Econ 210C Homework 3

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1. Sticky Wage Model

Instead of assuming that prices are sticky for one period, we now assume that nominal wages are sticky for one period,

$$W_1 = W_0$$

The short-run equilibrium ($t = 1$) is

$$\begin{aligned}
Y_1 &= A_1 N_1 \\
W_1 &= W_0 \\
\frac{W_1}{P_1} &= A_1 \\
Y_1 &= C_1 \\
\frac{M_1}{P_1} &= \zeta^{1/\nu}\left(1 - \frac{1}{Q_1}\right)^{-1/\nu} C_1^{\gamma/\nu} \\
1 &= \beta E_1 \left\{ Q_1 \frac{P_1}{P_2} \frac{C_2^{-\gamma}}{C_1^{-\gamma}} \right\}
\end{aligned}$$

The long-run equilibrium ($t \geqslant 2$) is

$$\begin{aligned}
Y_t &= A_t N_t \\
\frac{W_t}{P_t} &= A_t \\
\frac{W_t}{P_t} &= \frac{\chi N_t^{\varphi}}{C_t^{-\gamma}} \\
Y_t &= C_t \\
\frac{M_t}{P_t} &= \zeta^{1/\nu}\left(1 - \frac{1}{Q_t}\right)^{-1/\nu} C_t^{\gamma/\nu} \\
1 &= \beta E_t \left\{ Q_t \frac{P_t}{P_{t+1}} \frac{C_{t+1}^{-\gamma}}{C_t^{-\gamma}} \right\}
\end{aligned}$$

(a) Are firms on their labor curve? Explain.

Solution: Yes. The third equilibrium condition of the short run equilibrium is the labor demand curve $\frac{W_1}{P_1} = A_1$ of the firms.
In the class version of the RBC model with sticky prices in $t = 1$ ($P_1 = P_0$) firms were not in their labor demand curve: $\frac{W_1}{P_0} \neq A_1 = MPL_1$, and the equilibrium condition for labor demand curve was replaced by $P_1 = P_0$. Here, it is the labor supply of households that is being replaced by $W_1 = W_0$.

(b) Are households on their labor supply curve? Explain.

Solution: No. In the short-run equilibrium, the sticky wages assumption replaces the labor supply curve. Households will supply all labor being demanded by the firms in period 1, not depending on

the real wage.
Another way to think of it is that labor supply will be completely inelastic at N_1, the labor necessary to produce C_1 that depends on W_0.

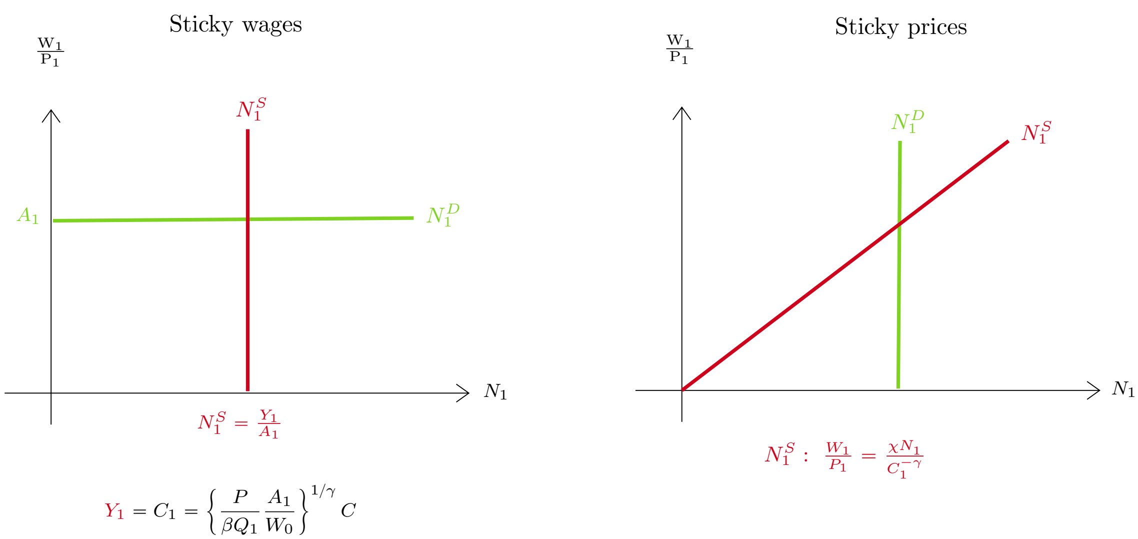
(c) How does the labor market clear?

Solution: In the class' model, labor market cleared as following:

- Firms demanded labor inelastically to supply the goods demanded at price P_0 (i.e. they hired $N_1 = \frac{C_1(P_0)}{A_1}$, $C_1(P_0)$ is just to expose the fact that C_1 depends on P_0)
- Wage was such that households were willing to offer N_1 units of labor.
- Therefore, the real wage could be found solving for $N_1 = C_1(P_0)/A_1$ on labor supply.

