Econ 210C Homework 4

Beatrice Allamand

June 6, 2024

Comment to revisor: I attached handwritten pages at the end of this document. Those pages include mathematical details from Question 1 (a), and the complete answer to Question 2 parts (b), (c) and (d).

1. Productivity Shocks in the Three Equation Model

The log-linearized NK model boils down to three equations:

$$\begin{aligned}
\hat{y}_t &= -\sigma[\hat{i}_t - E_t\{\hat{\pi}_{t+1}\}] + E_t\{\hat{y}_{t+1}\} \\
\hat{\pi}_t &= \kappa(\hat{y}_t - \hat{y}_t^{flex}) + \beta E_t\{\hat{\pi}_{t+1}\} \\
\hat{i}_t &= \phi_\pi \hat{\pi}_t + v_t
\end{aligned}$$

with $\hat{y}_t^{flex} = \frac{1+\varphi}{\gamma+\varphi}\hat{a}_t$.

For this part assume that $v_t = 0$ and that $\hat{a}_t = \rho_a \hat{a}_{t-1} + \epsilon_t$.

(a) Using the method of undetermined coefficients, solve for $\hat{y}_t$ and $\hat{\pi}_t$ as a function of $\hat{a}_t$.

Solution:

$$\begin{aligned}
\hat{y}_t &= \Psi_{ya}\hat{a}_t \\
\hat{\pi}_t &= \Psi_{\pi a}\hat{a}_t
\end{aligned}$$

With

$$\begin{aligned}
\Psi_{ya} &= \frac{\sigma(\rho_a - \phi_\pi)}{1-\rho_a}\Psi_{\pi a} \\
\Psi_{\pi a} &= \frac{\kappa(\rho_a - 1)}{(1-\rho_a)(1-\beta\rho_a) - \kappa(\rho_a - \phi_\pi)}\frac{1+\varphi}{\gamma+\varphi}
\end{aligned}$$

So

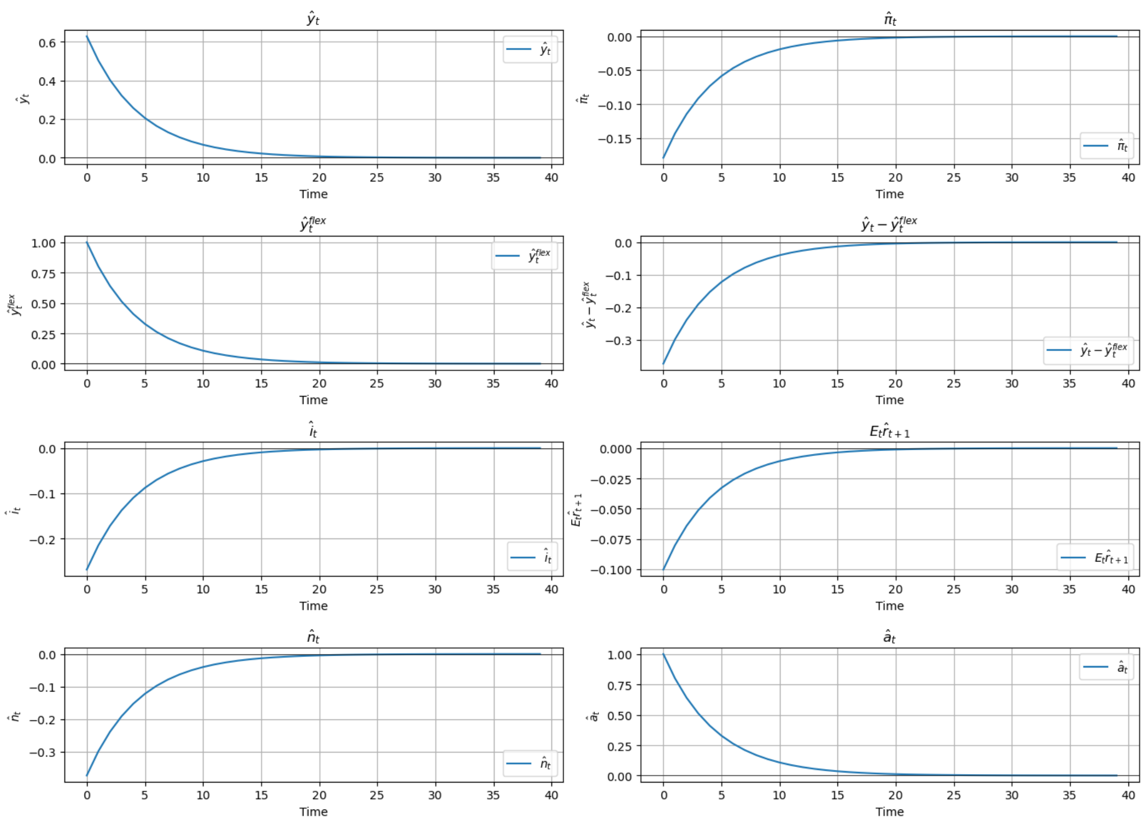
$$
\begin{aligned}
\hat{y}_t &= \frac{\sigma\kappa(\rho_a - \phi_\pi)(\rho_a - 1)(1+\varphi)}{(1-\rho_a)\left[(1-\rho_a)(1-\beta\rho_a) - \kappa(\rho_a - \phi_\pi)\right](\gamma+\varphi)}\hat{a}_t \\
\hat{\pi}_t &= \frac{\kappa(\rho_a - 1)(1+\varphi)}{\left[(1-\rho_a)(1-\beta\rho_a) - \kappa(\rho_a - \phi_\pi)\right](\gamma+\varphi)}\hat{a}_t
\end{aligned}
$$

