On a geometric interpretation of the $1/4$ factor in black hole entropy

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The Bekenstein–Hawking entropy $S = A/4\ell_{\rm P}^2$ contains a coefficient $1/4$ that has been calculated by many approaches to quantum gravity. We show that for any null causal boundary in a four-dimensional Lorentzian spacetime, including horizons, that factor would naturally arise from geometry. A heuristic accessibility argument based on null causal structure is supported by a geometry-only derivation using the canonical symplectic structure on T^*M and the double-cone structure of null covectors. No gravitational field equations or thermodynamic assumptions are invoked.

INTRODUCTION

The entropy of a black hole is

$$S = \frac{A}{4\,\ell_{\rm P}^2}, \qquad \ell_{\rm P}^2 = \frac{G\hbar}{c^3}. \tag{1}$$

The universality of the coefficient $1/4$ suggests a geometric origin independent of microscopic details, as it arises in diverse frameworks including semiclassical gravity, Noether-charge methods, Euclidean path integrals, and thermodynamic arguments [1–6].

We argue that this factor is the fraction of phase space accessible across any null causal boundary in a four-dimensional Lorentzian space-time, including black holes.

HEURISTIC ACCESSIBILITY ARGUMENT

Consider a closed spacelike 2-surface $\mathcal{S}$ that is a cross-section of a null hypersurface $\mathcal{N}$.

A local description of data near $\mathcal{S}$ involves four spacetime coordinates and conjugate momenta. We impose four kinematic restrictions associated with being confined to a null causal boundary. We restrict attention to four-dimensional Lorentzian spacetimes.

(1) Surface localization. Being on $\mathcal{N}$ fixes the transverse coordinate, removing one degree of freedom.

(2) Null constraint. Displacements along $\mathcal{N}$ satisfy $g_{ab}\,dx^a\,dx^b = 0$, relating the remaining coordinates and removing one independent combination.

(3) Generator parametrization. Motion along the null generators provides the evolution parameter. Using it as a clock removes one dynamical variable, leaving one canonical pair per surface element.

(4) Causal restriction. A null hypersurface is a one-way causal boundary. An observer on one side accesses only propagation in one time direction along the generators. Thus only half of the canonical phase-space measure is accessible.

Combining these steps yields an accessible fraction

$$\frac{1}{4} \tag{2}$$

of the original phase space per Planck-area cell.

This argument is heuristic; a geometry-only formal realization is given next.

GEOMETRY-ONLY FORMALIZATION

We distinguish three types of index contractions that appear below.

(i) The expression $X^\mu P_\mu$ denotes the natural dual pairing between a vector X^μ and a covector P_μ, independent of any metric structure.

(ii) The expression $g_{\mu\nu}X^\mu Y^\nu$ denotes contraction using the spacetime metric, i.e. an inner product determined by $g_{\mu\nu}$.

(iii) On the cotangent bundle T^*M, the canonical (tautological) one-form

$$\theta = p_\mu\,\mathrm{d}x^\mu \tag{3}$$

represents the natural pairing between the covector p_μ and the base-space component of a tangent vector to T^*M. This structure is metric-independent and depends only on the bundle geometry.

Throughout, indices are raised and lowered using $g_{\mu\nu}$ only when explicitly indicated.

We now formulate this reasoning using the geometry of null covectors.

Canonical phase space

Let (M, g) be a time-oriented Lorentzian manifold. On T^*M define

$$\theta = p_a\,\mathrm{d}x^a, \qquad \omega = \mathrm{d}\theta\,. \tag{4}$$

Define the quadratic function on T^*M

$$Q(x,p) = g^{ab}(x)\,p_a\,p_b\,. \tag{5}$$

The null covectors form the hypersurface

$$\mathcal{C} = \{Q = 0\} \subset T^*M\,. \tag{6}$$

The characteristic directions of this hypersurface correspond to null geodesics in M.

Space of null rays

The restriction $\theta|_{\mathcal{C}}$ defines a contact structure. Quotienting by the characteristic flow of Q yields the space $\mathcal{R}$ of null geodesics, which inherits a symplectic structure [7–10].

Dimension counting:

$$\dim T^*M = 8\,, \tag{7}$$

$$Q = 0 \Rightarrow 7\,, \tag{8}$$

$$\text{quotient} \Rightarrow 6\,. \tag{9}$$

The characteristic flow of Q generates reparametrizations along null generators. Quotienting by this one-dimensional gauge direction reduces the dimension by one, yielding the space of null rays.

Restriction to a cross-section

Let $\mathcal{N}$ be a null hypersurface and $\mathcal{S} \subset \mathcal{N}$ a closed spacelike 2-surface that serves as a cross-section of $\mathcal{N}$. Restrict to null rays intersecting $\mathcal{S}$. Fixing the intersection point removes two degrees of freedom, and quotienting by generator reparametrization removes another, leaving two dimensions—one canonical pair—per surface element.

Time orientation and exact halving

Time orientation splits the null cone into two components: future and past. The map $(x, p) \mapsto (x, -p)$ exchanges them and preserves the symplectic form.

Therefore the Liouville measure satisfies

$$\mu(\text{future}) = \mu(\text{past}) = \tfrac{1}{2}\,\mu(\text{full}). \tag{10}$$

An observer restricted to one side of a causal boundary accesses only one sector, yielding an exact factor 1/2.

ENTROPY

The geometric construction above assigns to each surface element of $\mathcal{S}$ the data of null rays intersecting it. After imposing the null constraint and quotienting by generator reparametrization, the reduced ray space carries two independent degrees of freedom per surface point, forming one canonical pair.

Time orientation divides the null directions into future and past components. A causal boundary allows propagation only in one temporal direction along the generators, restricting the accessible phase-space measure to one half of this pair.

Introducing a Planck-area cutoff, the surface contains $N = A/\ell_{\mathrm{P}}^2$ independent cells. Each cell therefore contributes one half of a canonical pair to the accessible phase space, corresponding to one phase-space dimension out of the two associated with a full pair.

Relative to the original four-dimensional phase-space data per spacetime point, the accessible fraction is thus

$$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}\,, \tag{11}$$

yielding the Bekenstein–Hawking entropy

$$S = \frac{A}{4\,\ell_{\mathrm{P}}^2}\,. \tag{12}$$

DISCUSSION

The coefficient 1/4 is a consequence of null causal structure and time orientation in four-dimensional Lorentzian spacetime. It is independent of the Einstein equations, of any gravitational action, of Newton's constant (except through the Planck length in defining the cell size), and of any particular spacetime solution.

The derivation requires three ingredients: a Lorentzian metric, null geodesics, and a closed 2-surface that acts as a one-way causal boundary. These are shared by every framework in which black hole entropy has been computed. The universality of the coefficient across semi-classical, string-theoretic, and loop gravity derivations is therefore not a coincidence to be explained by each framework separately, but a geometric fact that precedes all of them.

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