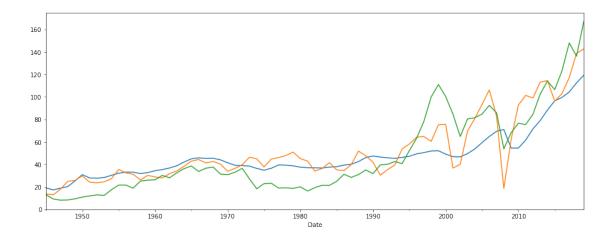
TimeSeries2021 VAR

March 24, 2021

```
[1]: %matplotlib inline
     import numpy as np
     import pandas as pd
     import matplotlib.pyplot as plt
[2]: plt.rc("figure", figsize=(16, 6))
[3]: div_url = "https://www.multpl.com/s-p-500-dividend/table/by-year"
     earn_url = "https://www.multpl.com/s-p-500-earnings/table/by-year"
     price_url = "https://www.multpl.com/inflation-adjusted-s-p-500/table/by-year"
[4]: divs = pd.read_html(div_url, header=0, index_col=0, parse_dates=True)[0]["Value__
     →Value"].sort_index().to_period("Y")["1945":"2019"].rename("divs")
     divs.head()
[4]: Date
     1945
              9.54
     1946
              8.69
     1947
              9.44
     1948
             10.15
             12.70
     1949
     Freq: A-DEC, Name: divs, dtype: float64
[5]: divs.tail()
[5]: Date
    2015
             48.25
     2016
             49.79
    2017
             52.21
     2018
             56.27
     2019
             59.61
     Freq: A-DEC, Name: divs, dtype: float64
[6]: earnings = pd.read_html(earn_url, header=0, index_col=0,_u
      →parse_dates=True)[0]["Value Value"].sort_index().to_period("Y")["1945":
      →"2019"].rename("earnings")
     earnings.head()
```

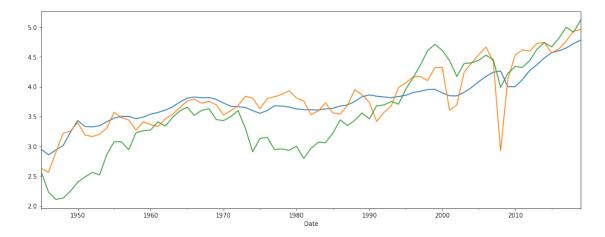
```
[6]: Date
      1945
              13.87
      1946
              12.97
      1947
              18.10
              24.99
      1948
      1949
              25.86
      Freq: A-DEC, Name: earnings, dtype: float64
[41]: earnings.tail()
[41]: Date
               96.22
      2015
      2016
              103.00
      2017
              117.23
      2018
              138.60
      2019
              142.75
     Freq: A-DEC, Name: earnings, dtype: float64
 [8]: prices = pd.read_html(price_url, header=0, index_col=0,__
       →parse_dates=True) [0] ["Value Value"] .sort_index().shift(-1,freq="D").
       →to_period("Y")["1945":"2019"].rename("prices")
      prices.head()
 [8]: Date
      1945
              260.41
      1946
              186.07
      1947
              164.58
      1948
              168.33
      1949
              188.92
      Freq: A-DEC, Name: prices, dtype: float64
[40]: prices.tail()
[40]: Date
      2015
              2129.95
      2016
              2464.14
      2017
              2960.28
              2724.46
      2018
      2019
              3342.29
     Freq: A-DEC, Name: prices, dtype: float64
[10]: plt.figure()
      (divs*2).plot()
      earnings.plot()
      (prices/20).plot().set_ylim(0,None)
[10]: (0, 175.058775)
```



```
[11]: logdivs = np.log(divs)
logearnings = np.log(earnings)
logprices = np.log(prices)
```

```
[12]: plt.figure()
    (logdivs+np.log(2)).plot()
    logearnings.plot()
    (logprices-np.log(20)).plot()
```

