Destributed Predictive Control and Estimation

Bachelor in Aerospace Engineering

Laboratory Work

Report 1

Group 3

Afonso Magalhães, 95765

Beatriz Pedroso, 95773

Miguel Isidoro, 95834

Teresa Gonçalves, 95826

Professors

João Miranda Lemos

Pedro Batista

The group of students identified above guarantees that the text of this report and all the software and results delivered were entirely carried out by the elements of the group, with a significant participation of all of them, and that no part of the work or the software and results presented was obtained from other people or sources.

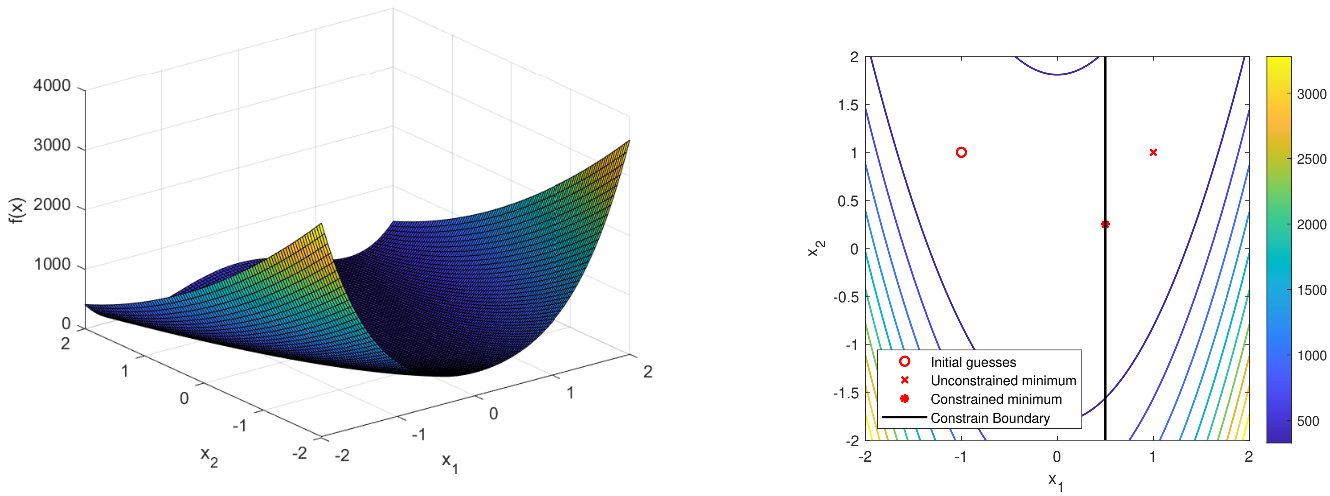
February 23, 2023

1 P1

In this part of the laboratory work the objective was to get acquainted with algorithms and software for function minimization.

1.1 Question 1

On this question, the Rosenbrock function given by $f(x) = 100\left(x_2 - x_1^2\right)^2 + (1 - x_1)^2$ will be analysed. The unconstrained minimum and the minimum that satisfies the constraint $x_1 \leq 0.5$ will be computed. It is important to notice that the domain considered was $x_1 = [-2, 2]$ and $x_2 = [-2, 2]$.



(a) 3D Representation.

(b) 2D Representation.

Figure 1: Plots of the function $f(x)$.

For both cases considered, the initial guess of the minimum was $x_0 = [-1, 1]^{\mathrm{T}}$. In figure 1a and 1b it can be observed that the Rosenbrock function is symmetric and has an unconstrained minimum in $x_1 = 1$ and $x_2 = 1$.

On the other hand, considering the constraint it is obtained the following constrained minimum $x_1 = 0.5$ and $x_2 = 0.25$.

