# MT1004 Linear Algebra

Thursday, October 21, 2021

**Course Instructors**

Mr. Amjad Hussain

Ms. Ghilmana Sarmad

Serial No:

**$1^{st}$ Mid Term Exam**

**Total Time: 1 Hour**

**Total Marks: 40**

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Signature of Invigilator

| Roll No | Section | Signature |
|---|---|---|
| | | |

**DO NOT OPEN THE QUESTION BOOK OR START UNTIL INSTRUCTED.**

**Instructions:**

1. Verify at the start of the exam that you have a total of two (2) questions printed on six (6) pages including this title page.
2. Attempt all questions on the question-book and in the given order.
3. The exam is closed books, closed notes. Please see that the area in your threshold is free of any material classified as 'useful in the paper' or else there may a charge of cheating.
4. Read the questions carefully for clarity of context and understanding of meaning and make assumptions wherever required, for neither the invigilator will address your queries, nor the teacher/examiner will come to the examination hall for any assistance.
5. Fit in all your answers in the provided space. You may use extra space on the last page if required. If you do so, clearly mark question/part number on that page to avoid confusion.
6. Use only your own stationery and calculator. If you do not have your own calculator, use manual calculations.
7. Use only permanent ink-pens. Only the questions attempted with permanent ink-pens will be considered. Any part of paper done in lead pencil cannot be claimed for checking/rechecking.

| | Q-1 | Q-2 | Total |
|---|---|---|---|
| **Total Marks** | 16 | 24 | **40** |
| **Marks Obtained** | | | |

**Vetted By:** ____________________ **Vetter Signature:** ____________________

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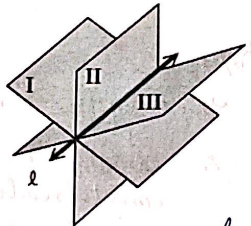
| | | | |
|---|---|---|---|
| Q#1 | | Points | 2+2+3+4+5=16 |

a) I, II and III are three planes corresponding to some system of linear equations as shown in the figure. $l$ is a line passing through the given planes. Identify either the given system of three planes has a unique solution, no solution or infinitely many solutions.




Infinite many solutions because of the infinite points on line $l$.

b) Find three vector that lies on the plane $x + 2y - 3z = 0$.

$x$ is the B.V and $y, z$ are f. variables

$x = -2y + 3z$
$y$ is free
$z$ is free

$$\Rightarrow \underline{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2y+3z \\ y \\ z \end{bmatrix}$$

$$= \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} y + \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix} z$$

$$\underline{x} = \underline{t}y + \underline{r}z$$

$$\underline{x} = t\underline{v} + r\underline{u} \quad , v, u \in$$

c) Let $w = \begin{bmatrix} -1 \\ 10 \end{bmatrix}$ and $S = \left\{ \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ -2 \end{bmatrix} \right\}$. Can $w$ be written as a linear combination of vectors in $S$? If yes, find that linear combination? Also write a vector that has no linear combination with the vectors in set $S$, if exist.

Yes, The vectors in $S$ are non-parallel vectors in $\mathbb{R}^2$. OR

$$\left[\begin{array}{cc|c} 2 & 3 & -1 \\ 1 & -2 & 10 \end{array}\right] \quad R_1 \leftrightarrow R_2$$

$$\sim \left[\begin{array}{cc|c} 1 & -2 & 10 \\ 2 & 3 & -1 \end{array}\right] \quad R_2 - 2R_1$$

$$\sim \left[\begin{array}{cc|c} 1 & -2 & 10 \\ 0 & 7 & -21 \end{array}\right] \quad R_2/7$$

The system is consistent

$\therefore$ $\underline{w}$ is a linear combination of vectors in $S$.

$$\sim \left[\begin{array}{cc|c} 1 & -2 & 10 \\ 0 & 1 & -3 \end{array}\right]$$

$$x_1 - 2x_2 = 10$$

$$\boxed{x_2 = -3}$$

$$\Rightarrow \boxed{x_1 = 4}$$

<u>Hence</u>

$$x_1 \begin{bmatrix} 2 \\ 1 \end{bmatrix} + x_2 \begin{bmatrix} 3 \\ -2 \end{bmatrix} = \begin{bmatrix} -1 \\ 10 \end{bmatrix}$$

$$4 \begin{bmatrix} 2 \\ 1 \end{bmatrix} - 3 \begin{bmatrix} 3 \\ -2 \end{bmatrix} = \begin{bmatrix} -1 \\ 10 \end{bmatrix}$$

is the required linear combination of $\underline{w}$.

