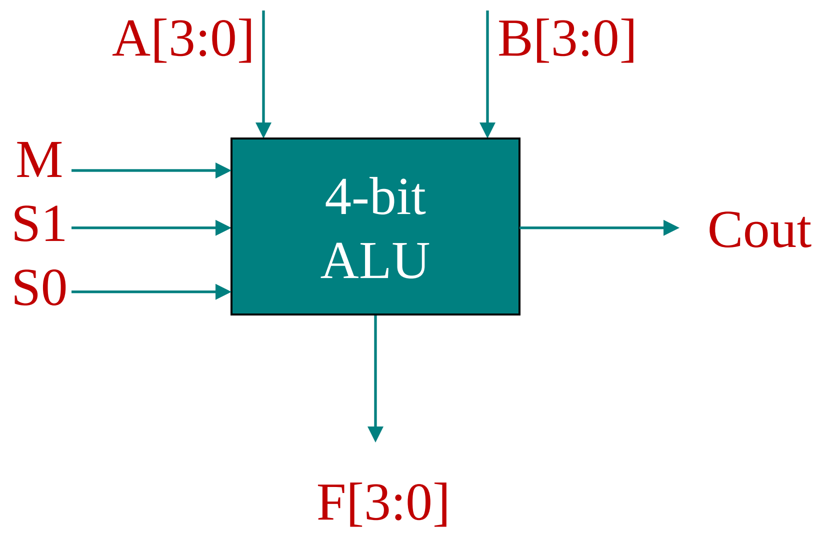
4-bit Arithmetic Logic Unit (ALU)

Operation:

M	S1	S0	Operation	F
0	0	0	Complement	A’
0	0	1	Logic AND	A AND B
0	1	0	Identity	A
0	1	1	Logic OR	A OR B
1	0	0	Decrement	A – 1
1	0	1	Addition	A + B
1	1	0	Subtraction	A + B’ +1
1	1	1	Increment	A + 1

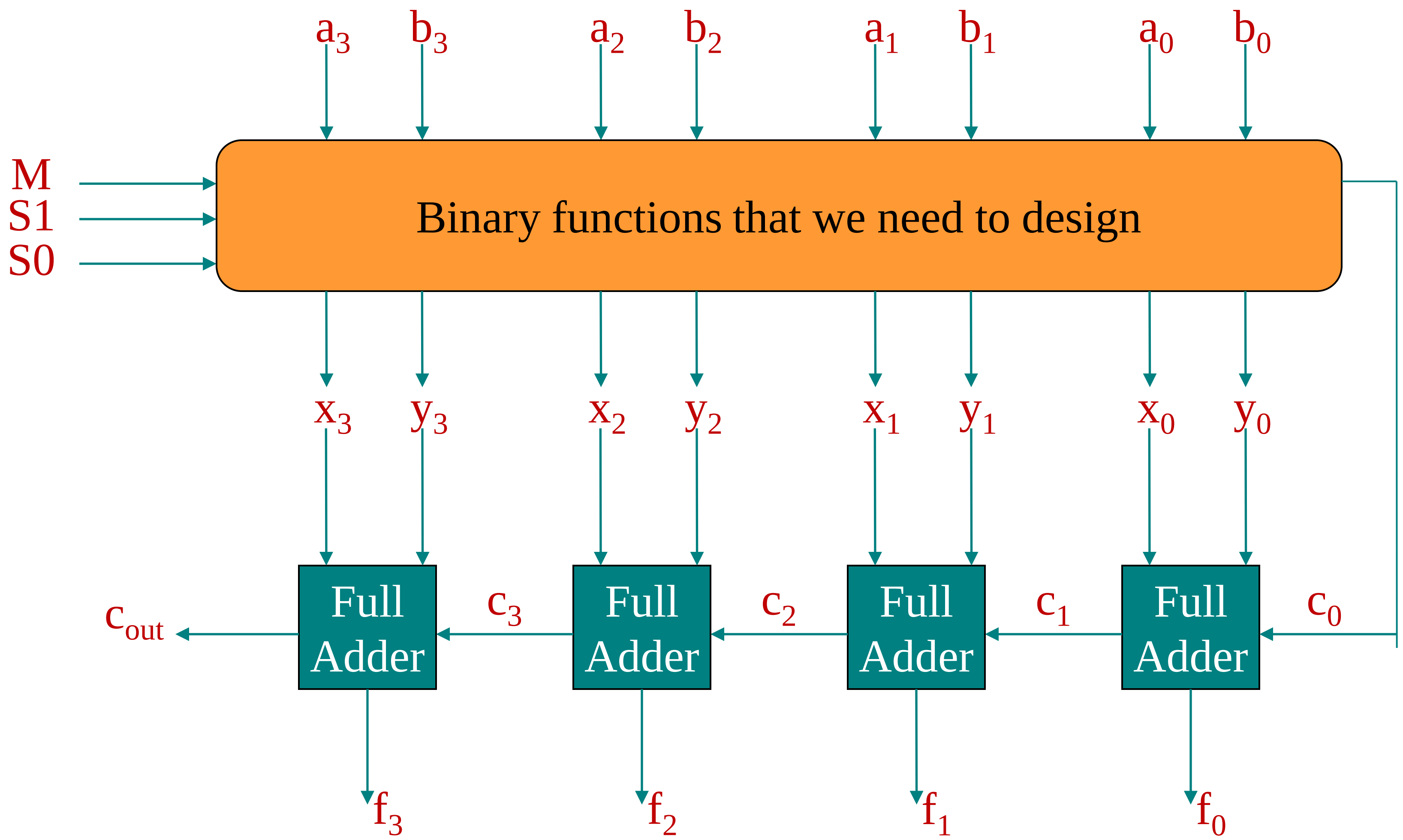
Symbol:



