National University

of Computer & Emerging Sciences-Faisalabad

Linear Algebra **Assignment # 2**

CLO 2: ***Demonstrate a comprehensive understanding of vectors and their properties, including linear combinations, spans, linear independence, vector spaces structure, subspaces, and their basis and dimension*.**

Note: ***Last date for the submission is October 21, 2024.***

Question # 1: **Consider**

$$A = \begin{bmatrix} 1 & 2 & 3 & 1 & 4 \\ 2 & 1 & 3 & -1 & 2 \\ 3 & 4 & 7 & -1 & 6 \\ 4 & 3 & 7 & 3 & 10 \end{bmatrix}$$

a) Find the order of transformation define by T(x) = Ax.
b) Find five vectors in Null space of A.
c) Find geometry of Null space of A.
d) Find basis for Null space of A.
e) Find dimension of Null space of A.
f) Find geometry of Column space of A.
g) Find five vectors in Column space of A.
h) Find basis for Column space of A.
i) Find dimension of Column space of A.
j) Find Rank of A.
k) Column space is a subspace of R^k, what is k?
l) Null space is a subspace of R^k, what is k?
m) Is $\begin{bmatrix} 1 \\ 4 \\ 2 \\ 1 \\ 0 \end{bmatrix}$ in Null space of A?
n) Is $\begin{bmatrix} 1 \\ 3 \\ 2 \\ 4 \end{bmatrix}$ in Column space of A?

i)

$$A=\begin{bmatrix}1 & 2 & 3 & 1 & 4\\ 2 & 1 & 3 & -1 & 2\\ 3 & 4 & 7 & -1 & 6\\ 4 & 3 & 7 & 3 & 10\end{bmatrix}$$

(a) The order of transformation ~~A~~ $T(x)=Ax$ is $T: \mathbb{R}^5 \longrightarrow \mathbb{R}^4$.

(b)

$$A \sim \begin{bmatrix}1 & 2 & 3 & 1 & 4\\ 0 & 1 & 1 & 1 & 2\\ 0 & 0 & 0 & 1 & 1\\ 0 & 0 & 0 & 0 & 0\end{bmatrix}$$

For Null space

$$Ax=0$$

$$\begin{bmatrix}1 & 2 & 3 & 1 & 4\\ 0 & 1 & 1 & 1 & 2\\ 0 & 0 & 0 & 1 & 1\\ 0 & 0 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\\ x_5\end{bmatrix}=\begin{bmatrix}0\\0\\0\\0\\0\end{bmatrix}$$

$$x_1+2x_2+3x_3+x_4+4x_5=0 \quad -(i)$$

$$x_2+x_3+x_4+2x_5=0 \quad -(ii)$$

$$x_4+x_5=0$$

$$\boxed{x_4=-x_5}$$

Put in (ii)

$$x_2+x_3-x_5+2x_5=0$$

$$\boxed{x_2=-x_5-x_3}$$

Put in (i)

$$x_1+2(-x_3-x_5)+3x_3-x_5+4x_5=0$$

$$x_1-2x_3-2x_5+3x_3+3x_5=0$$

$$x_1+x_3+x_5=0$$

$$\boxed{x_1=-x_3-x_5}$$

So,

$$\begin{bmatrix}x_1\\x_2\\x_3\\x_4\\x_5\end{bmatrix}=\begin{bmatrix}-x_3-x_5\\-x_3-x_5\\x_3\\-x_5\\x_5\end{bmatrix}=x_3\begin{bmatrix}-1\\-1\\1\\0\\0\end{bmatrix}+x_5\begin{bmatrix}-1\\-1\\0\\-1\\1\end{bmatrix}$$

Five vectors in Null space are.

