PH342 Philosophy of Mathematics
Term 2, 2023–2024

Lecturer

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  Please put "PH342" in the subject line of your email and sign your full name.

  During term time, advice and feedback hours take place on Tuesdays, 11:00–12:00, in my office (S2.58 in the Social Sciences building). Please feel free to drop by. If you can't come at this time, email me for an appointment.

Website

`https://moodle.warwick.ac.uk/course/view.php?id=58360`

Announcements and other materials will be posted at this address.

Course description

This course is a first introduction to philosophy of mathematics, via one of our most fascinating and perplexing concepts: *the infinite*. We encounter the concept of infinity in myriad ways. In Zeno's paradoxes of time, space, and motion, the idea of infinite division is used to argue in favour of a radical monism. The ancient atomists Leucippus and Democritus claimed that the universe consisted of an infinity of atoms moving in an infinite void, and contemporary cosmology still considers the issue of whether the universe is infinite to be an open question.

But what does it mean for something to be infinite? It is mathematics that offers us the precise definitions that let us begin to answer this question, and thus in mathematics that many of the most important questions concerning the infinite arise. Do the infinite structures that we talk about in mathematics really exist? If so, how can we have knowledge of them? Is it even coherent to talk about the truly infinite, or does it fall victim to paradox? This course will investigate these and other questions by engaging with the ideas of philosophers and mathematicians from across history, with a focus on the reception of Georg Cantor's theory of sets, and the crisis in the foundations of mathematics that it precipitated.

Prerequisites

PH136 (Logic 1) is recommended as a prerequisite. Otherwise, the module is designed to be as self-contained as possible. But you should be aware that several of the topics we will discuss are related to developments in mathematical logic (as treated in modules like Logic 2, Logic 3, and Set Theory) and also build on philosophical themes from metaphysics, epistemology, and the philosophy of language. A background in these subjects will therefore be helpful in fully engaging with the module content.

Lectures and seminars

- **Lectures** (weeks 1–5 and 7–10):
  - Tuesday 10:00–11:00 in S0.20 (Social Sciences).
  - Tuesday 14:00–15:00 in PLT (Physics).
- **Seminars** (weeks 2–5 and 7–10):
  - Group A: Tuesday 17:00–18:00 in S0.52 (Social Sciences).
  - Group B: Wednesday 10:00–11:00 in FAB1.09 (Faculty of Arts).
  - Group C: Wednesday 11:00–12:00 in FAB1.09 (Faculty of Arts).

Lectures will be recorded via lecture capture and made available on the module's Moodle page. I will upload lecture handouts ahead of time.

The lectures are intended to give you a broad introduction to the topics we are covering. In the seminars we will have more focused discussions of those topics, making reference to the core readings for that week.

Assessment

Assessment for this module is based on a two-hour exam (80%) as well as a 1000-word essay (20%). The deadline for the essay is Tuesday of Week 7 (20 February 2024). For details on extensions and how to request them, please see the Philosophy Department guidance page:

- Philosophy Handbook: Extensions and Mitigation.

More details and guidance on the assessed essay and the exam appear below.

Core texts

Much of the background and historical reading will be drawn from three books:

- *The Infinite* (2nd ed.) by A. W. Moore (Routledge, 2001).
  `https://webcat.warwick.ac.uk/record=b2893101~S1`
- *The Search for Certainty* by M. Giaquinto (Oxford University Press, 2002).
  `https://webcat.warwick.ac.uk/record=b2324098~S1`
- *Thinking About Mathematics* by S. Shapiro (Oxford University Press, 2000).
  `https://webcat.warwick.ac.uk/record=b2246715~S1`

There will also be a substantial use of original sources and recent scholarship. Many important papers can be found in the following collections.