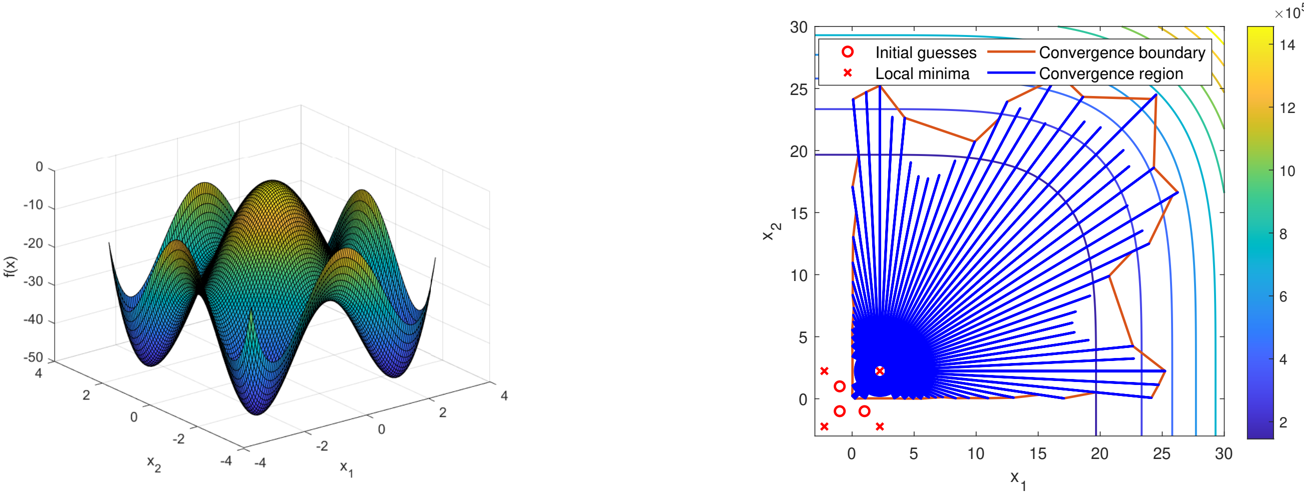
1.2 Question 2

On this question, the task was to analyze the function $f(x) = x_1^4 - 10x_1^2 + x_2^4 - 10x_2^2$. The plots obtained are shown in figure 2. The domain that was considered was $x_1 = [-3, 3]$ and $x_2 = [-3, 3]$ for the 3D plot and $x_1 = [-3, 30]$ and $x_2 = [-3, 30]$ to plot the convergence region.

Firstly, the task was to demonstrate that the function has several local minima. In fact, by observing the two plots, both show that not only the function is symmetric, but also that there is one local minima per quadrant. In figure 2b, these minima are marked with a red cross. For the upcoming considerations, it was considered the minimum with coordinates $x = [2.3361, 2.2361]^{\mathrm{T}}$, which corresponds to the minimum of the first quadrant.

Secondly, in order to find the boundary of its attraction basin, an algorithm was performed. This consisted in running the MATLAB function *fminunc* for several points and verifying if the minimum it converged to was the one in the first quadrant. These points were chosen by implementing two nested cycles, one of which increased the angle, which varied from 0 rad to 2π rad, and the other incremented the distance from the minimum taken in consideration, which commenced as a null value. Thus, these instructions allowed the verification of points in all directions and distances by using polar coordinates.

Furthermore, in the inside loop an *if* condition was created. In case the iterated point did converge to the minimum in the first quadrant, its coordinates were added to a vector that would, later on, be used to sketch the boundary, using the MATLAB function *boundary*. Otherwise, the inside loop finished.



(a) 3D Representation. (b) 2D Representation.

Figure 2: Plots of the function $f(x)$.

Nevertheless, as *fminunc* is a numerical function, the point to which it converges is not exactly the minimum, diverging a little from it. Hence, the inside loop would finish if the difference between the function output and the exact value of the minimum was greater than 0.001. It is to be noted that another value could have been chosen, which would result in a slightly different convergence zone.

The estimate of the attraction basin boundary was plotted as well, as it can be observed by the red line in figure 2b. By observing figure 2, specially the zone in blue, which corresponds to the initial values that converge to the minimum chosen, one concludes that although the method chosen by the group is efficient, it is not extremely exact.

2 P2

The main objective of this part of the laboratory work was to compare the state feedback gain obtained using the receding horizon strategy with the infinite horizon gain. In order to do that two different plants were used:

- open-loop unstable 1$^{\text{st}}$ order plant given by $x(t+1) = 1.2x(t) + u(t)$,
- open-loop stable 1$^{\text{st}}$ order plant given by $x(t+1) = 0.8x(t) + u(t)$.