$$(1)\begin{bmatrix}-1\\-1\\1\\0\\0\end{bmatrix}+(1)\begin{bmatrix}-1\\-1\\0\\-1\\+1\end{bmatrix}=\begin{bmatrix}-2\\-2\\1\\-1\\+1\end{bmatrix}$$

$$(2)\begin{bmatrix}-1\\-1\\1\\0\\0\end{bmatrix}+(1)\begin{bmatrix}-1\\-1\\0\\-1\\1\end{bmatrix}=\begin{bmatrix}-3\\-3\\2\\-1\\1\end{bmatrix}$$

$$(1)\begin{bmatrix}-1\\-1\\1\\0\\0\end{bmatrix}+(2)\begin{bmatrix}-1\\-1\\0\\-1\\+1\end{bmatrix}=\begin{bmatrix}-3\\-3\\1\\-2\\+2\end{bmatrix}$$

$$(1)\begin{bmatrix}-1\\-1\\1\\0\\0\end{bmatrix}+(-1)\begin{bmatrix}-1\\-1\\0\\-1\\+1\end{bmatrix}=\begin{bmatrix}0\\0\\1\\1\\-1\end{bmatrix}$$

$$(1)\begin{bmatrix}-1\\-1\\1\\0\\0\end{bmatrix}+(-2)\begin{bmatrix}-1\\-1\\0\\-1\\1\end{bmatrix}=\begin{bmatrix}1\\1\\-1\\2\\-2\end{bmatrix}$$

Geome

Since

Geometry.

Since Null space is generated by two vectors

$$\begin{bmatrix}-1\\-1\\1\\0\\0\end{bmatrix}, \begin{bmatrix}-1\\-1\\0\\-1\\1\end{bmatrix}$$

So it is a plane in $\mathbb{R}^5$.

(d) Since $\begin{bmatrix}-1\\-1\\1\\0\\0\end{bmatrix}, \begin{bmatrix}-1\\-1\\0\\-1\\1\end{bmatrix}$ generate Null space.

Also $\begin{bmatrix}-1 & -1\\-1 & -1\\1 & 0\\0 & -1\\0 & 1\end{bmatrix}$

$\sim \begin{bmatrix}1 & 1\\0 & 1\\0 & 0\\0 & 0\\0 & 0\end{bmatrix}$

So these are linearly independent.

Hence $\left\{\begin{bmatrix}-1\\-1\\1\\0\\0\end{bmatrix}, \begin{bmatrix}-1\\-1\\0\\-1\\1\end{bmatrix}\right\}$ is a basis

(e) Dimension, that is number of vectors in basis is 2.

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(f)

Since

$$A \overset{R}{\sim} \begin{bmatrix} 1 & 2 & 3 & 1 & 4 \\ 0 & 1 & 1 & 1 & 2 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

By deleting non-piviot colum. we have

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

So, Column space is generated by these vectors.

$$\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 6 \\ 9 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ -1 \\ 3 \end{bmatrix}$$

And hence geometry of Col A. is solid space in $\mathbb{R}^4$.

(g) Five vectors.

$$(1)\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + (1)\begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + (1)\begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ 1 \\ 0 \end{bmatrix}$$

$$(1)\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + (1)\begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + (-1)\begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ -1 \\ 0 \end{bmatrix}$$

$$(1)\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + (-1)\begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + (-1)\begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ -2 \\ -1 \\ 0 \end{bmatrix}$$

$$(-1)\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + (-1)\begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + (-1)\begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -4 \\ -2 \\ -1 \\ 0 \end{bmatrix}$$

$$(2)\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + (2)\begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + (2)\begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 8 \\ 4 \\ 2 \\ 0 \end{bmatrix}$$

Since Col A is generated by $\begin{bmatrix}1\\0\\0\\0\end{bmatrix}, \begin{bmatrix}2\\1\\0\\0\end{bmatrix}, \begin{bmatrix}1\\1\\1\\0\end{bmatrix}$

and $\begin{bmatrix}1 & 2 & 1\\0 & 1 & 1\\0 & 0 & 1\\0 & 0 & 0\end{bmatrix}$ is in row echelon form

Hence linearly independent.

So, Bases $= \left\{ \begin{bmatrix}1\\0\\0\\0\end{bmatrix}, \begin{bmatrix}2\\1\\0\\0\end{bmatrix} \begin{bmatrix}1\\1\\1\\0\end{bmatrix} \right\}$

(I) Dimension of Col A. is 3.

(J) The Rank of A = Dimension of Col A.