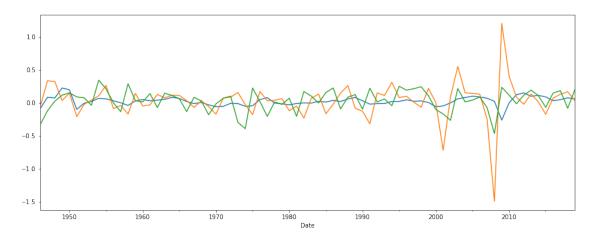
[12]: <matplotlib.axes._subplots.AxesSubplot at 0x12795a750>



```
[13]: divGrowth = logdivs.diff().dropna()
  earnGrowth = logearnings.diff().dropna()
  returns = logprices.diff().dropna().rename("returns")
```

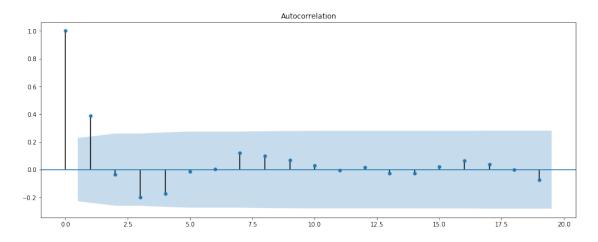
```
[14]: plt.figure()
    divGrowth.plot()
    earnGrowth.plot()
    returns.plot()
```

[14]: <matplotlib.axes._subplots.AxesSubplot at 0x127cb7a90>

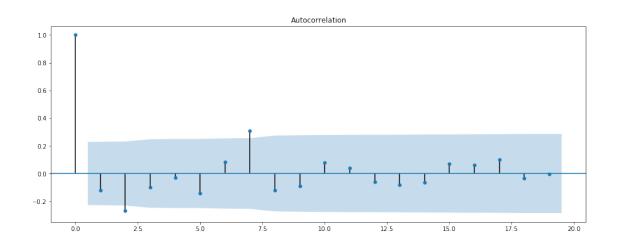


[15]: from statsmodels.graphics.tsaplots import plot_acf

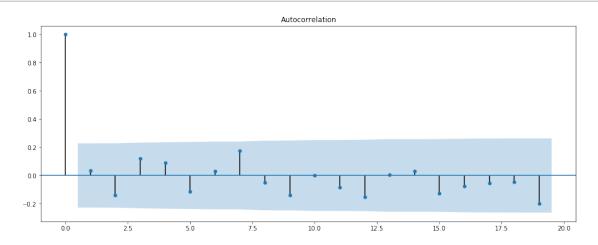
[16]: plot_acf(divGrowth);



[17]: plot_acf(earnGrowth);



[18]: plot_acf(returns);



[19]: df = pd.concat([divGrowth, earnGrowth, returns], axis=1) df.describe()

```
[19]:
                  divs
                          earnings
                                       returns
                         74.000000
             74.000000
                                    74.000000
      count
      mean
              0.024761
                          0.031505
                                     0.034489
              0.067610
                          0.286360
                                     0.162472
      std
      min
             -0.263294
                         -1.493037
                                    -0.465843
      25%
             -0.010887
                         -0.054911
                                    -0.062923
      50%
              0.022310
                          0.049897
                                     0.064437
      75%
              0.064197
                          0.131911
                                     0.145565
              0.224128
      max
                          1.204188
                                     0.342731
```

[20]: df.corr()

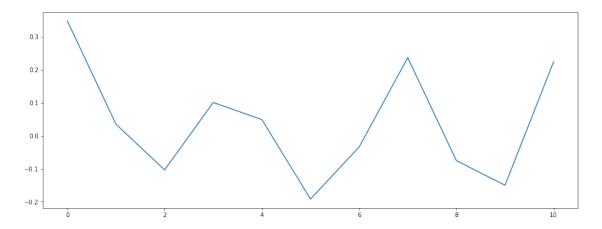
```
[20]: divs earnings returns divs 1.000000 -0.049263 0.139044 earnings -0.049263 1.000000 0.347752 returns 0.139044 0.347752 1.000000
```