(b) Since every row of coefficient matrix has a pivot, therefore, such vector will never exists that can't be written as linear combination of vectors of $S$.

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d) For what valu

d) For what values of $r$ and $s$, the following system of linear equations is inconsistent?

$$x_1 + 3x_2 = 1 + s$$
$$x_1 + rx_2 = 5$$

$$\left[\begin{array}{cc|c} 1 & 3 & 1+s \\ 1 & r & 5 \end{array}\right] R_2 - R_1$$

$$\sim \left[\begin{array}{cc|c} 1 & 3 & 1+s \\ 0 & r-3 & 4-s \end{array}\right]$$

The system will be inconsistent if $r-3=0$ and $4-s \neq 0$.

i.e $r=3$ and $s \neq 4$.

e) Determine if the set of vectors $\{u, v, w\}$ is linearly independent, where $a, b, c$ are non-zero real numbers

Where $u = \begin{bmatrix} a \\ b \\ c \end{bmatrix}, v = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, w = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

Augmented Matrix corresponding to $A\underline{x} = \underline{0}$

$$\left[\begin{array}{ccc|c} a & 0 & 1 & 0 \\ b & 1 & 1 & 0 \\ c & 0 & 0 & 0 \end{array}\right] R_1 \leftrightarrow R_3$$

$$\sim \left[\begin{array}{ccc|c} c & 0 & 0 & 0 \\ b & 1 & 1 & 0 \\ a & 0 & 1 & 0 \end{array}\right] R_1/c$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ b & 1 & 1 & 0 \\ a & 0 & 1 & 0 \end{array}\right] \begin{array}{l} R_2 - bR_1 \\ R_3 - aR_1 \end{array}$$

$$\sim \begin{bmatrix} \boxed{1} & 0 & 0 & | & 0 \\ 0 & \boxed{1} & 1 & | & 0 \\ 0 & 0 & \boxed{1} & | & 0 \end{bmatrix}$$

$$x_1 = 0$$
$$x_2 + x_3 = 0$$
$$x_3 = 0$$

$$\Rightarrow \quad x_1 = 0, \quad x_2 = 0, \quad x_3 = 0$$

trivial solution

$\Rightarrow$ vectors are linearly independent.

OR

$$\begin{bmatrix} a & 0 & 1 \\ b & 1 & 1 \\ c & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$a x_1 + x_3 = 0$$
$$b x_1 + x_2 + x_3 = 0$$
$$c x_1 = 0 \qquad \text{since } c \neq 0$$

$$\Rightarrow \boxed{x_1 = 0}$$

$$\Rightarrow \boxed{x_3 = 0}$$

$$\Rightarrow \boxed{x_2 = 0}$$

trivial solution

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| Q#2 | Points | 2+8+3+3+3+2+3=24 |
|---|---|---|

Consider the following system of equations.

$$\begin{aligned} x_1 - x_2 - 3x_3 + x_4 - x_5 &= 0 \\ -2x_1 + 2x_2 + 6x_3 - 6x_5 &= 0 \\ 3x_1 - 2x_2 - 8x_3 + 3x_4 - 5x_5 &= 0 \end{aligned}$$

a) Write the above system of linear equations in the form of $AX = b$.

