Mechanics Y&F Chapter 3, Lecture 1
Motion in two and three dimensions

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Objectives

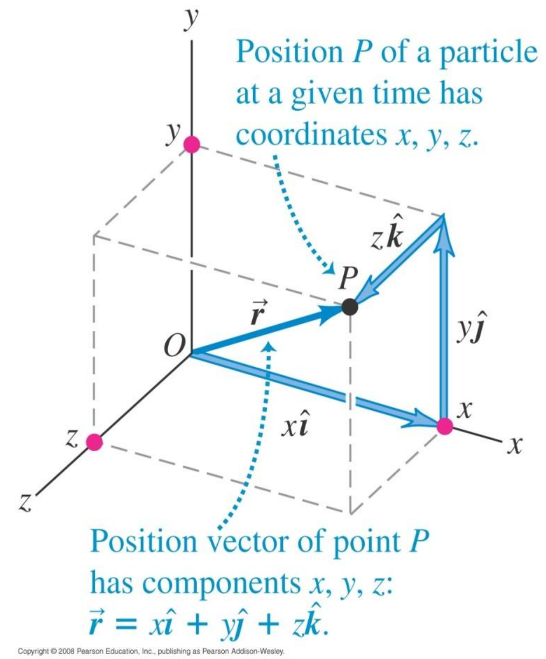
Relating position, velocity and acceleration in 3D

- $a_{\parallel}$ & $a_{\perp}$ to velocity

Uniform circular motion

Position & displacement in 3D

In three dimensions, we represent position by a vector ***r***.



Displacement is given by the vector difference between ***r***$_2$ and ***r***$_1$.

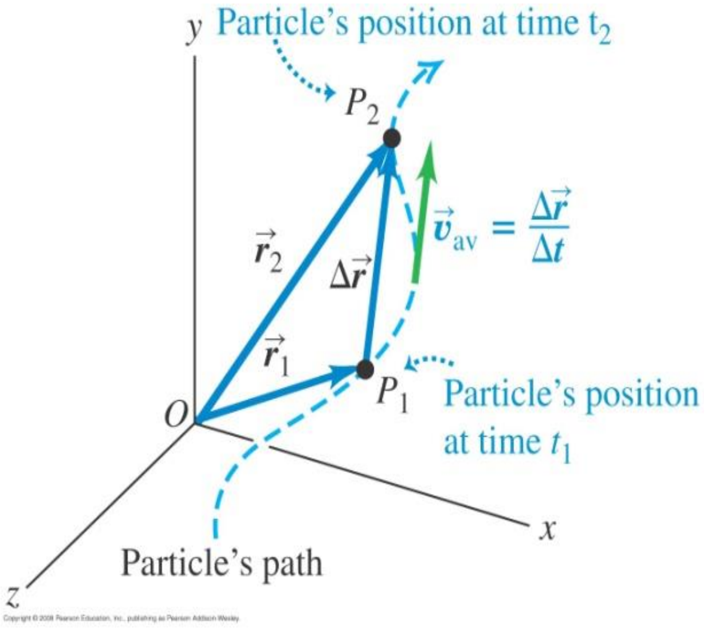
$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1 = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$

Displacement & velocity in 3D

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1 = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$

We can now consider velocity and acceleration in 3D. For average velocity:

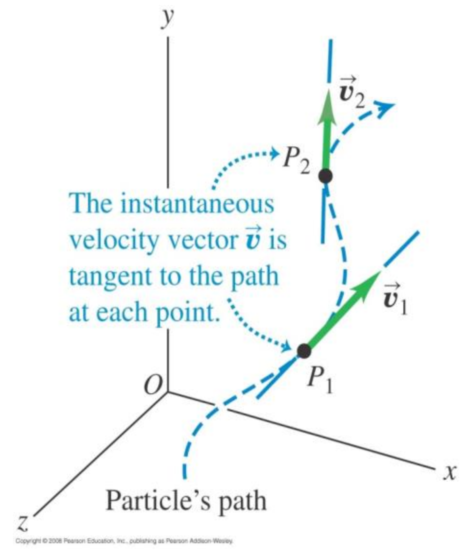
$$\vec{v}_{ave} = \frac{\Delta\vec{r}}{\Delta t}$$



