Modal Logic
Exercise Set 07

To be completed by Thursday 27 June

A constant domain **K** model is a constant domain first-order modal model in which the accessibility relation has no particular restrictions.

Which of the following sentences are valid in all constant domain **K** models, and which are not valid? In the former case, give a proof that the sentence is valid in all constant domain **K** models; in the latter, either construct an explicit countermodel or derive a contradiction from the assumption that it is valid in all such models.

1. $((\exists x)\Diamond P(x) \wedge \Box(\forall x)(P(x) \rightarrow Q(x))) \rightarrow (\exists x)\Diamond Q(x)$.
2. $(\forall x)\Box P(x) \rightarrow \Box(\forall x)P(x)$.
3. $\Box(\forall x)P(x) \rightarrow (\forall x)\Box P(x)$.
4. $(\exists x)\Box P(x) \rightarrow \Box(\exists x)P(x)$.
5. $\Box(\exists x)P(x) \rightarrow (\exists x)\Box P(x)$.
6. $(\exists x)\Diamond(\Box P(x) \rightarrow (\forall x)\Box P(x))$.
7. $(\exists x)\Diamond(P(x) \rightarrow (\forall x)\Box P(x))$.
8. $(\exists x)(\forall y)\Box R(x, y) \rightarrow (\forall y)\Box(\exists x)R(x, y)$.