(b) Plot the impulse response function for $\hat{y}_t, \hat{\pi}_t, \hat{y}_t^{flex}, \hat{y}_t - \hat{y}_t^{flex}, \hat{i}_t, \mathbb{E}_t\hat{r}_{t+1}, \hat{n}_t, \hat{a}_t$ to a one unit shock to $\hat{a}_t$. Use the following parameter values:

$\beta = 0.99, \sigma = 1, \kappa = 0.1, \rho_a = 0.8, \phi_\pi = 1.5$

Solution: See code hw4_1.b.ipynb.

Figure 1: IRF: Impulse of $\hat{a}_t$ and Response on $\hat{y}_t, \hat{\pi}_t, \hat{y}_t^{flex}, \hat{y}_t - \hat{y}_t^{flex}, \hat{i}_t, \mathbb{E}_t\hat{r}_{t+1}, \hat{n}_t, \hat{a}_t$



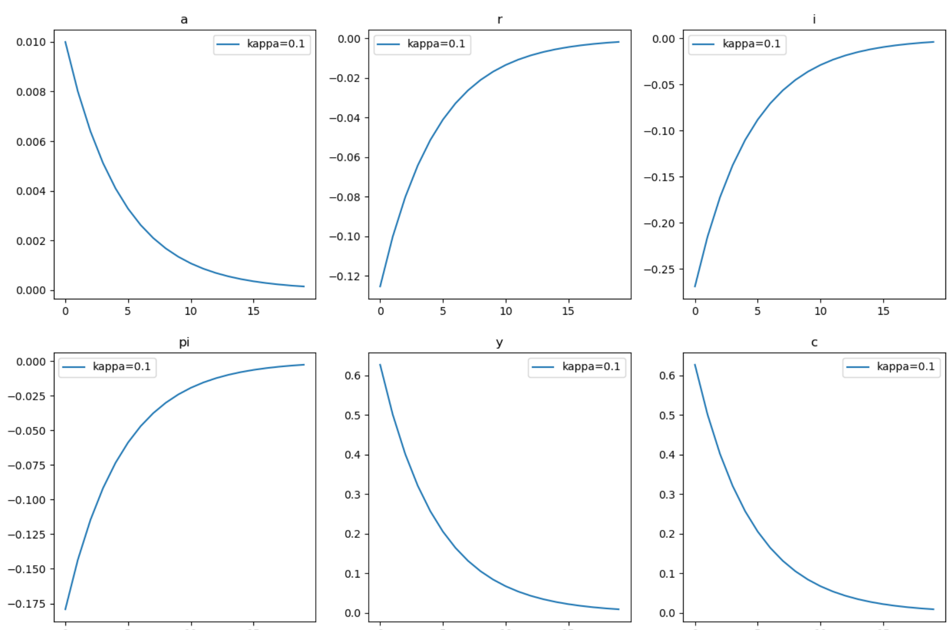
(c) Intuitively explain your results.