- *An Historical Introduction to the Philosophy of Mathematics: A Reader*, edited by R. Marcus and M. McEvoy (Bloomsbury, 2016).
  `https://webcat.warwick.ac.uk/record=b3821979~S1`
- *Philosophy of Mathematics: Selected Readings* (2nd ed.), edited by P. Benacerraf and H. Putnam (Cambridge University Press, 1983).
  `https://webcat.warwick.ac.uk/record=b2797754~S1`
- *From Frege to Gödel: A Source Book in Mathematical Logic, 1879–1931*, edited by J. van Heijenoort (Harvard University Press, 1967).
  `https://webcat.warwick.ac.uk/record=b1350155~S1`

- *From Kant to Hilbert: A Source Book in the Foundations of Mathematics*, volume I, edited by W. B. Ewald (Clarendon Press, 1996).
  https://webcat.warwick.ac.uk/record=b2922205~S1

- *From Kant to Hilbert: A Source Book in the Foundations of Mathematics*, volume II, edited by W. B. Ewald (Clarendon Press, 1996).
  https://webcat.warwick.ac.uk/record=b2920415~S1

- *From Brouwer to Hilbert: The Debate on the Foundations of Mathematics in the 1920s*, edited by P. Mancosu (Oxford University Press, 1998).
  https://webcat.warwick.ac.uk/record=b3348834~S1

An excellent handbook in philosophy of mathematics is:

- *The Oxford Handbook of Philosophy of Mathematics and Logic*, edited by S. Shapiro (Oxford University Press, 2005). https://webcat.warwick.ac.uk/record=b2663883~S1

Two good surveys of topics in mathematical logic that are relevant for this module are:

- *Modern Mathematical Logic* by Joseph Mileti (Cambridge University Press, 2022).
  https://webcat.warwick.ac.uk/record=b3861128~S15

- *Mathematical Logic* by J. R. Shoenfield (Addison-Wesley, 1967).
  https://webcat.warwick.ac.uk/record=b3517405~S1

Finally, the Stanford Encyclopedia of Philosophy (plato.stanford.edu) contains myriad excellent entries on a wide variety of philosophical topics. Specific entries are recommended below.

Schedule and list of readings

Schedule and list of readings

- **Week 1**: Introduction to the module. The infinite in ancient Greek thought.
  - *Core reading*: introduction and chapters 1 and 2 of Moore [2001].
  - *Supplementary reading*:
    - Early Greek thought: Curd [2020], Huffman [2018, 2019].
    - Zeno's paradoxes: Huggett [2019], Palmer [2021, 2020].
    - Ancient Greek atomism: Berryman [2016a,b,c].
    - Aristotle on infinity: Mendell [2019], sections 1 and 2 of Linnebo and Shapiro [2019], Hintikka [1966], Lear [1980].

- **Week 2**: Infinity in mathematics. Cantor's theory of sets.
  - *Core reading*: part I (pages 1–34) of Giaquinto [2002].
  - *Supplementary reading*:
    - Continuity and infinitesimals: Bell [2022], Gray [1992], Gray [2008], Haffner and Schlimm [2017], Yap [2014], Borovik and Katz [2012], Sinaçeur [1973] (in French).
    - Bolzano/Dedekind and visual proofs in analysis: chapters 4 and 8 of Moore [2001], chapters VI and VII of Hahn [1980], Russ [2004], Morscher [2018], Rusnock and Šebestik [2022], preface to 'Continuity and Irrational Numbers' in Dedekind [1963], Giaquinto [1994], Brown [1999], Giaquinto [2007], Giaquinto [2011], Reck [2012], Giaquinto [2020], Kitcher [1975], Mancosu [1999].

    * Early development of set theory: chapter 8 of Moore [2001], chapters 1–5 of Dauben [1979], introduction and chapter 1 of Hallett [1984], Ferreirós [1993], chapters I–VI of Ferreirós [2007], Ferreirós [2020].