[21]: from statsmodels.tsa.stattools import ccf

[22]: plt.plot(ccf(returns,earnGrowth)[0:11]) # 2nd series shift 0, 1, 2, 3, ... print(ccf(returns,earnGrowth)[0:11]) # 0nly shift 0 is significant 2/len(returns)**0.5 # 95% confidence bands +/- 2 / sqrt(n)

```
[ 0.34775194  0.0348649  -0.10358045  0.10063521  0.04882275  -0.19185602  -0.03416783  0.23642307  -0.07544957  -0.15003368  0.22437801]
```

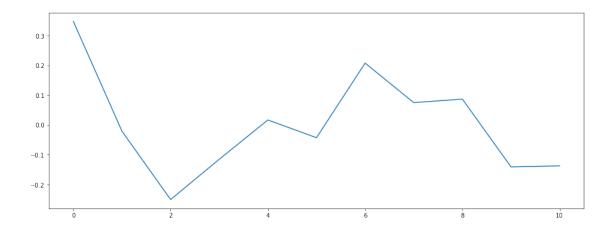
[22]: 0.23249527748763857



```
[23]: plt.plot(ccf(earnGrowth,returns)[0:11]) # 2nd series shift 0, 1, 2, 3, ...
print(ccf(earnGrowth,returns)[0:11]) # Shift 0 and shift 2 is significant_u
but not shift 1? Ignore shift 2?
2/len(returns)**0.5 # 95% confidence bands +/- 2 / sqrt(n)
```

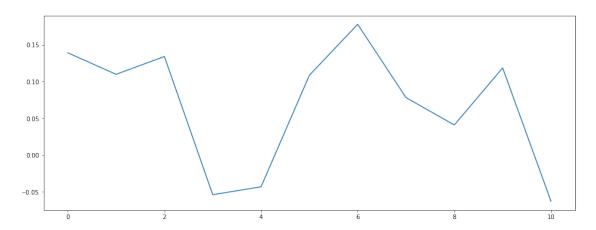
[0.34775194 -0.02100698 -0.25044343 -0.11514607 0.01676789 -0.0427807 0.20771867 0.0750129 0.08677466 -0.14086962 -0.13721904]

[23]: 0.23249527748763857



```
[24]: plt.plot(ccf(returns,divGrowth)[0:11]) # 2nd series shift 0, 1, 2, 3, ... print(ccf(returns,divGrowth)[0:11]) # Not signficant. 2/len(returns)**0.5 # 95% confidence bands +/- 2 / sqrt(n)
```

[24]: 0.23249527748763857



```
[25]: plt.plot(ccf(divGrowth,returns)[0:11]) # 2nd series shift 0, 1, 2, 3, ...

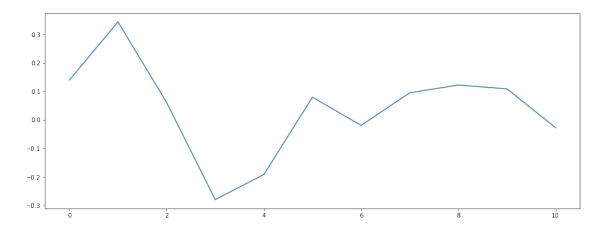
print(ccf(divGrowth,returns)[0:11]) # Shift 1 is signficant, i.e. div(t)

correl w/ returns(t-1)

# i.e. returns predict divs 1 year ahead

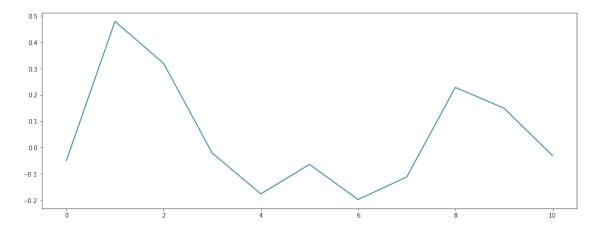
2/len(returns)**0.5 # 95% confidence bands +/- 2 / sqrt(n)
```