Solution: A unitary TFP shock affects $\hat{y}_t^{flex}$ 1:1, reflecting the fact that if prices were flexible, the output would follow productivity. However, due to price stickiness, we can see that output only partially reacts to the higher $\hat{a}_t$, and the net effect on output gap is a decrease in the difference between the two measures. A fraction of the firms cannot change their prices (which they would want to decrease), causing employment to decrease as we observed in the 1 period sticky prices model. This affects real wages downwards too, although we don't observe that in the 3 equation model. The real interest rate, defined by the Fisher equation, will capture the difference between the nominal interest rate and the expectation of future inflation adjustment. Here, because prices will not reduce in the amount they would under flexible prices, inflation reacts to the output gap and to the expected future inflation, both negatively related to $\hat{a}_t$ as showed in (a). Also, the fact that a Taylor rule governs monetary policy implies that the decrease in inflation will produce a further decrease in nominal interest rate, resulting in a decrease of real interest rate.

(d) Use the Jupyter notebook "newkeynesianlinear.ipynb" to check that your plots in (b) are correct.

Solution: It can be checked that the plots are in line with previous result. The code used can be found here. It adds a small modification to the original, so that the model can react to a shock in the required variable.

Figure 2: IRF: Using Jupyter program



2. Non-linear NK model in Jupyter

Implement the standard new Keynesian model in Jupyter. We will write all conditions recursively and let the Sequence-Space Jacobian (SSJ) routines do the differentiation for us. Note that the first order conditions for firms and households are exactly as we have written in the lectures.

(a) The real reset price equation for the firm is,

$$p_t^* \equiv \frac{P_t^*}{P_t} = (1+\mu)E_t \left\{ \sum_{s=0}^{\infty} \frac{\theta^s \Lambda_{t,t+s} Y_{t+s} (P_{t+s}/P_t)^{\epsilon-1}}{\sum_{k=0}^{\infty} \theta^k \Lambda_{t,t+k} Y_{t+k} (P_{t+k}/P_t)^{\epsilon-1}} \frac{W_{t+s}/P_t}{A_{t+s}} \right\}$$

Explain why this expression is not recursive.

> **Solution:** The first reason this expression is not recursive is it's non linearity. Additionally, we can argue that this expression cannot be expressed in a recursive manner because even if we were to linearize it using logs, the fact that it depends on two different summations to the future makes it impossible to express future values of the expression as the main variable in the future, as we would do, for example, in a recursive formulation of an objective function.

(b) We next show that we can write $B_t = E_t(F_{1t}/F_{2t})$, where both F_{1t}, F_{2t} are recursive. First, show that the denominator can be recursively written as,

$$\begin{aligned} F_{2t} &\equiv \sum_{k=0}^{\infty} \theta^k \Lambda_{t,t+k} Y_{t+k} (P_{t+k}/P_t)^{\epsilon-1} \\ &= Y_t + \theta \Pi_{t+1}^{\epsilon-1} \Lambda_{t,t+1} F_{2,t+1} \end{aligned}$$

noting that $\Lambda_{t,t+k} = \Lambda_{t,t+1}\Lambda_{t+1,t+k}$ for all $k \geqslant 1$.

> **Solution:** Question 2 parts (b), (c) and (d) are answered in handwritten part at the end of this PDF.

(c) Second, show that the numerator can be recursively written as,

$$\begin{aligned} F_{1t} &\equiv (1+\mu) \sum_{s=0}^{\infty} \theta^s \Lambda_{t,t+s} Y_{t+s} (P_{t+s}/P_t)^{\epsilon-1} \frac{W_{t+s}/P_t}{A_{t+s}} \\ &= (1+\mu) Y_t \frac{W_t/P_t}{A_t} + \theta \Pi_{t+1}^{\epsilon} \Lambda_{t,t+k} F_{1,t+1} \end{aligned}$$

noting that $\Lambda_{t,t+k}\Lambda_{t,t+1}\Lambda_{t+1,t+k}$ for all $k \geqslant 1$.

(d) Show that (gross) inflation can implicitly be written as

$$1 = \theta \Pi_t^{\epsilon-1} + (1-\theta) p_t^{*1-\epsilon}$$

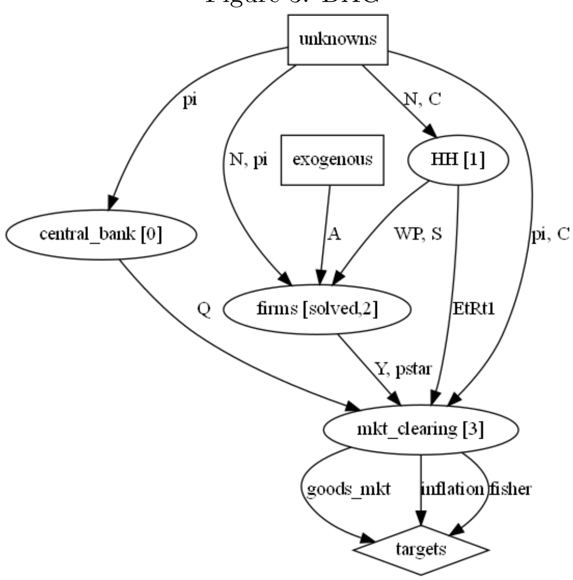
(e) Explain intuitively how when $p_t^* > 1$, then $\Pi_t > 1$.

