Modal Logic
Exercise Set 09

To be completed by Thursday 11 July

1. Give a constant domain tableau proof of a Converse Barcan formula.

2. Use a failed variable domain proof of a Converse Barcan formula to construct a countermodel. Prove that the Converse Barcan formula in question is not valid in the model.

3. The notion of *satisfiable set of prefixed first-order modal formulas* is a natural generalisation of the notion of a satisfiable set of prefixed propositional modal formulas.

   Definition 0.1. Suppose S is a set of prefixed formulas (where members of S may contain parameters). We say S is *satisfiable* in the varying domain model $M = (W, R, D, I)$ with respect to valuation v, if there is an assignment θ of the prefixes that occur in S to elements of W such that:

   (a) If σ and $\sigma.n$ both occur as prefixes in S, then $\theta(\sigma)R\theta(\sigma.n)$.
   (b) If the parameter p^σ occurs in S, then $v(p^\sigma) \in D(\theta(\sigma))$.
   (c) If $\sigma\ \varphi$ is in S, then $M, \theta(\sigma) \Vdash_v \varphi$.

   A branch is satisfiable if the set of prefixed formulas that occur on it is satisfiable; a tableau is satisfiable if it contains a satisfiable branch.

   Prove the universal case of the branch extension lemma for the variable domain tableau system.

   That is, show that if T is a satisfiable tableau, then the tableau T' obtained by applying either of the variable domain universal rules to a prefixed formula of the form $\sigma\ \forall x\varphi(x)$ or $\sigma\ \neg\exists x\varphi(x)$ is also satisfiable.

   (For details of the lemma, including a proof of the existential case, see p. 122–123 of Fitting and Mendelsohn [1998].)

References

M. Fitting and R. L. Mendelsohn. *First-Order Modal Logic.* Number 277 in Synthese Library. Springer, 1998.