[25]: 0.23249527748763857



[26]: plt.plot(ccf(divGrowth,earnGrowth)[0:11]) # 2nd series shift 0, 1, 2, 3, ... print(ccf(divGrowth,earnGrowth)[0:11]) # Shift 1,2 signficant, i.e. div(t) correl w/ earn(t-1), (t-2) # i.e. earnings predict divs 1- and 2-years ahead 2/len(returns)**0.5 # 95% confidence bands +/- 2 / sqrt(n)

[26]: 0.23249527748763857

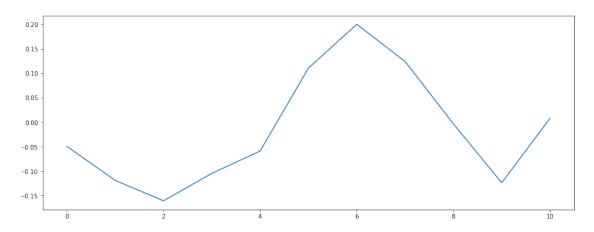


[27]: plt.plot(ccf(earnGrowth,divGrowth)[0:11]) # 2nd series shift 0, 1, 2, 3, ... print(ccf(earnGrowth,divGrowth)[0:11]) # Insignificant 2/len(returns)**0.5 # 95% confidence bands +/- 2 / sqrt(n)

[-0.04926259 -0.11874393 -0.16044432 -0.10452656 -0.05885002 0.11055937

0.20001561 0.12408141 -0.00339224 -0.12366642 0.00813759]

[27]: 0.23249527748763857



Some conclusions: change in earnings roughly contemporaneous with returns, and divs lag 1-2 years behind.

Also, Python handles data much more elegantly than (base) R. Perhaps an unfair comparison of pandas vs base R though, since R also its own add-on packages. Also R is ahead on time series analysis functionality and visualization (e.g. ggplot2 and the rest of the "tidyverse").

```
[28]: from statsmodels.tsa.api import VAR
[29]:
     model = VAR(df)
     results = model.fit(0) # vector random walk with drift
[31]:
     results.summary()
[31]:
       Summary of Regression Results
     Model:
                                   VAR
                                   OLS
     Method:
     Date:
                     Wed, 24, Mar, 2021
     Time:
                              14:25:45
     No. of Equations:
                              3.00000
                                         BIC:
                                                               -11.5494
     Nobs:
                              74.0000
                                         HQIC:
                                                               -11.6055
     Log likelihood:
                              118.778
                                         FPE:
                                                            8.78246e-06
     AIC:
                             -11.6428
                                         Det(Omega_mle):
                                                            8.43583e-06
     Results for equation divs
```

	coefficient	std. error	t-stat	prob
const	0.024761	0.007860	3.150	0.002
======	==========		=========	=========

Results for equation earnings

=======	coefficient	std. error	t-stat	prob
const	0.031505	0.033289	0.946	0.344

Results for equation returns

		coefficient	std. error	t-stat	prob
const 0.034489 0.018887 1.826 0.06	const	0.034489	0.018887	1.826	0.068