Instantaneous velocity in 3D

In 3D, For instantaneous velocity:

$$\vec{v} = \lim_{\Delta t \to 0} \Delta\vec{r} \Big/ \Delta t = \frac{d\vec{r}}{dt}$$



We can get ***v*** from the time derivatives of the r components:

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}$$

$$|\vec{v}| = v = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

3D Velocity Example

If Temoc the Comet moves from (2.0 m, 1.0 m, 0.0 m) to (3.0 m, 5.0 m, -4.0 m) in 4.0 seconds, what is his average velocity?

$$\vec{v}_{ave} = \frac{\Delta\vec{r}}{\Delta t}$$

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1 = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$

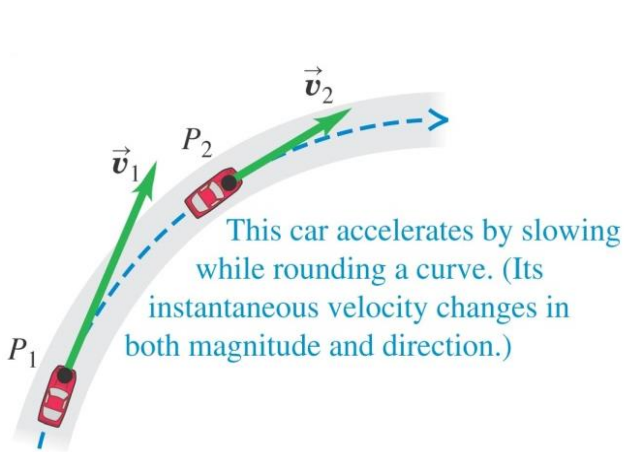
$$\Delta\vec{r} = (3.0m - 2.0m)\hat{\imath} + (5.0\ m - 1.0\ m)\hat{\jmath} + (-4.0\ m - 0.0\ m)\hat{k}$$

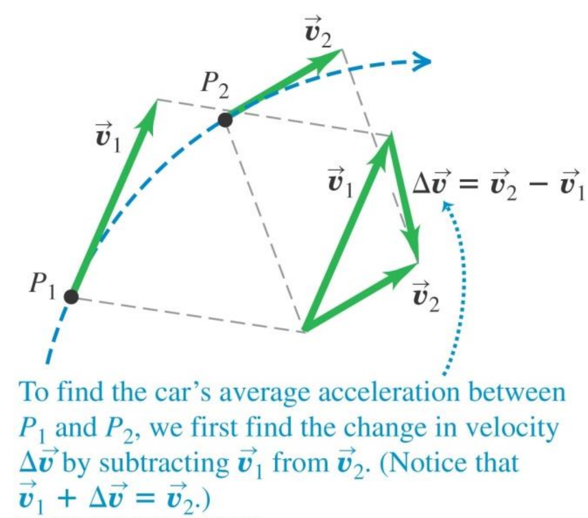
$$\Delta\vec{r} = (1.0\ m)\hat{\imath} + (4.0\ m)\hat{\jmath} + (-4.0\ m)\hat{k}$$

$$\vec{v}_{ave} = {\Delta\vec{r}}/{\Delta t} = \frac{(1.0\ m)\hat{\imath} + (4.0\ m)\hat{\jmath} + (-4.0\ m)\hat{k}}{4.0\ s} = (0.25\ m/s)\hat{\imath} + (1.0\ m/s)\hat{\jmath} + (-1.0\ m/s)\hat{k}$$