Notes: The logic operations are “bit-wise” operations. Cout is the carry or borrow of arithmetic operations.

When M=0, S1 and S0 choose among four logic operations, when M=1, they choose among four arithmetic operation.

Implementation based on 4-bit Adder



4-Bit ALU Design

Notation: $A[3:0] = a_3a_2a_1a_0$ $B[3:0] = b_3b_2b_1b_0$ $F[3:0] = f_3f_2f_1f_0$

Complement: For each bit a_i of the 4-bit number A[3:0], we will invert the bit making $x_i=a_i$' and we will add $y_i=0$ to it, with $c_0=0$, to obtain F[3:0]=A'[3:0]

Identity: For each bit a_i of the 4-bit number A[3:0], we will preserve the bit making $x_i=a_i$ and we will add $y_i=0$ to it, with $c_0=0$, to obtain F[3:0]=A[3:0]

Logic AND: For each pair of bits a_i , b_i of the 4-bit numbers A[3:0], B[3:0], we will make $x_i=a_i \cdot b_i$ and we will add $y_i=0$ to it, with $c_0=0$, to obtain F[3:0]=A[3:0] AND B[3:0] (bit-wise AND)

Logic OR: For each pair of bits a_i , b_i of the 4-bit numbers A[3:0], B[3:0], we will make $x_i=a_i + b_i$ and we will add $y_i=0$ to it, with $c_0=0$, to obtain F[3:0]=A[3:0] OR B[3:0] (bit-wise OR)

Partial ALU Definition

M	S1	S0	Operation	F	x_i	y_i	c_o
0	0	0	Complement	A’	a_i’	0	0
0	0	1	Logic AND	A AND B	$a_i \cdot b_i$	0	0
0	1	0	Identity	A	a_i	0	0
0	1	1	Logic OR	A OR B	$a_i + b_i$	0	0
1	0	0	Decrement	A – 1			
1	0	1	Addition	A + B			
1	1	0	Subtraction	A + B’ +1			
1	1	1	Increment	A + 1			

Based on the previous description we can fill out the table and obtain the binary functions for x_i, y_i, and c_0

4-Bit ALU Design (Cont'd)

Notation: $A[3:0] = a_3a_2a_1a_0$ $B[3:0] = b_3b_2b_1b_0$ $F[3:0] = f_3f_2f_1f_0$

Addition: For each pair of bits a_i , b_i of the 4-bit numbers A[3:0], B[3:0], we will make $x_i=a_i$ and we will add $y_i= b_i$ to it, with $c_0=0$, to obtain F[3:0]=A[3:0] + B[3:0]

Subtraction: For each pair of bits a_i , b_i of the 4-bit numbers A[3:0], B[3:0], we will make $x_i=a_i$ and we will add $y_i= b'_i$ to it, with $c_0=1$, to obtain F[3:0]=A[3:0] - B[3:0] (subtraction via 2's complement addition.)

