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$a \geq b$ means $b \leq a$

means ~~$b \leq a$~~

$R_1 = \{(1,1), (2,1), (1,-1), (2,2)\}$

$a \leq b$
$a \geq b$
$a = b$, $a = -b$
$a = b$
$a = b+1$
$a + b \leq 3$
$a = b+1$

$a < b$
$R_2 = \{(1,2)\}$

$R_3 = a = b$

$\{(1,1), (2,2)\}$

$R_4 = \{(1,1), (1,-1), (2,2)\}$

$R_5 = \{(1,2), \}$

$R_6 = \{1,1, (1,-1), (2,1) (1,2), \} (~~2,2~~)$

A relation is Reflexive if $(a,a) \in R$ for all a in the set of integer.

R_1 is Reflexive because $\forall a \in R$

$a \leq b$

$a < b$ $a = b$

where $b = a$

$\boxed{a = a}$

So R_1 is Reflexive

R_2 is not Reflexive beacuse $a < b$ means $a < a$ is never true.

R_3 is Reflexive because $a = a$ for all a.

R_4 Relexive because $a = a$ and $a = -a$ (only for 0) holds for all a.

R_5 is not Reflexsive beacus $a = b$ (not satisfies $a = a$

R_6 is reflexive $a + a \leq 3$ holds for $a = 1, 0, -1$ which are part of integers

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A ~~relation~~ is symmetric if $(a,b) \in R$ implies $(b,a) \in R$

R_1 = The relation is not symmetric because $(a,b) \in R$ for $a \geq b$ but for $(b,a) \in R$ $a \geq b$ does not hold.

Example $(2,1) \in R$ but $2 \geq 1$ but $(1,2) \in R$ ~~because~~ $1 \ngeq 2$

R_2 is not symmetric because $(a,b) \in R$ for $a < b$ then $(b,a) \in R$ $b < a$ does not hold. $1 < 2 \in R$ but $2 < 1$ does not hold.

R_3 is symmetric because $(a,b) \in R$ for $a = b$ and $(b,a) \in R$ for $a = b$ which is correct. $(1,1) \in R$

R_4 is symmetric because $(a,b) \in R$ when either $a = b$ or $a = -b$ but $(b,a) \in R$ also holds

example. $(1,-1) \in R$ and $(-(-1),1) \in R$

R_5 is not symmetric $(a,b) \in R$ when $a = b - 1$ but $(b,a) \in R$ then $b = a - 1$ which does not holds. $(1,2) \in R$ but $1 = 2 - 1$ $(2,1) \in R$.

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R_6 is symmetric as for $(a,b) \in R_6$
$a+b \leq 3$ then same $(b,a) \in R_6$ i.e.
for $b+a \leq 3$ which satisfies cond.
$(1,2) \in R$ $(2,1) \in R$.

A relation is anti symmetric if $(a,b) \in R$ and $(b,a) \in R$ ~~but~~ it implies that $a=b$.

1) R_1 is anti symmetric
$(a,b) \in R$ and, $(b,a) \in R$
$a \geq b$ which ~~holds~~ shows that $a=b$
Example $(1,1) \in R_1$ $1 \leq 1$ and $1=1$

$\boxed{2}$ R_2 is ~~not~~ anti-symmetric
as $(a,b) \in R$ and $(b,a) \in R$
doesn't not satisfies that $a=b$
and $a<b$ but $b \not< a$ so it is not anti symmetric

3) R_3 is anti-symmetric $(a,b) \in R$ and $(b,a) \in R$ where $a=b$ ~~that~~ holds for all condition for anti-symmetric

4) R_4 is not anti symmetric $(a,b) \in R$ and $(b,a) \in R$ but $a=b$ or $a=-b$
means $(1,-1) \in R$ and $(-1,1) \in R$
but $\boxed{1 \neq -1}$

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(5) R_5 is anti symmetric.

if $(a,b) \in R$ then $a = b-1$ which mean $(b,a) \in R$ then $b = a-1$ which is not possible

Example $((1,2) \in R, (2,1) \notin R)$

(6) R_6 is not anti symmetric

beause $(a,b) \in R$ for $a+b \le 3$ and $(b,a) \in R$ for $(\overset{b+a}{b+a}) \le 3$ which satifies for symmetric not for anti-symmetric.

Transitive

R_1 The relation is transitive if $(a,b) \in R_1$ $a \ge b$, $(b,c) \in R_1$ if $b \ge c$ then $a \ge c$ holds so it holds transitive property of inequality

R_2 R_2 is transitive

R_3 R_3 is transitive if $(a,b) \in R_3$ means $a = b$ $(b,c) \in R_3$ mean $b = c$ then $a = c$ which mean $(a,c) \in R$

R_4 R_4 is ~~not~~ transitive as $(a,b) \in R$ for $a = b$ ~~or $a = -b$~~ $(a,-b) \in R$ for $a = -b$

but

R_5 is not transitive as $(a,b) \in R$ if $a = b-1$ $(b,c) \in R$ which mean $b = c-1$ but $a = c-2$ mean $(a,c) \notin R_5$