3D Average Acceleration

$$\vec{a}_{ave} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1}$$

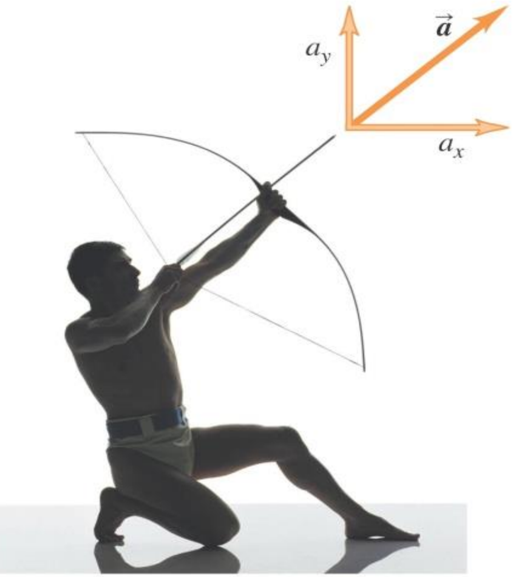




3D Instantaneous Acceleration

$$\vec{a} = \lim_{\Delta t \to 0} \Delta\vec{v} \Big/ \Delta t = \frac{d\vec{v}}{dt}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} + \frac{dv_z}{dt}\hat{k}$$



Example, Instantaneous **a**

(Based on Exercise 3.46) A bird flies in the xy-plane with a velocity vector given by:

$$\vec{v} = (\alpha - \beta t^2)\hat{\imath} + \delta t\hat{\jmath} = \left[2.4\frac{m}{s} - \left(1.6\frac{m}{s^3}\right)t^2\right]\hat{\imath} + \left[\left(4.0\frac{m}{s^2}\right)t\right]\hat{\jmath}$$

- *Calculate the acceleration vector as a function of time.*

$$\vec{a} = \frac{d\vec{v}}{dt} \quad \vec{a} = (-2\beta t)\hat{\imath} + \delta\hat{\jmath} = \left[-\left(3.2\frac{m}{s^3}\right)t\right]\hat{\imath} + \left[\left(4.0\frac{m}{s^2}\right)\right]\hat{\jmath}$$

Example, Instantaneous **a**, Part 2

(Based on Exercise 3.46) A bird flies in the xy-plane with a velocity vector given by:

$$\vec{v} = (\alpha - \beta t^2)\hat{\imath} + \delta t\hat{\jmath} = \left[2.4\frac{m}{s} - \left(1.6\frac{m}{s^3}\right)t^2\right]\hat{\imath} + \left[\left(4.0\frac{m}{s^2}\right)t\right]\hat{\jmath}$$

- *What is the value of the y-component of velocity when the bird is moving purely in y, when t > 0?*