Increment: For each bit a_i of the 4-bit number A[3:0] we will make $x_i=a_i$ and we will add $y_i= 0$ to it, with $c_0=1$, to obtain F[3:0]=A[3:0] + 1

Decrement: For each bit a_i of the 4-bit number A[3:0] we will make $x_i=a_i$ and we will add $y_i= 1$ to it, with $c_0=0$, to obtain F[3:0]=A[3:0] + 1 (addition of 2's complement of 0001=1111)

Complete ALU Definition

M	S1	S0	Operation	F	x_i	y_i	c_o
0	0	0	Complement	A’	a_i’	0	0
0	0	1	Logic AND	A AND B	$a_i \cdot b_i$	0	0
0	1	0	Identity	A	a_i	0	0
0	1	1	Logic OR	A OR B	$a_i + b_i$	0	0
1	0	0	Decrement	A – 1	a_i	1	0
1	0	1	Addition	A + B	a_i	b_i	0
1	1	0	Subtraction	A + B’ +1	a_i	b_i’	1
1	1	1	Increment	A + 1	a_i	0	1

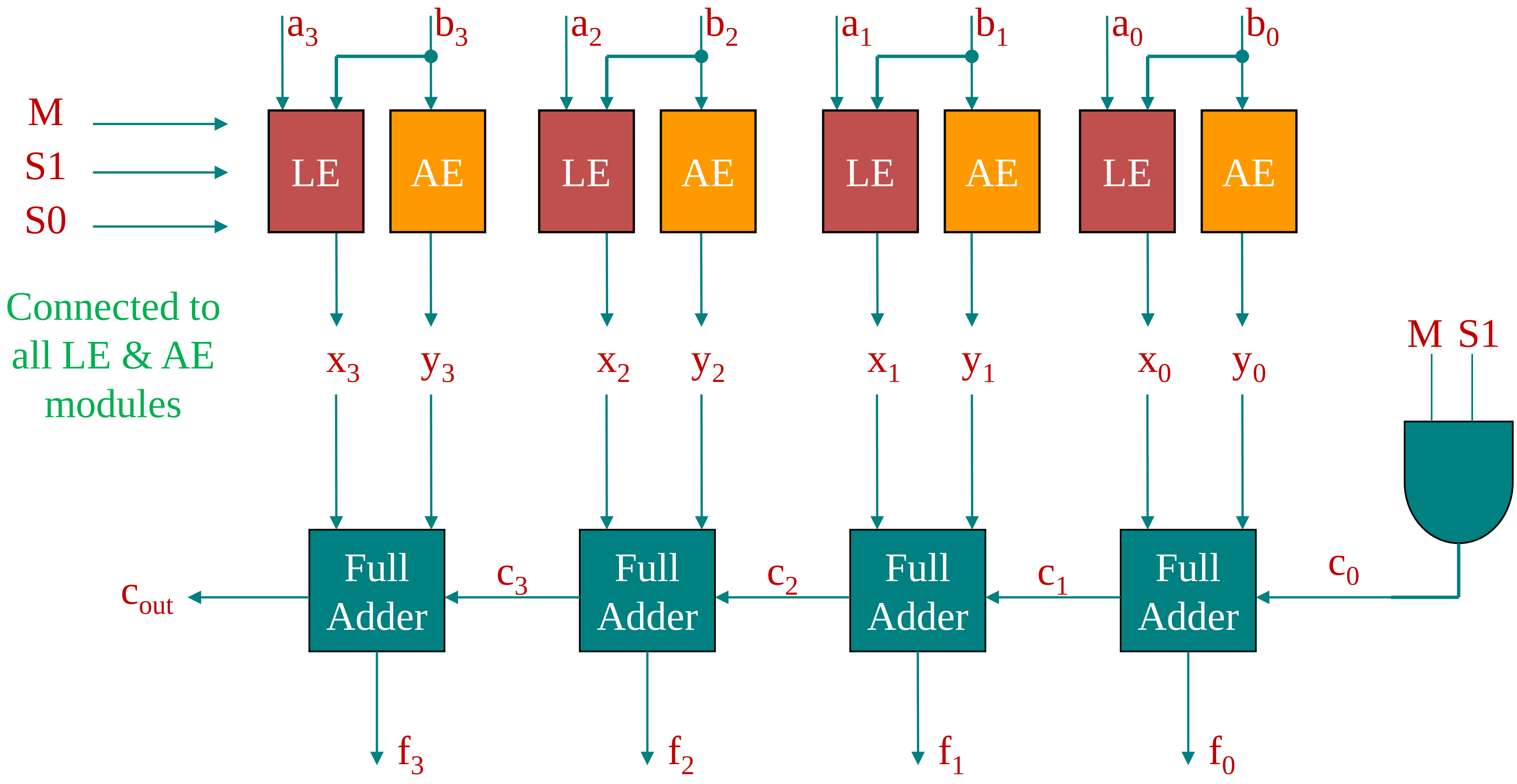
Binary Functions:

