By default both algorithms (heuristic and flags) are run in competition mode and the program will exit as soon as one of them finishes. If you wish to run only the heuristic method or only the flags method you can specify `-h` or `-f` respectively. Additionally, there is a built in memory and recursion depth sampling profiler which will log to `/tmp/mem.log` and `/tmp/dep.log`.

Optional flags and example usage of the tautology checker is shown below.

---

```

1 $ ./tc
2 Usage: ./tc [flags] inputfile
3       -f Only run flags algorithm
4       -h Only run heuristic algorithm
5       -m Log memory and recursion depth usage to /tmp/mem.log and /tmp/dep.log
6
7 $ ./tc benchmarks/Cover3_8_250.txt
8 Using algorithms: flag heur
9 Flags found it
10 Waiting for threads to join
11 Function is a tautology
12
13 $ ./cc benchmarks/Cover2_8_100.txt
14 Using algorithms: flag heur
15 Flags is printing complements
16 00001011
17 00011011
18 00111011
19 01011011
20 01011111
21 01101001
22 01101101
23 11000111
24 Number of missing covers: 8
25 Flags found it
26 Waiting for threads to join

```

---

There is also a `test.sh` script that will run `tc` against all benchmarks in the `./benchmarks/` folder. If `tc` indicates they are not a tautology, `cc` is run to generate the missing covers. It then concatenates the original benchmark and the new covers into a new file, updates the number of rows, and re runs `tc` to ensure that we now have all the missing covers.



```

=====Cover3_8_250===== [3/17]
Is tautology
=====Cover4_15_1000===== [4/17]
NOT a tautology, finding complement...
Using algorithms: flag heur
Flags is printing complements
Number of missing covers: 4463
Flags found it
Waiting for threads to join
Found 4463 /tmp/cc.txt complements
Adding complement to original cover...
Running tc against new cover
Complement is correct!

```

## 4 Results

The tables below summarize the results and performance of our tautology and complement checkers for each of the given benchmarks. The ‘Flags Memory’ column shows the heap allocation for the flags array. This is computed manually as it is static and based off the number of variables per cube. The ‘Heuristic Memory’ column shows the maximum size our algorithm allocated from the heap at once. This was obtained by using our built in memory profiler, which wraps malloc and free and logs all allocations and deallocations to a file. We then looked for time in our execution where our memory was the highest. The ‘Reported RSS Memory’ is the memory usage as reported by the standard Linux tools like `time`, `ps`, or `top`. The RSS memory is actually slightly deceiving as it also includes shared memory. The ‘Faster Algorithm’ column indicates which algorithm found the answer and finished first.

### 4.0.3 TC Performance

We can see from the table below that the flags algorithm seems to always be faster at identifying covers that are tautologies, with the exception of the really smaller cover files where the difference in time between the two algorithms is negligible. We can also see that our memory usage seems to climb pretty high for our heuristic algorithm. Looking at the profile plots on the following pages we can see that for some covers (like Cover 5 and 8) there appears to be a memory leak. Unfortunately we were unable to identify it.

Benchmark	Tautology	Execution Time(s)	Flags Memory	Heuristic Memory	Reported RSS Memory	Faster Algorithm
Cover1.8_10	NO	0.00	32B	-	3.360MB	Heuristic
Cover2.8_100	NO	0.00	32B	9KB	3.472MB	Flags
Cover3.8_250	YES	0.02	32B	22KB	3.808MB	Flags
Cover4.15_1000	NO	0.00	4KB	124KB	4.704MB	Heuristic
Cover5.15_10000	YES	0.24	4KB	1.449MB	14.928MB	Flags
Cover6.15_30000	YES	0.01	4KB	4.084MB	34.080MB	Flags
Cover7.20_10000	NO	0.05	131KB	1.513MB	10.816MB	Heuristic
Cover8.20_100000	YES	21.69	131KB	21.458MB	135.968MB	Flags
Cover9.20_1000000	YES	13.54	131KB	164.462MB	1.025GB	Flags
Cover_25_100000	NO	0.58	4MB	18.207MB	124.608MB	Heuristic
Cover_25_1000000	YES	449.00	4MB	226.092MB	1.320GB	Flags
Cover_25_10000000	YES	480.58	4MB	1.971GB	11.559GB	Flags
Cover_30_1000000	NO	9.38	134MB	210.996MB	1.805GB	Heuristic
Cover_30_10000000	YES	17981.09	134MB	2.636GB	16.566GB	Flags
Cover_30_100000000	YES	18656.40	134MB	14.552GB	125.263GB	Flags

### 4.0.4 CC Performance

From the table below we can see that the flags algorithm dominates when it comes to finding the missing covers fast. Like our `tc` program there appears to be a memory leak that manifests in some covers like Cover 4. Other covers, such as Cover 5 do not seem to cause leaks. We were unable to pinpoint where we were losing this memory.

