Concurrent Process algebra - syntax (B)

$\mathcal{P} \ni P, Q ::= K \mid \alpha.P \mid P+Q \mid 0 \mid P[f] \mid P \backslash L \mid P|Q$

where

$\alpha \in N \cup \bar{N} \cup \{\tau\}$ is an action

K is a collection of process names or process constants

$L \subseteq N$ is a set of labels

f is a function that renames actions s.t. $f(\tau) = \tau$ and $f(\bar{a}) = \overline{f(a)}$

Notation → $[f] = [a_1 \mapsto b_1, \dots, a_n \mapsto b_n]$, where $a_i, b_i \in N \cup \{\tau\}$

Exercise: Which are syntactically correct?

(11) $a.\bar{b}.A + B$

R: Correct

(12) $(a.0 + \bar{a}.A) \backslash \{\bar{a}, b\}$

R: Correct

(13) $(a.0 + \bar{a}.A) \backslash \{a, \tau\}$

$\tau \in L \notin N$

R: Incorrect

(14) $(a.0 + \bar{\tau}A) \backslash \{a\}$

$\bar{\tau}$ is not a thing

R: Incorrect

(15) $\tau.\tau.B + \bar{a}.0$

R: Correct

(16) $(0|0) + 0$

R: Correct

(17) $(a.B + b.B)[a \mapsto a, \tau \to b]$

R: Correct

(18) $(a.B + \tau.B)[b \mapsto a, b \mapsto a]$

?

(19) $(a.B + b.B)[a \mapsto b, b \mapsto \bar{a}]$

$\bar{a} \notin N$

R: Incorrect

(20) $(a.b.A + \bar{a}.0) | B$

R: Correct

(21) $(a.b.A + \bar{a}.0).B$

R: Incorrect, because $a.b.A + \bar{a}.0$ is not an action

2. $(A|b.a.B)+((b.A)[b\mapsto a]) \xrightarrow{a} A[b\mapsto a]$

$$\frac{\dfrac{\dfrac{}{b.A \xrightarrow{b} A}\ (act)}{(b.A)[b\mapsto a] \xrightarrow{a} A[b\mapsto a]}\ (rel)}{(A|b.a.B)+((b.A)[b\mapsto a]) \xrightarrow{a} A[b\mapsto a]}\ (sum\text{-}2)$$

Exercise: Draw the NFAs A and D.

$A = x.B + x.x.C$

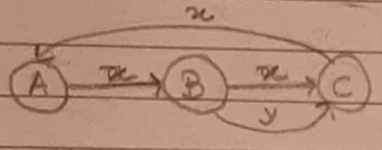
$B = x.x.A + y.C$

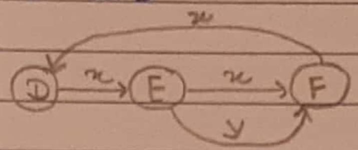
$C = x.A$

$D = x.x.x.D + x.E$

$E = x.F + y.F$

$F = x.D$





Observational Equivalence

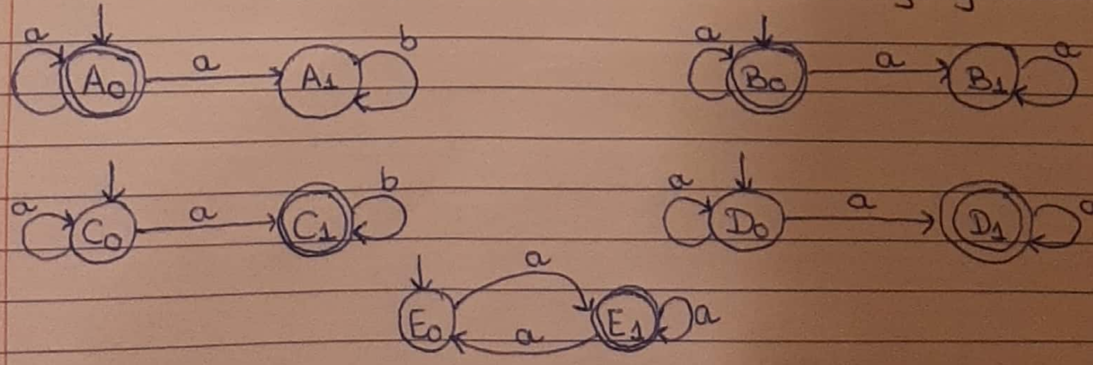
Two programs are observationally equivalent iff it is impossible to observe any difference in their behavior.

