LAB 2: Astronomy with the 21-cm Line and Waveguides

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1. Introduction

In this lab, we build upon the signal processing techniques of the previous lab, applying them to making astronomical measurements with a horn antenna tuned to frequencies around 1.4 GHz where we can measure 21-cm hyperfine emission from neutral hydrogen in the Milky Way. With luck, we should be able to see evidence of the spiral-arm structure of gas in our galaxy.

In the second half of this lab, we expand our understanding of how signals propagate with measurements from transmission lines and waveguides. Our first examination of statistical regression uses these measurements to fit a line that empirically determines the speed of light.

As before, your work should be oriented around producing a high-quality report that addresses the goals in §2. From your previous report, you will be familiar with the need to succintly describe your experimental setup and hardware. New to this report will be sections describing how you obtained your observations and the calibration and analysis that went into producing your key results.

We have a few more suggestions for how to organize your work, building on the suggestions of the previous lab:

- Develop (version-controlled) scripts for taking data from the command line. Have these scripts save data with **all the necessary metadata** needed to reconstruct your observation. Omitting critical information could mean you have to redo observations later.
- Test your reality. How do you know your rotation matrices give you the right pointing? How do you know if your gain calibration is right? To build your confidence, you need internal consistency checks.
- As you conduct observations, think critically about your assumptions and approximations. How well have you pointed the horn? How completely did you cover the aperture for the calibration run? The more you can capture as you do the work, the more resources you will have for critically analyzing your results and procedures in your report.
- By the end, you should have empirical numerical results to report. Remember that values without errors are meaningless. As you estimate errors, think about how well your model matches the data. How can you estimate the accuracy of your results? Are you confident enough in your analysis to bet money on your reported accuracy? If not, think about the sources of error (systematic and otherwise) that may be undermining your confidence.

2. Goals and Instructions for Your Report

As always, lab reports and analysis code must be written *individually*.

The electronic components are such an important part of this lab that you should include a block diagram of the telescope electronics in your lab report. You can prepare this either by either hand or computer. If by hand, you'll have to scan your drawing to get a file you can insert into your report's tex file. If by computer, you can use your favorite software for making line drawings. Although it is dated, `xfig` comes installed on Linux, so that's an option.

Below is a list of learning goals that your report should demonstrate mastery of.

- Learn about time. Demonstrate accurate conversion between UTC, PST, LST, and Julian Day.
- Learn about telescope pointing and use rotation matrices to convert among spherical coordinate systems.
- Measure a 21-cm line power spectrum from atomic hydrogen in the Milky Way at a defined and reproducible location.
- Calibrate your telescope observations to an absolute scale, remove systematic instrumental

effects in your spectra, and apply noise-reduction techniques.

- Learn about Doppler correction and produce spectra with velocities calibated relative to standard frames of reference.
- Fit spectra with Gaussian components to localize celestial structures.
- Demonstrate undestanding of transmission lines and waveguides, and how to minimize reflections with impedance matching.
- Derive propagation velocities of radio waves by measuring the wavelength of standing waves.
- Empirically explore the relationship between waveguide geometry and cutoff frequency.
- Use linear and non-linear least-squares methods to fit models to data and estimate error.

3. Schedule

This lab covers a lot of new territory, particularly in the realm of data analysis. Work hard to get good observations early, reserving plenty of time for analysis and, if necessary, re-observation. A suggested schedule follows.

1. *Week 1.* Finish §6 and §7, and understand §4. Be prepared to show work, software, and results in class.
2. *Week 2.* Finish §8, §9, and §10. Produce candidate plots and analysis results for class.
3. *Week 3.* Finish any re-observations needed for the first two weeks, then write your formal report. Remember to follow the report guidelines and address the specific goals in §2.

4. What Time Is It?

From relativity, we know that time depends on reference frame. Nothing drives this home like sitting on the surface of an orbiting, spinning sphere. To deal with this, humans have invented a surfeit of time standards. We will restrict our attention to those most useful for astronomy.