Solution: $p_t^* > 1 \implies P_t^* > P_t$. This means that firms that adjusted their price chose a reset price higher than the aggregate price level in that moment. Because the aggregate price level is a somehow weighted average of the current reset price and the past period price level, this exposes the fact that we also have $P_t^* > P_t > P_{t-1}$. Finally, note $P_t > P_{t-1}$ implies $\pi_t = \frac{P_t}{P_{t-1}} > 1$

(f) Implement the non-linear NK using your recursive equations in Python using the Sequence Space Jacobian toolbox. For now, ignore the dispersion of labor in production and write the aggregate production function as $Y_t = A_t N_t$. Use the following parameters: $\beta = 0.99, \gamma = 1, \varphi = 1, \chi = 1, \epsilon = 10, \rho_a = 0.8, \phi_\pi = 1.5, \phi_y = 0$ where $A_t = (A_{t-1})^{\rho_a} e^{\epsilon_t^a}$. Productivity is the only shock. Price stickiness is specified below.

Solution: The code hw4_2.f.ipynb implements the non-linear NK with θ fixed at 0.25. The DAG of the model is:

Figure 3: DAG

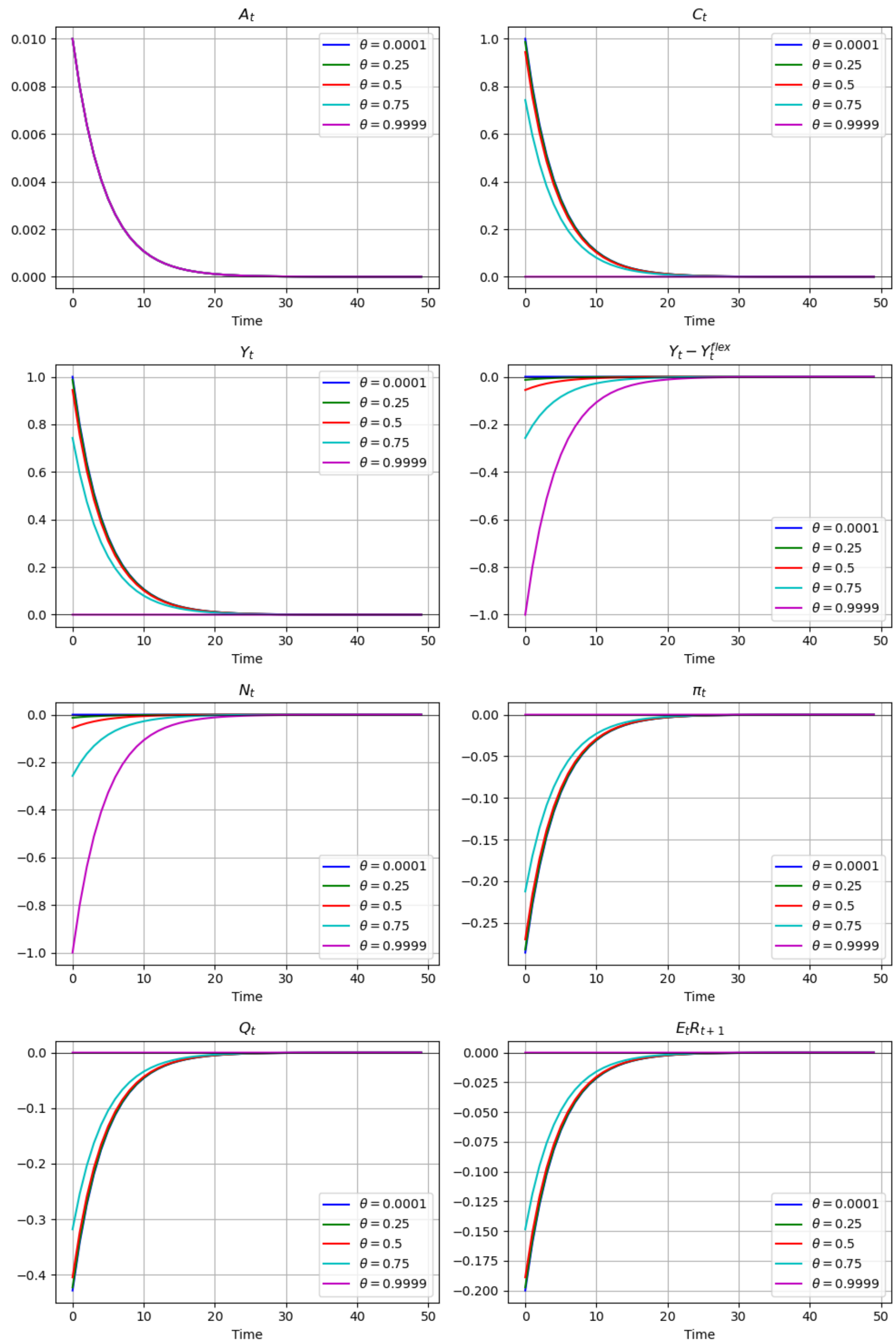


(g) Compute IRFs for $\theta \in \{0.0001, 0.25, 0.5, 0.75, 0.9999\}$ using a first order approximation to your non-linear equations.

Report the IRFs for consumption, the output gap, the level of output, employment, inflation, the markup, the nominal interest rate, and the ex-ante real interest rate. Your graph for each variable should contain all cases for θ, appropriately labelled.

Solution: The code hw4_2.g.ipynb implements a loop of the DAG on each value of theta and reports results in graphs. Note that the initial values for f1 and f2 depend on theta. For reporting IRF of output gap, we also have to run the flexible prices equilibrium.

Figure 4: IRF in the non linear model



(h) Intuitively explain how the impulse response functions depend on the value of θ.

