logo

Solution of 3 component decay

Computer Project 1 of NPRE 247

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I INTRODUCTION

1.1 BACKGROUNDS

Radioactive decays obey to the radioactive decay law: the decay probability of an unstable nucleus is independent of the past history and is the same for all radionuclides of the same type.



The number of atoms of a particular radionuclide depends on three aspects: (1) rates of production and decay, (2) initial values of it and its parents, (3) the rate at which it escapes from the sample. In this computer project, we consider a decay chain of three components, where  decays to  with decay constant ,  decays to  with decay constant  and  is stable.



We denote the number of these three types of particles at time  as , and decay constant of particle  as . The differential decay equations for these particles are



1.2 ANALYTICAL SOLUTIONS

The analytical solutions of the equations above can be attained through this way: Integrate the first equation we can get ; Plug it into the second equation and integrate, we can get; Plug it into the third equation and integrate, we can get . Thus, the solutions are



To obtain the time of maximal , we can let , which yields





Its solution is



Note that, to make the above solution meaningful, these following conditions should be satisfied,



They can be simplified as



which means, the number of particle  should increase at first. So the time of maximal  is



1.3 NUMBERICAL METHODS

In this section we show how to numerically solve the differential equations above with a forward difference approximation. For , we can approximate it as .If we denote the initial number of paritcle  as , and denote the number after time  as , then we can approximate  as  for time . Thus, the differential equations can be converted into the equations below,



If we give the initial number of these three types of particles , then we can get  using the above equations, and so on. Thus, by repeating this process, we can get the numbers at any time , with a time resolution of .

To use the numerical solutions above, we also need these initial values: (1) half life time  or decay constant  of particle A and B; (2) initial numbers of A, B and C; (3) time resolution  used to calculate; (4) final time , which is the end time of calculation.

In practice, the given  may yields a coarse solution, which is not stable enough. Thus, we may have to decrease  by half until the solution does not change significantly with further decreasing of .

1.4 FILE FORMAT DESCRIPTION

There are four output files for every decay chain, they are

(1) Coarse numbers of A, B and C;

(2) Medium numbers of A, B and C;

(3) Stable numbers of A, B and C;

(4) Time of maximal  vs. .

The first three files have the following format,

(First line)       

(Second line)  ‘Length of the data’

(Third line) data.

The fourth file has the following format,

(First line)       

(Second line)  calculated by analytical solution; ‘length of the data’

(Third line) data.

II DECAY CHAIN 1

In this section, we consider a given decay chain with the following initial values:

Table 2.1 Initial values of decay chain 1

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |  |
| 1h | 10h | stable | 100 | 0 | 0 | 1h | 50h |

Here, and  are half life time of A and B; *C* is stable. ,, are initial numbers of A, B and C. is the given initial time interval used for calculation.  is the end time of calculation.

With equations and in section I, we could get the analytical and numerical solutions. Here, we assume that if the maximal difference of numbers between results calculated from  and  is 0.01, the numerical solution is stable. This value could be smaller if needed.

The numerical solution shows: when , it gives a coarse solution; when , it gives a medium solution; when , it gives a stable solution, which means the change of numbers with further smaller  is within 0.01. These three situations and the analytical solution are shown in Fig. 2.1.



Fig. 2.1 Coarse, medium and stable number of .

The green line indicates the analytical solution.

For stable condition, the numbers of A, B and C and their sum are shown in Fig. 2.2.



Fig. 2.2 stable numbers of . The black dash line

indicates their sum, which is always .

Time of maximal  and dependence of  are shown in Fig. 2.3. The time value of the maximal  obtained from equation is also shown in it.



Fig 2.3 The time of maximal  vs. 

III DECAY CHAIN 2

In this section, we consider a given decay chain with the following initial values:

Table 3.1 Initial values of decay chain 2

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |  |
| 2min | 12s | stable | 100 | 0 | 0 | 10s | 10min |

The symbols used here has the same meaning as those in section II.

The numerical solution shows: when , it gives a coarse solution; when , it gives a medium solution; when , it gives a stable solution, which means the change of numbers with further smaller  is within 0.01. These three situations and the analytical solution are shown in Fig. 2.1.



Fig. 3.1 Coarse, medium and stable number of .

The green line indicates the analytical solution.

For stable condition, the numbers of A, B and C and their sum are shown in Fig. 2.2.



Fig. 3.2 stable numbers of . The black dash line

indicates their sum, which is always .

Time of maximal  and dependence of  are shown in Fig. 2.3. The time value of the maximal  obtained from equation is also shown in it.



Fig 3.3 The time of maximal  vs. 