Mechanics Y&F Chapter 3, Lecture 2
Motion in two and three dimensions

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Objectives

Projectile Motion

Relative Velocity

Projectile motion

What is a projectile?

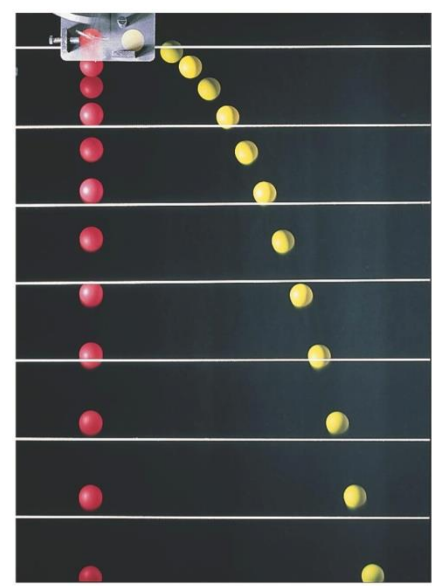
- Any body given an initial velocity and allowed to follow a path governed only by gravity and wind resistance
- Examples: baseball, football, fired cannonball

Trajectory: path followed by a projectile

Projectile motion

Projectile motion is at most a 2D problem, defined by the direction of the initial velocity vector. (Gravity can't move an object laterally.)

- X & Y coordinates can be treated <u>separately</u>
- X acceleration is always 0
- Y acceleration is –g (constant)
- Since acceleration is constant, we can use our table of equations from chapter 2…



Constant Acceleration Formulas

Equation	Included variables	Excluded variable
$v = v_0 + at$	v, v_0, a, t	Δx
$\Delta x = v_0 t + \frac{1}{2} a t^2$	$\Delta x, v_0, a, t$	v
$v^2 = v_0^2 + 2a\Delta x$	$\Delta x, v, v_0, a$	t
$\Delta x = \frac{1}{2}(v_0 + v)t$	$\Delta x, v, v_0, t$	a
$\Delta x = vt - \frac{1}{2} a t^2$	$\Delta x, v, a, t$	v_0

Projectile Motion

In the x-direction:

$$a_x = 0$$

$$v_x = v_{0x} \text{ (constant)}$$

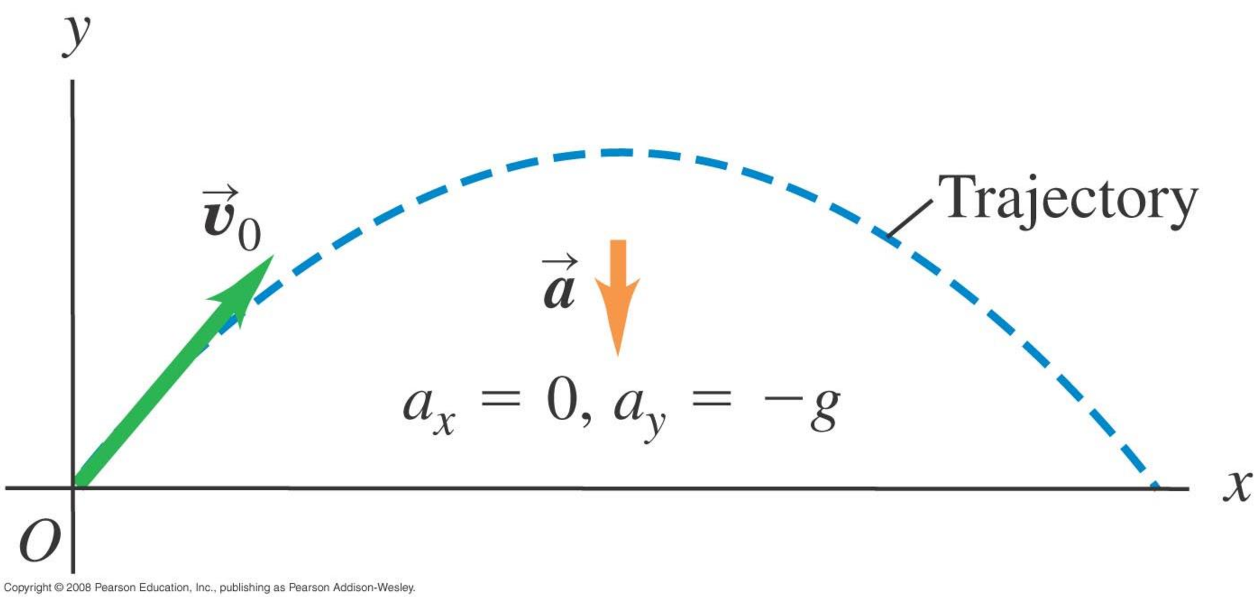
$$x = x_0 + v_{0x} t$$

In the y-direction:

$$a_y = -g$$

$$v_y = v_{0y} - gt$$

$$y = y_0 + v_{0y} t - \tfrac{1}{2} g t^2$$



Projectile Motion

For initial position at the origin, x_0=y_0=0

Fired at an initial speed v_0, angle θ_0:

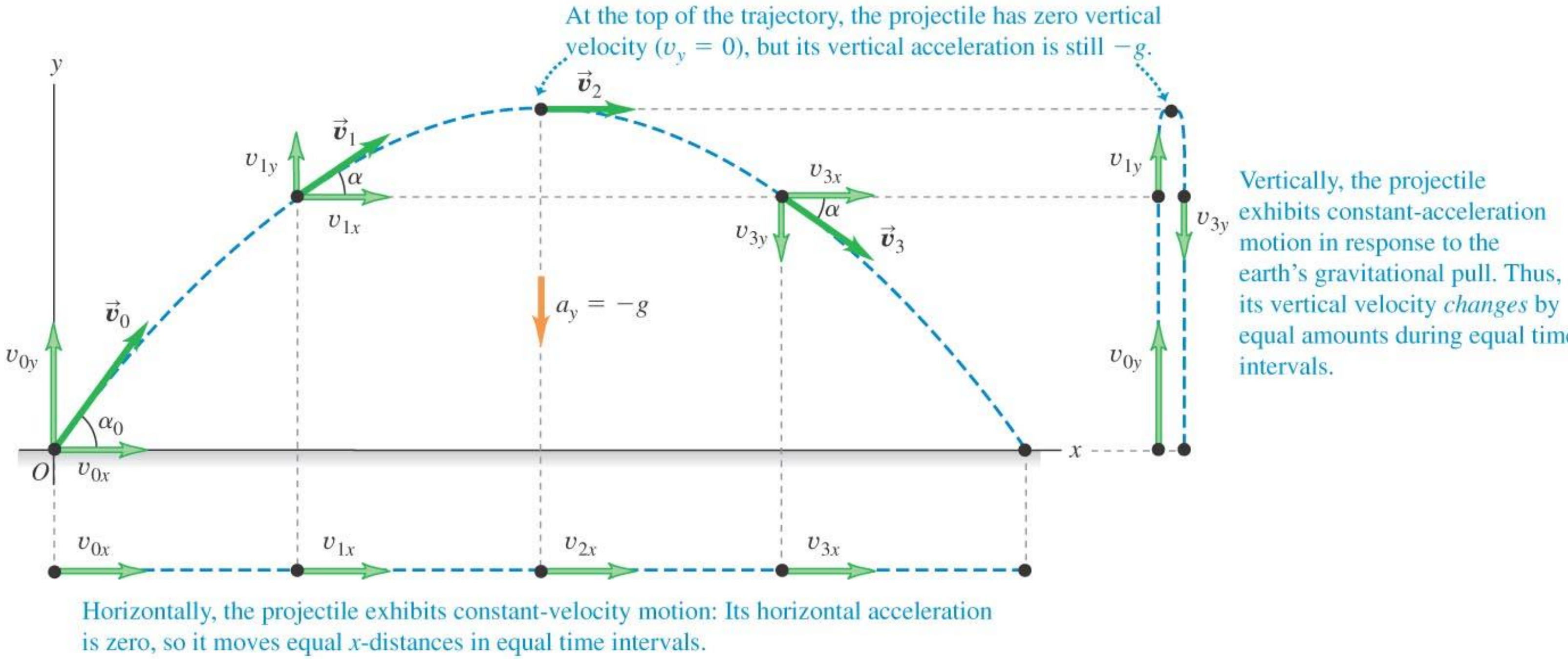
$$v_{0x} = v_0 \cos\theta_0 \qquad v_{0y} = v_0 \sin\theta_0 - gt$$