Solution: Because θ measures de degree of price stickyness, it also will affect the degree of non neutrality of the system of equations in equilibrium. Under an RBC model with classical dichotomy we would expect the TFP shock to drive the cycle, whereas on a completly sticky prices model we know TFP does not have contemporaneous effect in production and consumption, and it decreases N_t. While we tipically study how do NK models respond to monetary shocks, it is also relevant to see how does the introduction on non-neutrality with respect to money can cause the model to departure from dependance on real variables. This is exactly what we see. When exposed to a higher degree of stickyness (higher θ), a higher A_t will be less and less absorbed by R_{t+1} and have less effect on C_t and Y_t. In the limit, they do not respond at all, and the higher productivity is fully absorbed by lower employment.

(i) What would you expect to see from the same shock in an RBC model without capital? (No derivation should be necessary.)

Solution: The New Keynesian model nests the results and predictions of the RBC model. Every block of the model is the same except for the firm side. If we take $\theta \to 0$, so that all firms adjust at all times, we are back in the RBC model with flexible prices. We can expect the responses to be similar to the ones graphed under $\theta = 0.0001$. In particular, a shock to A_t will have 1 to 1 response in C_t, Y_t, R_{t+1}, the output gap is 0 because $Y_t = Y_t^{flex}$, N_t and nominal variables do not respond.

PART ①

The log-linearized NK model boils down to three equations:

$$\hat{y}_t = -\sigma[\hat{i}_t - E_t\{\hat{\pi}_{t+1}\}] + E_t\{\hat{y}_{t+1}\}$$
$$\hat{\pi}_t = \kappa(\hat{y}_t - \hat{y}_t^{flex}) + \beta E_t\{\hat{\pi}_{t+1}\}$$
$$\hat{i}_t = \phi_\pi \hat{\pi}_t + v_t$$

with $\hat{y}_t^{flex} = \frac{1+\varphi}{\gamma+\varphi}\hat{a}_t$.

For this part assume that $v_t = 0$ and that $\hat{a}_t = \rho_a \hat{a}_{t-1} + \epsilon_t$.

(a)

(a) Using the method of undetermined coefficients, solve for $\hat{y}_t$ and $\hat{\pi}_t$ as a function of $\hat{a}_t$.