R_6 - is not transitive $(a,b) \in R$ $a+b \leq 3$ and $(b,c) \in R$ $b+c \leq 3$ ~~but~~ $a+2b \not\leq 3$ or b $2a+b \not\leq 3$

for exampl

$(2,1) \in R_6$ $1+2 \leq 3$

$(1,2) \in R_6$

$(2,2) \notin R_6$

Q# 2

for equivalent relation a relation should by symmetric, reflexive, transitive

$(x,y) \in R$ $(x,x) \in R$ when $x \in Z$

$(x,y) : x-y$

$(x,x) = x-x$

$= 0$ is divisible by 6

(x,x) is reflexive

2) if $(x,y) \in R$ $(y,x) \in R$ where $(x,y) \in Z$

$= x-y$ is divisible by 6

$= -x+y = -(x-y)$ is divisible by 6

Symmetric

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Transitive if $(x,y) \in R$ $(y,z) \in R$ $\Rightarrow (x,z) \in R$

$x-y$ is divisible by 6

$y-z$ is divisible by 6

$x-y+y-z$ is divisible 6

$x-z$ is divisible by 6

$(x,z) \in R$ is transitive.

Question #4

i) $a_5 = 17$ and $a_9 = 37$

As we are given in Arithematic sequence

$$a_n = a_1 + (n-1)d$$

$a_5 = a_1 + 4d$	$a_9 = a_1 + 8d$
$17 = a_1 + 4d$ —(i)	$37 = a_1 + 8d$ —(ii)

subtracting (i) from (ii)

$$37 = \cancel{a_1} + 8d$$
$$-17 = \cancel{a_1} + 4d$$

$$20 = 4d$$

$$\boxed{d = 5}$$

~~$a_1 + 5 + 1$~~

putting $d = 5$ in (i)

$$17 = a_1 + 4(5)$$
$$17 = a_1 + 20$$
$$17 - 20 = a_1$$
$$\boxed{a_1 = -3}$$

$a_2 = a_1 + d$

$= -3 + 5$

$= 2$

$a_3 = a_2 + d$

$= 2 + 5 = 7$

ii) $3a_7 = 7a_4$ $\quad a_{10} = 33$

$a_7 = a_1 + 6d$

$a_4 = a_1 + 3d$

$3(a_1 + 6d) = 7(a_1 + 3d)$

$3a_1 + 18d = 7a_1 + 21d$

$3a_1 - 7a_1 = 21d - 18d$

$-4a_1 = 3d$

$\boxed{a_1 = -\frac{3d}{4}}$

$\boxed{d = 4}$

$a_1 = -\frac{3(4)}{4}$

$a_1 = -\frac{12}{4}$

$\boxed{a_1 = -3}$

$a_2 = a_1 + d$

$\boxed{a_2 = -3 + 4 = 1}$

$a_{10} = a_1 + 9d$

$\cancel{33 =}$

$33 = -\frac{3d}{4} + 9d$

$33 = \frac{-3d + 36d}{4}$

$33 = \frac{33d}{4}$

$4(33) = 33d$

$\boxed{d = 4}$

$a_3 = a_2 + d$

$= 1 + 4$

$\boxed{= 5}$

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Q# 5 $p(n) = 1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$

$$P(1) = \frac{1(1+1)(2(1)+1)}{6}$$

$$= \frac{(2)(2+1)}{6}$$

$$P(1) = \frac{2(3)}{6} = 1$$

NEXT for k

$$P(k) = \frac{k(k+1)(2k+1)}{6}$$

Next for $(k+1)$

$$P(k+1) = \frac{(k+1)(k+1+1)(2(k+1)+1)}{6}$$

$$= \frac{(k+1)(k+2)(2k+2+1)}{6}$$

$$= \frac{(k^2+2k+k+2)(2k+3)}{6}$$

$$= \frac{(k^2+3k+2)(2k+3)}{6}$$

$$= \frac{2k^3+3k^2+6k^2+6}{6}$$

$$= \frac{2k^3+9k^2+6\ 13k+6}{6}$$

$$= \frac{2k^3+9k^2+13k+6}{6}$$

$$P(k)+(k+1)^2 = \frac{k(k+1)(2k+1)}{6} + (k+1)^2$$

$$= \frac{2k^3+9k^2+13k+6}{6}$$

that's why we add 1

i) Out of 12 any one of them can 'win' so there are 12 different possibilites.

ii) once first place decided there are 11 remaining horses for second place

iii) After first and second there are 10 different way for third place

iv) so position first, second and third can be filled independently so total number of permutation is ${}^{n}P_{r}$

$\overline{1320}$

As we know a club has 25 members

a) we have to chose 4 fourl members of the club to serve on an executive commitee, so well will use combination

^nC_r

$n = 25 \quad r = 4$

$^{25}C_4$

$$\frac{25!}{4!(25-4)!} = \frac{25!}{4! \cdot 21!} = \frac{25 \cdot 24 \cdot 23 \cdot 22 \cdot \cancel{21!}}{4! \cdot \cancel{21!}}$$

$$= \frac{25 \cdot 24 \cdot 23 \cdot 22}{4 \cdot 3 \cdot 2 \cdot 1} = \frac{25 \cdot \cancel{24} \cdot 23 \cdot 22}{\cancel{24}} = \boxed{12650}$$

Hence the possible number of ways to chase in executive are 12,650.

b) As we have to choose in a proper way so we will use permutation

$^nP_r = \frac{n!}{(n-r)!}$ when $n = 25$ and $r = 4$

$$\frac{25!}{(25-4)!} = \frac{25 \cdot 24 \cdot 23 \cdot 22 \cdot \cancel{21!}}{\cancel{21!}} = 303600$$