$$x = (v_0 \cos\theta_0)t \qquad y = (v_0 \sin\theta_0)t - \tfrac{1}{2} g t^2$$

Projectile Motion

For initial position at the origin, $x_0=y_0=0$

Fired at an initial speed v_0, angle θ_0



Projectile Motion

At any given time, we can find the total displacement from the origin:

$$r(t) = \sqrt{(x(t))^2 + (y(t))^2}$$

And the speed:

$$v(t) = \sqrt{(v_x(t))^2 + (v_y(t))^2}$$

And the angle of the velocity with time:

$$\tan\theta = \frac{v_y}{v_x}$$

Projectile Motion

What is the shape of projectile motion? Can we find it mathematically?

$$x = (v_0 \cos\theta_0)t$$

So $\quad t = x/(v_0 \cos\theta_0)$

$$y = (v_0 \sin\theta_0)t - \tfrac{1}{2}gt^2$$

$$y = (v_0 \sin\theta_0)\frac{x}{v_0 \cos\theta_0} - \tfrac{1}{2}g\left(\frac{x}{v_0 \cos\theta_0}\right)^2$$

Projectile Motion

What is the shape of projectile motion? Can we find it mathematically?

$$y = x(\tan\theta_0) - (\frac{g}{2v_0^2\cos^2\theta_0})x^2$$

$$y = bx - cx^2$$ for b,c constants

It's a parabola!

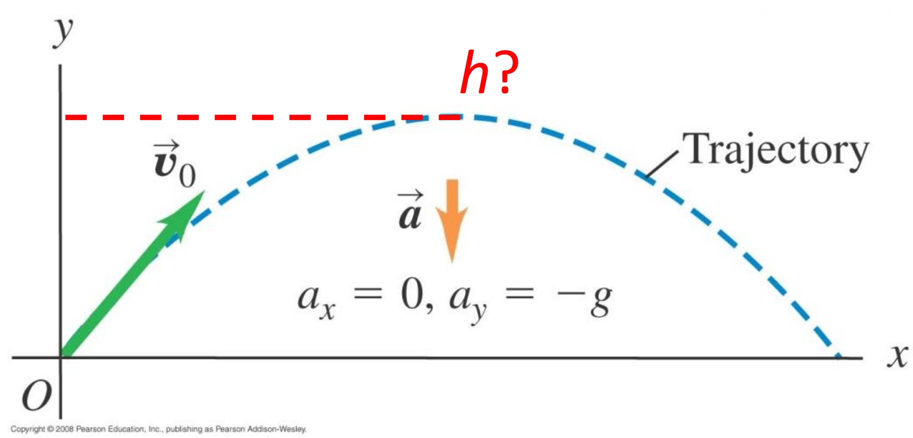
Projectile motion, max height

What is the maximum height *h* achieved by a projectile fired at angle α_0 & speed v_0?

