National University of Computer and Emerging Sciences, Chiniot-Faisalabad Campus

	Course Name:	**Discrete Structures**	**Course Code:**	**CS1005**
	Degree Program:	**BS(CS) | BS(SE) | BS(AI)**	**Semester:**	**Spring-2024**
	Exam Duration:	**60 Minutes**	**Total Marks:**	**50**
	Paper Date:	**Saturday, February 24, 2024**	**No of Page(s):**	**5**
	Sections:	**ALL**		
	Exam Term & Type:	**1st Sessional I Closed Book**	**Required Answer Book: NO**	
Course Instructor	**Dr. Muhammad Ahmad and Mr. Ali Hamza**			

Student Name:_______________________ **Roll No.**_______________ **Section:**_______ **Invigilator's Signature**

Q.Part#	Q.1 (Objective)	Q.2	Q.3	Q.4	Total	Examiner Signature and Date
Total Marks	**10**	**15**	**10**	**15**	**50**	
Obtained Marks						

Instruction/Notes:	Attempt all questions. Programmable calculators are not allowed.	
Vetted By		**Vetter Signature:**
Declaration by course instructor	The question paper has an 100% dissimilarity as compared to the question papers of the same subject from the last two years.	

(Objective Part)

QUESTION 1: CLO-3

Answer the following short questions. Partially cooked answers will not be considered. **[2*5 = 10 points]**

1. If the compound statement $A \rightarrow (A \rightarrow B)$ is false, then the truth values of A and B are,
   a) T, T b) F, T c) T, F d) F, F

2. Which of the following statements is correct?
   a) $p \lor q \equiv q \lor p$ b) $\neg(p \land q) \equiv \neg p \lor \neg q$
   c) $(p \lor q) \lor r \equiv p \lor (q \lor r)$ d) All of mentioned

3. $p \leftrightarrow q$ is logically equivalent to
   a) $(p \rightarrow q) \rightarrow (q \rightarrow p)$ b) $(p \rightarrow q) \lor (q \rightarrow p)$
   c) $(p \rightarrow q) \land (q \rightarrow p)$ d) $(p \land q) \rightarrow (q \land p)$

4. $(p \rightarrow r) \lor (q \rightarrow r)$ is logically equivalent to
   a) $(p \land q) \lor r$ b) $(p \lor q) \rightarrow r$
   c) $(p \land q) \rightarrow r$ d) $(p \rightarrow q) \rightarrow r$

5. $\neg (p \leftrightarrow q)$ is logically equivalent to
   a) $q \leftrightarrow p$ b) $p \leftrightarrow \neg q$
   c) $\neg p \leftrightarrow \neg q$ d) $\neg q \leftrightarrow \neg p$

(Subjective Part)

Question 2: CLO-3

Convert following English statement into expression and prove that this expression is tautology or not by using truth table. “If the sun is shining, then either both the temperature is above 25°C and people are wearing sunglasses, or the temperature is not above 25°C”. **[15]**

$$p \rightarrow ((q \wedge r) \vee \neg q)$$

p	q	r	$(q \wedge r)$	$\neg q$	$(q \wedge r) \vee \neg q)$	$p \rightarrow ((q \wedge r) \vee \neg q)$
T	T	T	T	F	T	T
T	T	F	F	F	F	F
T	F	T	F	T	T	T
T	F	F	F	T	T	T
F	T	T	T	F	T	T
F	T	F	F	F	F	T
F	F	T	F	T	T	T
F	F	F	F	T	T	T

This is not tautology

Question 3: CLO-3

Are the statements $p \rightarrow (q \vee r)$ and $(p \rightarrow q) \vee (p \rightarrow r)$ logically equivalent? Prove with help of law of logic and start from $p \rightarrow (q \vee r)$ statement. **[10]**

3. Are the statements $p \rightarrow (q \vee r)$ and $(p \rightarrow q) \vee (p \rightarrow r)$ logically equivalent?

▾ Solution.