Benchmark	Execution Time(s)	Flags Memory	Heuristic Memory	Reported RSS Memory	Faster Algorithm
Cover1.8_10	0.00	32B	1.656KB	3.680MB	Flags
Cover2.8_100	0.00	32B	6.679KB	3.808MB	Flags
Cover3.8_250	0.02	32B	15.789KB	3.504MB	Flags
Cover4.15_1000	0.11	4KB	120.775KB	4.640MB	Flags
Cover5.15_10000	0.23	4KB	899.670KB	9.904MB	Flags
Cover6.15_30000	0.28	4KB	2.682MB	22.464MB	Flags
Cover7.20_10000	4.08	131KB	577.593KB	13.536MB	Flags
Cover8.20_100000	11.20	131KB	10.938MB	66.592MB	Flags
Cover9.20_1000000	13.66	131KB	109.235MB	626.864MB	Flags
Cover_25_100000	161.77	4MB	21.771MB	168.016MB	Flags
Cover_25_1000000	452.42	4MB	128.959MB	895.152MB	Flags
Cover_25_10000000	480.53	4MB	1.288GB	8.765GB	Flags
Cover_30_1000000	6269.12	134MB	305.227MB	2.324GB	Flags
Cover_30_10000000	18074.69	134MB	1.484GB	9.277GB	Flags
Cover_30_100000000	19045.47	134MB	6.454GB	88.025GB	Flags