Coordinated Universal Time (UTC) is the Civil Time[1] in Greenwich, England, which is 8 hours ahead of Pacific Standard Time (PST). In the middle of this course, we switch from PST to Pacific Daylight Time (PDT); PST = UTC − 8 hr, while PDT = UTC − 7 hr). UTC measures solar time; 24 hours is the time it takes for the Sun to appear in the same position in the sky on neighboring days.

Another standard, common since the computer era, is **Unix Standard Time**, which measures the number of seconds since midnight 1 Jan., 1970, UTC. This clock is how your computer keeps track of time. All other computer times are derived from this one; it is what you get if you call `time.time()`. You phone/computer uses a quartz crystal oscillator to keep track of this time over short intervals. Over longer intervals, a time-exchange protocol called Network Time Protocol (NTP) is used to discipline the on-board clock to atomic clocks in timing centers around the world.

Sidereal Time is "star time"; it tells when distant stars (i.e. not the Sun) rise and set. The sidereal time period is shorter than the 24-hour Solar time period: 1/356.24 less, to be nearly exact. The difference, which corresponds to slightly less than 4 minutes per day, comes from how much the Earth's orbit around the Sun changes the position of the Sun in the sky. Sidereal time depends on the Earth's spin. The Earth is constantly slowing owing to tidal friction produced by the Moon.

[1]You use Civil Time when you set your alarm clock for getting up in the morning.

(Where does the angular momentum go?) The conversion between Civil Time and LST has to be adjusted periodically by inserting *leap seconds*.

Local Sidereal Time (LST) adjusts sidereal time by longitude so that a star of a given **right ascension** (RA; the “longitude” coordinate for celestial sources) transits the local meridian (the line of longitude that goes right overhead) at the moment that LST = RA.

The **Julian Day** is a sequential numbering of Solar days since 1 Jan. –4713. It uniquely specifies the date and time (as a fraction of a day) without involving months (with their nonsensical definitions[2]), leap years, etc. It begins at noon in Greenwich. This makes it 12 hours out of phase with UTC, but the international date line is 12 hours away from UTC, so JD is in sync with humankind’s definition of ‘when the day begins’. On computers, the Julian day is represented as double-precision float.

The **Modified Julian Day** is a shorter version of Julian Day. MJD is (sigh) 12 hours out of sync with JD, so it is in sync with UTC. MJD=0 corresponds to 0 hr UT on 17 Nov 1858.

4.1. Useful Python Procedures for Time Conversion

Python has several built-in modules for dealing with time, including `time` and `datetime`. Astronomers, however, have historically been the best time-keepers on the planet. For astronomy-quality time routines, `astropy.time`, part of the `astropy` package, is the industry standard. We have wrapped some of them into `ugradio.timing` for your convenience.

Calculating LST requires knowing your longitude. New Campbell Hall (NCH) is at `(lat,lon) = (37.8732, -122.2573)`, Note that `lon = -122.2573 = (360 - 122.2573) = 237.7427`. This information is stored in the `ugradio.nch` module. For all procedures mentioned below, this location is default

```
local_now = ugradio.timing.local_time() # current local time as a string
ut_now = ugradio.timing.utc() # current UTC as a string
ut_now = ugradio.timing.unix_time() # seconds since 1 January 1970
julian_now = ugradio.timing.julian_date() # current julian day (which \
        contains the current time, too--it's not just an integer \
        number.
lst_now = ugradio.timing.lst() # current LST at NCH
lst_julian = ugradio.timing.lst(jd) # LST for the specified Julian day
ut_julian = ugradio.timing.unix_time(jd) # seconds since 1/1/1970 for given JD
julian_ut = ugradio.timing.julian_date(ut) # julian day for given unix time
```

5. Handouts

The following handouts may be of use to you:

1. The complete story of producing calibrated spectral line shapes and intensities: *CALIBRATING THE INTENSITY AND SHAPE OF SPECTRAL LINES*. This handout is optional because the steps discussed in the lab instructions are sufficient, but if they are unclear or

[2] “Thirty days hath September. . . ”

you want to know more, this writeup has it all.

