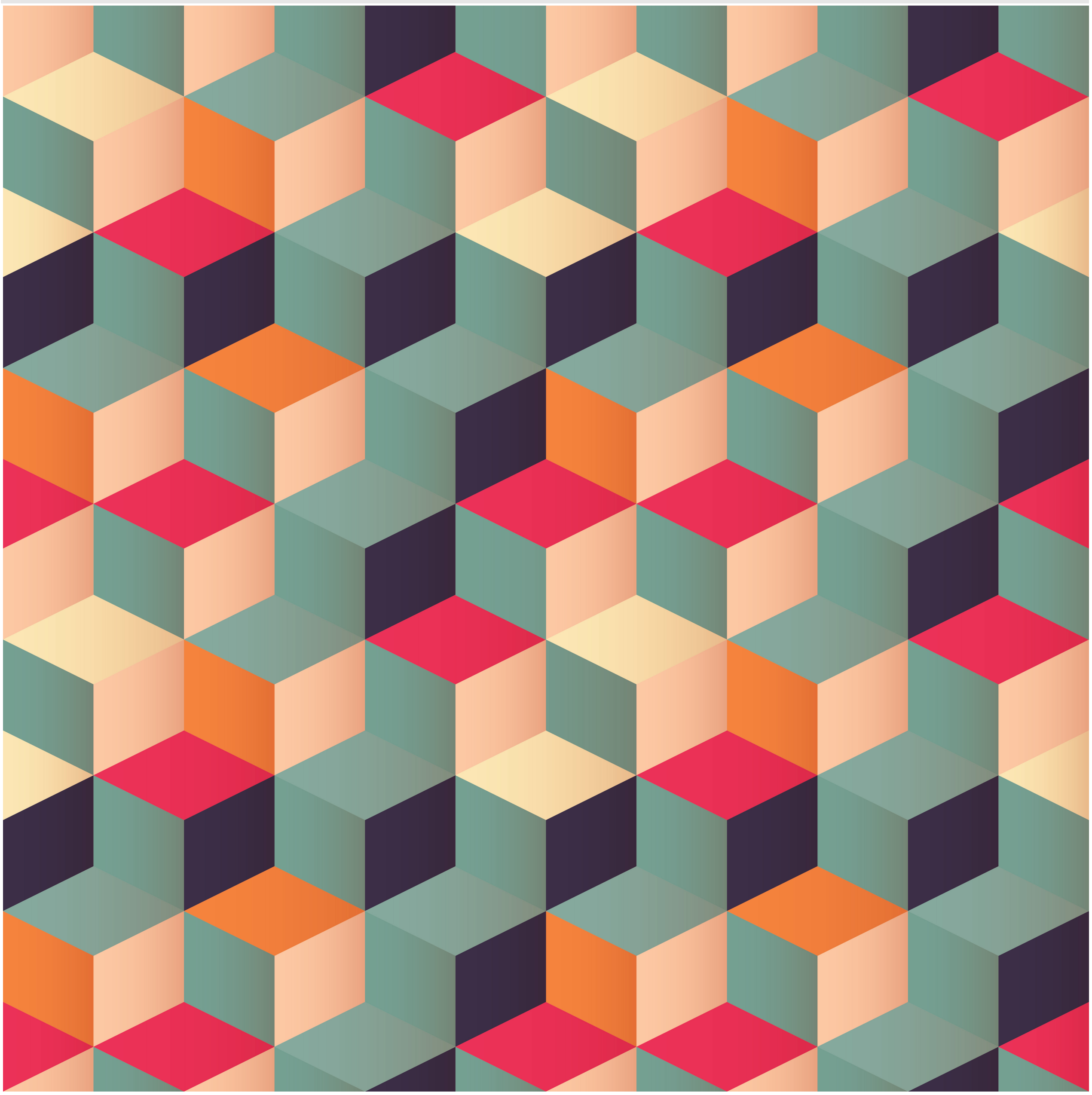
IB Analysis and Approaches
Math Internal Assesment
Year Two



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Rotation Using Matrix Math

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Introduction

As an individual, I am fascinated by 3d graphics and research the topic on my own time. Mathematical applications in computer graphics is a fascinating part of both computer science and math. The idea that math is able to describe entire virtual environments is remarkable. Among other important mathematical concepts, vectors and rotational transformations play a crucial role in the positioning of objects in three-dimensional spaces.

Although there are alternative systems of rotation that avoid problems such as Gimbal Lock, in this math Internal Assessment I decided to use rotation matrices and learn how they work using the unit circle. I decided to focus on rotation matrices because they allow us to use Euler angles and build up important knowledge of linear algebra. Knowing how these systems work is important because it allows the individual to make an end product more intuitively and to work in 3d space without any help or helper libraries.

This assessment will connect to trigonometry. I will utilize basic trigonometry to reason vector composition, and I will use the unit circle in order to visualize vector rotation. In order to break down the task, I will divide the problem into smaller, more manageable problems that build on each other. Not only will I explain the math behind these methods of rotation, but I will also use math I already know to implement the rotational algorithms in C++ and JavaScript on my own. During this project, I will be writing multiple visualization programs, making GIF's [1] of that demonstrate the math I learn. The final project that I will produce will be a simple program rotating a triangle in 3d space.

1. Problem

The problem for the math internal assessment is as follows. I need to derive a method to create rotations in three-dimensional space by using trigonometry and basic linear algebra.

[1] gifs are represented as image urls pointing to the animated gif, this is because free pdf viewers like firefox dont support most pdf animations and scripting

This problem is very important to everyday life powering many aspects of video games, CAD, and complex milling. The problem with rotation has an elegant system of solutions that builds up to more complicated and important problems.

Trigonometry (the study of angles) is integral in the rotation process. In rotation matrices, trigonometric functions are used in order to calculate rotation and position. Trigonometry is also utilized to more generally in linear algebra in order to calculate the angle and position of resultant vectors, which is integral to the process.

2. Math

NOT FINALIZED

In order to make the layout of this document easier to understand the math section will be broken into smaller chunks of a larger recipe. Eventually we will make a working demo in open Gl or three.js, but before we compute beautiful rotations I will have to clear the air on the math behind the magic and the process I used to find it first.

2.1 Defining 2d Transformation

In order to understand 3d rotation It would be much simpler to start with simpler problems such as transforming objects in 2d space. Part one will explore how to actually get 2d vectors to move.

To start making vectors move, we should first define what a vector is. Different areas of math define vectors differently, such as physics, who define vectors as a scalar with an angle attached to it. In a mathematical point of view, a vector can be represented in a diversity of formats. Since matrix operations will be performed on the vector in this context the vector is defined as a matrix that defines coordinates in space pointing away from the origin. Essentially, this vector is a matrix that contains two perpendicular vectors that add up to the final vector.

$$\vec{v} = \begin{bmatrix} x & y \end{bmatrix} \tag{1}$$

In order to construct the vector from the two coordinates, we can use trigonometry to find the angle of the vector and the Pythagorean theorem to find its magnitude.

$$\theta = \arctan\left(\frac{y}{x}\right) \qquad |\vec{v}| = \sqrt{x^2+y^2} \tag{2}$$

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