It is important to notice that the following results were performed for a fixed value of $Q = 1$ and that for both plants $B = 1$.

Lastly, for all the upcoming questions, K_{LQ} will mean *linear quadratic gain* and K_{RH} will mean *receding horizon gain*.

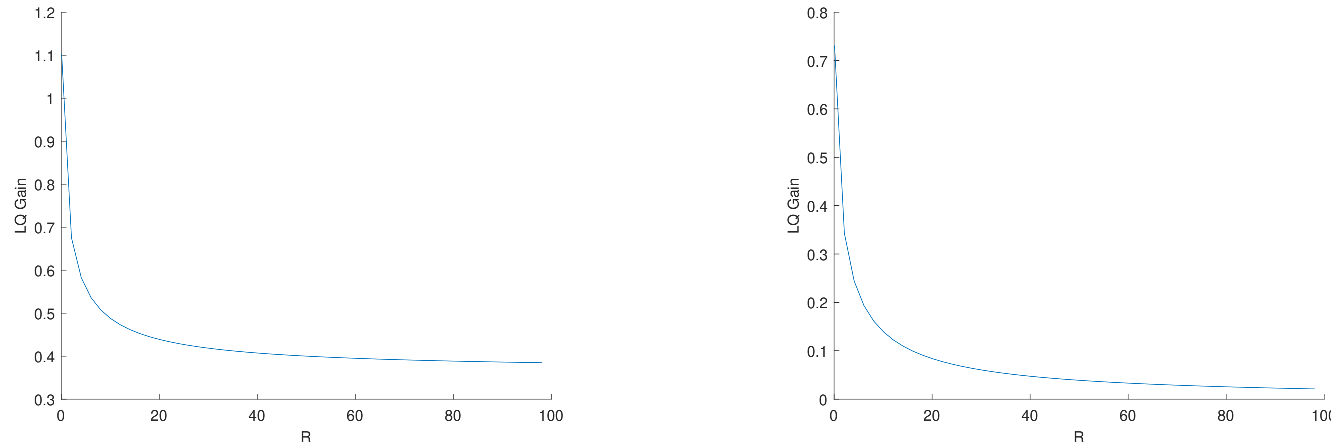
2.1 Question 1

The main goal of this question was to compute for both plants the K_{LQ}. Therefore, the MATLAB function *dlqr* was used to obtain different values of K_{LQ}, S, the positive definite solution of the Riccati equation and *lambda*, the vector of eigenvalues of the closed-loop system dynamics $A - BK_{LQ}$, being $A = 1.2$ for plant 1 and $A = 0.8$ for plant 2. Hence, as suggested in the laboratory handout, small and high values of R were tested in the range of $R = [0.1, 100]$.

In figures 3a and 3b, it can be observed that, for small values of R, there are higher values and variations of K_{LQ}, being almost vertical the slope of the curve. For increasing values of R the curve gradually drops. Both plants follow a similar behaviour.

In addition, given the optimal linear quadratic control equation $u(t) = -K_{LQ}x(t)$, it can be inferred that for a decreasing K_{LQ}, $u(t)$ also becomes lower.

Such result was expected as, from the quadratic cost function, one can conclude that for increasing values of R, the aggressiveness of the controller, u, is decreased.



(a) Plant 1.

(b) Plant 2.

Figure 3: LQ state feedback gain for varying values of R.

2.2 Question 2

In this question, the task was to compute the optimal K_{RH} for different values of the horizon, H. It is important to refer that H was tested for a range of $[1, 50]$. For each value of H, the value of R was also changed in the range of $[0.001, 1000]$. In addition, for each value of R, the values obtained for K_{RH} were superimposed with a line that corresponds to the value of K_{LQ}. It is to notice that, since H= [1, 50], the plots do not show values for H= [0, 1].

It is to be noted that the amplitude of the gains is different, being the gains of plant 2 the lowest. Such can be explained as a consequence of what was previously analysed in section 2.1. In fact, examining the figures 3a and 3b, one concludes that for each R, the correspondent K_{LQ} is dissimilar. For instance, for $R = 10$, the values of K_{LQ} for plant 1 and 2 are approximately 0.5 and 0.15, respectively.