$x_i = M' \cdot S_1' \cdot S_0' \cdot a_i' + M' \cdot S_1 \cdot S_0 \cdot b_i + S_o \cdot a_i \cdot b_i + S_1 \cdot a_i + M \cdot a_i$

$y_i = M \cdot S_1' \cdot b_i + M \cdot S_0' \cdot b_i'$

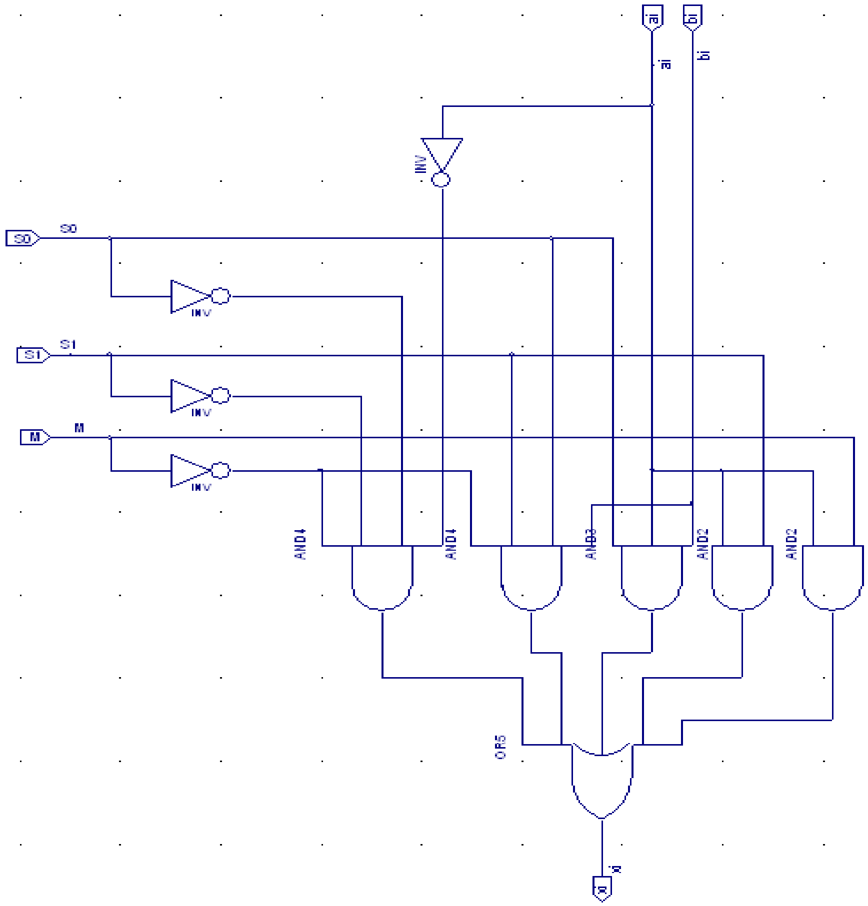
$c_o = M \cdot S_1$

ALU Design

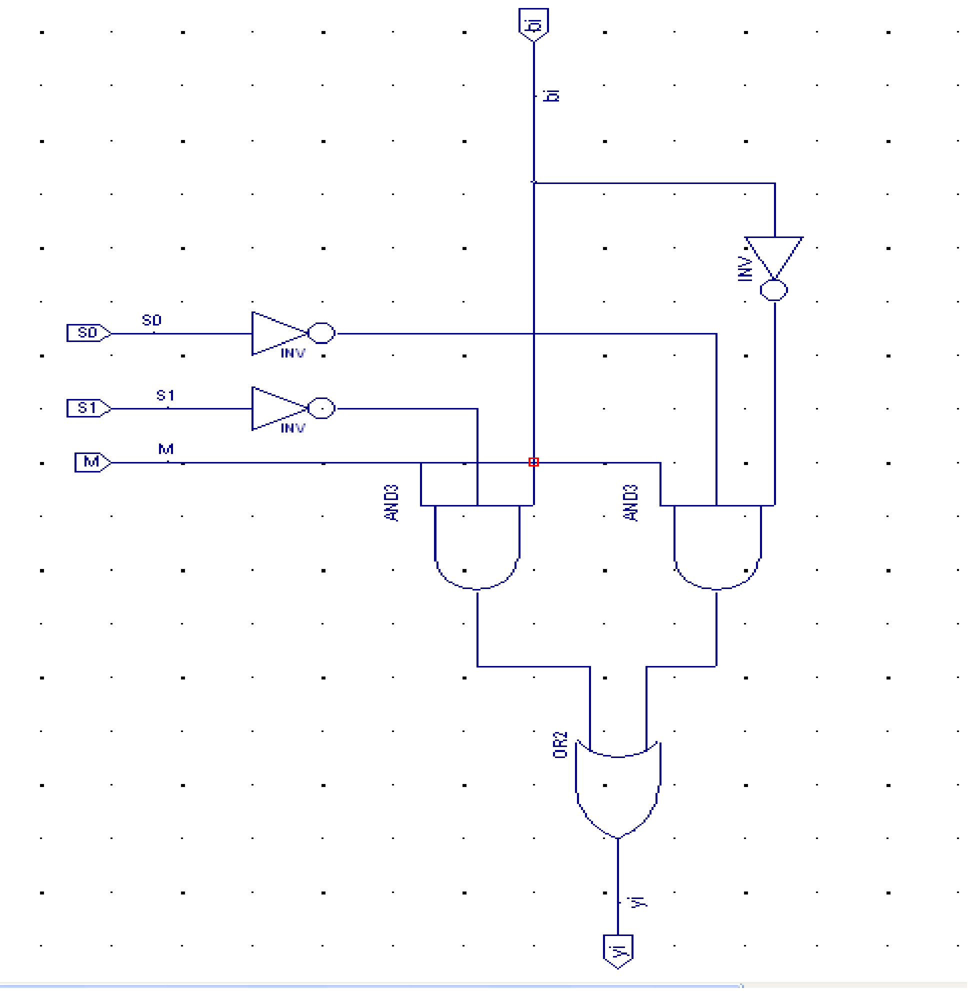


LE: Logic Extender AE: Arithmetic Extender

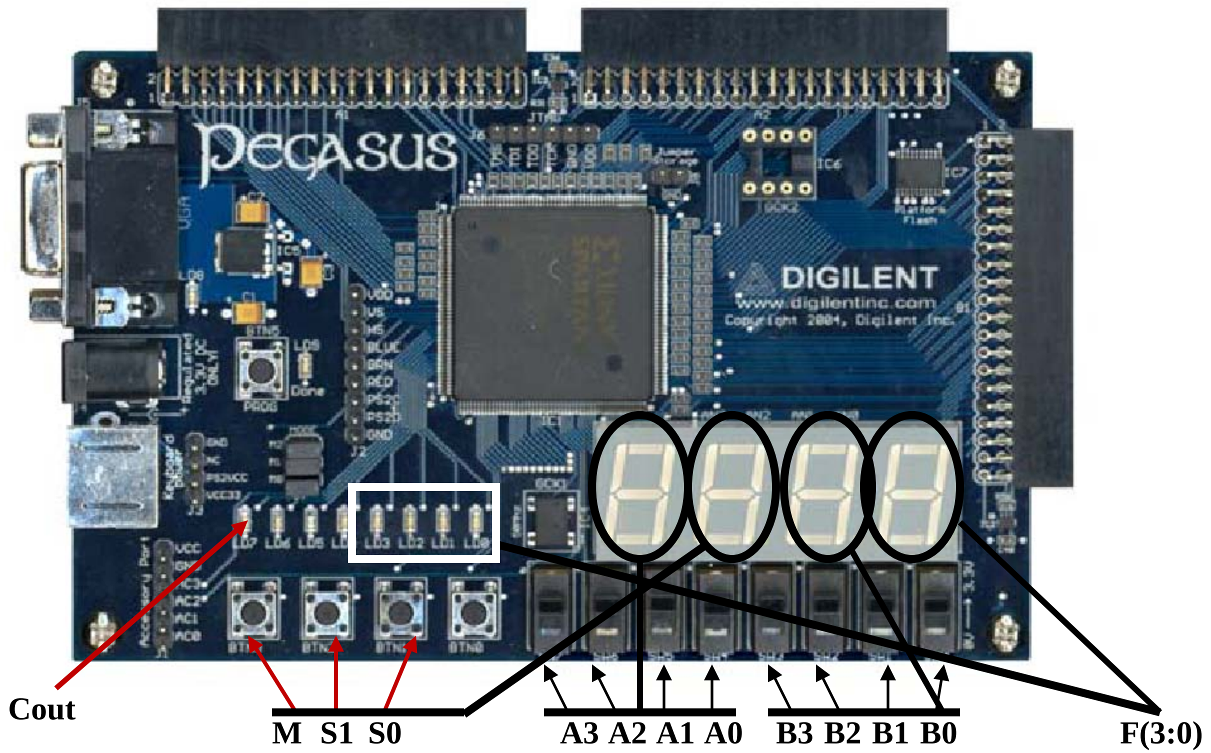
Logic Extender



Arithmetic Extender



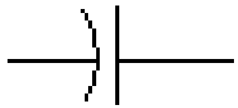
ALU – BOARD Interface



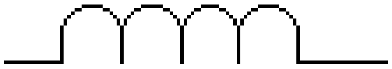
In order to interface the inputs/outputs of your circuit with the switches, LEDs, buttons, and 7-segment displays of the board, you will program the board through a User Constraint File (UCF). Details in Lab #2 description.

Transient DC Circuits

- Adding <u>inductors</u> and <u>capacitors</u> to a DC circuit will change its behavior.