Guess policy function: $\hat{y}_t = \psi_{ya}\hat{a}_t$

$\hat{\pi}_t = \psi_{\pi a}\hat{a}_t$

Note $E_t(\hat{a}_{t+1}) = E_t(\rho_a \hat{a}_t + \epsilon_{t+1}) = \rho_a \hat{a}_t$

$\Rightarrow$ $E_t\{\hat{\pi}_{t+1}\} = \psi_{\pi a} E_t\{\hat{a}_{t+1}\} = \psi_{\pi a}\rho_a \hat{a}_t$

$E_t\{\hat{y}_{t+1}\} = \psi_{ya} E_t\{\hat{a}_{t+1}\} = \psi_{ya}\rho_a\hat{a}_t$

(i) $\hat{i}_t = \phi_\pi \psi_{\pi a}\hat{a}_t$

(y)
$$\psi_{ya}\hat{a}_t = -\sigma\left[\phi_\pi\psi_{\pi a}\hat{a}_t - \psi_{\pi a}\rho_a\hat{a}_t\right] + \psi_{ya}\rho_a\hat{a}_t$$
$$\psi_{ya}\hat{a}_t = -\sigma\phi_\pi\psi_{\pi a}\hat{a}_t + \sigma\psi_{\pi a}\rho_a\hat{a}_t + \psi_{ya}\rho_a\hat{a}_t \qquad /:\hat{a}_t$$
$$\psi_{ya}(1-\rho_a) = -\sigma\phi_\pi\psi_{\pi a} + \sigma\psi_{\pi a}\rho_a$$
$$\psi_{ya} = \frac{\psi_{\pi a}\sigma(\rho_a - \phi_\pi)}{(1-\rho_a)}$$

(π)
$$\psi_{\pi a}\hat{a}_t = \kappa(\psi_{ya}\hat{a}_t - \hat{y}_t^{flex}) + \beta\psi_{\pi a}\rho_a\hat{a}_t \quad ; \quad \hat{y}_t^{flex} = \frac{1+\varphi}{\gamma+\varphi}\hat{a}_t$$
$$\psi_{\pi a}\hat{a}_t = \kappa\left(\psi_{ya}\hat{a}_t - \frac{1+\varphi}{\gamma+\varphi}\hat{a}_t\right) + \beta\psi_{\pi a}\rho_a\hat{a}_t \qquad /:\hat{a}_t$$
$$\psi_{\pi a}(1-\beta\rho_a) = \kappa\left(\psi_{ya} - \frac{1+\varphi}{\gamma+\varphi}\right)$$
$$\psi_{\pi a}(1-\beta\rho_a) - \kappa\frac{\psi_{\pi a}\sigma(\rho_a-\phi_\pi)}{(1-\rho_a)} = -\kappa\frac{1+\varphi}{\gamma+\varphi}$$
$$\psi_{\pi a}\left[1-\beta\rho_a - \frac{\kappa(\rho_a-\phi_\pi)}{1-\rho_a}\right] = -\kappa\frac{1+\varphi}{\gamma+\varphi}$$

we don't need φ

$$\psi_{\pi a} = -\frac{(1-\rho_a)}{(1-\rho_a)(1-\beta\rho_a)-\kappa(\rho_a-\phi_\pi)}\kappa\frac{1+\varphi}{\gamma+\varphi} = \frac{\kappa(\rho_a-1)(1+\cancel{\varphi})}{[(1-\rho_a)(1-\beta\rho_a)-\kappa(\rho_a-\phi_\pi)]\gamma+\varphi}$$

$\cancel{1+\varphi}$

$\hat{y}_t$ and $\hat{\pi}_t$ as functions of $\hat{a}_t$:

$$\hat{y}_t = \psi_{ya} \hat{a}_t$$
$$\hat{\pi}_t = \psi_{\pi a} \hat{a}_t$$

$$\psi_{ya} = \frac{\psi_{\pi a} \sigma (\rho_a - \phi_\pi)}{(1-\rho_a)}$$

$$\psi_{\pi a} = \frac{K(\rho_a - 1)(1+\varphi)}{[(1-\rho_a)(1-\beta\rho_a) - K(\rho_a - \phi_\pi)]\gamma + \varphi}$$

(b)

For (b) need to find $E_t \hat{r}_{t+1}$ and $\hat{n}_t$ as functions on $\hat{a}_t$.