When maximum height is achieved, v_y=0. Call this time t_h.

$$v_y = v_{0y} - g t_h = 0$$

$$t_h = \frac{v_{0y}}{g} = \frac{v_0 \sin\alpha_0}{g}$$



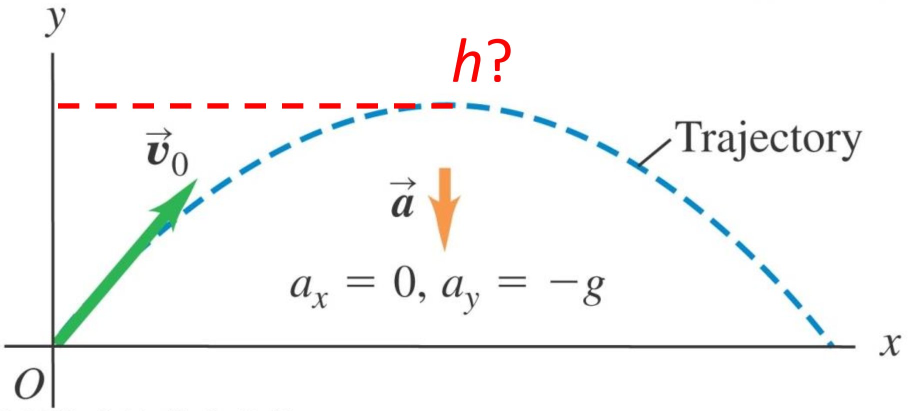
Projectile motion, max height

Now sub this t_h into the equation for y:

$$y = h = (v_0 \sin\alpha_0)t - \tfrac{1}{2}gt^2$$

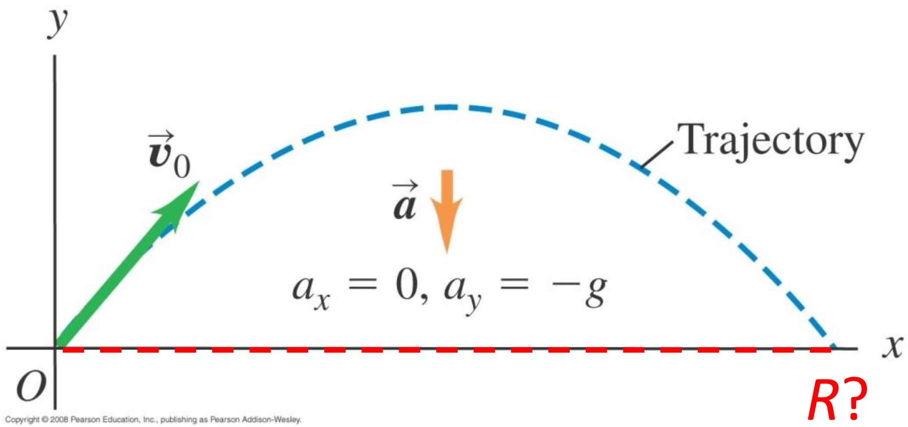
$$h = (v_0 \sin\alpha_0)\frac{v_0 \sin\alpha_0}{g} - \tfrac{1}{2}g\left(\frac{v_0 \sin\alpha_0}{g}\right)^2$$

$$\boxed{h = \frac{v_0^2 \sin^2\alpha_0}{2g}}$$



Projectile motion, max range

What is the maximum range R achieved by a projectile fired at angle α_0 and speed v_0?



The time for the projectile to return to the ground is given when *y*=0:

$$y = 0 = (v_0 \sin\alpha_0)t_R - \tfrac{1}{2} g t_R^2$$

Projectile motion, max range

What is the maximum range R achieved by a projectile fired at angle α_0 and speed v_0?

$$0 = (v_0 \sin\alpha_0)t_R - \tfrac{1}{2} g t_R^2 \quad \Rightarrow \quad t_R = \frac{2v_0 \sin\alpha_0}{g}$$

Sub t_R into x equation:

$$x = R = (v_0 \cos\alpha_0)t_R = (v_0 \cos\alpha_0)\frac{2v_0 \sin\alpha_0}{g}$$

$$\boxed{R = \frac{2v_0^2}{g}\sin\alpha_0 \cos\alpha_0 = \frac{v_0^2}{g}\sin 2\alpha_0}$$