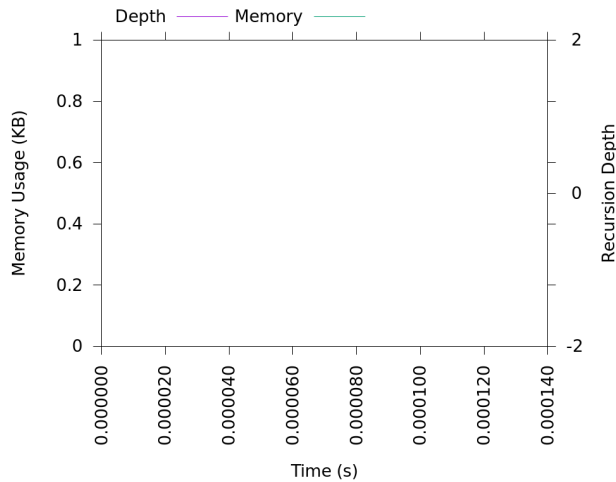


Figure 1: TC: Cover1.8\_10

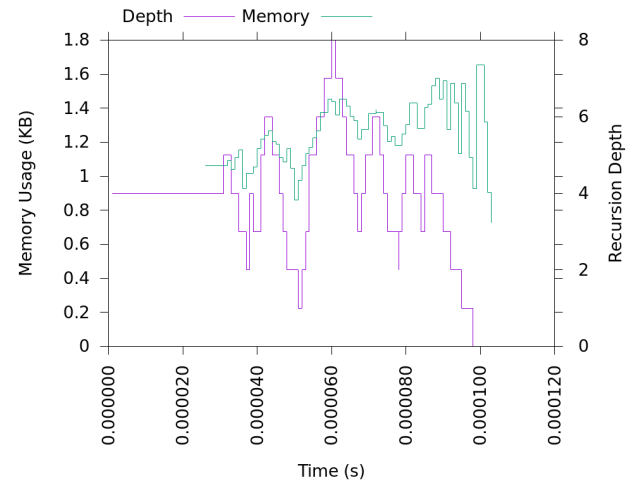


Figure 2: CC: Cover1.8\_10

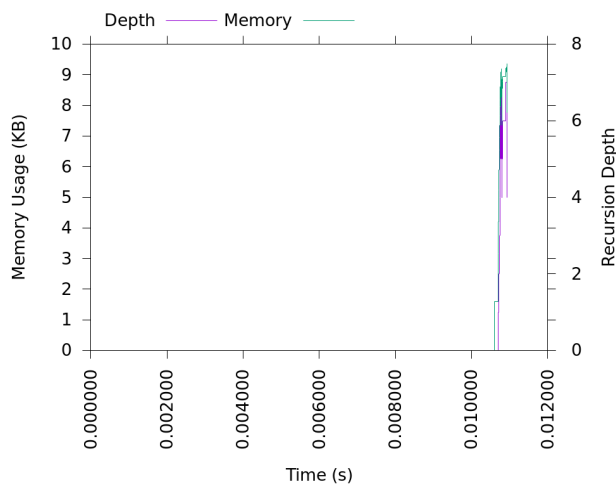


Figure 3: TC: Cover2.8\_100

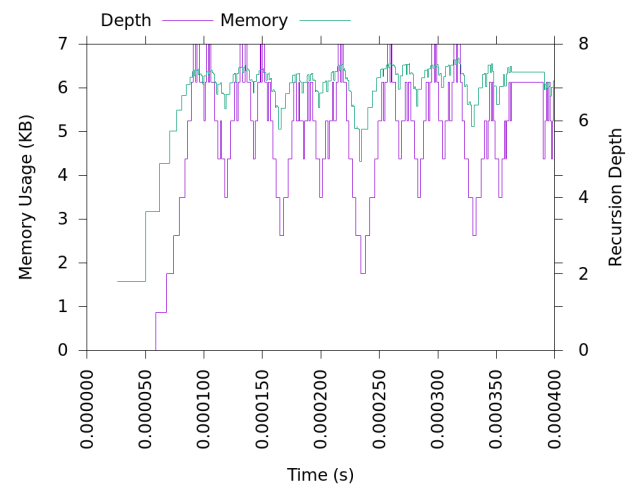


Figure 4: CC: Cover2.8\_100

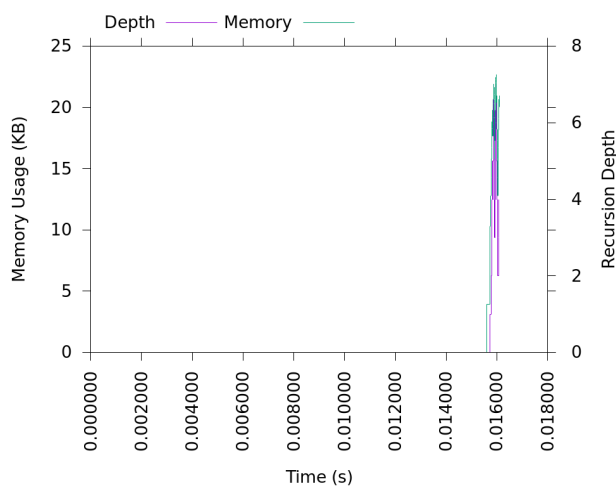


Figure 5: TC: Cover3.8\_250

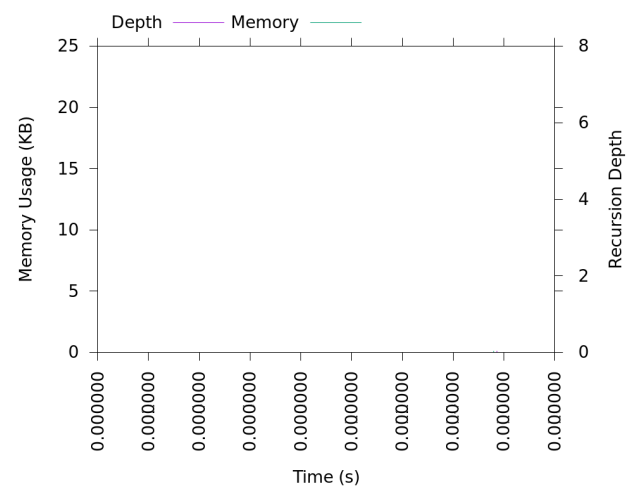


Figure 6: CC: Cover3.8\_250

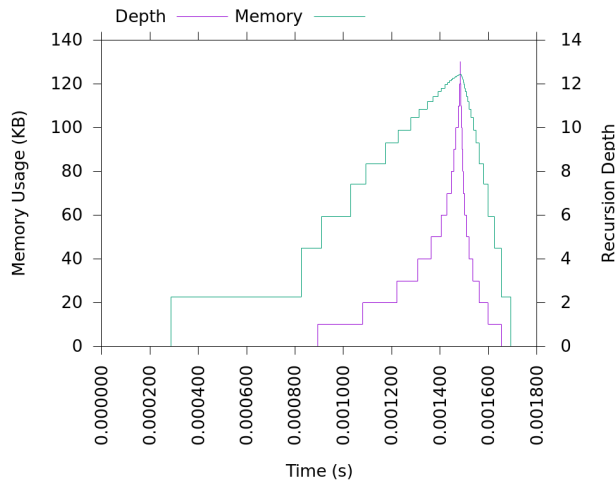


Figure 7: TC: Cover4.15\_1000

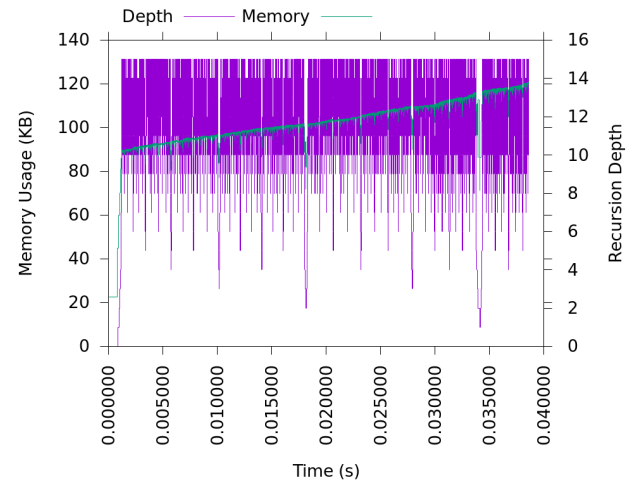


Figure 8: CC: Cover4.15\_1000

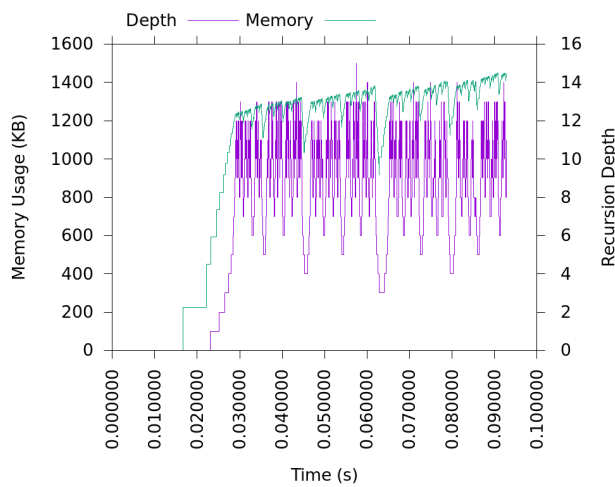