- **Week 3**: The class-theoretic paradoxes. Type theory and limitation of size.
  - *Core reading*:
    * Chapter 10 of Moore [2001].
    * Part II (pages 35–65) of Giaquinto [2002].
  - *Supplementary reading*:
    * Frege and logicism: Demopoulos and Clark [2005], Tennant [2017].
    * Class-theoretic paradoxes: Irvine and Deutsch [2020], Mortensen [2017].
    * Limitation of size: chapters 4–5 of Hallett [1984], chapters 6–11 of Dauben [1979].
    * Russell and type theory: Linsky and Irvine [2020], Coquand [2018], Gödel [1944].
- **Week 4**: The axiomatic method. Hilbertian finitism.
  - *Core reading*:
    * Chapters IV.3–4 of Giaquinto [2002].
    * Chapters 1 and 2 of Hilbert and Bernays [1968]. An English translation is available on the Moodle page.
  - *Supplementary reading*:
    * The axiomatic method: Torretti [2019], Webb [1995].
    * The Frege–Hilbert controversy: Blanchette [2018, 2012], Dummett [1991], Tappenden [1995].
    * Hilbert's finitism: Detlefsen [2005], Bernays [1930], Zach [2019].
    * Hilbert's program: Hilbert [1926] (up to page 384 in van Heijenoort [1967]), Franks [2017], Zach [2019, 2006].
    * Incompleteness: Franzén [2005], Smoryński [1977].
- **Week 5**: Constructivism in Brouwer and Heyting.
  - *Core reading*:
    * Chapter 7 of Shapiro [2000].
    * Chapter 1 ('Disputation') of Heyting [1971].
  - *Supplementary reading*: Brouwer [1912, 1949], van Atten [2017], Bridges and Palmgren [2018], Heyting [1971], Sundholm [1983], Troelstra and van Dalen [1988], Bishop and Bridges [1985], Beeson [1985], Dummett [1973, 2000], van Atten [2004], van Atten et al. [2008], Sundholm [2014], van Atten and Sundholm [2017], van Atten [2018].
- **Week 6**: **Reading week** (no lectures or seminars)
- **Week 7**: The Löwenheim–Skolem theorem. Skolem's paradox.
  - *Core reading*: Skolem [1922], chapters IV.1 and IV.2 of Giaquinto [2002].
  - *Supplementary reading*: Chapter 11 of Moore [2001], Putnam [1980], Benacerraf and Wright [1985], Hallett [2011], Bays [2006, 2014].
- **Week 8**: The continuum problem. Realism and indeterminacy in set theory.
  - *Core reading*: Gödel [1947], chapter VI.1 of Giaquinto [2002].

- *Supplementary readings*: Maddy [1988], Koellner [2009], Kanamori [1996], Feferman et al. [2000], Hamkins [2012], Steel [2014], Martin [2001].

- **Week 9**: Categoricity and determinacy. Structuralism.
  - *Core reading*: Benacerraf [1965], chapter 10 of Shapiro [2000].
  - *Supplementary reading*: Hellman [2005], Parsons [1990], Benacerraf [1996], Shapiro [1997], Reck and Price [2000], Reck [2003], Chihara [2004], Resnik [2019], Reck and Schiemer [2020].

- **Week 10**: Potential infinity revisited. Modality and potentiality.
  - *Core reading*: Linnebo and Shapiro [2019].
  - *Supplementary reading*: Linnebo [2013], Linnebo, Shapiro, and Hellman [2016], Hamkins and Linnebo [2019].

Revision sessions

There will be two revision sessions for the module in term 3.

Assessed essays

The general guidelines for writing your assessed essay are as follows.

- Your essay should be 1000 words ($\pm$ 10%).
- The submission deadline is 15:00 GMT on Thursday 16 February 2023.
- Essays are to be submitted via Tabula.
- Submission instructions can be found here.
- Marking criteria can be found here.

You can find some help in writing a good philosophy essay in the following guides.

- How to write a philosophy paper, from the Pink Guide to Philosophy.
- Guidelines on writing a philosophy paper, by Jim Pryor.
- How to write an essay, video series by Mark Jago.
- How to write a crap essay, by James Lenman.

The Philosophy department also has some writing support available.

- Philosophy Study Skills.

The list of pre-approved essay titles is as follows.

Pre-approved essay titles

1. Explain and evaluate Bolzano's argument for the claim that visual reasoning cannot play an ineliminable role in proofs of the intermediate value theorem.
2. Was Cantorian set theory undermined by Russell's paradox?
3. Was Hilbert right to say that consistency is the "criterion of truth and existence"?
4. Are non-constructive principles such as the Law of Excluded Middle valid principles of mathematical reasoning?

