

$$1) \frac{\frac{\langle p, \sigma \rangle \Downarrow z, \sigma'}{\langle \text{write}_n(p), \sigma \rangle \Downarrow n+z, \sigma'} \text{ (write)}}{\langle \text{write}_m(\text{write}_n(p)), \sigma \rangle \Downarrow m+n+z, \sigma'} \text{ (write)}$$

$$\frac{\langle p, \sigma \rangle \Downarrow z, \sigma'}{\langle \text{write}_{m+n}(p), \sigma \rangle \Downarrow m+n+z, \sigma'} \text{ (write)}$$

By the natural notion of equivalence provided,  $\text{write}_m(\text{write}_n(p)) \sim \text{write}_{m+n}(p)$

Other equivalences:

$$\rightarrow \text{write}_m(\text{write}_n(p)) \sim \text{write}_n(\text{write}_m(p))$$

$$\frac{\frac{\langle p, \sigma \rangle \Downarrow z, \sigma'}{\langle \text{write}_n(p), \sigma \rangle \Downarrow n+z, \sigma'} \text{ (write)}}{\langle \text{write}_m(\text{write}_n(p)), \sigma \rangle \Downarrow m+n+z, \sigma'} \text{ (write)} \quad \frac{\frac{\langle p, \sigma \rangle \Downarrow z, \sigma'}{\langle \text{write}_m(p), \sigma \rangle \Downarrow m+z, \sigma'} \text{ (write)}}{\langle \text{write}_n(\text{write}_m(p)), \sigma \rangle \Downarrow n+m+z, \sigma'} \text{ (write)}$$

$$\rightarrow p; \text{write}_m(q) \sim \text{write}_m(p); q$$

$$\frac{\frac{\langle p, \sigma \rangle \Downarrow z, \sigma' \quad \frac{\langle q, \sigma' \rangle \Downarrow r, \sigma''}{\langle \text{write}_m(q), \sigma' \rangle \Downarrow m+r, \sigma''} \text{ (write)}}{\langle p; \text{write}_m(q), \sigma \rangle \Downarrow z+m+r, \sigma''} \text{ (seq)}$$

$$\frac{\frac{\langle p, \sigma \rangle \Downarrow z, \sigma' \quad \langle q, \sigma' \rangle \Downarrow r, \sigma''}{\langle \text{write}_m(p); q, \sigma \rangle \Downarrow m+z+r, \sigma''} \text{ (seq)}}{\langle \text{write}_m(p), \sigma \rangle \Downarrow m+z, \sigma'} \text{ (write)}$$

→  $\text{write}_m(p); \text{write}_n(q) \sim \text{write}_{m+n}(p); q$

$\langle p, \sigma \rangle \Downarrow z, \sigma'$  (write)  $\langle q, \sigma' \rangle \Downarrow r, \sigma''$  (write)  
 $\langle \text{write}_m(p), \sigma \rangle \Downarrow m+z, \sigma'$  (seq:)  $\langle \text{write}_n(q), \sigma' \rangle \Downarrow n+r, \sigma''$   
 $\langle \text{write}_m(p); \text{write}_n(q), \sigma \rangle \Downarrow m+z+n+r, \sigma''$

$\langle p, \sigma \rangle \Downarrow z, \sigma'$   
 $\langle \text{write}_{m+n}(p), \sigma \rangle \Downarrow m+n+z, \sigma'$  (seq)  $\langle q, \sigma' \rangle \Downarrow r, \sigma''$   
 $\langle \text{write}_{m+n}(p); q, \sigma \rangle \Downarrow m+n+z+r, \sigma''$