Figure 9: TC: Cover5.15\_10000

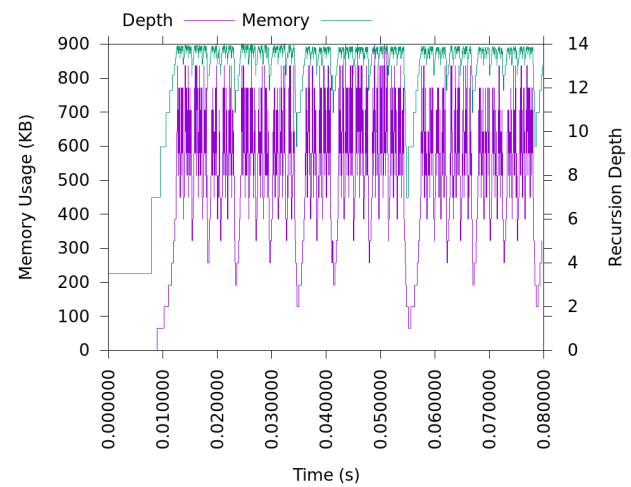


Figure 10: CC: Cover5.15\_10000

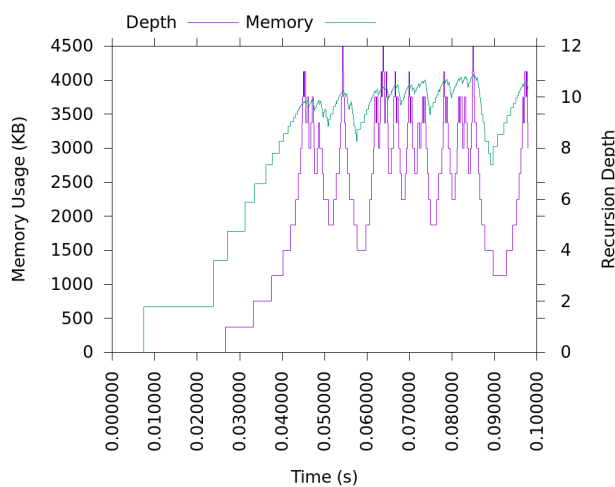


Figure 11: TC: Cover6.15\_30000

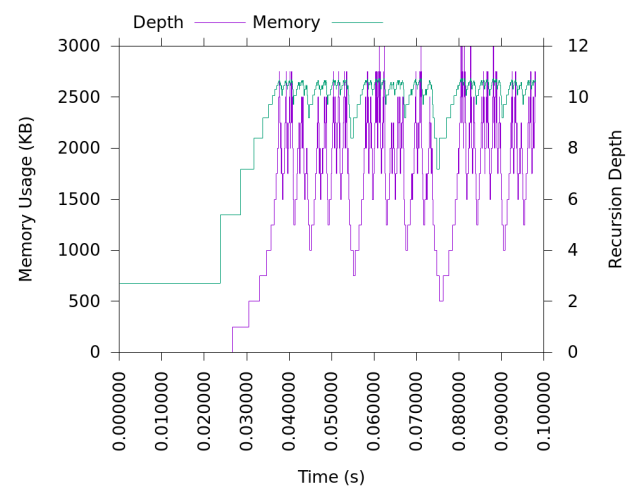


Figure 12: CC: Cover6.15\_30000



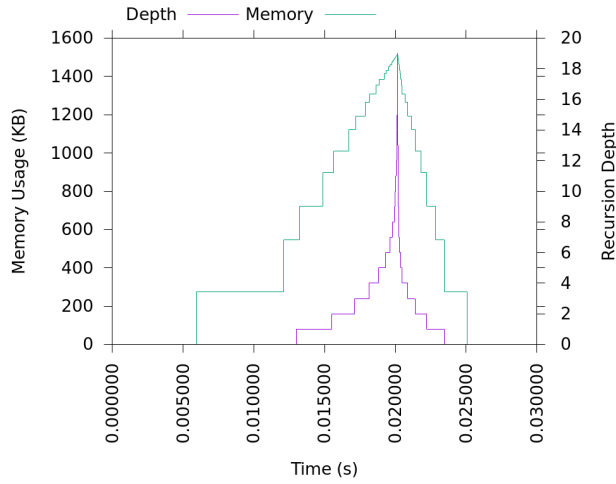


Figure 13: TC: Cover7\_20\_10000

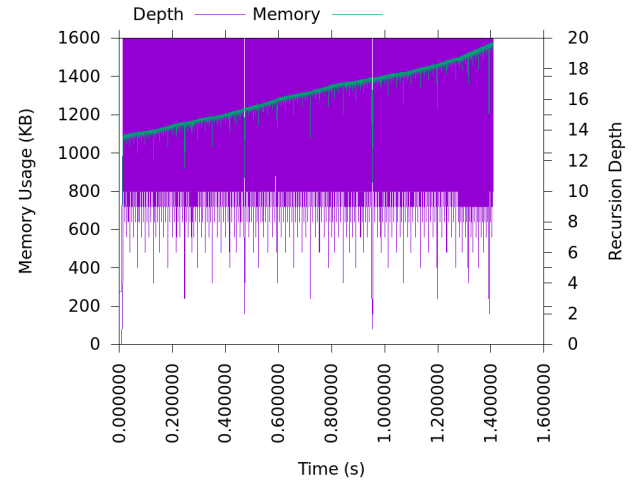


Figure 14: CC: Cover7\_20\_10000

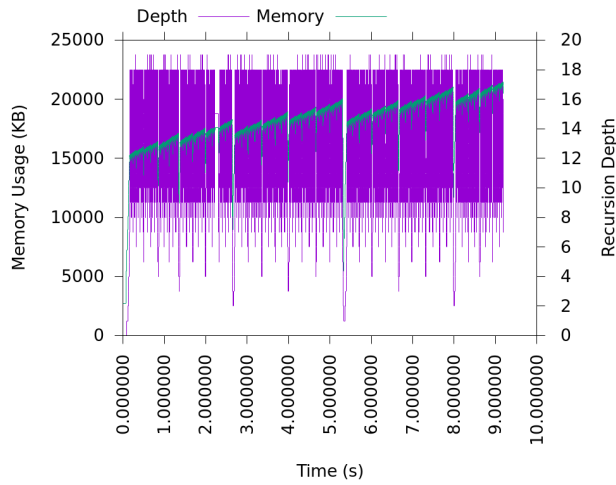


Figure 15: TC: Cover8\_20\_100000

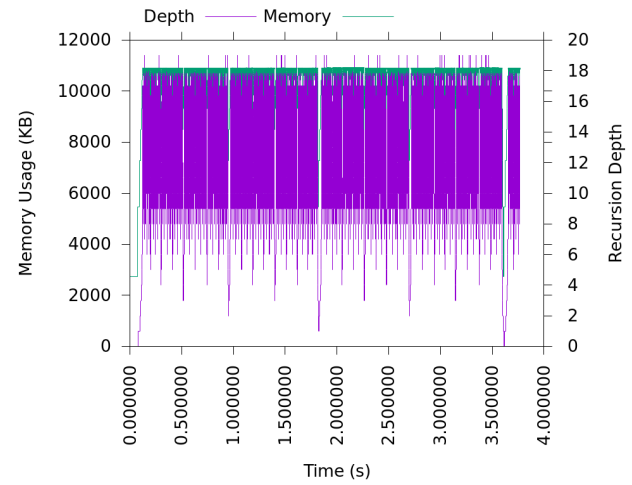


Figure 16: CC: Cover8\_20\_100000

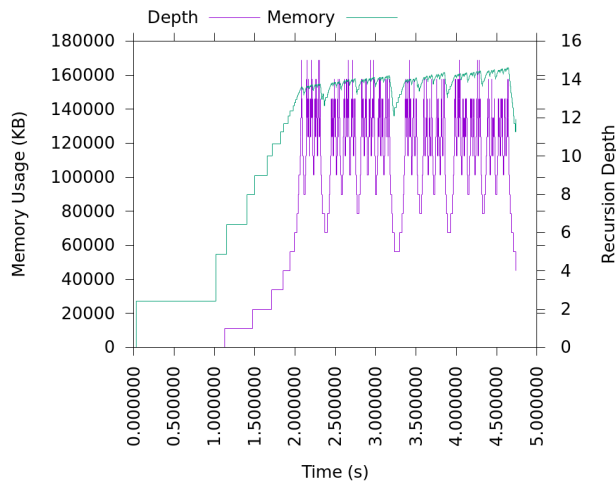


Figure 17: TC: Cover9\_20\_1000000