Exam

The module will be assessed (80%) via a two-hour online exam in the summer, via the Alternative Exams Portal. The exam format will be substantially the same as in previous years: you will need to answer 2 essay questions (out of 6). There is a 2500-word overall word limit. For more details please see the link below.

- Philosophy exam guidelines: essay-based exams in 2023–2024.

Past exam papers for this module are available, and are a good guide to the type and level of difficulty of the questions in this year's exam. However, since the module content changes to some extent from year to year, please be aware that past exam papers do not always offer a good guide to the content of this year's exam. Examinable topics are restricted to those which we have studied in the module.

- Past exam papers for PH342.

References

T. Bays. The mathematics of Skolem's paradox. volume 5 of *Handbook of the Philosophy of Science*, pages 615–648. Elsevier, 2006.

T. Bays. Skolem's Paradox. In E. N. Zalta, editor, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, winter 2014 edition, 2014.

M. J. Beeson. *Foundations of Constructive Mathematics*. Number 6 in Ergebnisse der Mathematik und ihrer Grenzgebiete (A Series of Modern Surveys in Mathematics). Springer-Verlag Berlin Heidelberg, 1985. doi:10.1007/978-3-642-68952-9.

J. L. Bell. Continuity and infinitesimals. In E. N. Zalta, editor, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, Spring 2022 edition, 2022.

P. Benacerraf. What Numbers Could Not Be. *The Philosophical Review*, 74(1):47–73, January 1965. doi:10.2307/2183530.

P. Benacerraf. Recantation *or* Any old ω-sequence would do after all. *Philosophia Mathematica*, 4(2):184–189, May 1996. doi:10.1093/philmat/4.2.184.

P. Benacerraf and H. Putnam, editors. *Philosophy of Mathematics: Selected Readings*. Cambridge University Press, Cambridge, 2nd edition, 1983. doi:10.1017/CBO9781139171519.

P. Benacerraf and C. Wright. Skolem and the skeptic. *Proceedings of the Aristotelian Society, Supplementary Volumes*, 59:85–137, 1985. doi:10.2307/4106752.

P. Bernays. Die Philosophie der Mathematik und die Hilbertsche Beweistheorie. *Blätter für deutsche Philosophie*, 4:326–367, 1930. Reprinted in Bernays [1976, pp. 17–61].

P. Bernays. *Abhandlungen zur Philosophie der Mathematik*. Wissenschaftliche Buchgesellschaft, 1976.

S. Berryman. Ancient atomism. In E. N. Zalta, editor, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, winter 2016 edition, 2016a.

S. Berryman. Democritus. In E. N. Zalta, editor, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, winter 2016 edition, 2016b.

S. Berryman. Leucippus. In E. N. Zalta, editor, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, winter 2016 edition, 2016c.

E. Bishop and D. Bridges. *Constructive Analysis*, volume 279 of *Grundlehren der mathematischen Wissenschaften*. Springer-Verlag, Berlin, 1985.

P. Blanchette. *Frege's Conception of Logic*. Oxford University Press, 2012.

P. Blanchette. The Frege–Hilbert controversy. In E. N. Zalta, editor, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, fall 2018 edition, 2018.

A. Borovik and M. G. Katz. Who gave you the Cauchy–Weierstrass tale? the dual history of rigorous calculus. *Foundations of Science*, 17:245–276, 2012. doi:10.1007/s10699-011-9235-x.

D. Bridges and E. Palmgren. Constructive mathematics. In E. N. Zalta, editor, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, summer 2018 edition, 2018.

L. E. J. Brouwer. "De onbetrouwbaarheid der logische principes" (The unreliability of the logical principles). *Tijdschrift voor Wijsbegeerte*, 2:152–158, 1908. doi:10.1016/B978-0-7204-2076-0.50009-X. English translation in van Atten and Sundholm [2017]. An older English translation is in Heyting [1975, pp. 107–111].

L. E. J. Brouwer. Intuitionism and formalism. In Benacerraf and Putnam [1983], pages 77–89. doi:10.1017/CBO9781139171519. Inaugural address at the University of Amsterdam, 1912.

