

* 24.06.2021

Tarefa básica

1) $A = \begin{bmatrix} x & 1 \\ 5 & 3 \end{bmatrix}$ $B = \begin{pmatrix} 3 & -1 \\ y & 2 \end{pmatrix}$ e $B' = \begin{pmatrix} x & 1 \\ 5 & 3 \end{pmatrix}$

B) $\begin{bmatrix} 3 & -1 \\ y & 2 \end{bmatrix}$ $x=2$, pois B_{22} era sua região inicial e $y=-5$, pois B_{21} é uma região final, sendo negativo, pois o sinal dele assim: não trocamos

$x+y \rightarrow 2+(-5)$
 $x+y = -3$ - letra C

2) $A = \begin{bmatrix} 1 & 0 & 1 \\ k & 1 & 3 \\ 1 & k & 3 \end{bmatrix}$ $A = \begin{bmatrix} 1 & 0 & 1 \\ k & 1 & 3 \\ 1 & k & 3 \end{bmatrix}$

$A = 1 \cdot \begin{bmatrix} 1 & 3 \\ k & 3 \end{bmatrix} - 0 \cdot \begin{bmatrix} k & 3 \\ 1 & 3 \end{bmatrix} + 1 \cdot \begin{bmatrix} k & 1 \\ 1 & k \end{bmatrix}$

$A = 3 - 3k + 0 + k^2 - 1$

$A = 2 - 3k + k^2$ - letra C

3) $A = \begin{pmatrix} 3 & 5 \\ 2 & 4 \end{pmatrix} \rightarrow \det A = 12 - 10 = 2$

$A' = \begin{pmatrix} 4 & -5 \\ 2 & 5 \end{pmatrix} \div 2$

અન્યથા B ને:

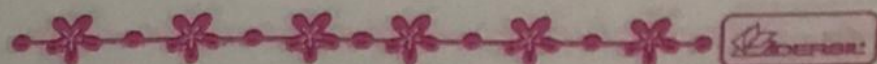
$B = \begin{pmatrix} 2 & -\frac{5}{2} \\ -1 & \frac{3}{2} \end{pmatrix} \det B = C$

4) $A = \begin{vmatrix} x & 1 & 2 & x & 1 \\ 3 & 1 & 2 & 3 & 1 \\ 10 & 1 & 2 & 10 & 1 \end{vmatrix}$ $\Delta = x^2 - 5x + 6$
 $\Delta = -5^2 - 4 \cdot 1 \cdot 6$
 $\Delta = 25 - 24$
 $\Delta = 1$

$x = \frac{5 \pm \sqrt{1}}{2} = \frac{5 \pm 1}{2}$
 $x \neq 3$
 $x \neq 2$

6) $(XA)^T)^T = B^T \rightarrow XA = B^T$
 $XA A^{-1} = B^T A^{-1} \rightarrow XT = B^T A^{-1}$
 $X = B^T A^{-1}$

det B



$$2+2+2=6$$

5) ~~$$\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 1 & 1 \\ 2 & 1 & -2 & 2 & 1 & 1 \\ 1 & 1 & -1 & 1 & 1 & 1 \end{array}$$~~
$$= 7 - 6 = 1$$

~~$$1+2+4=7$$~~

$$\begin{array}{ccc|ccc} -1 & -1 & 2 & 1 & 0 & 0 \\ 2 & 1 & -2 & 0 & 1 & 0 \\ 1 & 1 & -1 & 0 & 0 & 1 \end{array}$$

$$\begin{array}{l} (-1) \\ \downarrow \\ \downarrow \end{array} \begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & 0 \\ 2 & 1 & -2 & 0 & 1 & 0 \\ 1 & 1 & -1 & 0 & 0 & 1 \end{array}$$

$$\begin{array}{l} (-2) \\ \downarrow \\ \downarrow \end{array} \begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & 0 \\ 2 & 1 & -2 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & -1 & 0 \end{array}$$

$$\begin{array}{l} (-1) \\ \downarrow \\ \downarrow \end{array} \begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & -2 & -2 & -1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 \end{array}$$

$$\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & -1 & 2 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{array}$$

$$\begin{array}{ccc|ccc} -1 & -1 & 2 & 1 & 1 & 0 \\ 2 & 1 & -2 & 0 & -1 & 2 \\ 1 & 1 & 1 & 1 & 0 & 1 \end{array}$$

$$\begin{array}{ccc|ccc} 0 & 0 & 2 & & & \\ 2 & 0 & 0 & & & \\ 2 & 1 & 0 & & & \end{array}$$

Let's B

7) $\begin{vmatrix} 4 & 5 \\ 5 & 6 \end{vmatrix} = 24 - 25 = -1$

$\begin{vmatrix} 6 & -5 \\ -5 & 4 \end{vmatrix} \div 1$

$\begin{vmatrix} -6 & 5 \\ 5 & -4 \end{vmatrix}$ letra D

08) $A = \begin{pmatrix} 2 & k \\ -2 & 1 \end{pmatrix}$ → Valores de k para
 $\det A = \det A^{-1}$

$\det A = 2 - (-2k)$
 $\det A = 2 + 2k$

$\det A \neq 0$

$\det A = \det A^{-1} \rightarrow \det A = \frac{1}{\det A}$

$\rightarrow \det A^2 = 1 \rightarrow \det A = \pm 1$

então

$2 + 2k = 1$ ou $2 + 2k = -1$

$2k = 1 - 2$ $2k = -1 - 2$

$k = \frac{-1}{2}$ $k = \frac{-3}{2}$

9) Resolução:

$$a) (A+B) \cdot (A-B) = A^2 - AB + BA - B^2$$

$$b) (A+B)^2 = (A+B) \cdot (A+B) = A^2 + AB + BA + B^2$$

Assim sendo:

$$A^2 + AB + BA + B^2 = A^2 + 2AB + B^2 \Leftrightarrow AB = BA$$

c) Se A for uma matriz de ordem dois, então

$$\det(-A) = (-1)^2 \cdot \det A = \det A \neq 0.$$

$$\text{Logo: } \frac{\det(A)}{\det(-A)} = \frac{\det A}{\det A} = 1$$

$$d) \text{ Se B for a inversa de A, então } \det(AB) = 1 \rightarrow \det(A) \cdot \det(B) = 1 \rightarrow \det B = \frac{1}{\det A}$$

Respostas: a) $A^2 - AB + BA - B^2$

$$b) AB = BA$$

$$c) 1$$

$$d) \det B = \frac{1}{\det A}$$