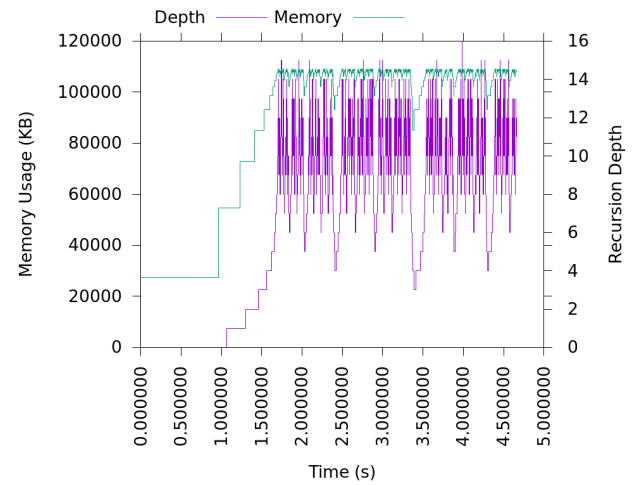


Figure 18: CC: Cover9\_20\_1000000

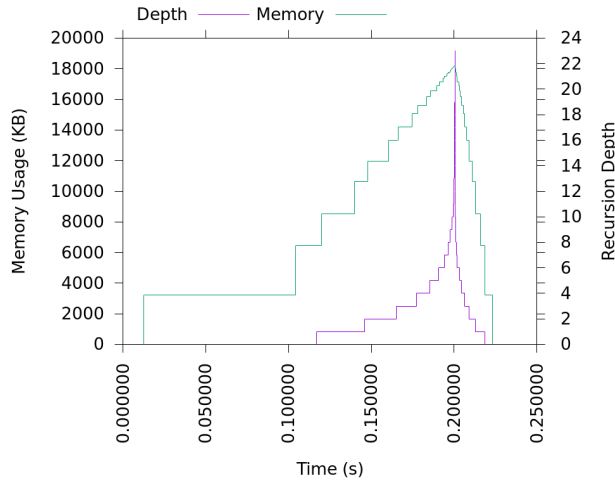


Figure 19: TC: Cover\_25\_100000

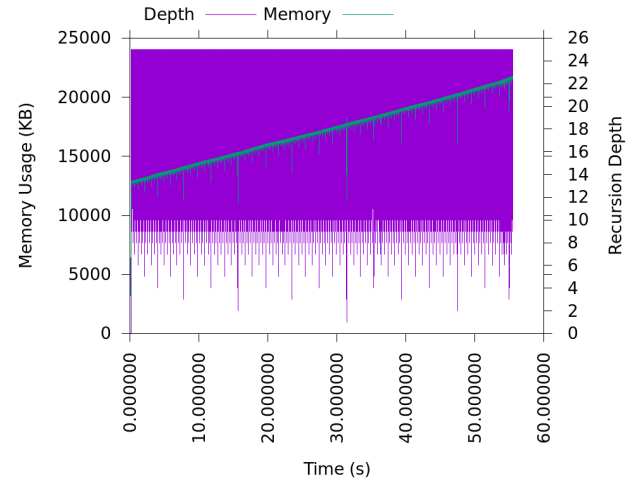


Figure 20: CC: Cover\_25\_100000

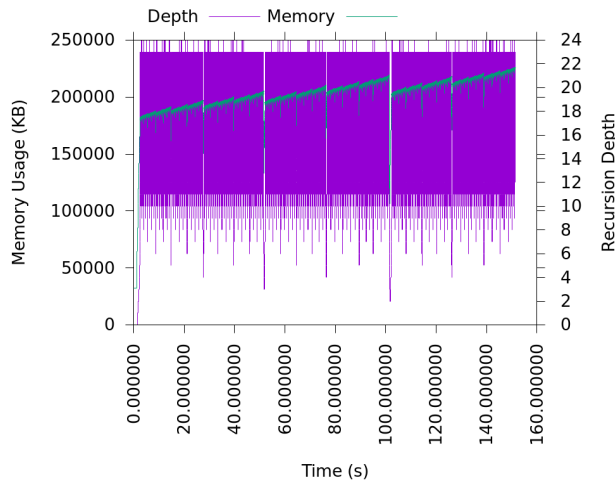


Figure 21: TC: Cover\_25\_1000000

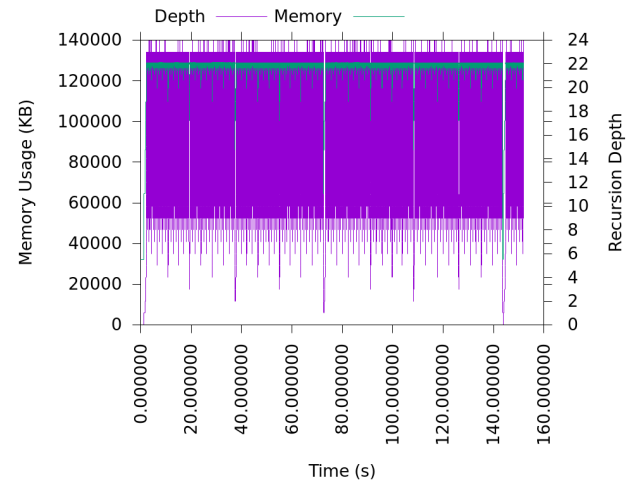


Figure 22: CC: Cover\_25\_1000000

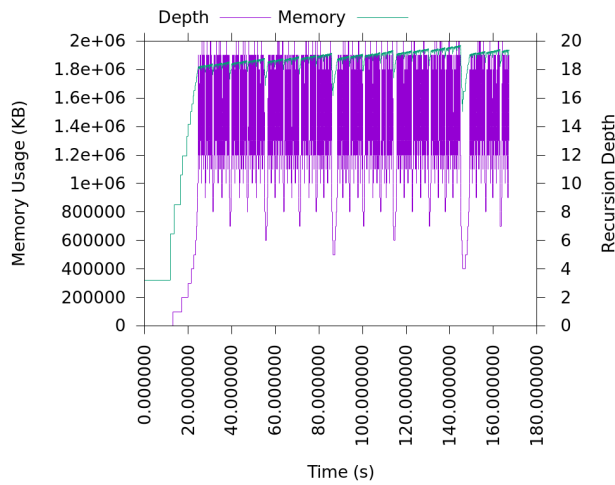


Figure 23: TC: Cover\_25\_10000000

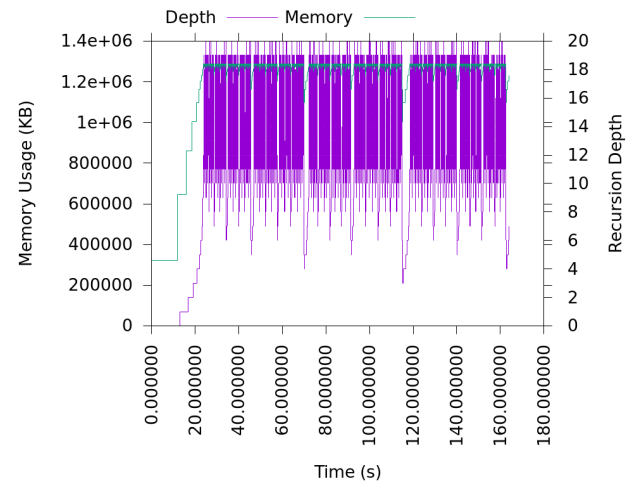


Figure 24: CC: Cover\_25\_10000000

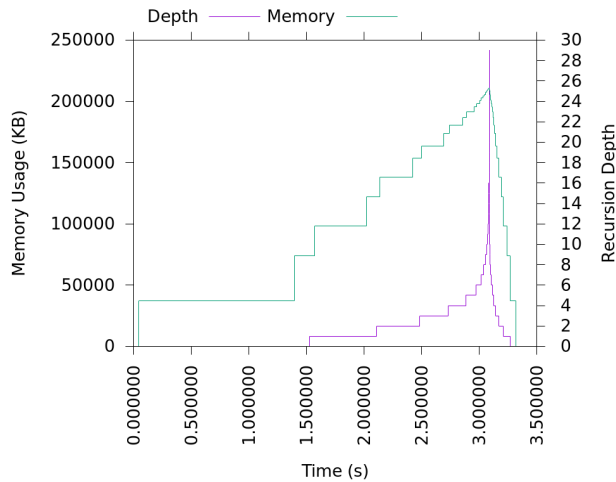


Figure 25: TC: Cover\_30\_1000000