Let’s start with the left-hand side and work towards the right to find out.

$$\begin{aligned}
p \rightarrow (q \vee r) &\equiv \neg p \vee (q \vee r) && \text{implication}\\
&\equiv \neg p \vee q \vee r && \text{associative; drop parens}\\
&\equiv \neg p \vee \neg p \vee q \vee r && \text{idempotent}\\
&\equiv \neg p \vee q \vee \neg p \vee r && \text{communitive}\\
&\equiv (\neg p \vee q) \vee (\neg p \vee r) && \text{associative}\\
&\equiv (p \rightarrow q) \vee (p \rightarrow r) && \text{implication}
\end{aligned}$$

Question 4: CLO-2

For each of these arguments, explain which rules of inference are used for each step. **[15]**

a) "Ali, a student in this class, knows how to write programs in Python. Everyone who knows how to write programs in Python can get a high-paying job. Therefore, someone in this class can get a high-paying job". **[5]**

$C(x)$ = "x is in this class"
$J(x)$ = "x knows how to write programs in JAVA"
$H(x)$ = "x can get a high paying job"

Premise 1	$C(\text{Doug})$
Premise 2	$J(\text{Doug})$
Premise 3	$\forall x(J(x) \to H(x))$
Conclude	$\exists x(C(x) \land H(x))$

Step		**Reason**
1	$\forall x(J(x) \to H(x))$	Premise 3
2	$J(\text{Doug}) \to H(\text{Doug})$	Universal Instantiation from (1)
3	$J(\text{Doug})$	Premise 2
4	$H(\text{Doug})$	Modus Ponens from (2) and (3)
5	$C(\text{Doug})$	Premise 1
6	$C(\text{Doug}) \land H(\text{Doug})$	Conjunction from (4) and (5)
$\therefore$	$\exists x(C(x) \land H(x))$	Existential generalization from (6)

b) "Each of the 56 students in this class owns a personal laptop. Everyone who owns a personal laptop can use a word processing program. Therefore, Ali a student in this class, can use a word processing program". **[5]**

"Each of the 93 students in this class owns a personal computer. Everyone who owns a personal computer can use a word processing program. Therefore, Zeke, a student in this class, can use a word processing program."

$C(x)$ = "x is in this class"
$P(x)$ = "x owns a personal computer"
$W(x)$ = "x can use a word processing program"

Premise 1	$\forall x(C(x) \to P(x))$
Premise 2	$\forall x(P(x) \to W(x))$
Premise 3	$C(\text{Zeke})$
Conclude	$W(\text{Zeke})$

Step		**Reason**
1	$\forall x(C(x) \to P(x))$	Premise 1
2	$C(\text{Zeke}) \to P(\text{Zeke})$	Universal Instantiation on (1)
3	$C(\text{Zeke})$	Premise 3
4	$P(\text{Zeke})$	Modus Ponens on (2) and (3)
5	$\forall x(P(x) \to W(x))$	Premise 2

c) "Everyone in New Jersey lives within 50 miles of the ocean. Someone in New Jersey has never seen the ocean. Therefore, someone who lives within 50 miles of the ocean has never seen the ocean". **[5]**

"Everyone in New Jersey lives within 50 miles of the ocean. Someone in New Jersey has never seen the ocean. Therefore, someone who lives within 50 miles of the ocean has never seen the ocean."

$J(x)$ = "x lives in New Jersey"
$O(x)$ = "x lives within 50 miles of the ocean"

$S(x)$ = "x has seen the ocean"

Premise 1	$\forall x(J(x) \rightarrow O(x))$
Premise 2	$\exists x(J(x) \wedge \neg S(x))$
Conclude	$\exists x(O(x) \wedge \neg S(x))$

Step		**Reason**
1	$\exists x(J(x) \wedge \neg S(x))$	Premise 2
2	$J(y) \wedge \neg S(y)$	Existential Instantiation on (1) (y is an element of the domain)
3	$J(y)$	Simplification on (2)
4	$\forall x(J(x) \rightarrow O(x))$	Premise 1
5	$J(y) \rightarrow O(y)$	Universal Instantiation on (4)
6	$O(y)$	Modus Ponens on (3) and (5)
7	$\neg S(y)$	Simplification on (2)
8	$O(y) \wedge \neg S(y)$	Conjunction on (6) and (7)
$\therefore$	$\exists x(O(x) \wedge \neg S(x))$	Existential generalizing on (8)