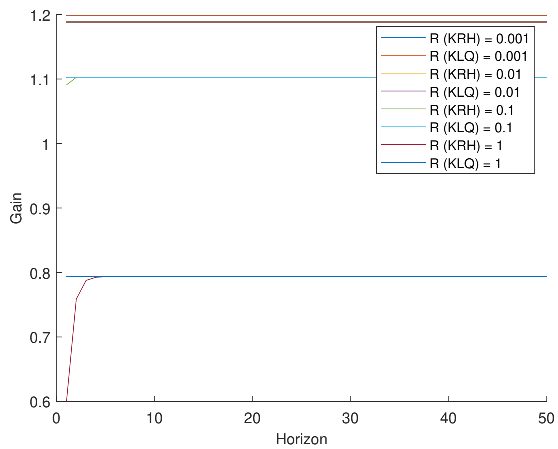
2.2.1 Plant 1

For the first plant, the results obtained are shown in figures 4a and 4b.

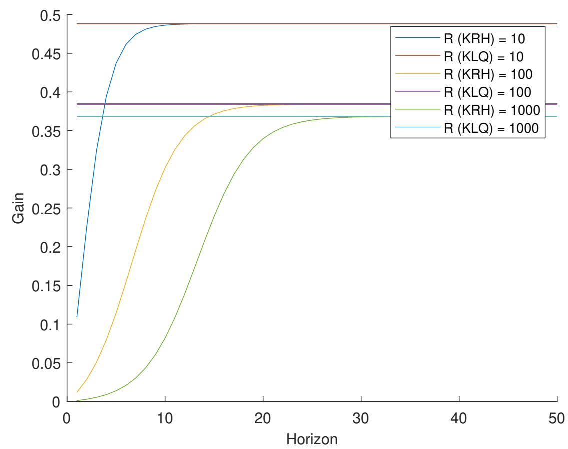
Firstly, it is clear that for every value of R, the K_{RH} tends to K_{LQ}. For lower values of R, this convergence commences for smaller horizons.

2.2.2 Plant 2

For the second plant, the results obtained are shown in figures 5a and 5b. As the behaviour of this system is extremely similar to the one studied in 2.2.1, no further comments will be made.

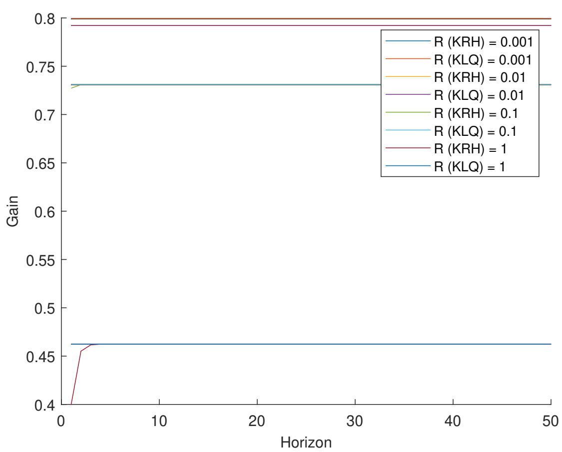


(a) Values for $R = [0.001, 0.01, 0.1, 1]$.

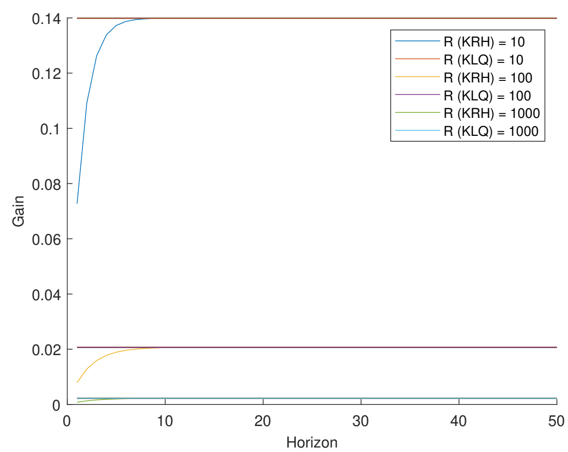


(b) Values for $R = [10, 100, 1000]$.

