Metal additive manufacturing processes

Local monitoring and defects prediction via Bayesian approach

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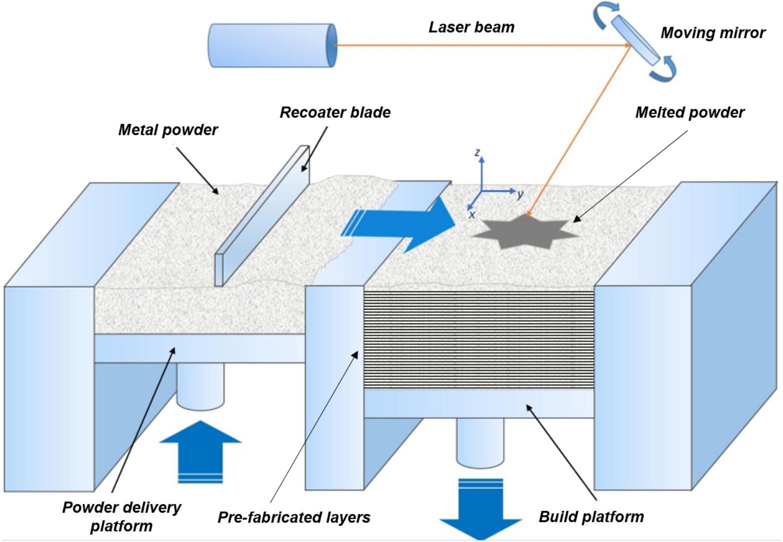
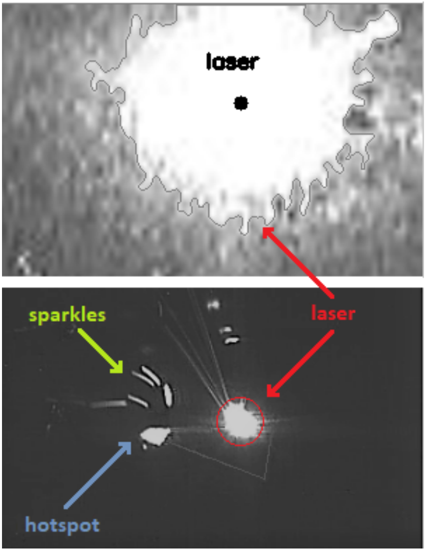
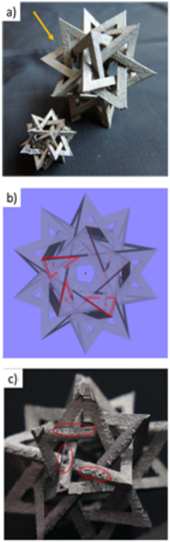


Laser beam
Moving mirror
Recoater blade
Metal powder
Melted powder
z
y
x
Powder delivery platform
Pre-fabricated layers
Build platform

1. SELECTIVE LASER MELTING

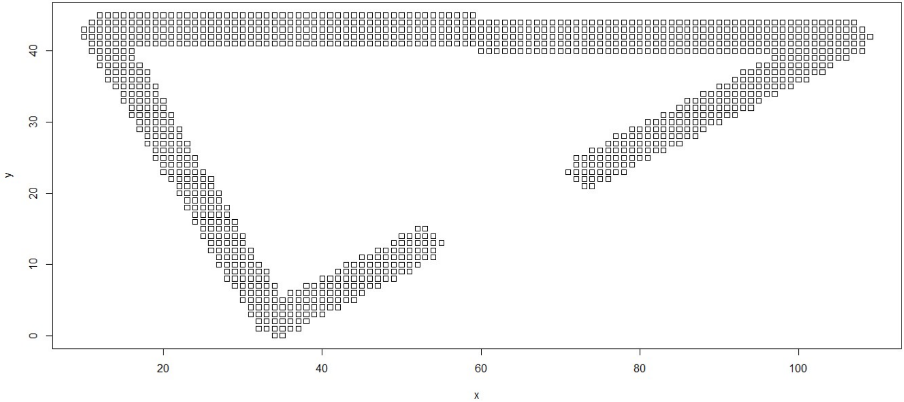
1.2 Images Acquisition and Preprocessing Phase