L. E. J. Brouwer. Consciousness, philosophy, and mathematics. In Benacerraf and Putnam [1983], pages 90–96. doi:10.1017/CBO9781139171519.

J. R. Brown. *Philosophy of Mathematics: An Introduction to the World of Proofs and Pictures*. Routledge, London, 1999.

C. S. Chihara. *A Structural Account of Mathematics*. Oxford University Press, 2004.

T. Coquand. Type theory. In E. N. Zalta, editor, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, fall 2018 edition, 2018.

P. Curd. Presocratic philosophy. In E. N. Zalta, editor, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, fall 2020 edition, 2020.

J. W. Dauben. *Georg Cantor: His Mathematics and Philosophy of the Infinite*. Princeton University Press, Princeton, New Jersey, 1979.

R. Dedekind. *Stetigkeit und irrationale Zahlen*. Vieweg, 1872. English translation in Dedekind [1901].

R. Dedekind. *Was sind und was sollen die Zahlen?* Vieweg, 1888. English translation in Dedekind [1901].

R. Dedekind. *Essays on the Theory of Numbers*. Open Court, 1901. English translations of Dedekind [1872] and Dedekind [1888], edited and translated by W. W. Beman.

R. Dedekind. *Essays on the Theory of Numbers*. Dover, 1963. Reprint of Dedekind [1901].

W. Demopoulos and P. Clark. The logicism of Frege, Dedekind, and Russell. In Shapiro [2005], pages 129–165. doi:10.1093/0195148770.001.0001.

M. Detlefsen. Formalism. In Shapiro [2005], pages 236–317. doi:10.1093/0195148770.001.0001.

M. Dummett. The philosophical basis of intuitionistic logic. In Benacerraf and Putnam [1983], pages 97–129. doi:10.1017/CBO9781139171519.

M. Dummett. *Frege: Philosophy of Mathematics*. Duckworth, London, 1991.

M. Dummett. *Elements of Intuitionism*. Oxford University Press, Oxford, 2000.

S. Feferman, J. W. Dawson, S. C. Kleene, G. H. Moore, R. M. Solovay, and J. van Heijenoort, editors. *Kurt Gödel: Collected Works. II: Publications 1938–1974*. Oxford University Press, 1990.

S. Feferman, H. M. Friedman, P. Maddy, and J. R. Steel. Does mathematics need new axioms? *The Bulletin of Symbolic Logic*, 6(4):401–446, 2000. doi:10.2307/420965.

J. Ferreirós. On the relations between Georg Cantor and Richard Dedekind. *Historia Mathematica*, 20(4):343–363, November 1993. doi:10.1006/hmat.1993.1030.

J. Ferreirós. *Labyrinth of Thought: A History of Set Theory and Its Role in Modern Mathematics*. Birkhäuser, 2nd edition, 2007.

J. Ferreirós. The early development of set theory. In E. N. Zalta, editor, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, summer 2020 edition, 2020.

C. Franks. Hilbert. In A. Malpass and M. Antonutti Marfori, editors, *The History of Philosophical and Formal Logic: From Aristotle to Tarski*, pages 243–265. Bloomsbury, London, 2017.

T. Franzén. *Gödel's Theorem: An Incomplete Guide to its Use and Abuse*. A. K. Peters, Wellesley, MA, 2005.

M. Giaquinto. Epistemology of visual thinking in elementary real analysis. *British Journal for the Philosophy of Science*, 45(3):789–813, September 1994. doi:10.1093/bjps/45.3.789.

M. Giaquinto. *The Search for Certainty: A Philosophical Account of the Foundations of Mathematics*. Oxford University Press, Oxford, 2002.

M. Giaquinto. *Visual Thinking in Mathematics: An Epistemological Study*. Oxford University Press, Oxford, 2007.

M. Giaquinto. Crossing curves: a limit to the use of diagrams in proofs. *Philosophia Mathematica*, 19(3):281–307, October 2011. doi:10.1093/philmat/nkr023.