Figure 4: K_{RH} for different values of H superimposed with the corresponding K_{LQ} for Plant 1



(a) Values for R= $[0.001, 0.01, 0.1, 1]$.



(b) Values for $R = [10, 100, 1000]$.

Figure 5: K_{RH} for different values of H superimposed with the corresponding K_{LQ} for Plant 2

2.3 Question 3

On this chapter, a plot of the absolute value of the closed-loop eigenvalue was computed in order to compare it with the stability boundary (line that defines the stable region). In the complex plane of eigenvalues the stable region is inside the unit circle.

2.3.1 Plant 1

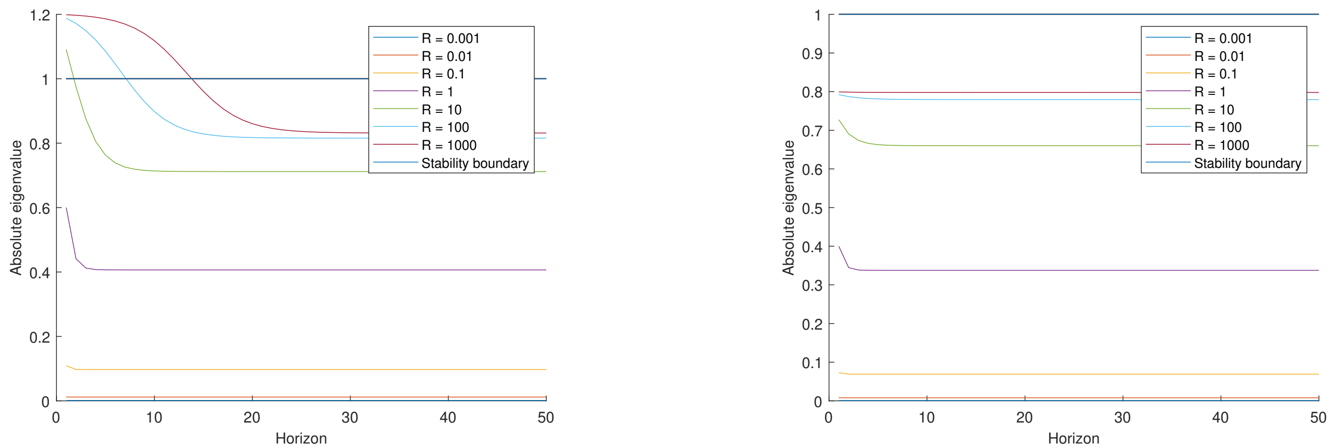
For plant 1, it can be observed in figure 6a that, for higher values of R ($R = [10, 100, 1000]$), there are some values that are outside of the stability boundary. This happens for lower values of the horizon.

On the other hand, if R is increased then from a certain value of the horizon (H) the plant is stable. For example, for $R = 1000$ the loss of stability happens if $H \in [0, 15]$ being stable for $H > 15$. The same analysis applies to the other values of R.

2.3.2 Plant 2

For plant 2, in figure 6b it is clear that for all R values tested, the absolute values of the closed-loop eigenvalue remain inside the unit circle so it means that is stable for all the horizons considered.

It is also to be noticed that for larger horizons the absolute value of the eigenvalues of the closed-loop tends to a certain value.

