Discrete Structures

Lecture # 1

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FAST -- National University of Computer and Emerging Sciences. CFD Campus

Material Collected by Prof. Dr. A.M. Khan, Slides Prepared by Mr. Hafiz Tayyeb Javed

About me

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Outline

- What is DS and when can it be used?
- Why study DS?
- Topics
- Grading
- Today's Lecture

Discrete Structures

- ❑ Study of $\underbrace{\text{Discrete}}_{\text{Consisting of Distinct Objects}}$ Objects

- ❑ Problems Solved Using Discrete Structures
  - How many ways are there to choose a valid password?
  - What’s the probability of winning a lottery?
  - How to encrypt a message?
  - What is the shortest path b/w two cities?
  - How to sort a list of integers?
  - How to prove that an algorithm works correctly?
  - .
  - .
  - .

Why Study Discrete Structures???

- ❑ Ability to understand and create mathematical arguments

- ❑ Gateway to more advanced courses

  - Algorithms
  - Database theory
  - Automata theory
  - Compiler theory
  - Computer security
  - Operating system

Topics we'll study

- Logic and Proofs
- Mathematical Induction
- Sequences and Recursion
- Set Theory
- Functions
- Relations
- Counting and Probability
- Graphs and Trees
- Machine Learning (Tentative)

Course organization

- **Class Schedule**
  - **Lecture # 1:** Check the Timetable ☺
  - **Lecture # 2:** Check the Timetable ☺

- Class will be conducted using Slides

- **Text Book**
  - Susana Epp, ***Discrete Mathematics with Applications***, (4th Edition)
  - Kenneth H. Rosen, ***Discrete Mathematics and Its Applications***, 4th Edition

Grading

- Two One Hour Mid Tests (30%)
- Final Exam (40%)
- Assignments (04)/Quizzes (04) (16%)
- Project (10%)
- Class Participation (4%)

Other Info

- **Cooperation Policy**
- **Bonus Points**
- **Cheating Policy**

Policies

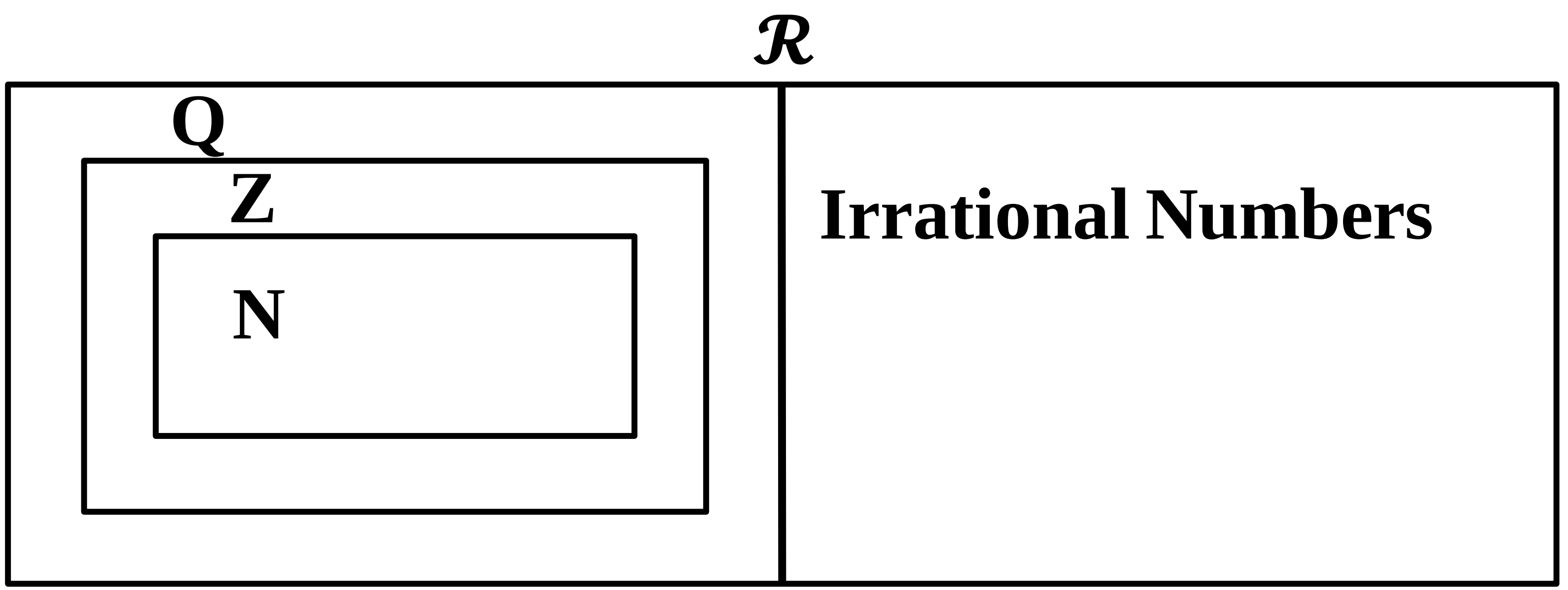
- No re-take of quizzes in any case except that the leave is officially approved by department
- Mid-term retakes are to be governed as per department policy
- Office hours will be announced in a couple of weeks. It is preferred to contact within office hours. Most of the problems can be dealt via email or in class.
- Cheating in one assignment= ZERO in two assignments
- Cheating in Quiz = ZERO in two quizzes
- Cheating in Project= ZERO in project
- Cheating in mid-terms and final exams are to be dealt as per department rules.