So, Rank of $A = 3$.

(K) Col A is generated by the vectors of $\mathbb{R}^4$

So Col A is subspace of $\mathbb{R}^4$ with $k=4$.

(L) Null A is generated by the vectors of $\mathbb{R}^5$

So, Null A is subspace of $\mathbb{R}^5$ with $k=5$.

(M) For a vector in Null A.

$An = 0$

$$\begin{bmatrix}1 & 2 & 3 & 1 & 4\\2 & 1 & 3 & -1 & 2\\3 & 4 & 7 & -1 & 6\\4 & 3 & 7 & 3 & 10\end{bmatrix}\begin{bmatrix}1\\4\\2\\1\\0\end{bmatrix} \neq \begin{bmatrix}0\\0\\0\\0\\0\end{bmatrix}$$

So, it is not in Null A.

(N) Let $v = \begin{bmatrix} 1 \\ 3 \\ 2 \\ 4 \end{bmatrix}$

$$[A \quad v] = \begin{bmatrix} 1 & 2 & 3 & 1 & 4 & 1 \\ 2 & 1 & 3 & -1 & 2 & 3 \\ 3 & 4 & 7 & -1 & 6 & 2 \\ 4 & 3 & 7 & 3 & 10 & 4 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 2 & 3 & 1 & 4 & 1 \\ 0 & 1 & 1 & 1 & 2 & -1/3 \\ 0 & 0 & 0 & 1 & 1 & 5/6 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Since pivot position is in last column of Augmented matrix. So, $\begin{bmatrix} 1 \\ 3 \\ 2 \\ 4 \end{bmatrix}$ is not in column space of A.

Question # 2: **Consider**

$$H = \left\{ \begin{bmatrix} r \\ s \\ t \end{bmatrix} ; 3r - 2 = 3s + t \right\}$$

a) **Find a vector that is in H.**
b) **Find geometry of H.**
c) **Find Basis for H.**
d) **Find dimension of H.**

Question # 02

$$H = \left\{ \begin{bmatrix} r \\ s \\ t \end{bmatrix} ; \ 3r-2 = 3s+t \right\}$$

$$3r - 2 = 3s + t$$

$$3r - 3s - t = 2$$

$$3r = 3s + t + 2$$

$$r = s + \frac{1}{3}t + \frac{2}{3}$$

$$\begin{bmatrix} r \\ s \\ t \end{bmatrix} = \begin{bmatrix} s + \frac{1}{3}t + \frac{2}{3} \\ 1s + 0t + 0 \\ 0s + 1t + 0 \end{bmatrix} = s\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + t\begin{bmatrix} 1/3 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 2/3 \\ 0 \\ 0 \end{bmatrix}.$$

a. $\begin{bmatrix} 5/3 \\ 1 \\ 0 \end{bmatrix} \in H$. $\quad s = 1, \ t = 0$

b. Geometry of H = plane in $\mathbb{R}^3$.

c. $\left\{ \begin{bmatrix} 5/3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right\}$ forms basis for H.

d. $\dim(H) = 2$

Question # 3: **Consider**

$$H = \left\{ \begin{bmatrix} r \\ s \\ t \end{bmatrix} ; \begin{matrix} 3r - 2 = 3s + t \\ 3r - 2s = t \end{matrix} \right\}$$

a) **Find a vector that is in H.**
b) **Find geometry of H.**
c) **Find Basis for H.**
d) **Find dimension of H.**

Question # 03

$$H = \left\{ \begin{bmatrix} r \\ s \\ t \end{bmatrix} ; \quad \begin{matrix} 3r - 2 = 3s + t \\ 3r - 2s = t \end{matrix} \right\}.$$

$$3r - 3s - t = 2$$
$$3r - 2s - t = 0$$

$$\left[\begin{matrix} 3 & -3 & -1 & 2 \\ 3 & -2 & -1 & 0 \end{matrix}\right].$$

$$\left[\begin{matrix} 3 & -3 & -1 & 2 \\ 0 & 1 & 0 & -2 \end{matrix}\right];$$

$$3r - 3s - t = 2$$
$$s = -2$$

$$3r + 6 - t = 2$$
$$3r - t = -4$$
$$3r = t - 4$$
$$r = \frac{1}{3}t - \frac{4}{3}$$