Projectile motion, max range

At what initial angle α_0 is range maximized?

$$R = \frac{v_0^2}{g}\sin 2\alpha_0 \quad \Rightarrow \quad \frac{dR}{d\alpha} = \frac{2v_0^2}{g}\cos 2\alpha_0 = 0$$

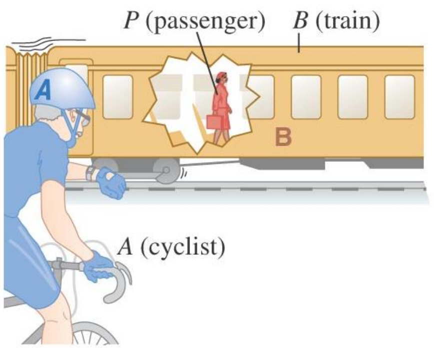
$$\cos(2\alpha_0) = 0$$

$$2\alpha_0 = 90^0$$

$$\boxed{\alpha_0 = 45^0}$$

Relative motion

If a person is walking at 1.0 m/s on a train that is moving at 3.0 m/s, how fast are they going?



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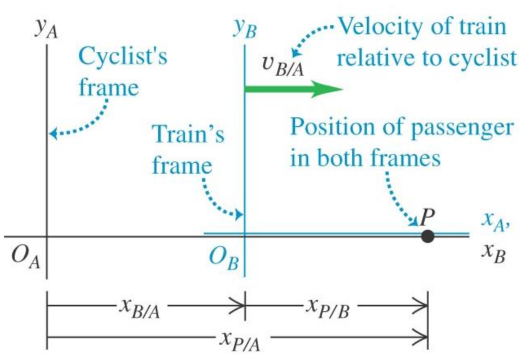


Depends on the *frame of reference*.

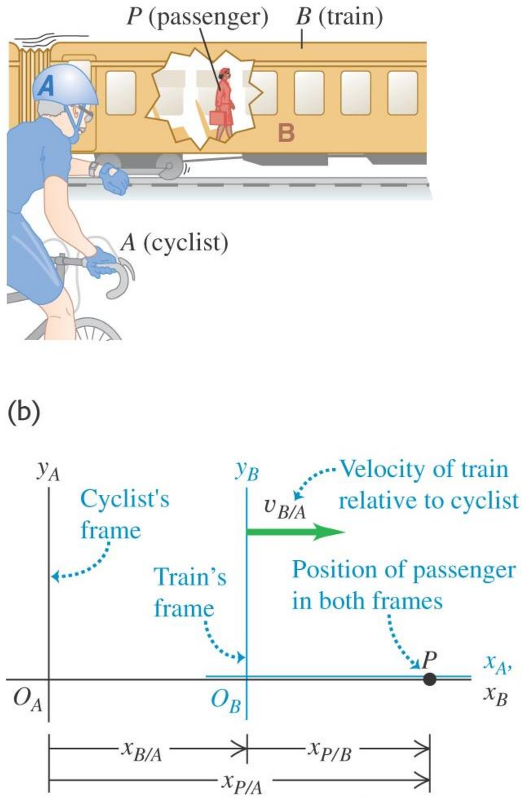
Consider a cyclist observing from outside the train.

Notation: $x_{p/a}$

" The position of p with respect to a"

$$x_{P/A} = x_{P/B} + x_{B/A}$$

Relative motion



$$x_{P/A} = x_{P/B} + x_{B/A}$$

$$\frac{dx_{P/A}}{dt} = \frac{dx_{P/B}}{dt} + \frac{dx_{B/A}}{dt}$$

$$v_{P/A} = v_{P/B} + v_{B/A}$$

Also: $\vec{v}_{P/A} = \vec{v}_{P/B} + \vec{v}_{B/A}$

So the cyclist sees the passenger moving at the speed of the passenger with respect to the train plus the speed of the train with respect to the cyclist.