Question 1:

a)

$R_1 = (1,1), (2,1), (1,-1), (2,2)$

$R_2 = (1,2)$

$R_3 = (1,1)\ (2,2)$

$R_4 = (1,1), (2,2), \cancel{(2,1)}, (1,-1)$

$R_5 = (1,2)$

$R_6 = (1,1), (1,2), (2,1), (1,-1)$

b) Reflexive

$R_1$ as $(a,b)$ where $a=b$ so $(a \geq a)$ i.e related to itself

$R_3$ $a=b$ so $a=a$

$R_4$ $a=-a$ for $a=0$

c) $R_3$ as $a=b$ and $b=a$ so it is symmetric

$R_4$ as if $a=-b$ or $a=b$ than $b=a$ or $b=-a$

d) $R_1$ if $a \geq b$ and $b \geq c$ than $a \geq c$

$R_3$ if $a=b$ and $b=c$ than $a=c$

$R_4$ if $a=b$ or $a=-b$ and $b=c$ or $b=-c$ than $a=c$ or $a=-c$

$R_2$ if $a<b$ and $b<c$ than $a<c$ so it is transitive

e) $R_1$ if $a \geq b$ and $b \geq a$ then $a = b$

$R_3$ if $a = b$ and $b = a$ than $a = b$

Q2 $R = \{(x,y) \mid x \in Z$ and $y \in Z, (x-y)$ is divisible by 6

Reflexive: for $x = y \rightarrow x - x = 0$ and 0 is divisible by 6 so reflexive

Symmetric:

$x - y = 6k$ for some integers if $-(x-y) = -6k$

$= y - x = -6k$ which is also divisble by 6 so symmetric

Transitive:

for $x - y = 6k$ there exists $y - z = 6m$ adding these we get

$(x+y) + (y-z) = 6k + 6m$

$(x - z) = 6(k+m)$

$\hookrightarrow$ is divisible by 6

$k+m$ is an integer so Transitive

Hence Proved.

Q3

a) when $x = -5$

Include all integers that are congruent to 4 modulus 5

(i) $[4]_{-5} = \{\ldots, -6, -1, 4, 9, 14\}$

$4 + (-5)k$

(ii) $[4]_9 = 4 + 9k$

$\{\ldots -5, 4, 9, 13, 22, 31\}$

b) $3 + 7k$

(i) $[3]_7 = \{\ldots, -11, -4, 3, 10, 17 \ldots\}$

(ii) $[6]_7 = \{\ldots -8, -1, 6, 13, 20 \ldots\}$

$6 + 7k$

c)

(i) $\sum_{k=m}^{n} a_k + 2 \cdot \sum_{k=m}^{n} b_k = \sum_{k=m}^{n} (k+1) + \sum_{k=m}^{n} (2k-2)$

$= \sum_{k=m}^{n} (3k-1)$

(ii) $\prod_{k=m}^{n} a_k \cdot \prod_{k=m}^{n} b_k = \prod_{k=m}^{n} (k+1) \cdot \prod_{k=m}^{n} (k-1)$

$= \prod_{k=m}^{n} (k^2-1)$

Q4 Q5

(i) $a_5 = 17$ and $a_9 = 37$

$$-a \cancel{+} - 4d = -17$$
$$a + 8d = 37$$
$$4d = 20$$
$$d = 5$$

$a = -3$

$a_1 = -3 + (5) = 2$

$a_2 = -3 + 2(5) = 7$

$$a + 4(5) = 17$$
$$a = 17 - 20$$
$$a = -3$$

(ii) $a + 9d = 33$

$$3a_7 = 7a_4$$
$$3(a + 6d) = 7(a + 3d)$$
$$3a + 18d = 7a + 21d$$
$$4a - 3d = 0$$
$$a + 9d = 33$$
$$-12a \cancel{+} - 9d = 0$$
$$11a = 33$$
$$\boxed{a = -3}$$
$$-3 + 9(d) = 33$$
$$\boxed{d = 4}$$

$a = -3$

$a + d = -3 + 4 = 1$

$a + 2(d) = -3 + (2.4) = 5$

Q5

$$1^2 + 2^2 + \ldots n^2 = (n(n+1)(2n+1))/6$$

a) $n = 1$

$$\frac{1(2)(3)}{6} = \frac{6}{6} = 1 \quad \text{True}$$

b) $$\frac{k(k+1)(2k+1)}{6}$$

c) $$\frac{(k+1)(k+2)\,2(k+1)+1}{6}$$

$$= \frac{(k+1)(k+2)(2k+3)}{6}$$

$$1^2 + 2^2 + \ldots k^2 + (k+1)^2 = \frac{(k+1)(k+2)(2k+3)}{6}$$

$$\frac{k((k+1)(2k+1) + 6(k+1)^2}{6} = \frac{(k+1)k(2k+1) + 6(k+1)}{6}$$

$$= \frac{(k+1)k(2k+1) + 6k + 6}{6} = \frac{(k+1)(2k^2 + 7k + 6)}{6}$$

$$= \frac{(k+1)(k+2)(2k+3)}{6}$$

Hence Proved

Q6

Pigeonhole Principle

State : Student

1 : 2

50 : $x$

$x = 100$

but one state must have more than 2 students coming to uni.

$50 \times (100 - 1) + 1 = 4951$ Students

Q7

$12 \times 11 \times 10 = 1320$ possible ways

Q8 (i)

$^{25}C_4 = 12650$ ways

(ii)

$^{25}P_4 = 303600$ ways

Q9

Base Case : $a \times n = 0$

where $n = 0$

Recursive :

$a + a \times (n-1)$

up until $n > 0$

10

$L(\lambda) = 0$ Length of empty string

for any string $x$

$L(xy) = L(x) + L(y)$

$L(x\lambda) = L(x) + L(\lambda)$

$= L(x) + 0$

$L(x) + L(\lambda) = L(x)$

hence the base case holds

Inductive:

$x \in \Sigma$

$L(xy) = L(x) + L(y)$

assume string $= (xa)y$ $\therefore$ $a$ is an additional character that adds in string

$L(xa)y = L(xa) + L(y)$

$L(xa) = L(x) + 1$

substituting

$L(xa)y = L(x) + 1 + L(y)$

$L(xa) + L(y) = L(x) + 1 + L(y)$

$L(x) + L(a) + L(y) = L(x) + L + L(y)$

$L(x) + 1 + L(y) = L(x) + 1 + L(y)$ $\therefore$ $L(xy) = L(x) + L(y)$ holds.