(1.a) Since K_1 and K_2 are kernels over $\mathbb{R}^d \times \mathbb{R}^d$,

$\forall Z \in \mathbb{R}^d$, such that
$Z^T K_1 Z + Z^T K_2 Z = Z^T(K_1+K_2)Z = Z^T(K_1^T+K_2^T)Z = Z^T K Z = Z^T K^T Z \geq 0$
for any finite set $\{x^{(1)}, x^{(2)}, \dots, x^{(m)}\}$
based on Mercer Theorem $\Rightarrow K = K_1+K_2$ is a valid kernel

(1.b) Consider a counter example where $K_2 = 2K_1$
then $\forall Z \in \mathbb{R}^d$, we can get
$Z^T K Z = Z^T(K_1 - K_2)Z = -Z^T K_1 Z \leq 0$
for any finite set $\{x^{(1)}, x^{(2)}, \dots, x^{(m)}\}$

based on Mercer Theorem $\Rightarrow K = K_1 - K_2$ is not a valid kernel

(1.c) Since K_1 is kernel and $a \in \mathbb{R}^+$
then $\forall Z \in \mathbb{R}^d$, we can get
$Z^T K Z = Z^T(aK_1)Z = aZ^T K_1 Z = aZ^T K_1^T Z = Z^T K^T Z \geq 0$
for any finite set $\{x^{(1)}, x^{(2)}, \dots, x^{(m)}\}$

based on Mercer Theorem $\Rightarrow K = aK_1$ is a valid kernel

(1.d) Consider a counter example where $a = 1$
then $\forall Z \in \mathbb{R}^d$, we can get
$Z^T K Z = -Z^T K_1 Z \leq 0$ for any finite set $\{x^{(1)}, x^{(2)}, \dots, x^{(m)}\}$
based on Mercer Theorem $\Rightarrow K = -aK_1$ is not a valid kernel

1.e. since K_1 and K_2 are kernels, $\exists \phi_1 : \mathbb{R}^n \to \mathbb{R}^d$, $\exists \phi_2 : \mathbb{R}^n \to \mathbb{R}^d$,

such that: $K_1(X,Z) = \phi_1^T(X)\phi_1(Z) = \sum_{k=1}^{d} \phi_1^{(k)}(X)\phi_1^{(k)}(Z)$

$$K_2(X,Z) = \phi_2^T(X)\phi_2(Z) = \sum_{k=1}^{d} \phi_2^{(k)}(X)\phi_2^{(k)}(Z)$$

where $\phi_1^{(k)}(X)$ is the kth coordinate of $\phi_1(X)$

$\phi_2^{(k)}(X)$ is the kth coordinate of $\phi_2(X)$

so we can further get

$$K(X,Z) = K_1(X,Z)K_2(X,Z) = \sum_{k=1}^{d}\phi_1^{(k)}(X)\phi_1^{(k)}(Z) \sum_{k=1}^{d}\phi_2^{(k)}(X)\phi_2^{(k)}(Z)$$

$$= \sum_{i=1}^{d}\sum_{j=1}^{d} \phi_1^{(i)}(X)\phi_1^{(i)}(Z)\phi_2^{(j)}(X)\phi_2^{(j)}(Z)$$

$$= \sum_{i=1}^{d}\sum_{j=1}^{d} \left(\phi_1^{(i)}(X)\phi_2^{(j)}(X)\right)\left(\phi_1^{(i)}(Z)\phi_2^{(j)}(Z)\right)$$

$$= \psi^T(X)\psi(Z)$$

where $\psi^{(k)}(X) = \phi_1^{(k)}(X)\phi_2^{(k)}(X)$ $k=1,\dots,d$, is the kth coordinate of $\psi(X)$

$\Rightarrow K(X,Z)$ is a valid kernel

1.f. since $f(X)$ is scalar, then $f^T(X) = f(X)$

let $\phi(X) = f(X)$, then $K(X,Z) = f(X)f(Z)$

$$= f^T(X)f(Z)$$

$$= \phi^T(X)\phi(Z)$$

$\Rightarrow K(X,Z)$ is a kernel

1.g. Since K_3 is kernel over $\mathbb{R}^P \times \mathbb{R}^P$, we can conclude that its kernel matrix obtained over $\{\phi(X^{(1)}), \phi(X^{(2)}), \dots, \phi(X^{(m)})\}$ where $\phi: \mathbb{R}^d \to \mathbb{R}^P$ is symmetric and positive definite $\Rightarrow K(X,Z) = K_3(\phi(X), \phi(Z))$ is a valid kernel.

1.h. since $p(x)$ is a polynomial over x with positive coefficients.

from 1.a, we proved that sum of kernels is a valid kernel

from 1.c, we proved that product of kernel with positive scalar is a valid kernel

from 1.e, we proved that product of kernels is a valid kernel

from 1.f. we proved that real valued constant term is a valid kernel.

Since $p(K_1(X,Z))$ can be re-written as a summation of kernels (with kernels represented by 1.a, 1.c, 1.e, 1.f) with positive coefficient $\Rightarrow p(K_1(X,Z))$ is a valid kernel