Here,

- Firms demand labor by labor demand $A_1 = \frac{W_1}{P_1}$.
- Because MPL does not depend on N_1, the choice of how many workers to hire at wage A_1 (set from labor demand), is to meet goods demand C_1 at fixed wage W_0, then, as before $N_1 = \frac{C_1(W_0)}{A_1}$.
- Households supply labor inelastically, in the quantity necessary to meet labor demand fixed W_0.
- The equilibrium wage will be A_1.



(d) Solve for the long-run steady state.

Solution: The long run steady state on the sticky wage model is equivalent to the RBC model, and to the RBC with sticky prices.
Assumptions: all exogenous variables constant for $t \geqslant 2 : A_t = A \; M_t = M$

$$Y = AN \tag{1}$$
$$\frac{W}{P} = A \tag{2}$$
$$\frac{W}{P} = \frac{\chi N^{\varphi}}{C^{-\gamma}} \tag{3}$$
$$Y = C \tag{4}$$
$$\frac{M}{P} = \zeta^{1/\nu}\left(1 - \frac{1}{Q}\right)^{-1/\nu} C^{\gamma/\nu} \tag{5}$$
$$1 = \beta\left\{Q\frac{P}{P}\frac{C^{-\gamma}}{C^{-\gamma}}\right\} \tag{6}$$

From (6) $\frac{1}{Q} = \beta$.
From (3)

$$\frac{W}{P} = \frac{\chi N^{\varphi}}{c^{-\gamma}} = \frac{\chi N^{\varphi}}{(AN)^{-\gamma}} = \chi A^{\gamma} N^{\varphi+\gamma}$$
$$(2) \Rightarrow \quad A = \chi A^{\gamma} N^{\varphi+\gamma}$$

$$\frac{A^{1-\gamma}}{\chi} = N^{\varphi+\gamma}$$
$$N = \left(\frac{A^{1-\gamma}}{\chi}\right)^{\frac{1}{\varphi+\gamma}} \Rightarrow Y = AA^{\frac{1-\gamma}{\varphi+\gamma}}\left(\frac{1}{\chi}\right)^{\frac{1}{\varphi+\gamma}}$$
$$C = Y = \left[\frac{A^{1+\varphi}}{\chi}\right]^{\frac{1}{\varphi+\gamma}}$$

From (5)

$$\frac{M}{P} = \zeta^{1/\nu}(1-\beta)^{-1/\nu} Y^{\gamma/\nu}$$

The steady state is defined by

$$N = \left(\frac{A^{1-\gamma}}{\chi}\right)^{\frac{1}{\varphi+\gamma}} \tag{1}$$
$$C = Y = \left[\frac{A^{1+\varphi}}{\chi}\right]^{\frac{1}{\varphi+\gamma}} \tag{2}$$
$$\frac{M}{P} = \zeta^{1/\nu}(1-\beta)^{-1/\nu} Y^{\gamma/\nu} \tag{3}$$

(e) Does the Classical Dichotomy hold in the long-run? Explain.

Solution: Yes. Any change in M causes a proportional change in P, leaving Y and C unchanged.

(f) Solve for output and the money market equilibrium in the short-run.

Solution: First replace long run variables for $t = 2$. The short-run equilibrium is

$$Y_1 = A_1 N_1 \tag{1}$$
$$W_1 = W_0 \tag{2}$$
$$\frac{W_1}{P_1} = A_1 \tag{3}$$
$$Y_1 = C_1 \tag{4}$$
$$\frac{M_1}{P_1} = \zeta^{1/\nu}\left(1 - \frac{1}{Q_1}\right)^{-1/\nu} C_1^{\gamma/\nu} \tag{5}$$
$$C_1 = \left\{\frac{1}{\beta Q_1}\frac{P}{P_1}\right\}^{1/\gamma} C \tag{6}$$

Solving the system for output market

$$\text{(2) and (3)}: \; P_1 = \frac{W_0}{A_1}, \text{ in (6)}: \; Y_1 = C_1 = \left\{\frac{P}{\beta Q_1}\frac{A_1}{W_0}\right\}^{\frac{1}{\gamma}} C$$
$$\text{by (1)}: \; N_1 = \frac{Y_1}{A_1}$$

Now for the money market

$$\frac{M_1}{P_1} = \zeta^{1/\nu}\left(1 - \frac{1}{Q_1}\right)^{-1/\nu} Y_1^{\gamma/\nu}$$

$$M_1 \frac{A_1}{W_0} = \zeta^{1/\nu}\left(1 - \frac{1}{Q_1}\right)^{-1/\nu} Y_1^{\gamma/\nu}$$

(g) Does the Classical Dichotomy hold in the short-run?

Solution: No. Although P_1 is a free price, the fact that W_1 is fixed at W_0 implies P_1 is fixed at W_0/A_1. Then, P_1 will not follow M_1. Money market clearing under a money supply shock will not be adjusted by P_1 but by Q_1, ultimatly affecting C_1 either way. This example exposes the nature of the non neutrality, because a monetary shock has effects on the output market.

(h) Explain intuitively (in words) how an increase in the money supply affects output in the short-run.