$$\begin{bmatrix} r \\ s \\ t \end{bmatrix} = \begin{bmatrix} \frac{1}{3}t - \frac{4}{3} \\ -2 \\ t \end{bmatrix} = t \begin{bmatrix} 1/3 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} -4/3 \\ -2 \\ 0 \end{bmatrix}.$$

a) $t = 1$, $\begin{bmatrix} -1 \\ -2 \\ 1 \end{bmatrix} \in H$

b) Geometry of H = Line in $\mathbb{R}^3$.

c) $\left\{ \begin{bmatrix} -1 \\ -2 \\ 1 \end{bmatrix} \right\}$ forms basis for H

d) $\dim(H) = 1$

Question # 4:

Show that the following set forms the basis for R^3

$$\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$$

Q # 04

Show that the following set forms basis for $\mathbb{R}^3$.

$$\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$$

Sol

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}.$$

$$\sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

$$\text{Span}\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\} = \mathbb{R}^3.$$

given Vectors are Linearly independent

so given Vectors forms basis for $\mathbb{R}^3$.

Question # 5:

Determine the basis for the following plane in R^3

$$X - 2y + 5z = 0$$

Q # 05

Determine the basis for the following plane in $\mathbb{R}^3$.

$$x - 2y + 5z = 0$$

Sol

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2y - 5z \\ 1y + 0z \\ 0y + 1z \end{bmatrix} = y\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} + z\begin{bmatrix} -5 \\ 0 \\ 1 \end{bmatrix}.$$

$\left\{ \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -5 \\ 0 \\ 1 \end{bmatrix} \right\}$ forms basis for given plane.

Question # 6:

Let H be the set of all vectors in R^4 whose coordinates satisfy the equations

$$A - 2b + 5c = d \text{ and } c - a = b.$$

a. Show that H is a subspace of R^4.
b. Find all vectors in H.
c. Find basis for H.
d. Find dimension of H.

Q.4

$$H = \left\{ \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} ; \quad \begin{array}{r} a-2b+5c-d=0 \\ -a+c-b=0 \end{array} \right\}.$$

$$\left.\begin{array}{r} a-2b+5c-d=0 \\ -a-b+c+0d=0 \end{array}\right\}.$$

$$\begin{bmatrix} 1 & -2 & 5 & -1 & 0 \\ -1 & -1 & 1 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} \boxed{1} & -2 & 5 & -1 & 0 \\ 0 & \boxed{-3} & 6 & -1 & 0 \end{bmatrix} \quad R_2+R_1$$

$$\left.\begin{array}{r} a-2b+5c-d=0 \\ -3b+6c-d=0 \end{array}\right\}.$$

$$\left.\begin{array}{rl} a-2b &= -5c+d \\ -3b &= -6c+d \end{array}\right\}. \quad \begin{array}{l} \longrightarrow (1) \\ \longrightarrow (2) \end{array}$$

from (2)

$$b = -\frac{6}{-3}c + \frac{-1}{3}d$$

$$\boxed{b = 2c - \frac{1}{3}d}$$

(1) $\Rightarrow$

$$a-2\left[2c-\frac{1}{3}d\right] = -5c+d.$$

$$a-4c+\frac{2}{3}d = -5c+d.$$

$$a = -5c + 4c + d - \frac{2}{3}d$$

$$\boxed{a = -c + \frac{1}{3}d}$$

$$\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} -c + \frac{1}{3}d \\ 2c - \frac{1}{3}d \\ c \\ d \end{bmatrix} = c\begin{bmatrix} -1 \\ 2 \\ 1 \\ 0 \end{bmatrix} + d\begin{bmatrix} 1/3 \\ -1/3 \\ 0 \\ 1 \end{bmatrix}$$

So H is plane in $\mathbb{R}^4$.

$\boxed{a}$ Show that H is subspace of $\mathbb{R}^4$.

$\boxed{\text{S.1}}$

i. $0 \in H$.