M. Giaquinto. The epistemology of visual thinking in mathematics. In E. N. Zalta, editor, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, spring 2020 edition, 2020. URL `https://plato.stanford.edu/archives/spr2020/entries/epistemology-visual-thinking/`.

K. Gödel. Russell's mathematical logic. In P. A. Schilpp, editor, *The Philosophy of Bertrand Russell*, pages 125–153. Northwestern University, Evanston, IL, 1944. Reprinted in Feferman et al. [1990, pp. 119–141].

K. Gödel. What is Cantor's continuum problem? *The American Mathematical Monthly*, 54(9): 515–525, 1947. doi:10.2307/2304666.

J. Gray. The nineteenth-century revolution in mathematical ontology. In D. Gillies, editor, *Revolutions in Mathematics*, pages 226–248. Clarendon Press, Oxford, 1992.

J. Gray. *Plato's Ghost: The Modernist Transformation of Mathematics*. Princeton University Press, Princeton and Oxford, 2008.

E. Haffner and D. Schlimm. Dedekind on continuity. To appear in *The Continuous*, G. Hellman and S. Shapiro, eds., 2017.

H. Hahn. *Empiricism, Logic and Mathematics. Philosophical Papers*. Number 13 in Vienna Circle Collection. D. Reidel, Dordrecht, 1980. doi:10.1007/978-94-009-8982-5.

M. Hallett. *Cantorian Set Theory and Limitation of Size*. Number 10 in Oxford Logic Guides. Oxford University Press, Oxford, 1984.

M. Hallett. Absoluteness and the Skolem Paradox. In D. DeVidi, M. Hallett, and P. Clarke, editors, *Logic, Mathematics, Philosophy, Vintage Enthusiasms*, number 75 in The Western Ontario Series in Philosophy of Science. Springer, Dordrecht, 2011. doi:10.1007/978-94-007-0214-1_10.

J. D. Hamkins. The set-theoretic multiverse. *The Review of Symbolic Logic*, 5(3):416–449, September 2012. doi:10.1017/S1755020311000359.

J. D. Hamkins and O. Linnebo. The modal logic of set-theoretic potentialism and the potentialist maximality principles. *The Review of Symbolic Logic*, pages 1–36, 2019. doi:10.1017/S1755020318000242.

G. Hellman. Structuralism. In Shapiro [2005], pages 536–562. doi:10.1093/0195148770.001.0001.

A. Heyting. *Intuitionism: An Introduction*. Number 17 in Studies in Logic and the Foundations of Mathematics. North-Holland, Amsterdam, 3rd edition, 1971.

A. Heyting, editor. *L. E. J. Brouwer: Collected Works. I: Philosophy and Foundations of Mathematics*. North-Holland, Amsterdam, 1975.

D. Hilbert. Über das Unendliche. *Mathematische Annalen*, 95:161–190, December 1926. doi:10.1007/BF01206605. Lecture given in Münster, 4 June 1925. English translation in van Heijenoort [1967, pp. 367–392].

D. Hilbert and P. Bernays. *Grundlagen der Mathematik*, volume I. Springer-Verlag, Berlin, Heidelberg, New York, 2nd edition, 1968.

J. Hintikka. Aristotelian infinity. *The Philosophical Review*, 75(2):197–218, April 1966. doi:10.2307/2183083.

C. Huffman. Pythagoras. In E. N. Zalta, editor, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, winter 2018 edition, 2018.

C. Huffman. Pythagoreanism. In E. N. Zalta, editor, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, fall 2019 edition, 2019.

N. Huggett. Zeno's paradoxes. In E. N. Zalta, editor, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, winter 2019 edition, 2019.

A. D. Irvine and H. Deutsch. Russell's paradox. In E. N. Zalta, editor, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, winter 2020 edition, 2020.

A. Kanamori. The mathematical development of set theory from Cantor to Cohen. *The Bulletin of Symbolic Logic*, 2(1):1–71, March 1996. doi:10.2307/421046.

P. Kitcher. Bolzano's ideal of algebraic analysis. *Studies in History and Philosophy of Science Part A*, 6(3):229–269, October 1975. doi:10.1016/0039-3681(75)90024-2.