Solution:

Money market:

- An increase in M_1 will not be absorbed by a proportional increase in P_1, because P_1 is fixed in the level that allows $A_1 = \frac{W_0}{P_1}$.
- Because Y_1 does not directly respond to M_1, the only endogenous variable in the money market equation that can absorb an increase in M_1 is Q_1. So Q_1 decreases so that money market clears.

Goods market:

- Because Q_1 decreases, C_1 increases.
- Higher Y_1 follows directly from higher C_1. For producing more output, N_1 also increases. Wage is fixed at A_1.
- Because Y_1 went up, the Q_1 that equates the money market is smaller than initially noted.

The only difference with the sticky prices model is the labor market response. In sticky prices, increasing employment implied increasing wages (movement along the labor supply curve). Here, more workers are hired at fixed wage A_1.

(i) How does productivity affect output? Explain intuitively.

Solution: A higher productivity increases consumption, and therefore also output. Although nominal wage is fixed, real wages (always equal to A because firms are in labor demand) increase by a decrease in P_1.

$$Y_1 = C_1 = \left\{ \frac{P}{\beta Q_1} \frac{A_1}{W_0} \right\}^{\frac{1}{\gamma}} C$$

Departing from the sticky prices result, where an increase in productivity reduced employment (and also nominal wage W_1 and real wage W_1/P_0) but did not affect consumption or output, here, at the same time productivity and real wage rises, employment needs to go up to produce more output.

(j) Derive the labor wedge. Is it procyclical or countercyclical?

Solution: If no distortions existed, the real wage will be such that labor market cleared: $W_t/P_t = MRS_t = MPL_t$. With:

- MRS_t: Marginal rate of substitution between consumption and hours.
- MPL_t: Marginal product of labor.

Given this doesn't happen, the labor wedge τ_t^N is defined as

$$(1-\tau_t^N) = \frac{MRS_t}{MPL_t} = \frac{\chi A_1^{\gamma} N_1^{\gamma+\varphi}}{A_1} = \chi A_1^{\gamma-1} N_1^{\gamma+\varphi}$$

$$\tau_t^N = 1 - \chi A_1^{\gamma-1} N_1^{\gamma+\varphi}$$

Note that τ_1^N is decreasing in N_1

$$\frac{\delta \tau_1^N}{\delta N_1} = -(\gamma+\varphi)\chi A_1^{\gamma-1} N_1^{\gamma+\varphi-1} \leqslant 0$$

And depends of A_1

$$\begin{aligned} \frac{\delta \tau_1^N}{\delta A_1} = -(\gamma-1)\chi A_1^{\gamma-2} N_1^{\gamma+\varphi} &\leqslant 0 \text{ if } \gamma \geqslant 1 \\ &\geqslant 0 \text{ if } \gamma \leqslant 1 \end{aligned}$$

So if we follow the recession by measuring N_t, then the labor wedge is counter cyclical. As the cycle goes up (N_t increases) the labor wedge decreases. If we believe productivity shocks drive the cycle, labor wedge is counter cyclical as long as $\gamma \geqslant 1$. If the oppisite holds, and productivity shocks drive the cycle, the labor wedge can be pro cyclical.

(k) What moments of the data would you use to discriminate between the predictions of the sticky price and the sticky wage model?

Solution: First, lets collect the different predictions of both models:

- **Different predictions due to productivity shocks:** Productivity shocks have clearly different effects on output, consumption, wages and prices in both models. The accuracy of both predictions can be tested by comparing the data driven IRFs to productivity shocks.
  - If we analyse the response of output or consumption, the sticky prices model would predict no effect, while the sticky wages model would predict an immediate increase in output and consumption.
  - If we study the response of the labor market, sticky prices predict a reduction in employment and wages, while sticky wages predict an increase in employment, and of course an increase in real wage that directly follows the productivity level.
  - The price level prediction also differs. In the sticky prices model, by assumption, P_1 will not respond to a productivity shock because it is fixed in P_0. By contrast, the price level should fall, if predicted by sticky wages model.
- **Different predictions due to monetary shock:** When exposed to a monetary shock, the sticky prices and wages models goods and money markets react similarly, however, the labor

market exposes differences. The accuracy of both predictions can be tested by comparing the data driven IRFs to monetary shocks on this variables.

  - In sticky prices, a monetary shock increase consumption implies higher employment and higher wages. In sticky wages, the real wage is fixed at A_1, so although employment increases to produce more output, it is demanded at constant wage A_1. Then, the response of wages should be positive in sticky prices, and flat in sticky wages.

- **Moments in the data:** The previous difference can be tested by studying the following time series moments

  - Inflation's volatility (variance) and persistence (autocorrelation). Higher volatility/smaller persistence is consistent with sticky wages, less volatility and higher persistence, sticky prices.
  - Output and employment volatility and persistence. Since output and employment not only react to monetary but also to productivity shocks in the sticky wages model, we can expect this model to be consistent with higher volatility and less persistance in this time series, compared to the sticky prices model. Specially if the believe technology shocks are relevant.