$$\begin{bmatrix} 1 & -1 & -3 & 1 & -1 \\ -2 & 2 & 6 & 0 & -6 \\ 3 & -2 & -8 & 3 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$A \quad \underline{X} = \underline{b}.$$

b) Find the general solution of the above equation

$$\left[\begin{array}{ccccc|c} 1 & -1 & -3 & 1 & -1 & 0 \\ -2 & 2 & 6 & 0 & -6 & 0 \\ 3 & -2 & -8 & 3 & -5 & 0 \end{array}\right] \begin{array}{l} R_2 + 2R_1 \\ R_3 - 3R_1 \end{array}$$

$$\sim \left[\begin{array}{ccccc|c} 1 & -1 & -3 & 1 & -1 & 0 \\ 0 & 0 & 0 & 2 & -8 & 0 \\ 0 & 1 & 1 & 0 & -2 & 0 \end{array}\right] \quad R_2 \leftrightarrow R_3$$

$$\sim \left[\begin{array}{ccccc|c} 1 & -1 & -3 & 1 & -1 & 0 \\ 0 & 1 & 1 & 0 & -2 & 0 \\ 0 & 0 & 0 & 2 & -8 & 0 \end{array}\right] \quad \frac{R_3}{2}$$

$$\sim \left[\begin{array}{ccccc|c} \boxed{1} & -1 & -3 & 1 & -1 & 0 \\ 0 & \boxed{1} & 1 & 0 & -2 & 0 \\ 0 & 0 & 0 & \boxed{1} & -4 & 0 \end{array}\right] \Rightarrow \begin{array}{l} x_1 - x_2 - 3x_3 + x_4 - x_5 = 0 \\ x_2 + x_3 - 2x_5 = 0 \\ x_4 - 4x_5 = \end{array}$$

$x_3, x_5$ are free.

$x_1, x_2, x_4$ are B.V

$x_3, x_5$ are f.v.



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general solution

$\Rightarrow x_1 = x_2 + 3x_3 - x_4 + x_5$

$x_2 = -x_3 + 2x_5$

$x_3$ is free

$x_4 = 4x_5$

$x_5$ is free.

OR $x_1 = -x_3 + 2x_5 + 3x_3 - 4x_5 + x_5$

$x_1 = 2x_3 - x_5$

$x_2 = -x_3 + 2x_5$

$x_3$ is free

$x_4 = 4x_5$

$x_5$ is free

$$\underline{X} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 2x_3 - x_5 \\ -x_3 + 2x_5 \\ x_3 \\ 4x_5 \\ x_5 \end{bmatrix} = x_3 \begin{bmatrix} 2 \\ -1 \\ 1 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -1 \\ 2 \\ 0 \\ 4 \\ 1 \end{bmatrix} x_5$$

c) Write its corresponding parametric general solution.

$$\underline{X} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 2x_3 - x_5 \\ -x_3 + 2x_5 \\ x_3 \\ 4x_5 \\ x_5 \end{bmatrix} = x_3 \begin{bmatrix} 2 \\ -1 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} -1 \\ 2 \\ 0 \\ 4 \\ 1 \end{bmatrix}$$

$$\underline{X} = t\underline{u} + r\underline{v} \quad , \quad t, v \in \mathbb{R}$$

d) Discuss the *Span* of the columns of matrix *A*?

Every row has a pivot, span {A} is a $\mathbb{R}^3$ itself.

e) Discuss the *Span*{*X*}? (Give geometric description of solution).

Since $\underline{u}$ and $\underline{v}$ are not scalar multiples of each other, therefore

Also.

$$\begin{bmatrix} 2 & -1 \\ -1 & 2 \\ 1 & 0 \\ 0 & 4 \\ 0 & 1 \end{bmatrix} \sim \begin{bmatrix} \boxed{1} & 0 \\ 0 & \boxed{2} \\ 0 & -1 \\ 0 & 4 \\ 0 & 1 \end{bmatrix}$$

2 pivots. in $\mathbb{R}^5$

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Span $\{\underline{x}\}$ = plane in $\mathbb{R}^5$.

f) Is *Span* obtained in "c" and "d" are same?

NO.

g) Write the linear dependence relation among the columns of matrix *A*.

Let $x_3 = 1$, $x_5 = 1$

$x_1 = 2(1) - 1 = 1$

$x_2 = -1 + 2 = 1$

$x_3 = 1$

$x_4 = 4$

$x_5 = 1$

Linear dependence Relation:

$c_1\underline{v_1} + c_2\underline{v_2} + c_3\underline{v_3} + c_4\underline{v_4} + c_5\underline{v_5} = 0$.

$\underline{v_1} + \underline{v_2} + \underline{v_3} + 4\underline{v_4} + \underline{v_5} = 0$

one of many (infinite) Linear dependence relations.