Today's Lecture

- **Integers:**
  - Arithmetic Properties
  - Powers
  - Divisibility
  - Primes of Composite Numbers
- **Rational Numbers**
  - Equivalent fractions
  - Operating with fractions
  - Decimals
- **Irrational Numbers**
- **Real Numbers**
  - Square roots
  - N-th roots
  - Logarithms
  - Inequalities
- **Oder of Operations**

Numbers

- **N**= {1,2,3, . . .} The set of Natural Numbers
- **Z**= {. . . , -2,-1,0,1,2, . . .} The set of Integers
- **Q**= {p/q | p ϵ Z, and q ≠ 0} The set of rational numbers
- $\mathcal{R}$, the set of real number. e.g. Real Space



Integers

❑ Simple rule of Addition

- For an integer a,
- $0+a = a+0=a$
- $a+(-a)=0$, and $(-a)+a=0$
- -a is the **additive inverse** of a.

We use "**Minus a**" rather than "**Negative a**"

Integers

- **Rules of Addition**
  - **Commutativity**
    - If a and b are integers, then
      - $a + b = b + a$
  - **Associativity**
    - If a , b and c are integers, then
    - $(a + b) + c = a + (b + c)$

Integers

Rules of Addition

- If $a + b = 0$, then $b = -a$ and $a = -b$

- Proof

$$a + b = 0$$

Add $-a$ to both sides

$$-a + a + b = 0 - a$$

$$0 + b = 0 - a$$

$$\mathbf{b = -a}$$

As desired.

Similarly we can find ---- $a = -b$

Integers

- **Rules of Addition**
  - If a , b are positive integers, then a + b is also positive integer.
  - If a , b are negative integers, then a + b is also negative integer.

  - If we have the relationship b/w three integers.
    - a + b = c

  Then we can drive other relationships b/w them.

  $$\mathbf{a = c - b} \qquad \mathbf{b = c - a}$$

- Example: Solve for x.

$$\mathbf{x + 3 = 5}$$

$$\mathbf{x = 5 - 3}$$

$$\mathbf{x = 2}$$

Integers

- **Rules of Addition**

  - Cancellation rule for addition

    - If $a + b = a + c$, then $b = c$

Exercise:

Prove that if $a + b = a$, then $b = 0$?

Integers

- **Rules of Multiplication**
  - **Commutativity**
    - If a and b are integers, then
      - $a * b = b * a$
  - **Associativity**
    - If a , b and c are integers, then
      - $(a * b) * c = a * (b * c)$
  - For any integer a
    - $1 * a = a$ and $0 * a = 0$

Integers

- **Rules of Multiplication**

  - **Distributivity**

    - **$a * (b + c) = a * b + a * c$**
    - **$(b + c) * a = b * a + c * a$**

Using all these properties

- **$-1 * a = -a$**
- **$-(a * b) = (-a) * (b)$ or $-(a * b) = a * (-b)$**
- **$(-a) * (-b) = a * b$**

Integers

- **Powers**
  - An exponent is used to indicate repeated multiplication.
  - Tells how many times the base is used as a factor.
    - $\mathbf{a * a = a^2}$
    - $\mathbf{a * a * a = a^3}$

In general if n is a positive integer,

- $\mathbf{a^n = a * a * a \dots a}$ (product is taken n times)

We say a^n is the n-th power of a.