X component of velocity: $(2.4 - 1.6t^2)$ m/s

If moving purely in y, $v_x = 0$

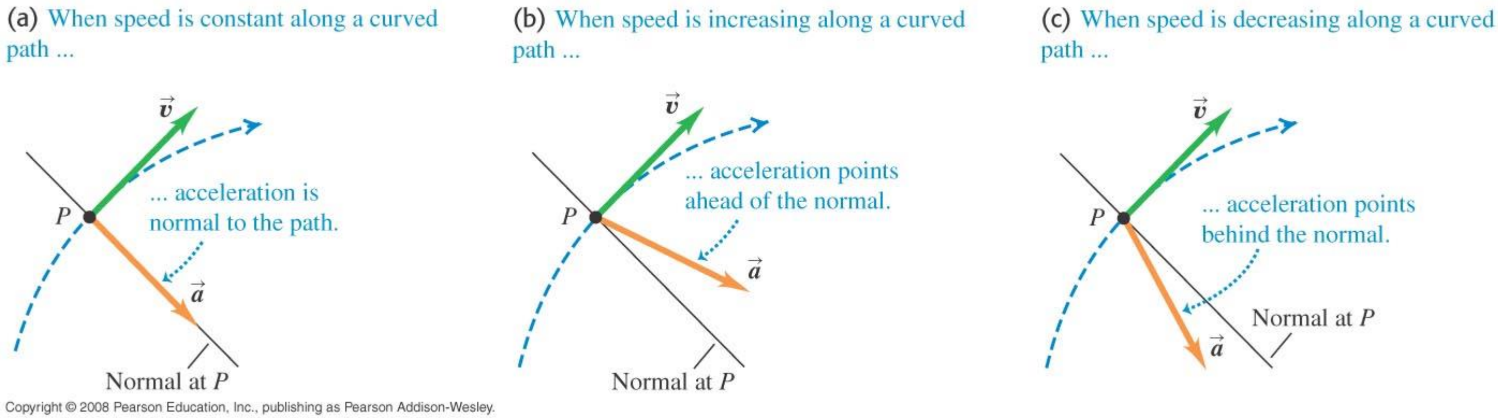
$2.4 - 1.6t^2 = 0$

$$t = \sqrt{\frac{2.4}{1.6}} = 1.2 \text{ s}$$

Y component of velocity at 1.2 s: (4.0 m/s^2)*1.2 s = **4.8 m/s**

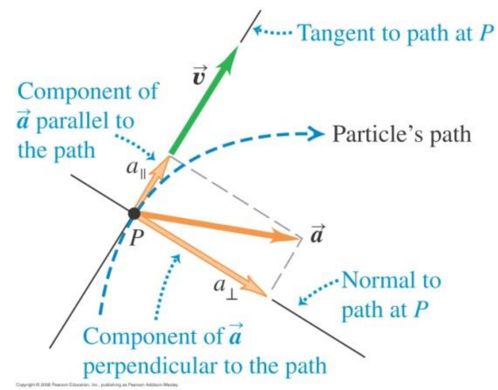
Motion along a curved path

Any object moving along a curved path is accelerating, even if the speed is constant!



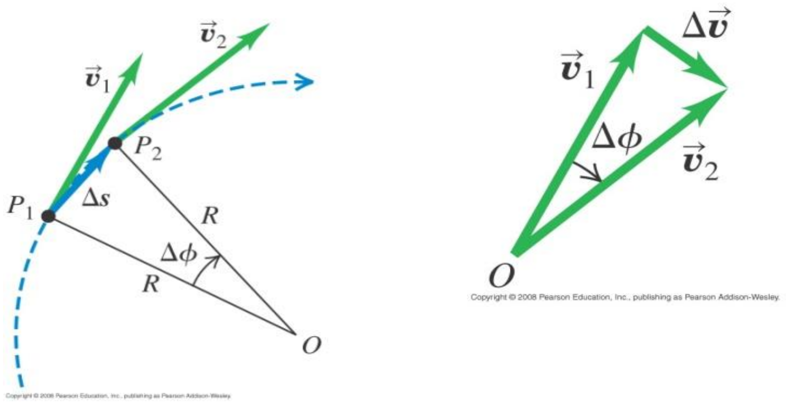
The **parallel component of acceleration** $\boldsymbol{a}_{||}$ tells us about changes in the object's **speed**.

The **perpendicular component of acceleration** $\boldsymbol{a}_{\perp}$ tells us about changes in the object's **direction of motion**.



Uniform Circular Motion

(a) A point moves a distance Δs at constant speed along a circular path.