Language Equivalence

Two automata A, B are language equivalent iff $L_A = L_B$ (i.e. if they can perform the same finite sequences of transitions).

Language equivalence applies when one can neither interact with a system, nor distinguish a slow system from one that has come to a stand still.

Exercise: Find pairs of automata with the same language.



B, D and E are language equivalent.

A and C as well.

(22) $(a.b.A + \bar{a}.0) + B$

R: Correct

Building an NFA

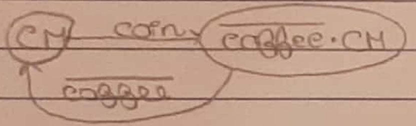
$$\text{(act)}\ \frac{}{\alpha.P \xrightarrow{\alpha} P} \qquad \text{(sum-1)}\ \frac{P_1 \xrightarrow{\alpha} P_1'}{P_1 + P_2 \xrightarrow{\alpha} P_1'} \qquad \text{(sum-2)}\ \frac{P_2 \xrightarrow{\alpha} P_2'}{P_1 + P_2 \xrightarrow{\alpha} P_2'}$$

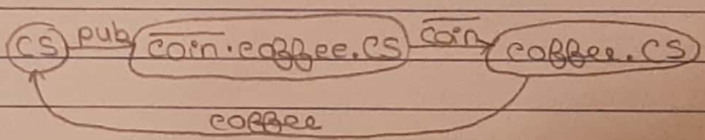
$$\text{(res)}\ \frac{P \xrightarrow{\alpha} P'}{P \backslash L \xrightarrow{\alpha} P' \backslash L}\ \alpha, \bar{\alpha} \notin L \qquad \text{(rel)}\ \frac{P \xrightarrow{\alpha} P'}{P[f] \xrightarrow{f(\alpha)} P'[f]}$$

$$\text{(com1)}\ \frac{P \xrightarrow{\alpha} P'}{P|Q \xrightarrow{\alpha} P'|Q} \qquad \text{(com2)}\ \frac{Q \xrightarrow{\alpha} Q'}{P|Q \xrightarrow{\alpha} P|Q'} \qquad \text{(com3)}\ \frac{P \xrightarrow{a} P' \quad Q \xrightarrow{\bar{a}} Q'}{P|Q \xrightarrow{\tau} P'|Q'}$$

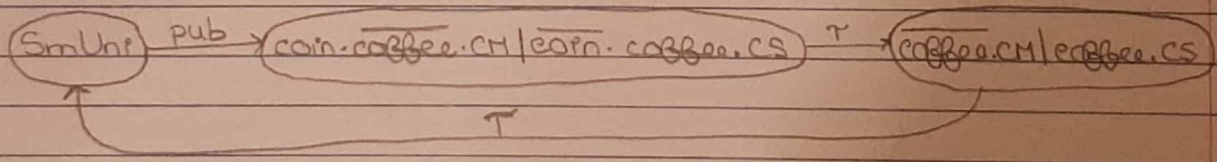
Exercise: Draw the transition systems.

$CM = coin.\overline{coffee}.CM$ $\qquad$ $CS = pub.\overline{coin}.coffee.CS$





$SmUni = (CM|CS) \backslash \{coin, coffee\}$



Exercise: Let $A = b.a.B$. Show that:

1. $(A|\bar{b}.0)\backslash\{b\} \xrightarrow{\tau} (a.B|0)\backslash\{b\}$

$$\dfrac{\dfrac{\dfrac{}{b.a.B \xrightarrow{b} a.B}\text{(act)} \qquad \dfrac{}{\bar{b}.0 \xrightarrow{\bar{b}} 0}\text{(act)}}{A|\bar{b}.0 \xrightarrow{\tau} a.B|0}\text{(com3)}}{(A|\bar{b}.0)\backslash\{b\} \xrightarrow{\tau} (a.B|0)\backslash\{b\}}\text{(res)}$$

$p \lesssim q \equiv \langle \exists R :: R$ is a simulation and $\langle p,q \rangle \in R \rangle$

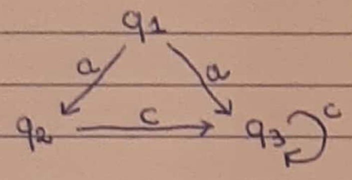
We say p is simulated by q.

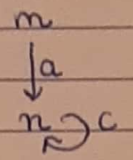
The similarity relation is a preorder (reflexive and transitive).

Bisimulation

Happens if p simulates q and q simulates p.

Exercise: Find bisimulations that include $\langle q_1, m \rangle$



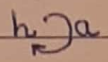


$R = \{(q_1,m),(q_2,n),(q_3,n)\}$

$R^o = \{(m,q_1),(n,q_2),(n,q_3)\}$

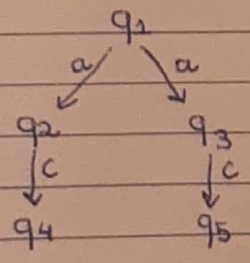
$\langle q_1, h \rangle$

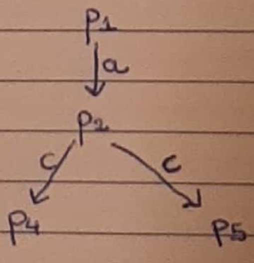
$q_1 \xrightarrow{a} q_2 \xrightarrow{a} q_3 \xrightarrow{a} \dots$



$R = \{(q_1,h),(q_2,h),(q_3,h)\}$

Exercise: Check if there is a bisim. that include $\langle q_1, p_1 \rangle$





$q \lesssim p = \{(q_1,p_1),(q_2,p_2),(q_3,p_2),(q_4,p_4),(q_4,p_5)\} = R$

$q_1 \xrightarrow{a} q_2 \quad p_1 \xrightarrow{a} p_2 \quad \langle q_1, p_1 \rangle$

$q_1 \xrightarrow{a} q_3 \quad p_1 \xrightarrow{a} p_2$

$q_2 \xrightarrow{c} q_4 \quad p_2 \xrightarrow{c} p_4 \quad \langle q_2, p_2 \rangle$

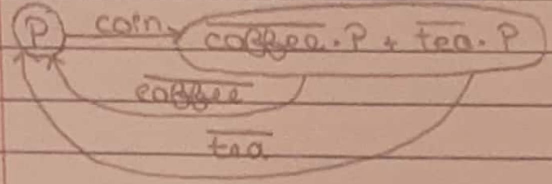
$q_3 \xrightarrow{c} q_5 \quad p_2 \xrightarrow{c} p_5 \quad \langle q_3, p_2 \rangle$

TO-DO: Prove R^o is a simulation $p \lesssim q$.

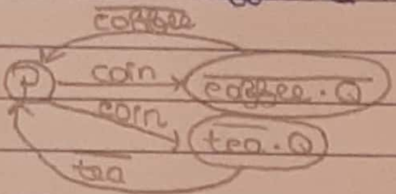
$R^o = \{(p_1,q_1),(p_2,q_2),(p_2,q_3),(p_4,q_4),(p_5,q_4)\}$

Exercise: Check if the processes are language equivalent.

$P = coin \cdot (\overline{coffee} \cdot P + \overline{tea} \cdot P)$



$Q = coin \cdot \overline{coffee} \cdot Q + coin \cdot \overline{tea} \cdot Q$



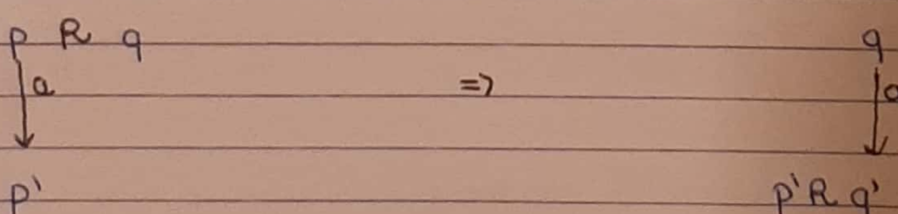
They are.