- As R is the symbol for a resistor, C and L are the symbols for capacitors and inductors.
- Capacitors and inductors are the other two passive elements.
- In a circuit with capacitors and inductors (and normally, also resistors), turning a DC power source on or off causes a brief, non-linear behavior of current in the circuit.
- Such circuits (usually referred to as RL, RC, or RLC circuits) are of great interest in electrical engineering, as is their transient behavior.



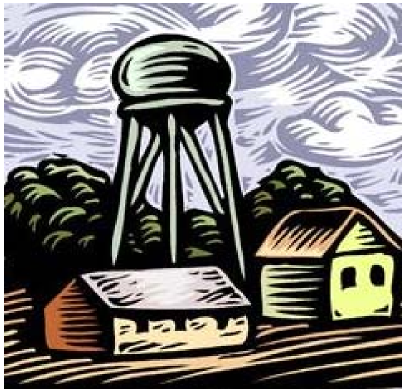
Capacitor (C)



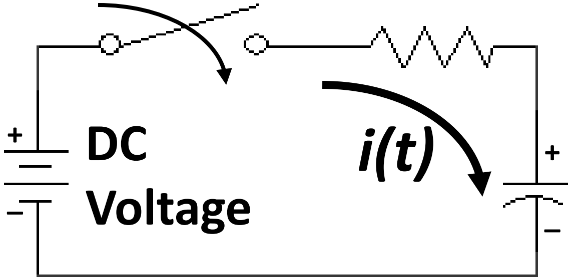
Inductor (L)

The Capacitor

- A capacitor consists of two conducting surfaces separated by a dielectric, or insulator.
- A capacitor stores electric charge when current flows due to an applied voltage, just as a water tank stores water.
- The capacitor develops an equal and opposite voltage as it collects charge.