2. How does the power we receive with a telescope relate to a source being observed? It's all about specific intensity, usually denoted I: "SPECIFIC INTENSITY: THE FOUNT OF ALL KNOWLEDGE!". You need to understand these topics to interpret your measurements.
3. Astronomical coordinate transformations with Rotation Matrices: "SPHERICAL/ASTRONOMICAL COORDINATE TRANSFORMATION". converting among Az/El, Ha/Dec, Ra/Dec, L/B. You need to know all of this stuff!
4. You can do this lab without doing any textbook-type reading... but then you wouldn't gain the insight from the highly readable and well-written text by Ramo, Whinnery, and van Duzer (RWvD), *Fields and Waves in Communication Electronics.* This is a great book because it contains commentary on applications to modern devices (e.g., the discussions of transmission lines carrying pulses from one piece of computer hardware to another are very nice). Reading this book is optional, but in any case see notes below in §11.

6. Your First 21-cm Measurement (Group Lab Activity, Week 1)

This week we will use the techniques we explored in Lab 1 to observe the 21-cm line of neutral hydrogen (colloquially, **HI**, pronounced "H-one"), measuring its line shape, velocity, and intensity using the big horn on the New Campbell Hall roof. Most of the measured power comes from noise in our own electronics, not the HI line. It's often called noise, and we need to calibrate out this instrumental contribution and the responses of our amplifiers and filters to obtain the correct line shape and intensity.

We'll measure the 21-cm line twice. The first time, the goal is to master the technical aspects and familiarize ourselves with the system and procedures, so instead of worrying about where to point the horn, we'll just take whatever position happens to be overhead. The second time (§8), we'll manually point the horn to a designated position and make a calibrated profile to compare with a well-established standard profile measurement.

6.1. The Receiving System

We will use a double-heterodyne system for the 21-cm line at 1420.4 MHz. **Double heterodyne** means we have two mixing stages.

the first mixer, we use the DSB technique. Suppose we use an LO (the first local oscillator) at 1230.0 MHz so that the difference frequency is 190.4 MHz and the sum frequency is 2650.4 MHz. We use a 20 MHz-wide bandpass filter centered at 190 MHz to excise the sum frequency, leaving us with one copy of the 21-cm line shifted to 190.4 MHz from 1420.4 MHz.

For the second mixer, we use the SSB technique. The second LO has frequency 190.0 MHz, so the output is at **baseband**, with positive and negative frequencies centered about zero. In the absence of Doppler shift, we are left with the line centered at 0.4 MHz. However, the line is shifted and broadened by Galactic rotation, the Earth's orbital velocity, and the Earth's spin, so it covers a range of frequency of, for example, 1420.2 ± 0.5 MHz. Use a low-pass filter with a cutoff frequency of $\sim$2 MHz to eliminate aliasing, sample the complex signal, and Fourier transform to calculate the power spectrum. We take many power spectra and average them to reduce the noise.

6.2. Your Measurement

Before taking data, do basic system checks: (1) make sure the signal levels are okay and (2) experimentally determine which way frequency increases on your power spectra.

Once you are convinced your signal is getting through, set the system up as you will be using it to observe. Point the horn to zenith to reduce interference and thermal noise. Take some data. How fast must you sample?

Look at the range of sample values using, e.g., `numpy.histogram`. As in Lab 1, check that you are not overly quantized or clipping. Adjust `volt_range` in the call to `pico.capture_data` accordingly. The histogram shape should look like a well-known function. Which function? Does it?

Next, insert a test signal (at a low level; ~ -40 dBm) so that it appears in the upper sideband and take some data; then change the test signal frequency so it's in the lower sideband. You'll use this to make sure that the SSB mixer is doing its job properly and to determine whether the frequency axis is flipped or not.

As a final note, remember that if it rains, the horn is a gigantic bucket. You won't see any astronomical signal unless you dump it!

6.2.1. Planning Observations

Having determined that the system basics work, it's time to do astronomy! For this first measurement the main goal is to master the technical aspects, so use whatever position happens to be overhead and point the horn straight up. The line will be strongest—strong enough to see visually—in the range LST = 19 – 6 hr.

It is most convenient to use temperature units for the power that we measure. Accordingly, the power that we measure is called the **system temperature** $T_{\rm sys}$. It's a function of frequency and has two kinds of behavior: the **continuum**, which is devoid of spectral features and changes very slowly with frequency; and the **line**, which in this case is the 21-cm line and it changes relatively rapidly with frequency—hence our desire to obtain the line shape.

