Description of Numerical Approach

The governing equations for the strain probability density functions $p_1(s,t)$, $p_2(s,t)$, and $p_3(s,t)$ are

$$\frac{\partial p_1}{\partial t} + \frac{dL}{dt}\frac{\partial p_1}{\partial s} = k_a \delta(s) U_{NR} P - \tilde{k}_d\, p_1 - \tilde{k}_1\, e^{-\alpha_1 s}\, p_1 + k_{-1}\, e^{+\alpha_1 s}\, p_2$$
$$\frac{\partial p_2}{\partial t} + \frac{dL}{dt}\frac{\partial p_2}{\partial s} = \tilde{k}_1\, e^{-\alpha_1 s}\, p_1 - k_{-1}\, e^{+\alpha_1 s}\, p_2 - \ k_2\, e^{-\alpha_2 s}\, p_2 + \tilde{k}_{-2}\, p_3$$
$$\frac{\partial p_3}{\partial t} + \frac{dL}{dt}\frac{\partial p_3}{\partial s} = \ k_2\, e^{-\alpha_2 s}\, p_2 - \tilde{k}_{-2}\, p_3 - \tilde{k}_3\, e^{\alpha_3 (s+s_3)^2} p_3. \tag{1}$$

These equations are simulated by discretizing the probability density variables on a grid

$$p_1(s,t) \approx P_1(n,t)$$
$$p_2(s,t) \approx P_2(n,t)$$
$$p_3(s,t) \approx P_3(n,t)$$

where $n \in (0, N)$ represents the index on the spatial grid and $P_1(n,t)$, $P_2(n,t)$, and $P_3(n,t)$ are discretized approximations of $p_1(s,t)$, $p_2(s,t)$, and $p_3(s,t)$. Since the velocities simulated in this study are always negative (associated with muscle shortening), the strain variable s is discretized over a range of negative values:

$$s = \ -nh, \quad n = 0, 1, \dots N$$

where h is the space step. Equation (1) is numerically integrated over a finite times step Δt by splitting the space and time operators into separate steps.

Specifically, the two operators are, an advection operator:

$$\frac{\partial P_1}{\partial t} + \frac{dL}{dt}\frac{\partial P_1}{\partial s} = 0$$
$$\frac{\partial P_2}{\partial t} + \frac{dL}{dt}\frac{\partial P_2}{\partial s} = 0$$
$$\frac{\partial P_3}{\partial t} + \frac{dL}{dt}\frac{\partial P_3}{\partial s} = 0 \tag{2}$$

and an ordinary-differential equation operator:

$$\frac{dP_1(n,t)}{dt} = k_a \delta(s) U_{NR} P - \tilde{k}_d\, P_1 - \tilde{k}_1\, e^{+\alpha_1 nh}\, P_1 + k_{-1}\, e^{-\alpha_1 nh}\, P_2$$
$$\frac{dP_2(n,t)}{dt} = \tilde{k}_1\, e^{+\alpha_1 nh}\, P_1 - k_{-1}\, e^{-\alpha_1 nh}\, P_2 - \ k_2\, e^{+\alpha_2 nh}\, P_2 + \tilde{k}_{-2}\, P_3$$
$$\frac{dP_3(n,t)}{dt} = \ k_2\, e^{+\alpha_2 nh}\, P_2 - \tilde{k}_{-2}\, P_3 - \tilde{k}_3\, e^{\alpha_3(-nh+s_3)^2} P_3. \tag{3}$$

For a finite time step Δt, intermediate estimates of strain densities are first calculated by integrating Equation (2) using an upwind differencing approach, with the time step

$$P_1^*(n,t) = P_1(n+1,t)$$
$$P_2^*(n,t) = P_2(n+1,t)$$
$$P_3^*(n,t) = P_3(n+1,t)$$

where the time step is set to match the space step based on the constant velocity: $\Delta t = -h/(dL/dt)$. Next the ordinary differential equations of Equation (3) are integrated using a differential equation integrator package over the time interval $(0, \Delta t)$ with initial condition $P_1^*(n,t)$, $P_2^*(n,t)$, $P_3^*(n,t)$ to yield $P_1(n, t+\Delta t)$, $P_2(n, t+\Delta t)$, $P_3(n, t+\Delta t)$.