P. Koellner. Truth in mathematics: The question of pluralism. In O. Bueno and Ø. Linnebo, editors, *New Waves in Philosophy of Mathematics*, New Waves in Philosophy, pages 80–116. Palgrave Macmillan, 2009.

J. Lear. Aristotelian infinity. *Proceedings of the Aristotelian Society*, 80(1):187–210, June 1980. doi:10.1093/aristotelian/80.1.187.

Ø. Linnebo. The potential hierarchy of sets. *The Review of Symbolic Logic*, 6(2):205–228, June 2013. doi:10.1017/S1755020313000014.

Ø. Linnebo and S. Shapiro. Actual and potential infinity. *Noûs*, 53(1):160–191, 2019. doi:10.1111/nous.12208.

O. Linnebo, S. Shapiro, and G. Hellman. Aristotelian continua. *Philosophia Mathematica*, 24 (2):214–246, June 2016. doi:10.1093/philmat/nkv024.

B. Linsky and A. D. Irvine. *Principia Mathematica*. In E. N. Zalta, editor, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, winter 2020 edition, 2020.

P. Maddy. Believing the axioms. I. *The Journal of Symbolic Logic*, 53(2):481–511, June 1988. doi:10.1017/S0022481200028425.

P. Mancosu. Bolzano and Cournot on mathematical explanation. *Revue d'histoire des sciences*, 52(3–4):429–456, 1999. doi:10.3406/rhs.1999.1364.

D. A. Martin. Multiple universes of sets and indeterminate truth values. *Topoi*, 20:5–16, January 2001. doi:10.1023/A:1010600724850.

H. Mendell. Aristotle and mathematics. In E. N. Zalta, editor, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, fall 2019 edition, 2019.

A. W. Moore. *The Infinite*. Routledge, London, 2nd edition, 2001.

E. Morscher. Bernard Bolzano. In E. N. Zalta, editor, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, Winter 2018 edition, 2018.

C. Mortensen. Inconsistent mathematics. In E. N. Zalta, editor, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, fall 2017 edition, 2017.

J. Palmer. Parmenides. In E. N. Zalta, editor, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, winter 2020 edition, 2020.

J. Palmer. Zeno of Elea. In E. N. Zalta, editor, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, summer 2021 edition, 2021.

C. Parsons. The structuralist view of mathematical objects. *Synthese*, 84(3):303–346, September 1990. doi:10.1007/BF00485186.

H. Putnam. Models and reality. *The Journal of Symbolic Logic*, 45(3):464–482, September 1980. doi:10.2307/2273415.

E. H. Reck. Dedekind's structuralism: an interpretation and partial defense. *Synthese*, 137(3): 369–419, 2003. doi:10.1023/B:SYNT.0000004903.11236.91.

E. H. Reck. Dedekind's contributions to the foundations of mathematics. In E. N. Zalta, editor, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, winter 2012 edition, 2012.

E. H. Reck and M. P. Price. Structures and structuralism in contemporary philosophy of mathematics. *Synthese*, 125(3):341–383, 2000. doi:10.1023/A:1005203923553.

E. H. Reck and G. Schiemer, editors. *The Prehistory of Mathematical Structuralism*. Logic and Computation in Philosophy. Oxford University Press, New York, 2020.

M. Resnik. Non-ontological structuralism. *Philosophia Mathematica*, 27(3):303–315, October 2019. doi:10.1093/philmat/nky002.

P. Rusnock and J. Šebestik. Bolzano's logic. In E. N. Zalta and U. Nodelman, editors, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, Fall 2022 edition, 2022.

S. Russ. *The Mathematical Works of Bernard Bolzano*. Oxford University Press, 2004.

S. Shapiro. *Philosophy of Mathematics: Structure and Ontology*. Oxford University Press, Oxford, 1997.

S. Shapiro. *Thinking About Mathematics. The Philosophy of Mathematics*. Oxford University Press, Oxford, 2000.

S. Shapiro, editor. *The Oxford Handbook of Philosophy of Mathematics and Logic*. Oxford University Press, Oxford, 2005. doi:10.1093/0195148770.001.0001.

