Modal Logic
Exercise Set 10

To be completed by Thursday 18 July

Recall that $\mathsf{E}(x)$ abbreviates $(\exists y)(y = x)$, where x is a free variable and y is a variable distinct from x.

1. Give variable domain tableau proofs of the following validities:
   (a) $\Diamond(x = y) \to (x = y)$.
   (b) $\neg(x = y) \to \Box\neg(x = y)$.
   (c) $((x = y) \wedge (y = z)) \to (x = z)$.
   (d) $(x = y) \to (\varphi(x) \leftrightarrow \varphi(y))$, where $\varphi(x)$ is a formula in which y does not occur, and $\varphi(y)$ is the result of substituting occurrences of y for free occurrences of x in $\varphi(x)$.
   (e) $((\forall x)\varphi(x) \wedge \mathsf{E}(z)) \to \varphi(z)$.
   (f) $((\forall x)\Diamond\mathsf{E}(x) \wedge (\exists x)\Box P(x)) \to \Diamond(\exists x)P(x)$.

2. Derive the *Parameter Nonexistence Rule*: A branch of a variable domain tableau containing $\sigma\ \neg\mathsf{E}(p^\sigma)$ closes.

3. Let $F = (W, R, D)$ be a variable domain frame. Prove that the following conditions are equivalent:
   (a) F is anti-monotonic;
   (b) $\Diamond\mathsf{E}(x) \to \mathsf{E}(x)$ is valid in every normal model based on F.