If m , n are positive integers, then

- $\mathbf{a^{m+n} = a^m * a^n}$

Integers

- **Powers**

  - $(a^m)^n = a^{m * n}$

  Some important formulas

  - $(a + b)^2 = a^2 + b^2 + 2ab$
  - $(a - b)^2 = a^2 + b^2 - 2ab$
  - $(a + b)(a - b) = a^2 - b^2$

Integers

- **Even and Odd integers**

  - An even integer is an integer which can be written in the form **2n** for some integer **n**
    - $2 = 2 * 1$
    - $4 = 2 * 2$
    - $6 = 2 * 3$
  - An odd integer is an integer that differs from an even integer by 1.
  - It can be written in the form **2m ± 1** for some integer **m.**
    - $1 = (2 * 1) - 1$
    - $3 = (2 * 2) - 1$
    - $7 = (2 * 3) + 1$

Integers

- **Theorem**

  - Let a , b be integers,

    - If a is even and b is also even, then a + b is also even

    - If a is even and b is odd, then a + b is odd

    - If a is odd and b is even, then a + b is odd

    - If a is odd and b is also odd, then a + b is also even

- **Exercise.**

  - Let’s prove the Second statement ??

Integers

- **Divisibility**

  - Given two integers a and b, with $a \neq 0$, we say that **a divides b**, or that **b is divisible by a** if there is an integer c, such that **b = a * c**.

  - Remember that every integer is divisible by 1 because we can always write

    - $n = 1 * n$

- Also, every positive integer is divisible by itself.

Rational Numbers

- By a rational numbers, we mean a fraction as $\frac{m}{n}$,

  where m and n are integers, $n \neq 0$.

  - m is called numerator
  - n is called denominator

- Improper fraction

  - m larger than or equal to n

- Proper fraction

  - m smaller than n

Rational Numbers

❑ Equivalent Fractions

❑ Two fractions that represent the same value.

- $\frac{\mathbf{1}}{\mathbf{2}} = \frac{\mathbf{2}}{\mathbf{4}}$

How can we know whether two fractions are equivalent?

❑ Rule for cross-Multiplication

- Let m ,n ,r ,s be integers and assume that n $\neq 0$ and s $\neq 0$. Then

- $\left(\frac{\boldsymbol{m}}{\boldsymbol{n}}\right) = \left(\frac{\boldsymbol{r}}{\boldsymbol{s}}\right)$, iff m * s = r * n

Rational Numbers

❑ Simplifying Fractions

- ❑ We can simplify four special fractions forms
  - ❑ Fractions that have the same numerator and denominator.
    - $1 = \frac{1}{1} = \frac{2}{2} = \frac{3}{3} = \frac{4}{4} = \ ...$
  - ❑ Fractions that have a denominator of 1.
    - $\frac{5}{1} = 5,\ \frac{24}{1} = 24,\ \frac{-6}{1} = -6$
  - ❑ Fractions that have a numerator of 0.
    - $\frac{0}{8} = 0,\ \frac{0}{71} = 0,\ \frac{0}{-10} = 0$
  - ❑ Fractions that have a denominator of 0
    - $\frac{7}{0} = \infty,\ \frac{-17}{0} = \infty,$ (∞=Infinity=Undefined Value)

Rational Numbers

- **Simplifying Fractions**
  - Cancellation Rule for Fractions
    - Let a be a non-zero integer. Let m , n be integers, and n ≠ 0, then
      - $\frac{am}{an} = \frac{m}{n}$
    - Proof: By applying the rule for cross-multiplication and using the associativity and commutativity laws.

Rational Numbers

- **Simplifying Fractions**

  - A fraction is in **simplest form** when the numerator and denominator have no common factors (or divisors) other than 1.

  - Theorem:

    - "Any positive rational number has an expression as a fraction in the lowest form."

Rational Numbers

❑ Operating with Fractions

- ❑ Addition (or Subtraction) with same denominator.

  - $\frac{\mathrm{a}}{\mathrm{d}} + \frac{\mathrm{b}}{d} = \frac{\mathrm{a+b}}{\mathrm{d}}$ **<u>or</u>** $\frac{\mathrm{a}}{\mathrm{d}} - \frac{\mathrm{b}}{d} = \frac{\mathrm{a-b}}{\mathrm{d}}$

- ❑ With different denominator:

  - $\frac{\mathrm{m.}}{\mathrm{n.}} + \frac{\mathrm{r}}{s} = \frac{\mathrm{ms+rn}}{\mathrm{ns}}$ **<u>or</u>** $\frac{\mathrm{m.}}{\mathrm{n.}} + \frac{\mathrm{r}}{s} = \frac{\mathrm{ms+rn}}{\mathrm{ns}}$

- ❑ Follows the same basic rules as addition of integers (commutativity and association)

Rational Numbers

- **Multiplication:**

  - Let $a = \frac{\mathrm{m}}{\mathrm{n}}$

    - Then for any positive integer k, such that

    - $$a^k = \left(\frac{m}{n}\right)^k = \frac{m^k}{n^k}$$

  - Follows the same basic rules as multiplication of integers.