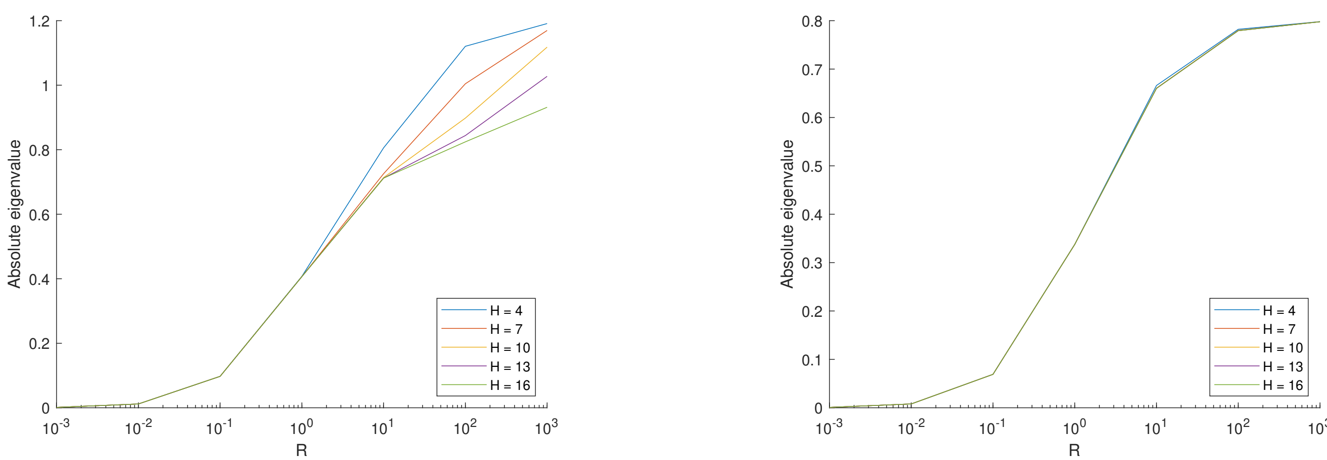


(a) Plant 1.

(b) Plant 2.

Figure 6: Absolute value of the closed-loop eigenvalue for different R values.

Also, the eignvalues were studied in function of R. The results are in picture 7a and 7b.



(a) Plant 1.

(b) Plant 2.

Figure 7: Absolute value of the closed-loop eigenvalue for different H values.

As it can be seen in figure 7a, for smaller values of R, the eigenvalues are not markedly influenced by the horizon. In figure 7b, there is no observable distance between the eigenvalues using different Hs. That goes in agreement with the conclusions obtained in figures 6a and 6b.

2.4 Question 4

The goal of this question is to compare K_{RH} and K_{LQ} and to discuss the advantages of enlarging the horizon H for plant 1 and plant 2.

Firstly, as it was previously regarded in section 2.2, the K_{LQ} is constant and K_{RH} tends to K_{LQ}, being always lower. Furthermore, for enlarged horizons, the difference between these two gains is considerably lower. Such is to expect, since LQ control optimizes in a fixed horizon, using the single (optimal) solution for the whole time horizon, whereas with MPC a new solution is computed often.

Moreover, plant 1, in comparison with plant 2, needs a higher horizon such that the difference between the gains do not have a observable difference.

2.5 Question 5

In consequence of the plots analyzed in question 3 (section 2.3), the system 1 is not always stable, which contrasts with system 2. In this way, it is straightforward to affirm that, to ensure stability with plant 1, there is a clear advantage in enlarging the horizon. In comparison with plant 2, the same cannot be said once the system is stable for every value of H that was tested.

3 P3

3.1 Question 1

The main goal of this question was to change the parameters of the MCP model and evaluate each influence of each change in the control and output.

For this, and for every variable that was changed, the others were kept at the default values, which can be observed in figure 8.

The plant that was analysed was:

$$\begin{cases} x(k+1) = 1.2x(k) + u(k), \\ y(k) = x(k). \end{cases} \quad (1)$$

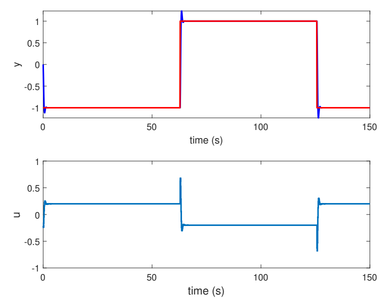
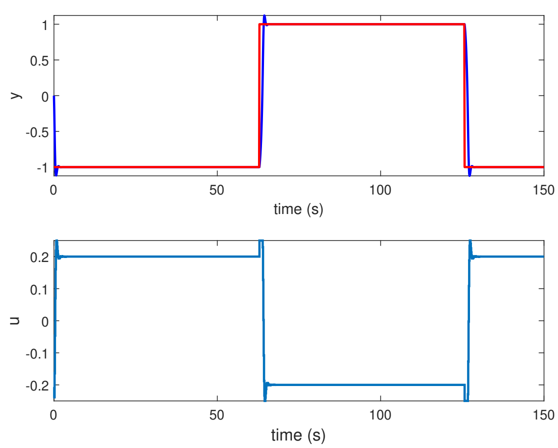