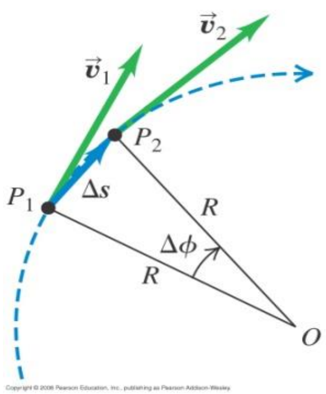
On the left, a particle moves at constant speed around a circular path. The change in path length Δs is simply the arclength, $R\Delta\phi$. If we redraw the velocity vectors from a common point, as in plot (b), the change in velocity is given by the opposite side. The triangles in (a) and (b) are similar triangles, and the ratios of the lengths of the sides are equal.

$$\frac{|\Delta\vec{v}|}{v_1} = \frac{\Delta s}{R} \qquad |\Delta\vec{v}| = \frac{v_1 \Delta s}{R}$$

$$a_{ave} = \frac{|\Delta\vec{v}|}{\Delta t} = \frac{v_1 \Delta s}{R\Delta t}$$

Uniform Circular Motion

(a) A point moves a distance Δs at constant speed along a circular path.



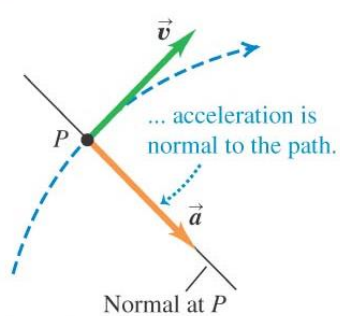
Instantaneous acceleration:

$$a = \lim_{\Delta t \to 0} \frac{v_1 \Delta s}{R \Delta t} = \frac{v_1}{R} \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t}$$

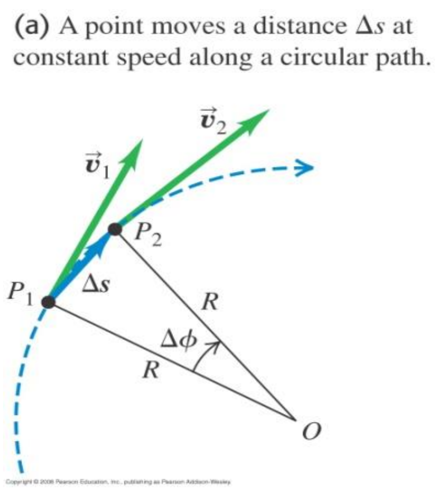
But the limit of $\Delta s/\Delta t$ is just the speed v_1.
So we find:

$$\boxed{a_{radial} = \frac{v^2}{R}, uniform\ circular\ motion}$$

The acceleration vector always points <u>radially inward</u> for uniform circular motion.



Uniform Circular Motion



The period of the revolution *T*, the time for the particle to traverse the circle, is given by the circumference divided by the speed:

$$T = \frac{2\pi R}{v}$$

$$a_{radial} = \frac{4\pi^2 R}{T^2}$$

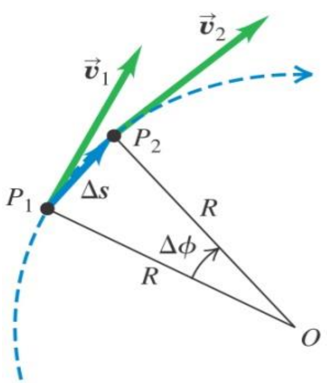
If there is non-uniform circular motion, that is, if the speed is changing, then there is also a tangential part to **a**:

$$a_{radial} = \frac{v^2}{R} \ and \ a_{tan} = \frac{d|\vec{v}|}{dt}, nonuniform\ circular\ motion$$

Example, Uniform Circular Motion

A commercial recently boasted that they had created the first "street tire" that could complete a "1 g" turn. How tight of a turn could this tire execute at 60. mi/hr? That is, what is the minimum radius of a circle that this tire could complete at this speed?

(a) A point moves a distance Δs at constant speed along a circular path.



$$a_{radial} = \frac{v^2}{R} \qquad R = \frac{v^2}{a_{radial}}$$

Now the acceleration due to gravity is 9.8 m/s^2. So:

v = 60. mi/hr * (1m/s)/(2.24 mi/hr) = 27 m/s

$$R = \frac{(27 m/s)^2}{9.8\ m/s^2} \qquad R = 74\ m$$