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Vagueness, lecture 3: Indeterminacy and degrees.

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1. Outline of the degree theory

- Maybe borderline cases aren't determinately true or false, but *partly true*.
- We could make this precise by introducing *degrees of truth*: a wholly true sentence gets value 1, a wholly false sentence gets value 0, and partly true/false sentences get values in between.
- In a sorites sequence $A_0 \to A_1, A_1 \to A_2, \cdots, A_n \to A_n + 1$ we assign degrees of truth lower than 1 to each conditional $A_i \to A_{i+1}$, $i \leq n$. This blocks the inference to A_{n+1}.

2. Precision and the degree theory

- Tye's objection is that the degree theory replaces vagueness with precision.

> *The problem [with the degrees of truth approach] in its most general form is that vagueness has been replaced by the most refined and incredible precision. Set membership, as viewed by the degrees of truth theorist, comes in precise degrees, as does predicate application and truth. The result is a commitment to precise dividing lines that is not only unbelievable but also thoroughly contrary to vagueness.* [Tye 1994, p. 190]

- If we assign degrees of truth to the conditionals above, in order to avoid the paradox, some conditional must have degree of truth < 1.
- But then there must be a first such conditional: a first borderline case.
- "Surely it is absurd to suppose that there is some single hair addition that divides the bald from the borderline bald" [Tye 1994, p. 191].

3. Tye's proposal

- Tye argues for the adoption of *truth-value gaps*, or the assignment of *indefinite* truth values to the conditionals in a sorites.

- Tye's theory uses Kleene's three-valued logic, with the following truth table for the conditional.

$P \to Q$	T	I	F
T	T	I	F
I	T	I	I
F	T	T	T

- An *extensionally vague monadic predicate* P is assigned an extension S (the set of objects of which P is true) and an anti-extension S' (the set of objects of which P is false).

- But the extension and anti-extension are not classical, sharp sets but *vague sets*.

> *Consider the set of tall men. Men who are over [2 metres] are certainly members of this set and men who are under [1.7 metres] are certainly not. Intuitively, however, some men are borderline members: there is no determinate, objective fact of the matter about whether they are in the set or outside it. Are there any remaining men? To suppose that it is true that this is the case is to postulate more categories of men than are demanded by our ordinary, non-philosophical conception of the set of tall men and hence to involve ourselves in gratuitous metaphysical complications. It is also to create the need to face a potentially endless series of such questions one after the other as new categories of men are admitted. On the other hand, to suppose that it is false that there are any remaining men is to admit that every single man fits cleanly into one of the three categories so that there are sharp partitions between the men in the set, the men on the border, so to speak, and the men outside. And intuitively, pretheoretically it is not true that there are any sharp partitions here. What, I think, we should say, then, is that it is objectively indeterminate as to whether there are any remaining men. In the ways I have just described, the set of tall men is, I maintain, a vague set.* [Tye 1994, p. 195]

- **Tye's definition of vagueness**: a property P is vague iff

  (a) P *could* have borderline instances, and

  (b) there is no determinate fact of the matter about whether there *could* be objects that are neither instances, borderline instances, nor non-instances.

- *Truth conditions for vague singular sentences*: for any individual constant c, let i_c be the object assigned to c. Then $P(c)$ is *true* iff i_c belongs to S; $P(c)$ is

false iff i_c belongs to S'; and $P(c)$ is *indefinite* iff there is no determinate fact of the matter about whether i_c belongs to S (or to S').

- Is this definition *viciously circular*? It defines indefiniteness in terms of there being "no determinate fact of the matter", so one might worry that it uses the very notion it's attempting to define.
  - Tye claims not: "there is no determinate fact of the matter about whether" is supposed to be a *sentential operator* (i.e. a connective, like 'not' or 'possibly'), not a predicate.
- The concept of a vague set is governed by the logic Tye presents, but the logic is itself couched in terms of vague sets, so one might also worry that one cannot understand the logic without understanding the concept of a vague set, and conversely.
  - Tye responds that the concept of a vague set can be understood in an intuitive, pretheoretical way: grasping the concept "does not itself presuppose a full understanding of the metalinguistic semantics specifying the conditions of application of the truth-value predicates for vague sentences" [Tye 1994, p. 198].

4. Tye's responses to the sorites paradoxes

- Since the initial premise A_0 is true and the conclusion A_n is false, it is not the case that the inductive premise is true or that it is false: rather, it is indefinite.
- For the sorites based on a series of conditionals, Tye holds that not all of them are true.
- The problem of the existence of a first borderline case is handled by holding that it is indefinite that there is a first conditional that is not true. "Thus, there are true conditionals initially, and indefinite conditionals later, but it is not true that there is a sharp transition from the former to the latter" [Tye 1994, p. 201].

References

M. Tye. Sorites paradoxes and the semantics of vagueness. *Philosophical Perspectives*, 8, Logic and Language:189–206, 1994. doi:10.2307/2214170.