Rational Numbers

- **Division:**
  - If a is a rational number and $a \neq 0$, then there exists $(\exists)$ a rational number, denoted by
    - a^{-1} such that
    - $a^{-1} * a = a * a^{-1} = 1$
  - Note that if $a = \frac{m}{n}$ then $a^{-1} = \frac{n}{m}$
  - a^{-1} is called the multiplicative inverse of a.

Rational Numbers

Decimals:

- Finite decimals (and periodic) give us examples of rational numbers.

  - $1.4 = \frac{14}{10}$
  - $1.41 = \frac{141}{100}$
  - $0.2 = \frac{1}{5}$
  - $0.75 = \frac{3}{4}$

Irrational Numbers

- A number that cannot be expressed as fraction of $\frac{p}{q}$ for any integers p and q.

- Have decimal expressions that neither terminate nor become periodic

  - $\sqrt[2]{2} = 1.41421356237\ldots$
  - $\sqrt[2]{3} = 1.73205080757\ldots$
  - $\pi = 3.14159265359\ldots$
  - $\ldots$

Irrational Numbers

- Is $\sqrt[2]{25}$ an irrational number?
  - No!
  - Because $\sqrt[2]{25} = \pm 5$
- Is $\sqrt[2]{-1}$ an irrational number?
  - No or Yes??? In both cases HOW?

Real Numbers

- Integers, Rational and Irrational Numbers are part of a larger system.

- Real Numbers can be described as all the numbers that consist of a decimal expansion, possibly infinite.

Real Numbers

- **Properties of Real Numbers:**
  - Addition:
    - $a + b = b + a$
    - $a + (b + c) = (a + b) + c$
    - For all (∀) real numbers a, b, and c.
  - Multiplication
    - $a * b = b * a$
    - $a * (b * c) = (a * b) * c$
    - ∀ real numbers a, b , c.
  - Also
    - $a * (b + c) = a * b + a * c$
    - $(b + c) * a = b * a + c * a$

Real Numbers

- ❑ **Absolute Value**

- ❑ The non-negative values of a real number without regard to it sign.

- $|a| = a$ for a positive a.
- $|a| = -a$ for a negative a
  (in which case –a is positive).
- $|0| = 0$

Real Numbers

- **Square Roots**
  - If a > 0, then there exists (∃) a number b such that (s.t).
    - $b^2 = a$
- **N-th Roots**
  - There exists a unique real number r such that
    - $r^n = a$

      It is called the n-th root of a, and is denoted by
    - $a^{1/n}$ or $\sqrt[n]{a}$

Logarithms

- Can be seen as the reverse operation of the exponentiation.
- The logarithm of a number is the exponent to which another fixed value, the **Base** must be raised to produce that number.
    - $\log_{10}(10000) = 4$, because $10^4=10000$
    - $\log_2(16) = 4$, because $2^4=16$
    - $\log_3\left(\frac{1}{3}\right) = -1$, because $3^{-1}= \frac{1}{3}$

Logarithms

- **Properties of Logarithms:**
  - Product:
    - $\log_b(x * y) = \log_b(x) + \log_b(y)$
  - Quotient:
    - $\log_b\left(\frac{x}{y}\right) = \log_b(x) - \log_b(y)$
  - Power:
    - $\log_b(x^p) = p * \log_b(x)$
  - Change of Base:
    - $\log_b(x) = \frac{\log_k(x)}{\log_k(b)}$

Inequalities

Symbol	Meaning	Example
$>$	Greater Than	$(X + 3) > 2$, for any X
$<$	Less Than	$(7X) < 28$, $X= \{\ldots, -2, -1, 0 , 1 , 2 , 3\}$
$\geq$	Greater Than or Equal	$5 \geq (X - 1)$, $X= \{\ldots, -2, -1, 0, 1, \ldots, 5 ,6\}$
$\leq$	Less Than or Equal	$(2Y + 1) \leq 7$, $Y= \{\ldots, -2, -1, 0, 1, 2, 3\}$

❑ Let a, b, c be real numbers,

- If $a > b$ and $b > c$ then $a > c$. (Transitivity)
- If $a > b$ and $c > 0$ then $a*c > b*c$.
- If $a > b$ and $c < 0$ then $a*c < b*c$.