① $E_t \hat{r}_{t+1}$:
$$E_t(R_{t+1}) = E_t\left(Q_t \frac{P_t}{P_{t+1}}\right)$$

$$E_t \hat{r}_{t+1} = \hat{i}_t - E_t(\hat{\pi}_{t+1}) //$$

② $\hat{n}_t$

From class "Monopolistic competition and sticky prices"

Firms' production function : $y_t(i) = A_t N_t(i)$

mkt clearing $\Rightarrow$ $C_t(i) = Y_t(i) \wedge C_t = Y_t$

In Gali
$$Y_t(i) = A_t N_t(i)^{1-\alpha}$$

$$y_t = a_t + (1-\alpha) n_t$$

here $y_t = a_t + n_t$

PART 2

(b)

$$P_t^* = (1+\mu)E_t\left\{\sum_{s=0}^{\infty}\frac{\theta^s \Lambda_{t,t+s}\, y_{t+s}\,(P_{t+s}/P_t)^{\epsilon-1}}{\sum_{k=0}^{\infty}\theta^k \Lambda_{t,t+k}\, y_{t+k}\,(P_{t+k}/P_t)^{\epsilon-1}}\;\frac{W_{t+s}/P_t}{A_{t+s}}\right\}$$

$$F_{2t} = \sum_{k=0}^{\infty}\theta^k\,\Lambda_{t,t+k}\,y_{t+k}\,(P_{t+k}/P_t)^{\epsilon-1}$$

$$F_{2t+1} = \sum_{k=0}^{\infty}\theta^k\,\Lambda_{t+1,t+1+k}\,y_{t+1+k}\,(P_{t+1+k}/P_{t+1})^{\epsilon-1}$$

$$F_{2t} = \Lambda_{t,t}\,y_t\,(P_t/P_t)^{\epsilon-1} + \theta\,\Lambda_{t,t+1}\,y_{t+1}\left(\frac{P_{t+1}}{P_t}\right)^{\epsilon-1} + \sum_{k=2}^{\infty}\theta^k\,\Lambda_{t,t+k}\,y_{t+k}\left(\frac{P_{t+k}}{P_t}\right)^{\epsilon-1}$$

- $\Lambda_{t,t+k} = \beta^k\, E_t\left[\frac{u'(c_{t+k})}{u'(c_t)}\right]$
- $\Lambda_{t,t} = E_t\left[\frac{u'(c_t)}{u'(c_t)}\right] = 1$

$$F_{2t} = y_t + \theta\,\Lambda_{t,t+1}\,\underline{y_{t+1}}\,\Pi_{t+1}^{\epsilon-1} + \sum_{k=2}^{\infty}\theta^k\,\Lambda_{t,t+k}\,y_{t+k}\left(\frac{P_{t+k}}{P_t}\right)^{\epsilon-1}$$

$$F_{2t} = y_t + \underline{\theta\,\Lambda_{t,t+1}\,y_{t+1}\,\Pi_{t+1}^{\epsilon-1}} + \sum_{k=2}^{\infty}\theta^k\,\Lambda_{t,t+1}\,\Lambda_{t+1,t+k}\,y_{t+k}\left(\frac{P_{t+k}}{P_t}\right)^{\epsilon-1}$$

Note: $F_{2t+1} = \sum_{k=0}^{\infty}\theta^k\,\Lambda_{t+1,t+1+k}\,y_{t+1+k}\,(P_{t+1+k}/P_{t+1})^{\epsilon-1}$

$$F_{2t+1} = y_{t+1} + \sum_{k=1}^{\infty}\theta^k\,\Lambda_{t+1,t+1+k}\,y_{t+1+k}\,(P_{t+1+k}/P_{t+1})^{\epsilon-1}$$

$$\Rightarrow\; y_{t+1} = F_{2t+1} - \sum_{k=1}^{\infty}\theta^k\,\Lambda_{t+1,t+1+k}\,y_{t+1+k}\,(P_{t+1+k}/P_{t+1})^{\epsilon-1}$$

$$\underline{\theta\,\Lambda_{t,t+1}\,y_{t+1}\,\Pi_{t+1}^{\epsilon-1}} = \theta\,\Lambda_{t,t+1}\,F_{2t+1}\,\Pi_{t+1}^{\epsilon-1} - \underbrace{\theta\,\Lambda_{t,t+1}\,\Pi_{t+1}^{\epsilon-1}\sum_{k=1}^{\infty}\theta^k\,\Lambda_{t+1,t+1+k}\,y_{t+1+k}\,(P_{t+1+k}/P_{t+1})^{\epsilon-1}}$$

$$\sum_{k=1}^{\infty}\theta^{k+1}\,\Lambda_{t,t+1}\,\Lambda_{t+1,t+1+k}\,y_{t+1+k}\left(\frac{P_{t+1+k}}{P_{t+1}}\;\frac{P_{t+1}}{P_t}\right)^{\epsilon-1}$$

$$\sum_{k=1}^{\infty}\theta^{k+1}\,\Lambda_{t,t+1}\,\Lambda_{t+1,t+1+k}\,y_{t+1+k}\left(\frac{P_{t+1+k}}{P_t}\right)^{\epsilon-1}$$

$$= \theta\,\Lambda_{t,t+1}\,F_{2t+1}\,\Pi_{t+1}^{\epsilon-1} - \sum_{k=2}^{\infty}\theta^k\,\Lambda_{t,t+1}\,\Lambda_{t+1,t+k}\,y_{t+k}\left(\frac{P_{t+k}}{P_t}\right)^{\epsilon-1}$$

so $\boxed{F_{2t} = y_t + \theta\,\Lambda_{t,t+1}\,F_{2t+1}\,\Pi_{t+1}^{\epsilon-1}}$