- When the voltage on the capacitor = the applied voltage, current flow ceases.
- Charge cannot cross the dielectric barrier of a capacitor.
- Voltage cannot appear instantaneously across a capacitor.



Water Tower

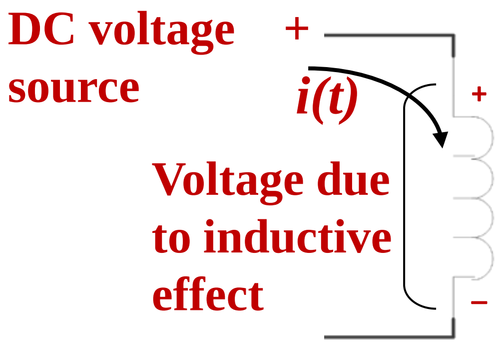


Charging a Capacitor

The Inductor

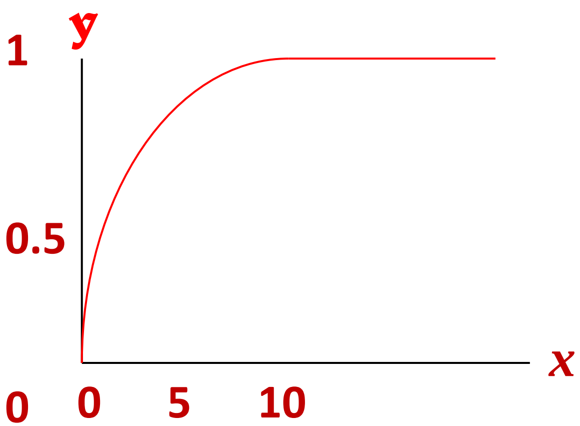
- The inductor has the property of electrical inertia.
- Physical inertia is the property of mass that resists a change in motion (acceleration). If at rest, an object resists moving; if moving, it resists a change in speed.
- Similarly, an inductor resists a change in current flow. If no current flows, it resists the start of current. If current is flowing, it resists a decline in current.
- Just as a voltage cannot instantaneously appear across a capacitor, current cannot flow instantaneously in an inductor.

A massive truck would have high resistance to rapid acceleration or braking.



Exponential Behavior

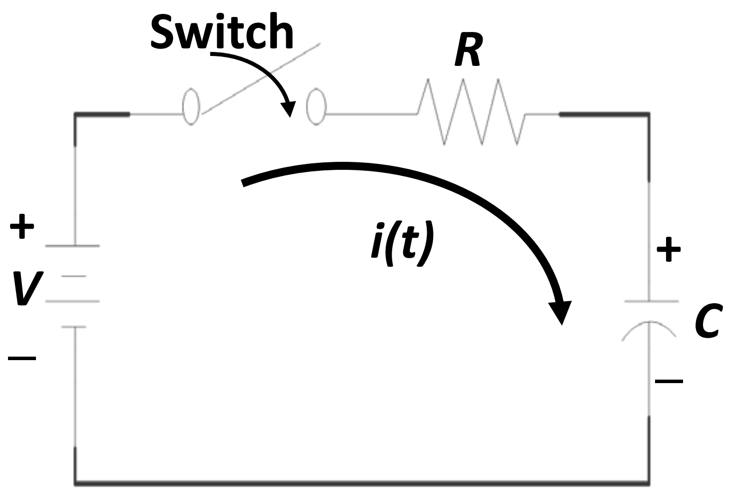
- Exponential behavior is mathematical behavior such that one of the variables is an exponent.
- Some functions have an exponential behavior that involves *e*, the base of natural logarithms.
- Some exponential behavior is asymptotic; it approaches a value but never reaches it. Such a behavior is exhibited in the equation to the right.
- DC transient circuit behavior is characterized by this mathematical description.



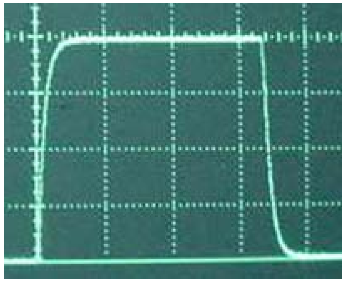
Plot of $1-e^{-x}$

Behavior of an RC Circuit

- Asymptotic, transient behavior is exhibited in an RC circuit.
- When the switch is closed, current flows into the capacitor.
- Current flow ceases when charge collected on the capacitor produces a voltage equal and opposite to *V*.
- An equation describing the behavior is shown; it is both exponential and asymptotic.



$$v_C(t) = V\left(1 - e^{-(t/RC)}\right)$$



Time Constant in RC circuit

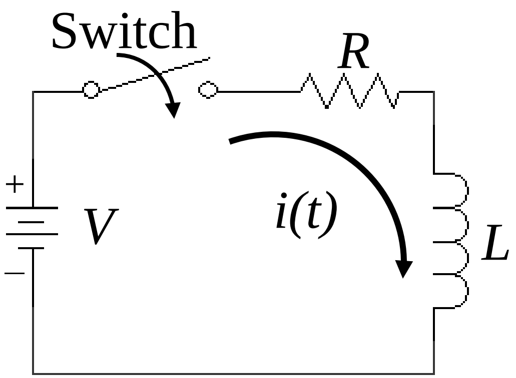
- In the equation shown, as time passes, $v_c(t) \rightarrow V$, as the value of $e^{-t/RC} \rightarrow 0$.
- In the equation, the value RC is called τ.
- Clearly, as τ grows smaller, transient behavior disappears much faster.
- Since τ determines how quickly the transient response of the circuit dies, it is called the time constant.
- Note: For R = 1000 Ω, C= 0.05 μF, then $\tau \approx$ 0.00005 sec. Transient effects last a very short time.

$$v_C(t) = V\left(1 - e^{-(t/RC)}\right)$$

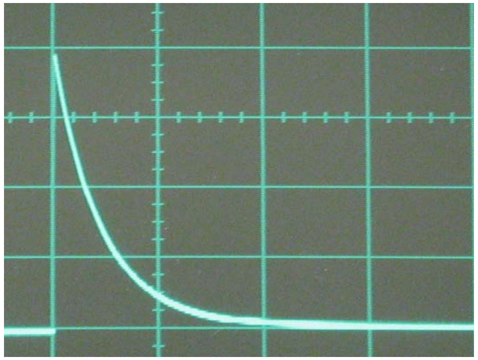
The time constant in an RC circuit is sometimes referred to as "the RC time constant."