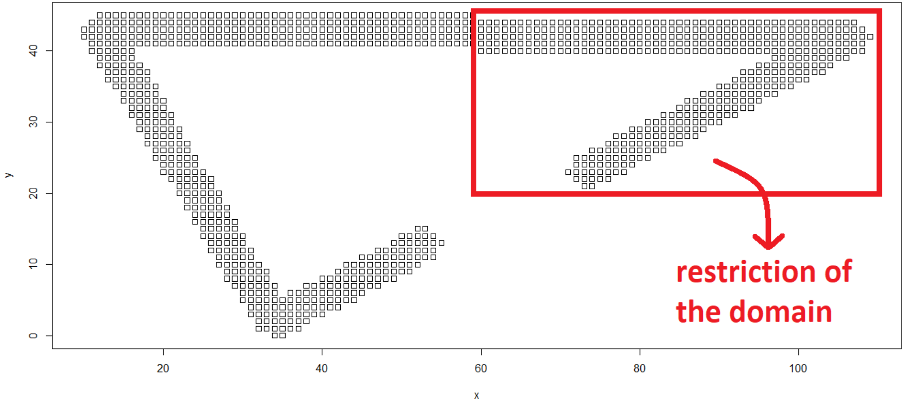
(BIG) COMPUTATIONAL PROBLEM:

Intensity values related to **996 pixels** ⇒ TOO LARGE AMOUNT OF DATA



(BIG) COMPUTATIONAL PROBLEM:

Intensity values related to ~~**996 pixels**~~ $\Rightarrow$ **420 pixels**



PROBLEM:

Simulate **missing values** (NA=sparkles), for *each* pixel

- **Variables:** $(\mathtt{V_{p1}}, \mathtt{V_{p2}}, ..., \mathtt{V_{pN}}) \quad \forall p = 1, ..., 996, \; N = 329$
- **Likelihood:** $\mathtt{V_{p1}}, ..., \mathtt{V_{pN}} | \mu_p \overset{\text{iid}}{\sim} \mathcal{N}(\mu_p, \sigma_p^2)$
- **Prior distributions:**

$$\mu_p \sim \mathcal{N}\left(\mu_{0_p}, \tau^2\right)$$

where:

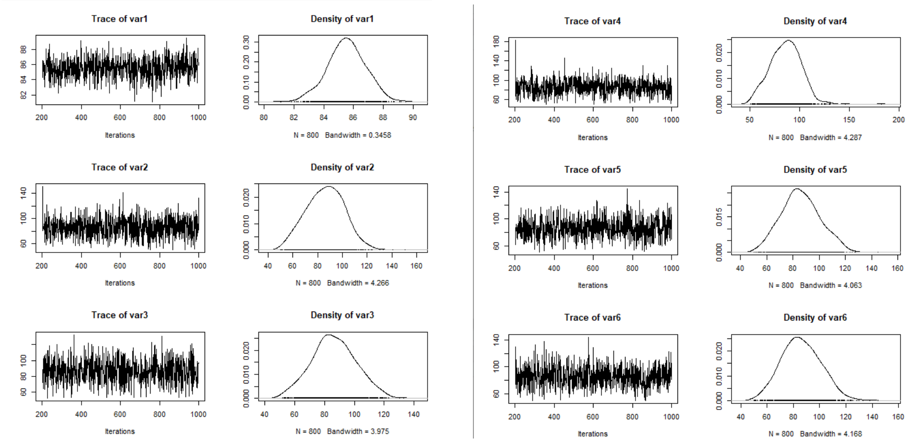
$\mu_{0_p} = \bar{\mathtt{V}}_\mathtt{p}$,

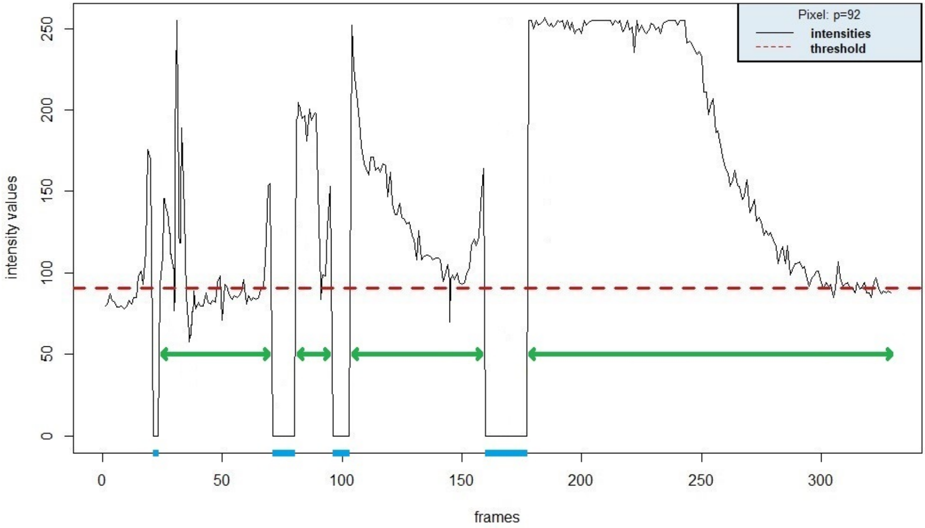
$\sigma_p^2 = Var(\mathtt{V_p})$

τ^2 fixed

- **Posterior distributions:**

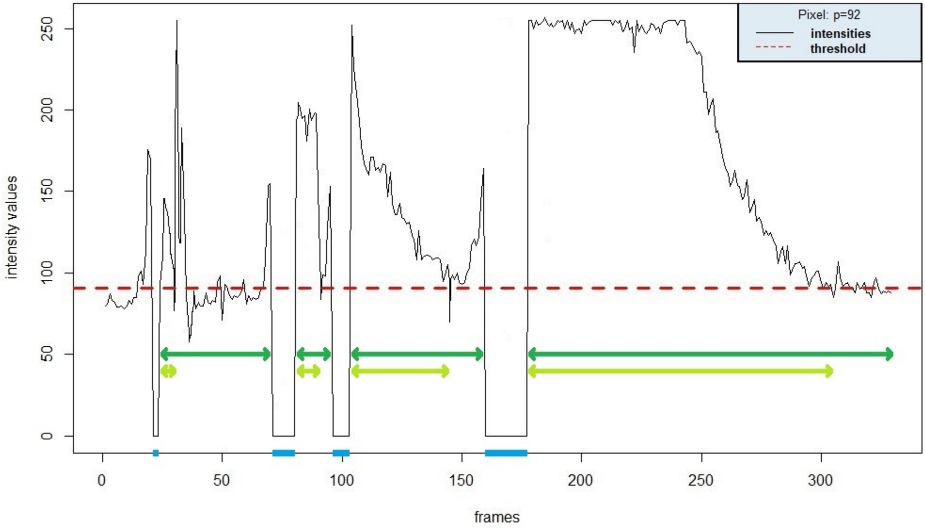
$$\mu_p \,|\, \mathtt{V}_{\mathtt{p1}}, \dots, \mathtt{V}_{\mathtt{pN}} \sim \mathcal{N}\left(\mu_{1_p}, \tau_1^2\right)$$



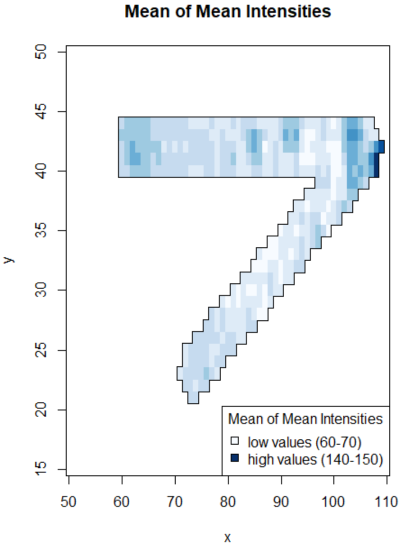


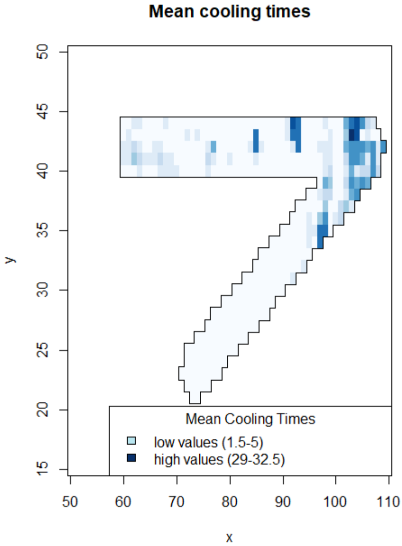
Pixel: p=92
intensities
threshold
intensity values
0
50
100
150
200
250
0
50
100
150
200
250
300
frames

2. DATASET
2.3 Data extraction



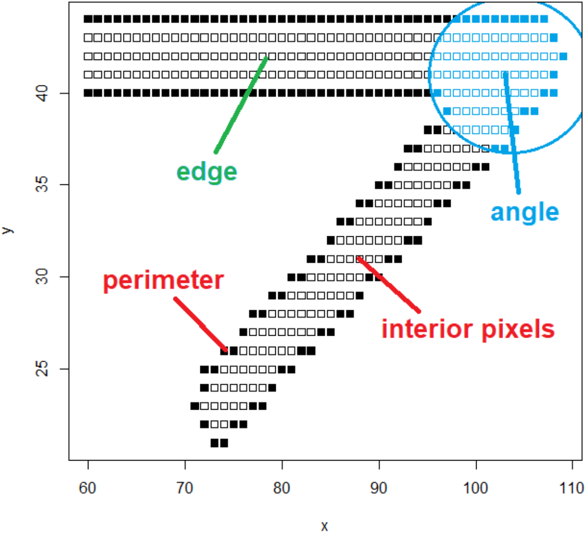
- $\bar{V}_{pt}$
- T_{pt}
- L_{pt}
- N_{pt}





Geometrical properties:

- G_{1_p}
- G_{2_p}



BAYESIAN GAUSSIAN HIERARCHICAL MODEL

- 420 groups (one for each pixel)
- each group has $n_{p,TOT}$ observations
- response variable: $\boldsymbol{V}_p$
- covariates: $\boldsymbol{x}_p = [\mathbf{1}, \mathrm{N_p}, \mathrm{L_p}, \mathrm{T_p}]$

For each **fixed** pixel $p = 1, ..., 420$:

$$\boldsymbol{V}_p = \boldsymbol{x}_p^t \boldsymbol{\beta} + \boldsymbol{\epsilon}_p \quad \boldsymbol{\epsilon}_p \sim \mathcal{N}(\boldsymbol{0}, Q_p)$$

$$\boldsymbol{V}_p | \boldsymbol{\beta}, Q_p \overset{\text{ind}}{\sim} \mathcal{N}_{n_p}\left(\boldsymbol{x}_p^t \boldsymbol{\beta}, Q_p\right)$$

$$\pi(\boldsymbol{\beta}, Q_p, \delta_p) = \pi(\boldsymbol{\beta})\pi(Q_p|\delta_p)\pi(\delta_p)$$

$$\boldsymbol{\beta} \overset{\text{iid}}{\sim} \mathcal{N}_4\left(\boldsymbol{0}, B\right)$$

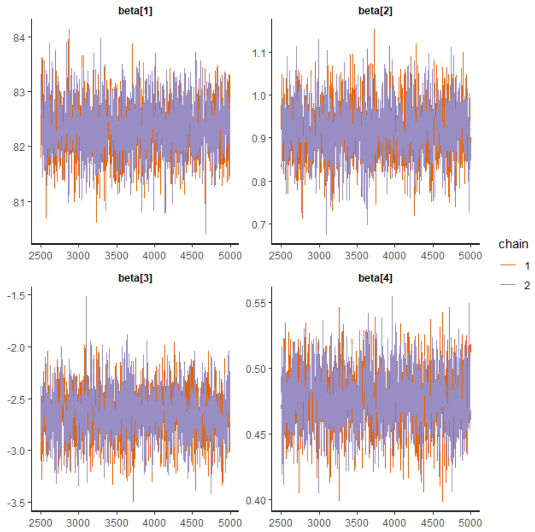
$$Q_p | \delta_p \sim \delta_p IW\left(\eta_0, \frac{1}{\sigma_0^2} I_{n_p}\right) + (1 - \delta_p)\sigma_0^2 I_{n_p}$$

$$\delta_p | \xi_p \sim Be(\xi_p)$$

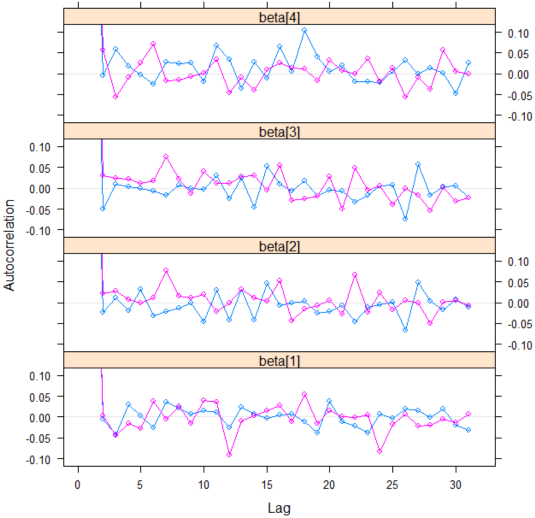
$\xi_p = \Lambda(\mathtt{G_p}^t \gamma)$: **logit model**

η_0, σ_0^2, B: fixed **hyperparameters** (using frequentist estimates)

- Model implemented in `RStan`
- Hence, δ_p integrated out
  $\Rightarrow$ direct use of ξ_p in the expression of the mixture on $Q_p|\delta_p$
- iterations: 2000
- thinning: 2
- chains: 2

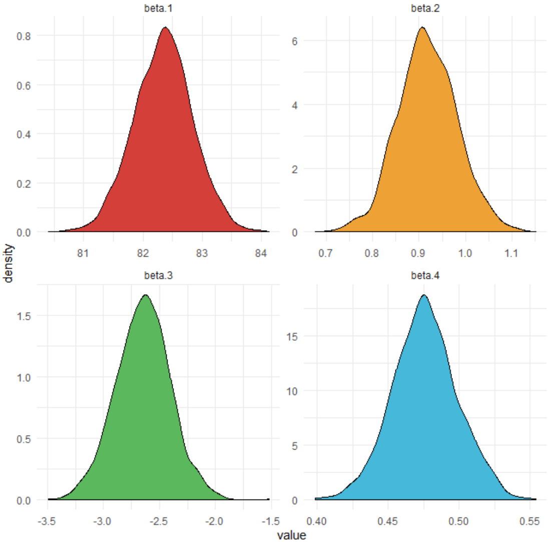


beta[1]
84
83
82
81
2500
3000
3500
4000
4500
5000
beta[2]
1.1
1.0
0.9
0.8
0.7
2500
3000
3500
4000
4500
5000
beta[3]
-1.5
-2.0
-2.5
-3.0
-3.5
2500
3000
3500
4000
4500
5000
beta[4]
0.55
0.50
0.45
0.40
2500
3000
3500
4000
4500
5000
chain
1
2


beta[4]
beta[3]
beta[2]
beta[1]
Autocorrelation
Lag
0.10
0.05
0.00
-0.05
-0.10
0
5
10
15
20
25
30

3. FIRST MODEL PROPOSAL

3.3 Results



LONGITUDINAL DATA (AR(1))

- each observation $\bar{\mathtt{V}}_{pt}$ depends *only* on the previous one
- $p = 1, \ldots, 420$ and $t = 1, \ldots, n_{p,TOT}$

$$\bar{\mathtt{V}}_{\mathtt{pt}} = \alpha_p \bar{\mathtt{V}}_{\mathtt{p,t-1}} + \epsilon_{pt}, \quad \epsilon_{pt} \overset{\text{iid}}{\sim} \mathcal{N}(0, \sigma^2)$$

$$
\begin{cases}
\bar{\mathtt{V}}_{\mathtt{p1}} \sim \mathcal{N}_1(\mu_0, \sigma^2) \\
\bar{\mathtt{V}}_{\mathtt{p2}} | \bar{\mathtt{V}}_{\mathtt{p1}} \sim \mathcal{N}_1(\alpha_p \bar{\mathtt{V}}_{\mathtt{p1}}, \sigma^2) \\
\bar{\mathtt{V}}_{\mathtt{p3}} | \bar{\mathtt{V}}_{\mathtt{p2}} \sim \mathcal{N}_1(\alpha_p^2 \bar{\mathtt{V}}_{\mathtt{p2}}, \sigma^2) \\
\qquad \vdots \\
\bar{\mathtt{V}}_{\mathtt{p},\mathtt{n}_{\mathtt{p},\mathtt{TOT}}} | \bar{\mathtt{V}}_{\mathtt{p},\mathtt{n}_{\mathtt{p},\mathtt{TOT}}-1} \sim \mathcal{N}_1(\alpha_p^{n_{p,TOT}-1} \bar{\mathtt{V}}_{\mathtt{p},\mathtt{n}_{\mathtt{p},\mathtt{TOT}}-1}, \sigma^2)
\end{cases}
$$

$$\Downarrow$$

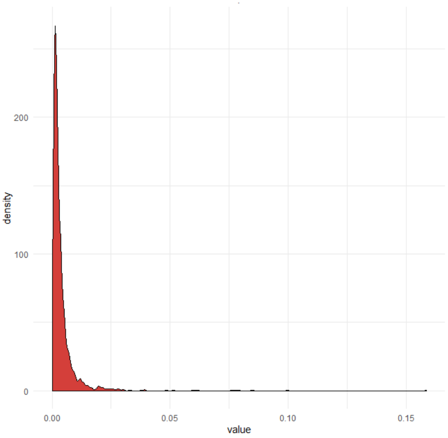
$$\underline{\bar{\mathtt{V}}}_{\mathtt{p}} | \alpha_p, \sigma^2 \sim \mathcal{N}_{n_{p,TOT}}(\boldsymbol{\mu}_p, V_p)$$

$$\alpha_p = \frac{2}{1+e^{-\xi_p}} - 1 = \tanh \frac{\xi_p}{2}$$

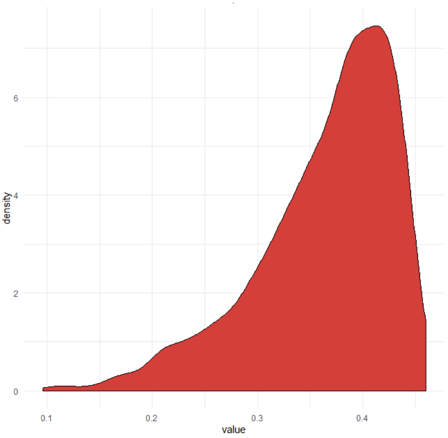
$$\boldsymbol{\mu}_p = \begin{bmatrix} \mu_0 \\ \alpha_p \mu_0 \\ \alpha_p^2 \mu_0 \\ \vdots \\ \alpha_p^{n_{p,TOT}-1} \mu_0 \end{bmatrix} \quad \text{and} \quad V_p \overset{\text{sym}}{=} \begin{bmatrix} \sigma^2 & \alpha_p & \alpha_p^2 & \alpha_p^3 & \dots & \alpha_p^{n_{p,TOT}-1} \\ & \sigma^2 & \alpha_p & \alpha_p^2 & \dots & \alpha_p^{n_{p,TOT}-2} \\ & & \sigma^2 & \alpha_p & \dots & \alpha_p^{n_{p,TOT}-3} \\ & & & \ddots & \ddots & \vdots \\ & & & & \ddots & \alpha_p \\ & & & & & \sigma^2 \end{bmatrix}$$

μ_0 and σ^2: fixed **hyperparameters**

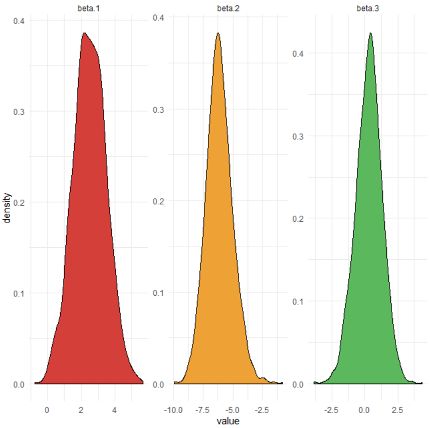
α_{147} ($p = 147$: 'defective' pixel)



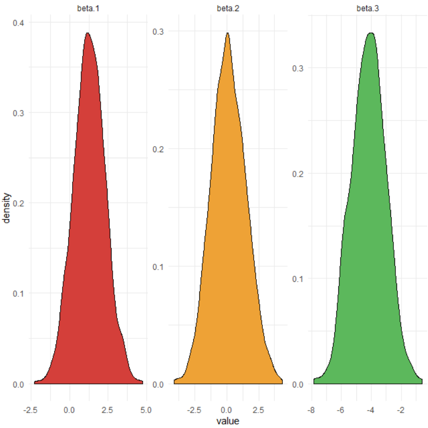
α_{188} ($p = 188$: 'normal' pixel)



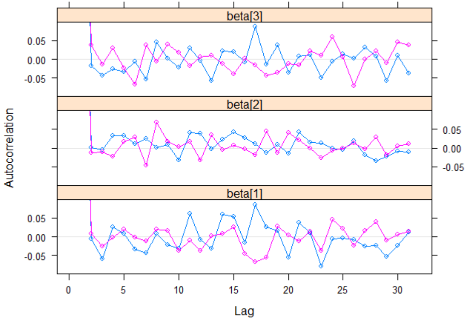
logit parameters for $p = 147$



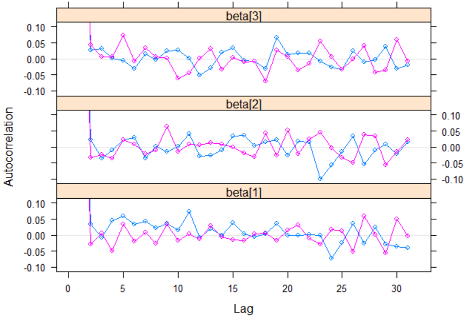
logit parameters for $p = 188$



autocorrelation plot for $p = 147$



autocorrelation plot for $p = 188$



5. BIBLIOGRAPHY

- Dataset provided by Matteo Bugatti (Mechanical Engineer and PhD student at Polimi and ESA)

AddMe.Lab

- M. Grasso, V. Laguzza, Q. Semeraro, B. M. Colosimo, *In-Process Monitoring of Selective Laser Melting: Spatial Detection of Defects Via Image Data Analysis*, 2017