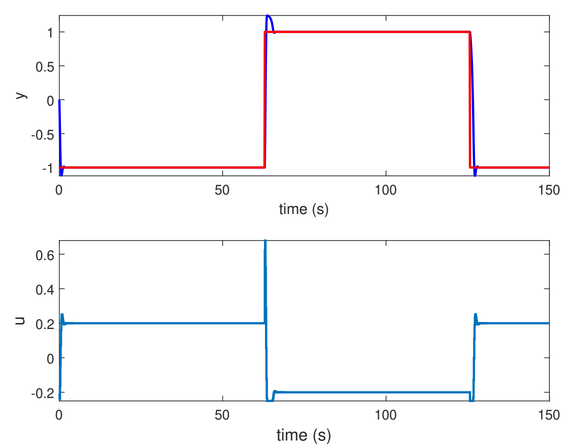
Figure 8: Default values for constraints.

3.1.1 Effect of the input

As it was tested, the system could only follow the reference for input constraint values approximately greater than 0.24 as the absolute value, for the default parameters. Thus, it was first analysed the result of imposing both maximum and minimum values of the input by 0.25. Then, so that a comparison could be established, the maximum input value was limited to 10. These situations are represented in figures 9a and 9b, respectively.



(a) u_{min} = -0.25, u_{max} = 0.25.



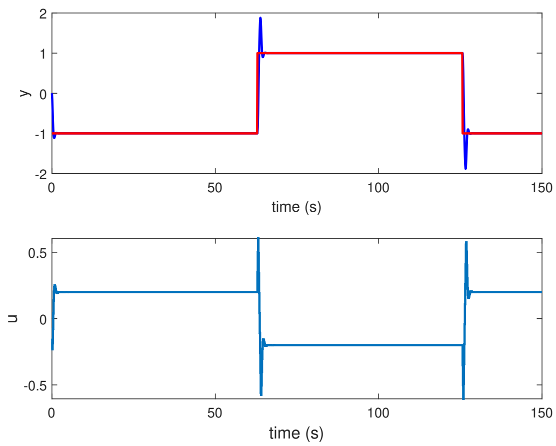
(b) u_{min} = -0.25, u_{max} = 10.

Figure 9: Constraints on the input.

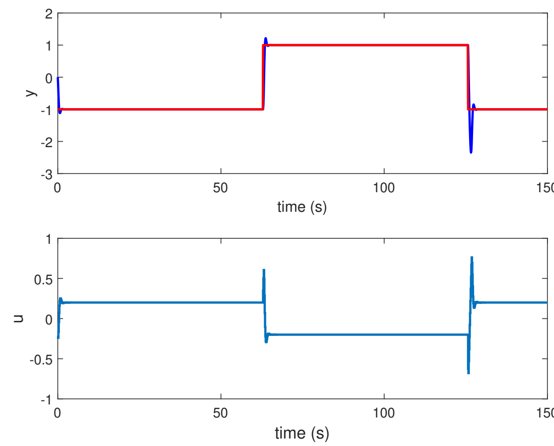
As it can be observed, from the first situation to the second, the overshoot increases significantly. Nevertheless, the peak and rise times follow the opposite tendency. There is not much influence on the settling time.

3.1.2 Effect of the input rate of change

In regards of the effect of the input rate of change, du, there are major impacts. Two constraints were analysed: the first, represented in figure 10a, has -0.15 and 0.15 as its limits and the second, represented in figure 10b, has -10 and 0.15 has its constraints.



(a) du_{min} = -0.15, du_{max} = 0.15.