$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \in H$.

ii. let $\begin{bmatrix} -1 \\ 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1/3 \\ -1/3 \\ 0 \\ 1 \end{bmatrix} \in H$.

$$\begin{bmatrix} -1 \\ 2 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 1/3 \\ -1/3 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -2/3 \\ 5/3 \\ 1 \\ 1 \end{bmatrix} \in H.$$

iii. All multiples of $\begin{bmatrix} -1 \\ 2 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 1/3 \\ -1/3 \\ 0 \\ 1 \end{bmatrix}$ are in H

So H is a subspace of $\mathbb{R}^4$.

$\boxed{b}$ Find all vectors in H.

$\boxed{Sol}$ $\begin{bmatrix} -1/2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1/3 \\ -1/3 \\ 0 \\ 1 \end{bmatrix}$ are in H

& all multiples of these vectors in H.

$\boxed{c}$ $\left\{ \begin{bmatrix} -1 \\ 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1/3 \\ -1/3 \\ 0 \\ 1 \end{bmatrix} \right\}$ forms basis for

$\boxed{d}$ $\dim(H) = 2$.

Question # 7:

$$\text{Let } A = \begin{bmatrix} 1 & 2 & 3 & -2 \\ 2 & 1 & 3 & -1 \\ 3 & 4 & 7 & -4 \end{bmatrix}$$

a) Find all vectors in Null (A).
b) Find geometry of Null (A).
c) Find basis for Null (A).
d) Find dimension of Null (A).
e) Find all vectors in Col (A).
f) Find geometry of Col (A).
g) Find basis for Col (A).
h) Find dimension of Col (A).
i) Is $\begin{bmatrix} 1 \\ 3 \\ 2 \\ 3 \end{bmatrix}$ in Null (A)?
j) Is $\begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}$ in Col (A)?

a) Find all Vectors in Null (A) ?

$Ax = 0$

$$\begin{bmatrix} 1 & 2 & 3 & -2 \\ 2 & 1 & 3 & -1 \\ 3 & 4 & 7 & -4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left.\begin{aligned} x_1 + 2x_2 + 3x_3 - 2x_4 &= 0 \\ 2x_1 + x_2 + 3x_3 - x_4 &= 0 \\ 3x_1 + 4x_2 + 7x_3 - 4x_4 &= 0 \end{aligned}\right\}.$$

$$\begin{pmatrix} 1 & 2 & 3 & -2 & 0 \\ 2 & 1 & 3 & -1 & 0 \\ 3 & 4 & 7 & -4 & 0 \end{pmatrix}$$

$$\sim \begin{pmatrix} \boxed{1} & 2 & 3 & -2 & 0 \\ 0 & \boxed{1} & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

x_1 & x_2 are basic, x_3 & x_4 are free.

$$\left.\begin{aligned} x_1 + 2x_2 + 3x_3 - 2x_4 &= 0 \\ x_2 + x_3 - x_4 &= 0 \end{aligned}\right\}.$$

$$\left.\begin{aligned} x_1 + 2x_2 &= -3x_3 + 2x_4 \\ \boxed{x_2 = -x_3 + x_4} \end{aligned}\right\}.$$

$$x_1 + 2\left(-x_3 + x_4\right) = -3x_3 + 2x_4$$

$$x_1 - 2x_3 + \cancel{2x_4} = -3x_3 + \cancel{2x_4}$$

$$\boxed{x_1 = -x_3}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -x_3 \\ -x_3 + x_4 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -x_3 + 0x_4 \\ -x_3 + 1x_4 \\ 1x_3 + 0x_4 \\ 0x_3 + 1x_4 \end{bmatrix} = x_3\begin{bmatrix} -1 \\ -1 \\ 1 \\ 0 \end{bmatrix} + x_4\begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}.$$

a $\begin{bmatrix} -1 \\ -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}$ are in Null(A).

& all multiples of these vectors are in Null (A).

b Null (A) is a plane in $\mathbb{R}^4$.

c $\left\{\begin{bmatrix} -1 \\ -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}\right\}$ forms basis for Null(A).

d ~~for Col(A)~~. dim(Null A) = 2 ..