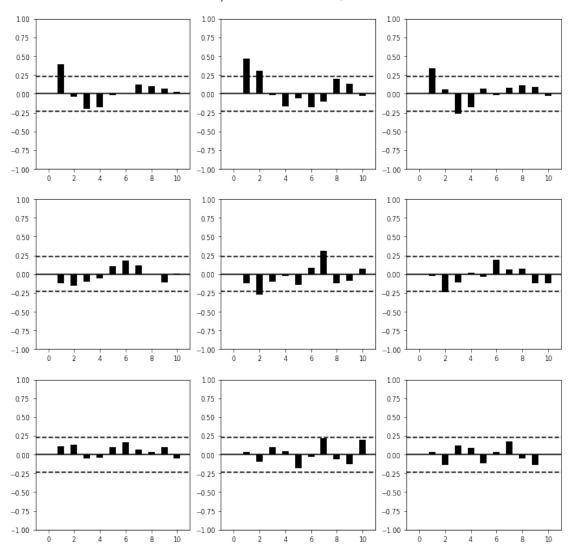
Correlation matrix of residuals

divs earnings returns divs 1.000000 -0.049263 0.139044 earnings -0.049263 1.000000 0.347752 returns 0.139044 0.347752 1.000000

[32]: results.plot_acorr();

- # How to intrepret this: The 1st row, 2nd col is the ccf of divs and earnings, # i.e. corr(divs(t), earnings(t-k)), k > 0
- # The signficant correlations at lag 1 and 2 mean earnings lead dividends at $_{\mbox{\tiny L}}$ +those lags.
- # Important: In these plots, the column leads the row.

ACF plots for residuals with $2/\sqrt{T}$ bounds



[33]: results = model.fit(1) # VAR(1)

[34]: results.summary()

[34]: Summary of Regression Results

Model: VAR
Method: OLS
Date: Wed, 24, Mar, 2021
Time: 14:25:49

No. of Equations: 3.00000 BIC: -11.7039 Nobs: 73.0000 HQIC: -11.9303 Log likelihood: 142.187 FPE: 5.67148e-06 AIC: -12.0804 Det(Omega_mle): 4.83273e-06

Results for equation divs

______ coefficient std. error t-stat prob const 0.011638 0.006479 1.796 0.072 0.396305 0.090444 4.382 0.000 L1.divs L1.earnings 0.105299 0.022560 4.667 0.000 L1.returns 0.055538 0.040380 1.375 0.169

Results for equation earnings

______ coefficient std. error t-stat 0.048233 0.036480 1.322 0.186 L1.divs -0.555327 0.509210 -1.091 0.275 -1.152 L1.earnings -0.146343 0.127017 0.249 L1.returns 0.085108 0.227341 0.374 0.708 ______

Results for equation returns

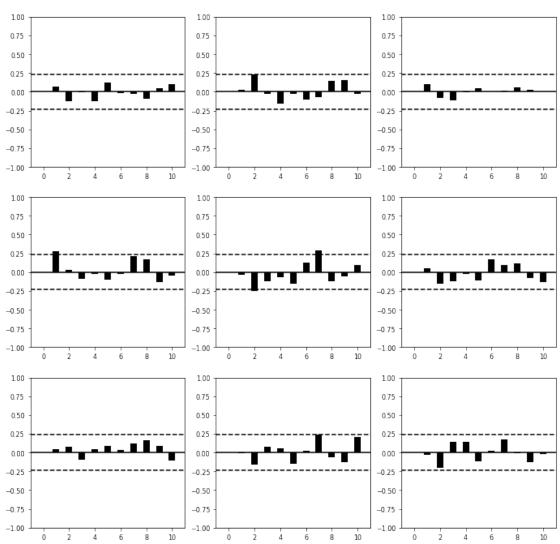
=========	coefficient	std. error	t-stat	prob
const	0.032255	0.020136	1.602	0.109
L1.divs L1.earnings	0.263855 0.021179	0.281075 0.070111	0.939 0.302	0.348 0.763
L1.returns	0.007089	0.125488	0.056	0.955

Correlation matrix of residuals

divsearningsreturnsdivs1.0000000.0639230.029378earnings0.0639231.0000000.377156returns0.0293780.3771561.000000

[35]: results.plot_acorr();





[]: