1. Viscous terms in the 3D dynamics corotational formulation

We add a viscous term $\boldsymbol{F}_v$ that acts as an external force. At the discrete nodal level this gives the following equilibirum equation

$$\boldsymbol{R}(\boldsymbol{d}, \dot{\boldsymbol{d}}, \ddot{\boldsymbol{d}}, t) = \boldsymbol{T}_{int} + \boldsymbol{T}_k - \boldsymbol{F}_v = \boldsymbol{0} \tag{1}$$

where we have omitted body, contact forces and any other external forces for simplicity. The viscosity term is proportional to the translation and rotational velocities. We just readapt equation (98) of the beam contact paper, i.e.

$$\boldsymbol{F}_v = -\mu_v \boldsymbol{E} \int_0^{l_0} (A_\rho \boldsymbol{H}_1^T \boldsymbol{R}_e^T \dot{\boldsymbol{u}}_0 + \boldsymbol{H}_2^T \bar{\boldsymbol{I}}_\rho \boldsymbol{R}_e^T \dot{\boldsymbol{w}}_0)\, ds \tag{2}$$

please notice the cancellation of the negative sign when inserting equation (2) into (1). Also, the parameter μ_v can be kept at low values (I think between 0 and 1), as A_ρ and $\bar{\boldsymbol{I}}_\rho$ take care of the scaling through the material parameters. Using the same approximation as in equation (101) of the beam contact paper, we can derive the variation of $\boldsymbol{F}_v$ by ignoring the contribution from the displacements, i.e.

$$\delta \boldsymbol{F}_v \simeq \boldsymbol{C}_v \delta \dot{\boldsymbol{d}}, \tag{3}$$

which yields

$$\boldsymbol{C}_v = -\mu_v \boldsymbol{E} \left(\int_{l_0} A_\rho \boldsymbol{H}_1^T \boldsymbol{H}_1 + \boldsymbol{H}_2^T \bar{\boldsymbol{I}}_\rho \boldsymbol{H}_2 \, ds \right) \boldsymbol{E}^T = -\mu_v \hat{\boldsymbol{M}} \tag{4}$$

, with $\hat{\boldsymbol{M}}$ being the two first terms of the mass matrix (see equation (102) in beam contact paper). Again, keeping in mind the cancellation of the negative sign, this yields the following tangent matrix (see equation (142) in beam contact paper):

$$\tilde{\boldsymbol{K}} = (1+\alpha)\boldsymbol{K}_{int} + \frac{1}{\beta \Delta t^2} \boldsymbol{M} + \frac{\gamma}{\beta \Delta t} (\boldsymbol{C}_k + \mu_v \hat{\boldsymbol{M}}), \tag{5}$$

therefore this is very similar to the Rayleigh damping, just keeping in mind that we keep the matrix $\boldsymbol{C}_k$ as it was and we need to use the force vector $\boldsymbol{F}_v$.