2.(c)

$$F_{1t} = (1+\mu) \sum_{s=0}^{\infty} \theta^s \Lambda_{t,t+s} \, y_{t+s} \left(\frac{P_{t+s}}{P_t}\right)^{\epsilon-1} \frac{W_{t+s}/P_t}{A_{t+s}}$$

$$F_{1t} = (1+\mu) \, y_t \, \frac{W_t/P_t}{A_t} + (1+\mu) \sum_{s=1}^{\infty} \theta^s \Lambda_{t,t+s} \, y_{t+s} \left(\frac{P_{t+s}}{P_t}\right)^{\epsilon-1} \frac{W_{t+s}/P_t}{A_{t+s}}$$

$$F_{1t+1} = (1+\mu) \sum_{s=0}^{\infty} \theta^s \Lambda_{t+1,t+1+s} \, y_{t+1+s} \left(\frac{P_{t+1+s}}{P_{t+1}}\right)^{\epsilon-1} \frac{W_{t+1+s}/P_{t+1}}{A_{t+1+s}} \quad / \cdot \; \theta \Pi_{t+1}^{\epsilon} \Lambda_{t,t+1}$$

$$= (1+\mu) \sum_{s=0}^{\infty} \theta^{s+1} \Lambda_{t,t+1} \Lambda_{t+1,t+1+s} \left(\frac{P_{t+1+s}}{P_t}\right)^{\epsilon-1 \, (\star)} \frac{W_{t+1+s}/P_t}{A_{t+1+s}}$$

$$\star : \left(\frac{P_{t+1+s}}{P_{t+1}}\right)^{\epsilon-1} \cdot \left(\frac{P_{t+1}}{P_t}\right)^{\epsilon-1} \left(\frac{\cancel{P_{t+1}}}{P_t}\right)^{1} \frac{W_{t+1+s}}{\cancel{P_{t+1}}} \cdot \frac{1}{A_{t+1+s}}$$

$$\left(\frac{P_{t+1+s}}{P_t}\right)^{\epsilon-1} \left(\frac{W_{t+1+s}}{P_t}\right) \frac{1}{A_{t+1+s}}$$

$$\theta \Pi_{t+1}^{\epsilon} \Lambda_{t,t+1} F_{1t+1} = (1+\mu) \sum_{s=1}^{\infty} \theta^s \Lambda_{t,t+1} \Lambda_{t+1,t+s} \left(\frac{P_{t+s}}{P_t}\right)^{\epsilon-1} \frac{W_{t+s}/P_t}{A_{t+s}}$$

$$\Rightarrow \boxed{F_{1t} = (1+\mu) \, y_t \, \frac{W_t/P_t}{A_t} + \theta \Pi_{t+1}^{\epsilon} \Lambda_{t,t+1} F_{1t+1}} \; //$$

Conclusion:

$$p_t = E_t(F_{1t} / F_{2t})$$

2. (d) Gross π can be written as:

$$1 = \theta \Pi_t^{\epsilon-1} + (1-\theta) p_t^{*\,1-\epsilon} \qquad p_t^* = \frac{P_t^*}{P_t}$$

$$P_t = \left[\theta P_{t-1}^{1-\epsilon} + (1-\theta) P_t^{*\,1-\epsilon} \right]^{\frac{1}{1-\epsilon}} \qquad / (\ )^{1-\epsilon}$$

$$P_t^{1-\epsilon} = \theta P_{t-1}^{1-\epsilon} + (1-\theta) P_t^{*\,1-\epsilon} \qquad / : P_t^{1-\epsilon}$$

$$1 = \theta \left(\frac{P_{t-1}}{P_t}\right)^{1-\epsilon} + (1-\theta) \left(\frac{P_t^*}{P_t}\right)^{1-\epsilon}$$

$$\boxed{1 = \theta \Pi_t^{\epsilon-1} + (1-\theta) p_t^{*\,1-\epsilon}}$$