Similarity

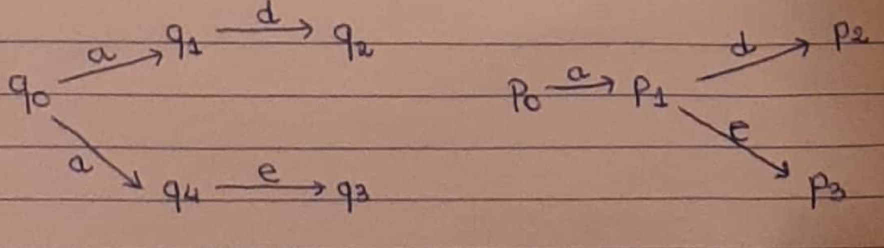
A state q simulates another state p if every transition from q is corresponded by a transition from p and this capacity is kept along the whole life of the system to which state space q belongs to.

Given NFA A_1 and A_2 over N with states S_1 and S_2 respectively, a relation $R \subseteq S_1 \times S_2$ is a simulation iff, for all $\langle p,q \rangle \in R$ and $a \in N$.

$$(1)\ p \xrightarrow{a}_1 p' \Rightarrow \langle \exists q' : q' \in S_2 : q \xrightarrow{a}_2 q' \wedge \langle p', q' \rangle \in R \rangle$$



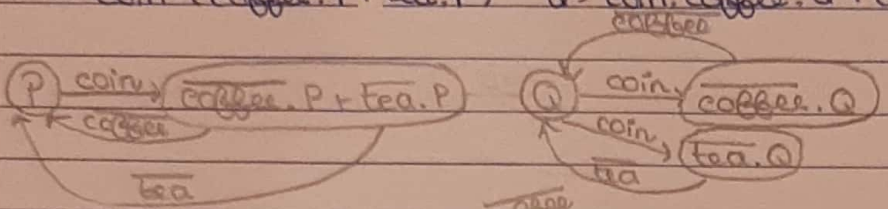
Exercise Find simulations.

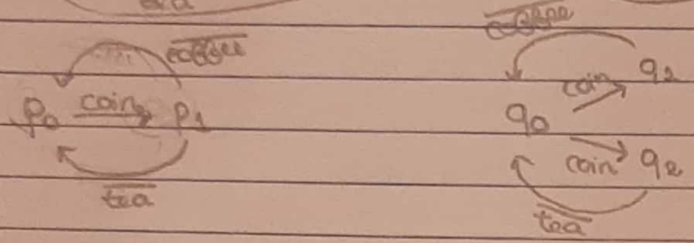


$q_0 \lesssim p_0$ cf. $\{\langle q_0,p_0\rangle, \langle q_1,p_1\rangle, \langle q_2,p_2\rangle, \langle q_4,p_1\rangle, \langle q_3,p_3\rangle\}$

$\langle P, Q \rangle$

$P = coin.(\overline{coffee}.P + \overline{tea}.P)$ $\quad$ $Q = coin.\overline{coffee}.Q + coin.\overline{tea}.Q$





$p_0 \xrightarrow{coin} p_1$ $\quad$ $q_0 \xrightarrow{coin} q_1$

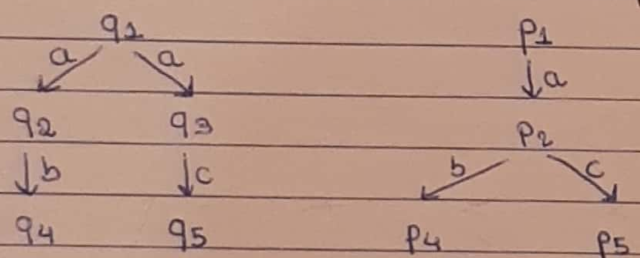
$p_1 \xrightarrow{\overline{coffee}} p_0$ $\quad$ $q_0 \xrightarrow{coin} q_2$

$p_1 \xrightarrow{\overline{tea}} p_0$ $\quad$ $q_1 \xrightarrow{\overline{coffee}} q_0$

$q_2 \xrightarrow{\overline{tea}} q_0$

It's not possible. In the pair $(p_1, ?)$, q must have a correspondence that can perform $\overline{coffee}$ and $\overline{tea}$, which doesn't occur.

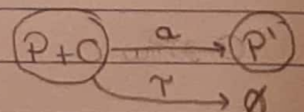
$\langle q_1, p_1 \rangle$



Not possible. p_2 has the ability to perform b and c, and neither q_2 or q_3 are able to perform them both.

Exercise Check if, for any process P $\quad P \sim P + 0$

$P \xrightarrow{a} P'$



?