H. Sinaçeur. Cauchy et Bolzano. *Revue d'histoire des sciences*, 26(2):97–112, April 1973. doi:10.3406/rhs.1973.3315.

T. Skolem. Some remarks on axiomatized set theory. In van Heijenoort [1967], pages 290–301.

C. Smoryński. The incompleteness theorems. In J. Barwise, editor, *Handbook of Mathematical Logic*, number 90 in Studies in Logic and the Foundations of Mathematics, pages 821–865. North-Holland, Amsterdam, 1977.

J. R. Steel. Gödel's program. In J. Kennedy, editor, *Interpreting Gödel: Critical Essays*, pages 153–179. Cambridge University Press, Cambridge, 2014.

G. Sundholm. Constructions, proofs and the meaning of logical constants. *Journal of Philosophical Logic*, 12(2):151–172, May 1983. doi:10.1007/BF00247187.

G. Sundholm. Constructive recursive functions, Church's thesis, and Brouwer's theory of the creating subject: Afterthoughts on a Parisian joint session. In J. Dubucs and M. Bourdeau, editors, *Constructivity and Computability in Historical and Philosophical Perspective*, number 34 in Logic, Epistemology, and the Unity of Science, pages 1–35. Springer, Dordrecht, 2014. doi:10.1007/978-94-017-9217-2_1.

J. Tappenden. Geometry and generality in Frege's philosophy of arithmetic. *Synthese*, 102(3): 319–361, March 1995. doi:10.1007/BF01064120.

N. Tennant. Logicism and neologicism. In E. N. Zalta, editor, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, winter 2017 edition, 2017.

R. Torretti. Nineteenth century geometry. In E. N. Zalta, editor, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, fall 2019 edition, 2019.

A. S. Troelstra and D. van Dalen. *Constructivism in Mathematics: An Introduction*, volume I of *Studies in Logic and the Foundations of Mathematics*. North-Holland, Amsterdam, 1988.

M. van Atten. *On Brouwer*. Wadsworth Philosophers Series. Wadsworth/Thomson Learning, Belmont, CA, 2004.

M. van Atten. The Development of Intuitionistic Logic. In E. N. Zalta, editor, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, spring 2017 edition, 2017.

M. van Atten. The Creating Subject, the Brouwer–Kripke Schema, and infinite proofs. *Indagationes Mathematicae*, 29(6):1565–1636, December 2018. doi:10.1016/j.indag.2018.06.005.

M. van Atten and G. Sundholm. L. E. J. Brouwer's 'Unreliability of the Logical Principles': A New Translation, with an Introduction. *History and Philosophy of Logic*, 38(1):24–47, 2017. doi:10.1080/01445340.2016.1210986. English translation of Brouwer [1908].

M. van Atten, P. Boldini, M. Bourdeau, and G. Heinzmann, editors. *One Hundred Years of Intuitionism (1907–2007): The Cerisy Conference*. Publications des Archives Henri Poincaré / Publications of the Henri Poincaré Archives. Birkhäuser, Basel, 2008.

J. van Heijenoort, editor. *From Frege to Gödel: A Source Book in Mathematical Logic, 1879–1931*. Harvard University Press, Cambridge, MA, 1967.

J. Webb. Tracking contradictions in geometry: The idea of a model from Kant to Hilbert. In J. Hintikka, editor, *From Dedekind to Gödel. Essays on the Development of the Foundations of Mathematics*, number 251 in Synthese Library, pages 1–20. Kluwer Academic Publishers, Dordrecht, 1995. doi:10.1007/978-94-015-8478-4_1.

A. Yap. Dedekind and Cassirer on mathematical concept formation. *Philosophia Mathematica*, 2014. doi:10.1093/philmat/nku029.

R. Zach. Hilbert's program then and now. In D. Jacquette, editor, *Philosophy of Logic*, volume 5 of *Handbook of the Philosophy of Science*, pages 411–447. Elsevier, 2006.

R. Zach. Hilbert's program. In E. N. Zalta, editor, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, fall 2019 edition, 2019.