Modal Logic
Exercise Set 04

To be completed by Thursday 23 May

We will work through these exercises (and possibly some others as well) during the problem class. Exercises marked with a (△) are a little more challenging, and those marked with a (▲) are more difficult still.

1. Use the tableau proof system for **K** to prove the following.
   (a) $(\Box P \wedge \Diamond Q) \to \Diamond(P \wedge Q)$
   (b) $(\Box\Diamond P \wedge \Box\Box Q) \to \Box\Diamond(P \wedge Q)$
   (c) $(\Box P \vee \Box Q) \to \Box(P \vee Q)$
2. (△) Show that for any formula A, there is a tableau proof of $\Box A$ if and only if there is a tableau proof of A.
3. Use the tableau proof system for **T** to prove $\Diamond(P \to \Box P)$.
4. Use the tableau proof system for **K4** to prove $(\Box P \wedge \Box Q) \to \Box(\Box P \wedge \Box Q)$.
5. Use the tableau proof system for **S4** to prove $(\Box\Diamond P \wedge \Box\Diamond Q) \to \Box\Diamond(\Box\Diamond P \wedge \Box\Diamond Q)$.
6. Use the tableau proof system for **S5** to prove $\Box P \vee \Box(\Box P \to Q)$.
7. Use the tableau proof system for **S5** to prove $\Diamond(P \wedge \Box Q) \leftrightarrow (\Diamond P \wedge \Box Q)$.
8. (△) We define a new tableau system as follows. Take prefixes to just be natural numbers n, k. The rules for the truth-functional logical connectives are as usual. The only modal rules are the following. If the prefix k is new to the branch, the possibility rules

$$\frac{n\ \Diamond A}{k\ X} \quad \text{and} \quad \frac{n\ \neg\Box A}{k\ \neg A}$$

   may be applied. If the prefix k already appears on the branch, the necessity rules

$$\frac{n\ \Box A}{k\ X} \quad \text{and} \quad \frac{n\ \neg\Diamond A}{k\ \neg A}$$

   may be applied.

   Prove that the theorems of this tableau system are exactly those of the tableau system **S5** given by the standard modal rules together with the rules T, 4, and 4r.