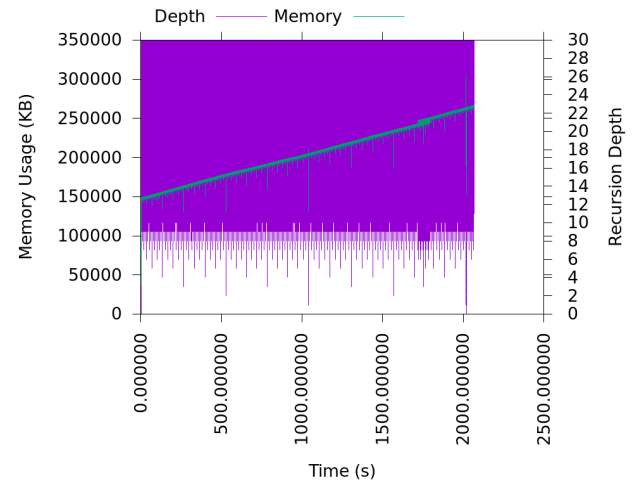


Figure 26: CC: Cover\_30\_1000000

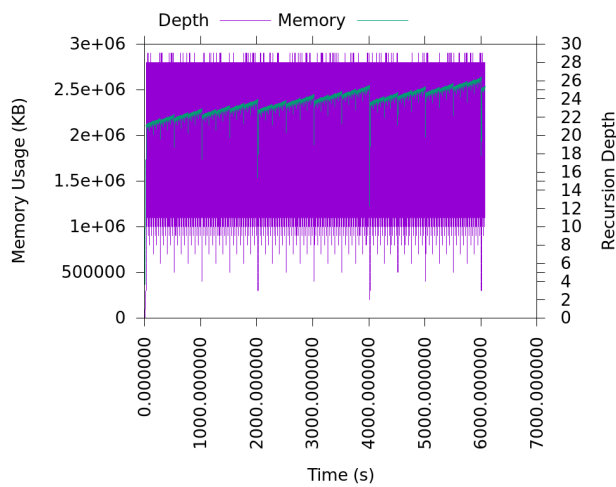


Figure 27: TC: Cover\_30\_10000000

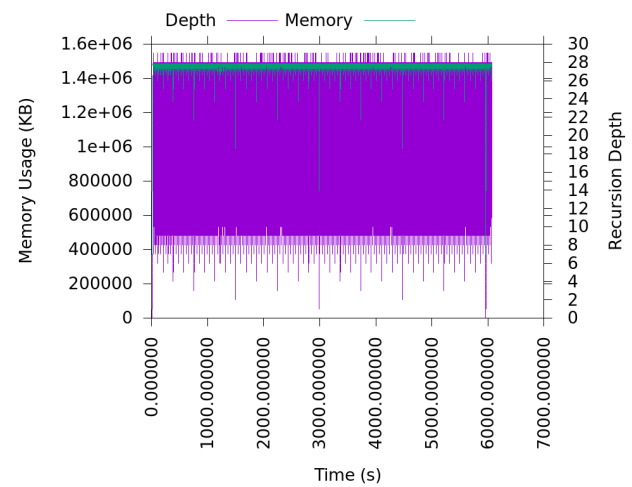


Figure 28: CC: Cover\_30\_10000000

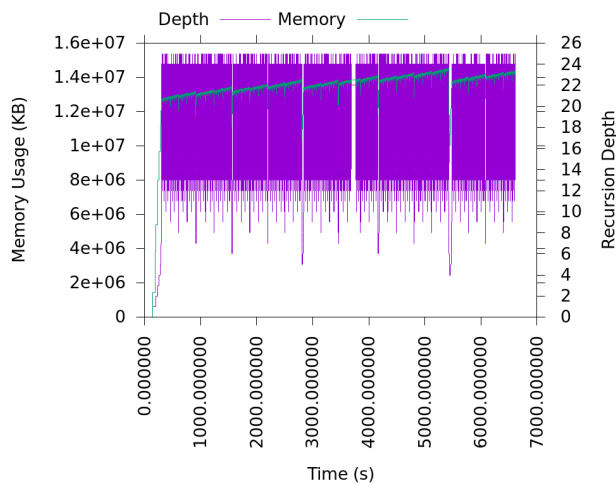


Figure 29: TC: Cover\_30\_100000000

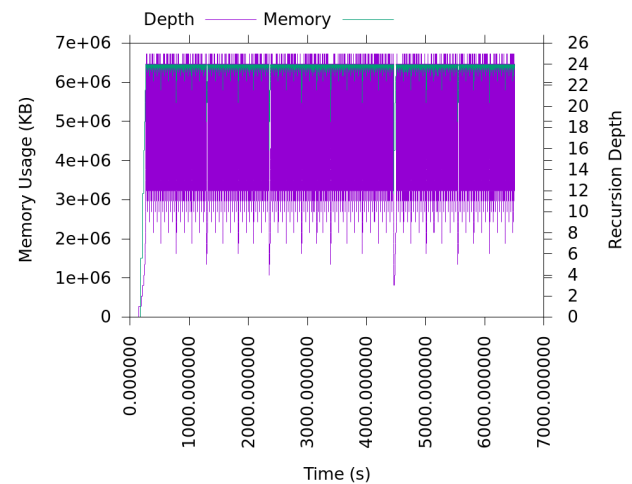


Figure 30: CC: Cover\_30\_100000000

## 5 Retrospective

Overall we feel that our results are acceptable. We can run all but the two biggest `Cover_30` in acceptable time and system requirements. Having the flags algorithm run in parallel provided a really nice speed boost and the simplicity of it (only 170 lines!) makes it a no brainer to implement. We think it would be interesting to extend the idea of parallelism into the heuristic algorithm since in both `tc` and `cc` we pretty much split the matrix in two right away.

There are definitely improvements to be made in our implementation. For one, there seems to still be some memory leaks in both our programs. We were able to use `valgrind` to find and fix a lot of leaks but there are still a few that remain. Looking at the profile plots in the above section you can clearly see an upward trend in memory even though the recursion depth is not increasing, `CC Cover 7` is a good illustration of this. Additionally, we spend a lot of CPU cycles re-finding the most unate and binate columns. A better strategy would be to only search for them once at the beginning and then just recalculate where they are every time we modify the matrix instead of re-searching.

In the end we were able to successfully run all the provided benchmarks and our `test.sh` script proves that we can take a non tautology cover, find the complements, put those complements back into the original cover, and then prove that it is now a tautology. Given this fact we are confident in saying our implementation was successful.