The system temperature has two contributions: the dominant contribution from our electronics, which we call the **receiver temperature** $T_{\rm rx}$; and the contribution our antenna picks up from the sky, the **sky temperature** $T_{\rm sky}$. (This is also sometimes called the **antenna temperature**). Thus $T_{\rm sys} = T_{\rm rx} + T_{\rm sky}$, and above 300 MHz, usually $T_{\rm rx} \gg T_{\rm sky}$.

Our horn is equipped with an old, noisy first amplifier so $T_{\rm rx} \sim 300$ K; in contrast, our Leuschner telescope is much better, with $T_{\rm rx} \sim 50$ K. The sky temperature comes from the Cosmic Microwave Background, with brightness temperature ($T_{\rm CMB} = 2.7$ K); from interstellar/intergalactic space, with brightness temperature $T_{\rm IGM}$ (usually no more than a few K in the continuum and up to about 100 K in the HI line); and the Earth's atmosphere with brightness temperature $T_{\rm atm}$, perhaps a few K at the HI line frequency. So off of the HI line we have $T_{\rm sky} \sim 10$ K, while on the HI line, in the Galactic plane where it is strongest, we have $T_{\rm sky} \sim 100$ K.

We'll take two sets of data:

1. a long integration to measure the line's *shape*, and
2. a short integration so we can calibrate the line's *intensity.*

6.2.2. *Two Frequency-Switched Line Measurements*

The measured power spectrum shape is dominated by the frequency-limiting filters acting on the system temperature. To see the line, which is weak, we need to correct for these filter shapes, which we do by obtaining a spectrum containing no line. Accordingly, to get the shape we take two spectra: one with the line present (the on-line spectrum $s_{\rm on}$) and one with the line not present (the off-line spectrum $s_{\rm off}$).

We could obtain the on-line spectrum by centering the line and making a measurement; then changing the first LO frequency so that the line shifts either completely outside the band (this is called **frequency switching**), or partway over but is still in the band (in-band frequency switching). The latter is better because you are always looking at the line; you end up with more measurements and better signal/noise.

To accomplish this, take a spectrum with the line roughly in the upper half of the baseband spectrum, and another with it centered roughly in the lower half. Use the first as the on-line and the second as the off-line spectrum for the upper half. Similarly, for the lower half, use the second as the on-line and the first for the off-line. (The HI line frequency is 1420.4058 MHz).

The line is weak, and you'll need to take lots of spectra. Recall that `pico.capture_data` obtains a specified number of blocks (`nblocks`), each of which has 16000 datapoints and writes them all out into a single file. Suppose you decide to obtain 10000 spectra. You might be tempted to do it in one fell swoop by setting `nblocks=10000`. That may work, but be aware that it will take about 5 minutes or more. You might want to instead take ten sets with `nblocks=1000` so you can do a sanity check on your data without waiting forever.

6.2.3. *Two Frequency-Switched Reference Measurements*

Intensity calibration requires a second set of (short) measurements with your telescope aimed toward a source of known temperature. Easiest is probably to take one with the horn looking at a known blackbody and one looking at the cold sky. What's a convenient blackbody? You and your friends! So take one short measurement with the horn pointing straight up at the cold sky and the other with as many people as you can find standing in front of it to fill the aperture. Call these spectra $s_{\rm cold}$ and $s_{\rm cal}$, respectively.

7. Analysis (Individually at Home, Week 1)

7.1. Take a Suitable Average/Median

Consider, first, the $s_{\rm on}$ and $s_{\rm off}$ spectra, from which you can find the line shape. There are many individual spectra, 10000 in our above example. You need to combine these to make a single spectrum for each measurement. You can do this by averaging the power spectra (use `numpy`'s `mean` function) or by taking the medians (use `numpy`'s `median` function). The former gives a less noisy result, but the latter handles time-variable interference better; use both and compare the results.

Even after combining the 10000 spectra, the resulting spectrum will look noisy. You can reduce the noise by averaging over channels—by 'smoothing' the spectrum. This reduces the noise, but degrades the spectral resolution, so you have to make a compromise on how many channels to smooth over. To decide, realize that the HI line is never narrower than about 1 km/s, so it's OK to degrade the frequency resolution to, say, 1 or 2 kHz. Again, you do the smoothing by averaging (by using `numpy.mean`) or medianing (by using `numpy.median`), and again try both to see what happens.

In the smoothed $s_{\rm on}$ spectrum you might not be able to see the HI line, because the instrumental bandpass dominates the spectrum shape. The instrumental bandpass is determined mainly by the low-pass filter, which should fall smoothly to zero as the frequency increases. Does it? If not, should you worry about aliasing?

7.2. Get the Line Shape

You can remove the instrumental bandpass to get the shape of the line $s_{\rm line}$ (but not the intensity), by taking the ratio

$$s_{\rm line} = \frac{s_{\rm on}}{s_{\rm off}} \ . \tag{1}$$

This is the first factor (i.e., the shape) in equation (13) in the spectral-line handout.

7.3. Get the Line Intensity

To get the line intensity in temperature units, we need to multiply terms of the calibration noise source, multiply the shape spectrum by the **gain**—the second factor in the handout's equation (13). We obtain the gain, G, by using a known difference in system temperature between the the calibration ($T_{\rm sys,cal}$) and the cold sky ($T_{\rm sys,cold}$) measurements, and seeing how much that known temperature difference changes the measured values in your spectra, as per equation (15) in the handout:

$$G = \frac{T_{\rm sys,cal} - T_{\rm sys,cold}}{\sum(s_{\rm cal} - s_{\rm cold})} \sum s_{\rm cold} \tag{2}$$

Here, a sum is over all channels in a spectrum. All this equation does is to convert measured units, which are digital numbers from the system, into physically meaningful units, i.e. Kelvin. Here, $T_{\rm sys,cal} = 300$ K, because that's the thermal power you injected by standing in front of the horn. Since $T_{\rm sys,cold} \ll T_{\rm sys,cal}$, to a first approximation you can neglect it (which is what we did in the handout). Then the final, intensity-calibrated spectrum is equation (16) in the handout, namely

$$T_{\rm line} = s_{\rm line} \times G \tag{3}$$

7.4. Plotting Intensity vs. Frequency—and Velocity

First, plot your final calibrated spectrum versus the r.f. frequency. Next, plot it versus the Doppler velocity[3]. Remember that, by astronomical convention, positive velocity means motion away (remember the expansion of the Universe!), so

$$\frac{v}{c} \approx -\frac{\Delta\nu}{\nu_0}, \tag{4}$$

[3]Astronomers usually express velocities in km s^{-1}.

where c is the speed of light and $\Delta\nu$ is the frequency offset from the line frequency ν_0[4].

7.5. Finally, Choose a Reference Frame

You may think we've done it all by this point, but we haven't! We need to correct the observed velocity for the orbital velocity of the Earth, and also the Earth's spin. And when observing the Galaxy, it is customary to express velocities with respect to the **Local Standard of Rest (LSR)**, so that's yet another correction.

Calculate the Doppler correction using `ugradio.doppler.get_projected_velocity` Correct the velocities and compare the spectrum for the observing frame and the LSR frame, which is approximately the frame that would rotate around the Galaxy in a circular orbit. Correcting to the LSR involves many components, including primarily: the rotation of the observatory around the center of the Earth, the orbit of the Earth around the barycenter of the solar system, and the peculiar velocity of our Sun with respect to other stars in the neighborhood. There are higher-order corrections (which are in the `barycorrpy` package we are using in `ugradio.doppler`, see (Wright & Eastman, 2014), including 1) special relativistic treatment of velocity, 2) general relativistic effects from the influence of the gravitational fields of all bodies in the solar system, and 3) the proper motion of the target source.

Notes on `ugradio.doppler`:

1. You need the celestial coordinates of the source, (ra, dec). How to find these for the horn pointing straight up? You could use rotation matrices. However, when looking straight up you don't need this powerful technique because, quite simply, the Dec is equal to the Latitude and RA=LST.
2. You need the Julian day of the observation; see §4.
3. You need the observatory coordinates (north latitude and west longitude) in degrees; you could enter them with the pair of optional input parameters `(obs_lat, obs_lon)`, but you don't have to because the default values are Campbell Hall's values (which are `lat`$= 37.873199$ `lon`$= -122.2573$ degrees).

8. Your Second 21-cm Line Measurement (Group Lab Activity, Week 2)

Obtain a fully-calibrated spectrum for the horn pointing at Galactic coordinates $(l, b) = (120°, 0°)$. How do you know where to point the horn? You definitely do need to use rotation matrices! See the handout "Spherical Coordinate Transformation" (§5). Plot your spectrum versus both the observed and the LSR velocity.

Your spectrum will probably look something like that in the top panel of Figure 1. There is a zero offset, a curvy baseline, and one or more spectral peaks. The curvy baseline comes from nonflat instrumental response before the first mixer, and can be removed with a low-order polynomial fit to the off-line channels (the approximate frequency ranges $-1.2 \rightarrow -0.6$ and $0.6 \rightarrow 1.2$ MHz). The dashed line is a second-order polynomial fit to the off-line channels and the bottom panel has this fit subtracted off. Here, the multiple peaks come from HI clouds having different velocities.

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[4]Radio astronomers, being frequency-oriented, use this equation; optical astronomers, of course, use something different: $\frac{v}{c} = \frac{\Delta\lambda}{\lambda_0}$ At high redshifts the difference becomes significant; the optical definition is the usual standard.

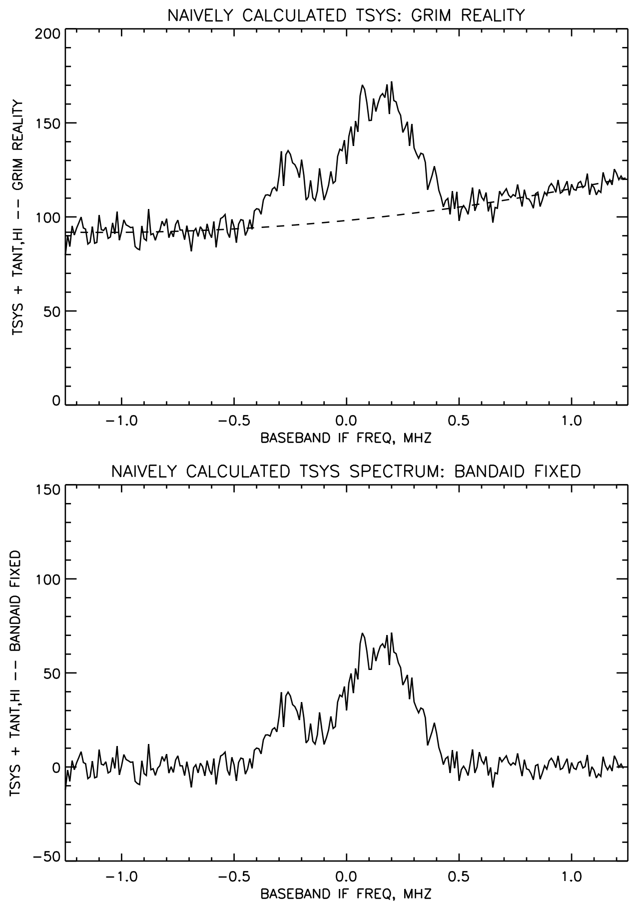

Fig. 1.— A typical raw spectrum with a curvy baseline and multiple velocity components.

The peaks are usually represented as Gaussians with appropriate central intensities, velocities, and widths. Fitting Gaussians requires specifying initial guesses for the component parameters. These guesses need not be highly accurate. For example, in the bottom panel of Figure 1, initial guesses for the two heights, centers, and widths (FWHM) could be [20, 50], [-0.3, 0.2] MHz, and [0.01, 0.03] MHz. After fitting, check to see if the result looks good: if it does not, then it's usually because you need one or more additional components. It is common to require a low-level, broad component that produces no obvious peak; here, the initial guesses might be [20, 50, 10], [-0.3, 0.2, 0.] MHz, and [0.01, 0.03, 0.07] MHz.

The procedures you need for these least-squares fits are:

1. For the polynomial fit, use `numpy.polyfit`
2. For fitting multiple Gaussians, use our home-grown `gaussfit`; for evaluating the Gaussians from the parameters produced by `gaussval`, use `gaussval`.

In class, we will compare the results from all groups. How reproducible is our science?

9. Measuring the Speed of Light with Waveguides (Group Lab Activity, Week 2)

How do you move power or electronic information from one place to another? *Transmission lines* (cables) and *waveguides* are indispensable. It sounds easy—just connect two things with a wire. But does this really work? We'll explore cables, waveguides, and reflections by measuring the Voltage Standing Wave Ratio (VSWR), which results from interference of incident and reflected waves.

The experimental work and measurements described below should be done by groups. The analysis in §10 should be done by individuals. The analysis is nontrivial, so don't delay with the measurements!

9.1. The coax slotted line at C-band (~ 3 GHz)

Set up the slotted coaxial line with the wide-range HP 83712B synthesizer. With the far end of the slotted line open, find the wavelength in the slotted line λ_{sl} by measuring the positions of *all* nulls as accurately as you can. Do the same with the far end shorted and note the differences. In particular, note how the positions of the nulls change when the slotted line is terminated with an open versus when it is shorted. Why is this?

From your measurements calculate the velocity [what does "velocity" mean?] in the slotted line by comparing λ_{sl} and the frequency. First do a quick calculation using your measurements. Then do it as accurately as possible by using least-squares fitting and following the advice in §10.1. *Note:* As it happens, velocity is independent of frequency for a coax cable—in contrast to a waveguide. If you are so motivated, you can check this experimentally.

9.2. The X-band waveguide (7 to 12 GHz)

Now we'll do the same as we did in §9.1, but for X-band waveguide. The cutoff frequency for X-band waveguide is about 7 GHz. Pick a suitable frequency and, with the far end of the waveguide completely open, measure the VSWR; then short the end of the waveguide, maybe with a piece of aluminum foil or a metal plate, and repeat. Why is an open ended waveguide different from an open ended coax cable?

With the end shorted, measure the velocity in the slotted waveguide by comparing the wavelength and the frequency; do this for several (half-dozen?) frequencies, including one or two near cutoff, and compare with theory. These frequencies should be reasonably closely separated, say by no more than 1 GHz. From these measurements, derive the cutoff wavelength (and thus the waveguide width) as accurately as you can. Also measure the waveguide dimensions, and compare with the width you derived above.

10. Statistical Error Analysis (Individually At Home, Week 2)

10.1. The coax slotted-line wavelength

For the measurements of §9.1, you sampled M cycles of the standing wave in the slotted line. You can calculate the wavelength λ_{sl} from the distances between a single null pair. While this calculation is a good estimate, each of your measurements has an error. You have measured the distance between $(M-1)$ null pairs. What's the best way to combine these measurements so as to obtain the most accurate λ_{sl}?

One way is to calculate the wavelength from each neighboring null *pair* and take the average of the $(M-1)$ *pair*s. You might then apply the usual rule to obtain the uncertainty in the average, i.e. the uncertainty is $\sigma/\sqrt{M-1}$. However, this would *not be correct*. Why? (Hint: The answer you get by averaging the $(M-1)$ pairs is identical to that obtained from a *single* well-chosen non-neighbor pair. Which one?). Actually, this method gives *almost* the best estimate of the wavelength. But you can do better!

Alternately, use the fact that the positions of the nulls should increase linearly with distance. So if x_m is the position of null number m, we should have

$$x_m = A + m\frac{\lambda_{sl}}{2} \ , \tag{5}$$

You know x_m and m from measurement and you would like to derive A and λ_{sl}. This is a classic least squares problem; you're fitting a first-degree polynomial, where the unknowns are A and λ_{sl}. You can use `numpy.polyfit`.

10.2. The X-band waveguide

RWvD provide a beautiful discussion of the TE_{10} mode in common rectangular waveguide (in which the ratio of side lengths equals 2). Most important is their equation 11, which gives the **guide wavelength**, λ_g; we reproduce it here:

$$\lambda_g = \frac{v_p}{f} = \frac{\lambda_{fs}}{[1-(\lambda_{fs}/2a)^2]^{1/2}} \ , \tag{6}$$

where λ_{fs} is the free-space wavelength. The guide wavelength is the wavelength within the waveguide, and combining it with the frequency gives the propagation velocity in the guide.

Here we want to do a least-squares fit of equation 6 to solve for the waveguide width a. Now, you've already measured a with the caliper. But here the idea is to compare your measurement of the a with what theory (that's Equation 6) predicts. So you imagine that a is unknown and determine it from your data on λ_g versus the free-space wavelength λ_{fs}. Doing this is not as straightforward as above in §10.1 because λ_g depends *nonlinearly* on a. The easiest way to handle this problem is by the brute force method, which we will discuss in class.

From your above solution, determine the best-fit value of a and compare it with the value you measured with the caliper.

11. Reference Reading

You can do this lab without doing any reading, so this section, which recommends parts of the highly readable and well-written text by Ramo, Whinnery, and van Duzer (RWvD), *Fields and Waves in Communication Electronics*, is optional. If you have the time and inclination to read, we suggest that you concentrate on the starred items below (***). The notes below refer to the second edition, which should reside in our lab's bookshelf.

- Theory of Coaxial Cable

  - *** *RWvD §5.2: Introduction: what transmission lines do.* Basic equations for transmission lines. Note equation (14) for the characteristic impedance Z_0, which is the ratio of voltage to current in the line. This is expressed in ohms, but it is not "ordinary resistance" because voltage and current are out of phase. Example 5.2 calculates Z_0 for a coax line. RG58/U, done in the example, has 50 ohms; so do the cables we use in our system (in contrast, the TV standard is 75 ohms). Because of the dielectric in the cable, the wave (phase) velocity is ~ 0.7 that in free space, making the wavelength in the cable shorter than that in free space; one needs to account for this when using cables to produce delays in a signal, as in an interferometer or a quadrature circuit.
  - *** *RWvD §5.4: Reflection and transmission at a resistive discontinuity.* The purpose of a transmission line is to take power from a source to a load. The load should absorb the power and therefore should be resistive. If its resistance matches the line impedance, then all the power transmitted by the line is absorbed by the load. If not, some is reflected; see their equations (4) and (6). We need to tune our devices so that they are "matched" and reflect no power.
- The Theory of Waveguide
  - *** *RWvD §8.1: Introduction: general description of waveguides.*
  - *** *RWvD §8.8: The TE_{10} Mode in a Rectangular Guide.* A nice, complete discussion of the TE_{10} mode in common rectangular waveguide (in which the ratio of side lengths equals 2). In this mode, E is perpendicular to the long side of the guide (and also to the longitudinal axis of the guide). We speak of the polarization of the waveguide: the guide transmits linear polarization that is perpendicular to the long side of a waveguide. When we couple power into a guide from coax, we do so with a probe that is parallel to the internal electric field. When we use a sliding probe to measure VSWR in a waveguide, the probe extends through a long slot in the broad side of the waveguide and the probe is parallel to the E field inside the guide, so it samples the internal E field. Note **Figure 8.8**, which shows the current flow in the waveguide walls; note that the current flow down the center of the broad side of the guide is parallel to the axis of the guide—that is, parallel to the slot that we use for the probe. The slot, being parallel to the current flow, does not interrupt any current flow and has no effect on the internal distribution of fields in the guide. Clever!
  - *RWvD §8.10.* Makes the analogy between coax cable and waveguide. A coaxial line can be considered as a waveguide containing walls not only the outside but also on the inside. The usual mode that propagates in coax is the TEM mode. A TE_{10}-like waveguide mode enters for wavelengths smaller than that given by their equation (4); at this point, coax lines become basically unusable. For type N connectors, we are beginning to approach this limit in our 12 GHz system!
  - *RWvD §8.11.* Brief description of how coax lines are coupled to waveguide. The method depends on which mode you wish to excite. For the TE_{10} mode, method (b) in Figure 8.11 works well and is almost universally the one used.