Modal Logic
Exercise Set 08

To be completed by Thursday 4 July

1. Instances of the Barcan formula are either of the form

   (B1) $\qquad (\forall x)\Box\varphi \to \Box(\forall x)\varphi$

   or of the form

   (B2) $\qquad \Diamond(\exists x)\varphi \to (\exists x)\Diamond\varphi.$

   Show that every formula of form B1 is equivalent to one of form B2, and conversely.

2. Instances of the Converse Barcan formula are either of the form

   (CB1) $\qquad \Box(\forall x)\varphi \to (\forall x)\Box\varphi$

   or of the form

   (CB2) $\qquad (\exists x)\Diamond\varphi \to \Diamond(\exists x)\varphi.$

   Show that every formula of form CB1 is equivalent to one of form CB2, and conversely.

3. In the lecture we saw that a variable-domain augmented frame $F = (W, R, D)$ is monotonic if and only if every instance of the Converse Barcan formula is valid in every model M based on F.

   Prove that a variable-domain augmented frame F is *anti-monotonic* if and only if every instance of the *Barcan formula* is valid in every model M based on F.

   Hint: the proof is very similar to the proof given in the lecture, and in Fitting and Mendelsohn [1998, pp. 111–2], of the corresponding theorem concerning monotonic frames.

References

M. Fitting and R. L. Mendelsohn. *First-Order Modal Logic*. Number 277 in Synthese Library. Springer, 1998.