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Vagueness, lecture 9: Higher-order vagueness.

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https://extralogical.net/teaching/vagueness2019.html

1. Definiteness operator and gap principles

- We use a primitive definitely operator 'D' to represent that some sentence φ is definitely the case, $D\varphi$.
- Together with a truth predicate $T(x)$, this lets us formulate the following *gap principle*

$$(D\Phi(x) \wedge D\neg\Phi(y)) \rightarrow \neg R(x, y).$$

  where 'Φ' is a vague predicate and 'R' stands for a *Sorites relation* for that predicate, e.g. "being one millimetre shorter than" when Φ is "is tall".
- The 'D' operator lets us reformulate disagreements about the sense in which it is correct to say that vague predicates are gappy, in terms of disagreements about how to understand the 'D' operator.
  - It could, for example be construed *pragmatically*, *epistemically*, or *semantically.*
- Fara [2004] argues that accepting gap principles leads to contradiction in the case of higher-order vagueness, at least when the definitely operator is understood semantically.

2. Higher-order vagueness

> [A] theory of definiteness [. . .] is required simultaneously to be both a theory of what it is to fall in the grey area, i.e. cause us to hedge, and a theory of our inability to locate a boundary. [¶] But we are no more able to locate a point marking a transition from the definitely tall to the *not* definitely tall . . . The push to accept higher-order vagueness comes from requiring that the very same explanation be given for this inability as was given for the first: just as we cannot locate a boundary dividing the tall from the not tall because there are men who do not fall definitely into either category [. . .] we cannot locate a boundary dividing the definitely tall from the not definitely tall because there are men who do not fall definitely into either of these categories. . . So although there is no obvious phenomenon of second-order hedging [. . .] a theory of definiteness is now on this picture required to accommodate there being not only first-order borderline cases, men who are neither definitely tall nor definitely not tall, but also 'second-order borderline cases', men who are neither definitely definitely tall nor definitely not definitely tall.
>
> [Fara 2004, p. 198]

- This leads to higher-order gap principles like

$$(DT(x) \wedge D\neg T(x)) \to \neg R(x, y)$$

  and

$$(DDT(x) \wedge D\neg DT(x)) \to \neg R(x, y)$$

  and

$$(DDDT(x) \wedge D\neg DDT(x)) \to \neg R(x, y)$$

  and so on.

- More generally we can say that, letting D^n denote n iterations of the definitely operator,

$$DD^n\Phi(x) \to \neg D\neg D^n\Phi(x')$$

  where x' is the successor of x in a Sorites series.

- The logic of the D operator includes at least the following axioms and rules:

  (T) $D\varphi \to \varphi$,

  (K) $D(\varphi \to \psi) \to (D\varphi \to D\psi)$,

  (Nec) if $\vdash \varphi$ then $\vdash D\varphi$.

- Epistemicists will object to, but gap theorists will typically endorse, a rule of D-introduction:

(D-intro) if $\Gamma \vdash \varphi$ then $\Gamma \vdash \varphi$.

  Fara [2004, p. 200] writes that "Standard supervaluationist semantics and conceptions of validity such as that developed by Kit Fine [1975] and endorsed more recently by Rosanna Keefe [2000] do indeed yield that D-introduction is validity-preserving".

3. Fara's argument

$T(1)$	Premise
$DT(1)$	D-intro
$D^2T(1)$	D-intro
$\vdots$	
$D^{m-1}T(1)$	D-intro
$\neg T(m)$	Premise
$D\neg T(m)$	D-intro
$\neg DT(m-1)$	Gap principle for $T(x)$
$D\neg DT(m-1)$	D-intro
$\neg D^2T(m-2)$	Gap principle for $DT(x)$
$D\neg D^2T(m-2)$	D-intro
$\neg D^3T(m-3)$	Gap principle for $D^2T(x)$
$\vdots$	
$\neg D^{m-1}T(1)$	Gap principle for $D^{m-2}T(x)$

We have thus derived a contradiction, $D^{m-1}T(1)$ and $\neg D^{m-1}T(1)$.

- What does this argument show?

- What doesn't it show?

4. Supervaluationism

- On a number of gap theories, arguments by conditional introduction, or contraposition, or *reductio ad absurdum*, are not acceptable.
- Here supervaluationism seems superior, since it preserves a greater degree of classicality.
- It's hard to say precisely what is meant here: what features of classical logic do we take to be essential or constitutive?
- Fara argues that supervaluationist semantics involves a failure of closure under contraposition, conditional-introduction, and *reductio ad absurdum.*
  - Therefore, it's not classical.
- These failures initially look like they're restricted to sentences involving the definitely operator.
- But Fara argues that some classically valid conditionals not involving the definitely operator, but involving "penumbral connections" such as that between the predicates 'is tall' and 'is short', need not be true on the supervaluationist view (p. 214).

References

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