Behavior of an RL Circuit

- We also see asymptotic, transient behavior in an RL circuit.
- When the switch is closed, current flow is inhibited as the inductor develops an opposite voltage to the one applied.
- Current slowly begins to flow, as the inductor voltage falls toward 0.
- As the transient effect dies, current flow approaches *V/R*.
- An equation describing the behavior is shown.



$$v_L(t) = Ve^{-(t/[L/R])} = Ve^{-(R/L)t}$$



Time Constant in RC circuit

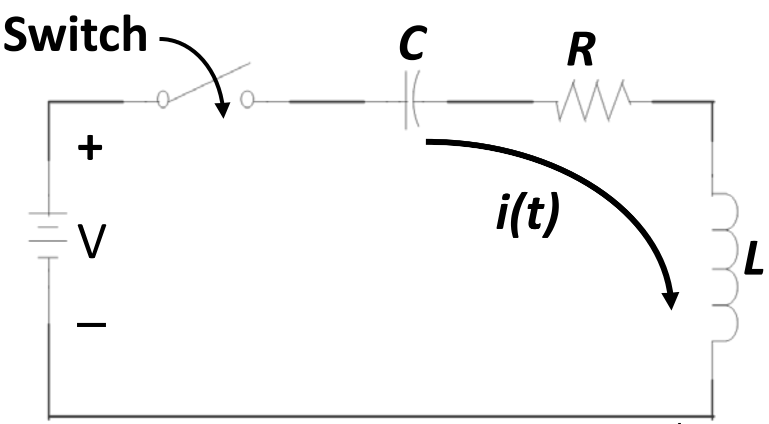
- The time constant τ in an *RL* circuit is defined as $\tau = L/R$.
- In the equation shown, as time passes, $v_L(t) \rightarrow 0$, as the value of $e^{-t/L/R} = e^{-(R/L)t} \rightarrow 0$.
- As τ grows smaller, transient behavior disappears much faster, as in the *RC* case.

$$v_L(t) = Ve^{-(t/[L/R])} = Ve^{-(R/L)t}$$

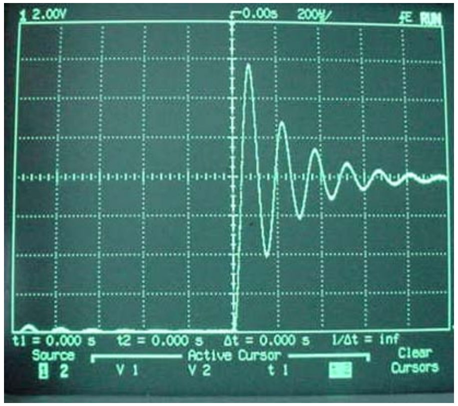
The time constant in an RL circuit is often referred to as "the RL time constant."

Behavior of RLC Circuit

- A circuit with *R*, *L*, and *C* can exhibit oscillatory behavior if the components are chosen properly.
- For many values of *R-L-C*, there will be no oscillation.
- The expression that describes this behavior is shown at right.
- The parameter ω_d is the radian frequency ($\omega_d = 2\pi f$, *f* the frequency in Hz), which depends on the values of *R* and *C*.
- α is the damping factor, which determines the rate at which the oscillation dies out.



$$v_c(t) = V(1 - [\cos\omega_d t]e^{-\alpha t})$$



Behavioral Components of RLC Circuit

- In the formula for $v_C(t)$, the radian frequency of oscillation, ω depends on R, L, and C.
- Note that in general, the smaller L and C, the higher frequency the oscillation. Also, if R is too large the quantity under the square root is negative, which means there is no oscillation.
- Note that α is very similar to τ. In fact the value of α is exactly ½ the value of τ for an RL circuit.

$$v_c(t) = V(1 - [\cos \omega_d t]e^{-\alpha t})$$

$$\alpha = R/2L$$

$$\omega_d = \sqrt{(1/LC) - (R/2L)^2}$$