Modal Logic
Exercise Set 03

To be completed by Thursday 16 May

We will work through these exercises (and possibly some others as well) during the problem class. Exercises marked with a (△) are a little more challenging, and those marked with a (▲) are more difficult still.

1. Prove that the following frame conditions are equivalent.
   (a) R is an equivalence relation (i.e. it is reflexive, symmetric, and transitive);
   (b) R is reflexive and euclidean;
   (c) R is serial, symmetric, and euclidean;
   (d) R is serial, symmetric, and transitive.
2. The class of transitive and converse well-founded frames **GL** is defined by the Löb axiom $\Box(\Box A \to A) \to \Box A$. Prove that every instance of the Löb axiom is valid in every frame $F \in \mathbf{GL}$.
3. Prove that if every instance of the Löb axiom is valid in a frame F then F is transitive and converse well-founded.
4. (▲) A frame (W, R) is *converse well-quasi-ordered* if the converse accessibility relation R^{-1} is a *well-quasi-ordering*: it is well-founded and contains no infinite antichains. Find a modal scheme defining the class of converse well-quasi-ordered frames, or prove that no such scheme exists.