Modal Logic
Exercise Set 06 (mid-term assignment)

To be completed by Thursday 13 June

This week's exercise sheet forms small a mid-term assignment, and must be *completed in advance* (i.e. before the exercise class on Thursday 13 June). You can either

1. Email it as a PDF file to Benedict.Eastaugh@lrz.uni-muenchen.de,

   or

2. Leave it in my pigeonhole in Ursula Danninger's office (Ludwigstr. 31, room 226).

Please make sure that your **name** and **email address** are written clearly at the top of every page. You can either type up the solution (using LaTeX or any word processing program), or write it by hand and then scan it (as long as I can read your handwriting!).

The assignment *will not count* towards your final grade for this course, and is just so I can see how you are all doing with the material and plan the second half of the course.

The assignment is to *prove the soundness of the class of models based on reflexive and transitive frames for the tableau system **S4***. In order to prove this, it is sufficient to prove a modified version of the branch extension lemma, as follows.

Branch extension lemma for S4. Suppose that T is a tableau that is satisfiable in a model based on a frame that is reflexive and transitive. Then any tableau T' obtained from T by applying one of the **S4** branch extension rules (i.e. those of **K** plus the special necessity rules (T) and (4)) is also satisfiable by a model based on a frame that is reflexive and transitive.

1. Prove the branch extension lemma for **S4**.

2. Briefly explain (1 paragraph) why proving this modified version of the branch extension lemma for **K** is sufficient to prove the soundness of the **S4** tableau system with respect to the class of reflexive and transitive frames.