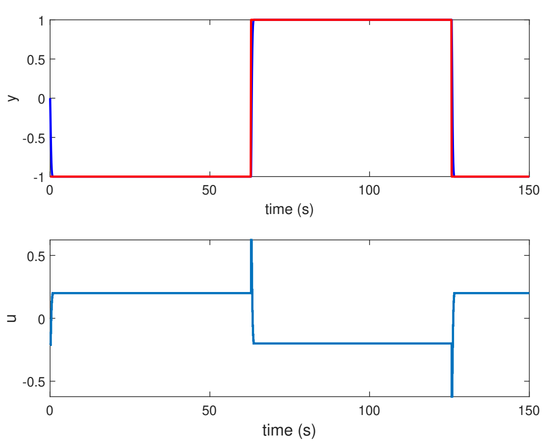
(b) du_{min} = -10, du_{max} = 0.15.

Figure 10: Constraints on the input rate of change.

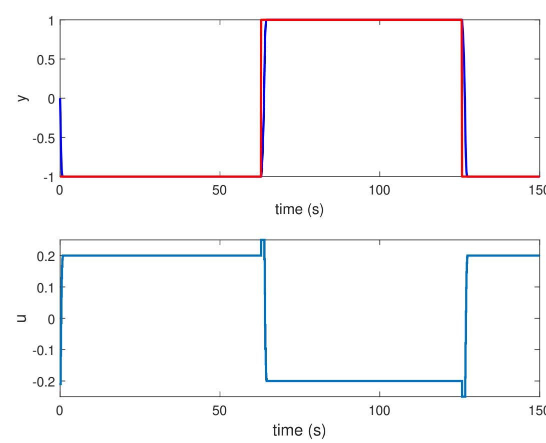
In fact, the overshoot of the output vividly decreases from the first to the second case when the reference changes from -1 to 1 and increases slightly when the opposite change of the reference occurs. This final behaviour is due to the du_{min}. Thus, if both limits of the input rate were augmented there would be a better reference tracking. The settling time remains constant through the analysis. Furthermore, as expected, the amplitude of the input is affected.

3.1.3 Effect of the state

By restricting the output constrains, which in this case is equal to restricting the state, the most evident conclusion is that by doing this, the overshoot was largely limited, which is a positive impact in comparison to the default values. Furthermore, an exception was made: the group also chose to limit the control constrains (figure 11b).



(a) z_{min} = -1, z_{max} = -1.



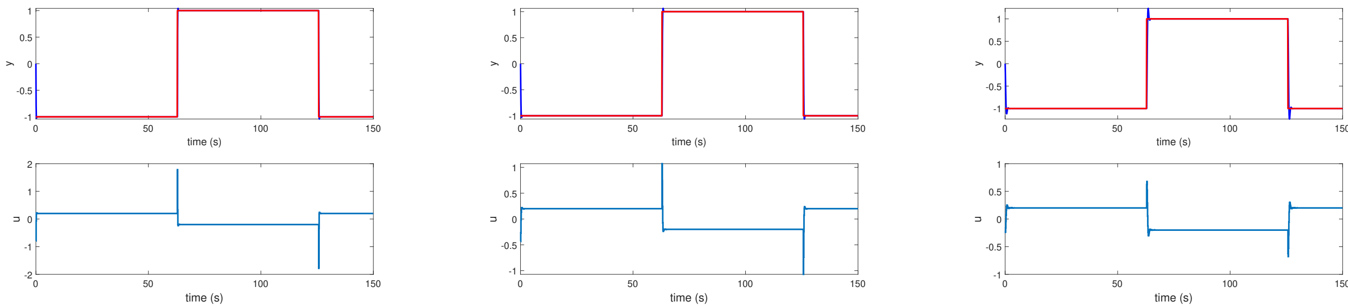
(b) z_{min} = -1, z_{max} = -1; u_{min} = -0.25, u_{max} = -0.25.

Figure 11: Effect of the state.

One can observe that by tightening the control constrains and the state constrains, the system became slower (the settling time increased).

3.1.4 Effect of the cost weights, R and Q

As it is known, from the quadratic cost function, Q defines the weights on the states while R defines the weights on the control input. As so, the group opted for testing several situations, including $R < Q$, $R = Q$ and $R > Q$. In fact, these conditions were chosen since only the ratio of R and Q have influence on the minimization of J, being the result independent of its concrete values.



(a) $Q = 1$, $R = 0.1$. (b) $Q = 1$, $R = 1$. (c) $Q = 1$, $R = 10$.

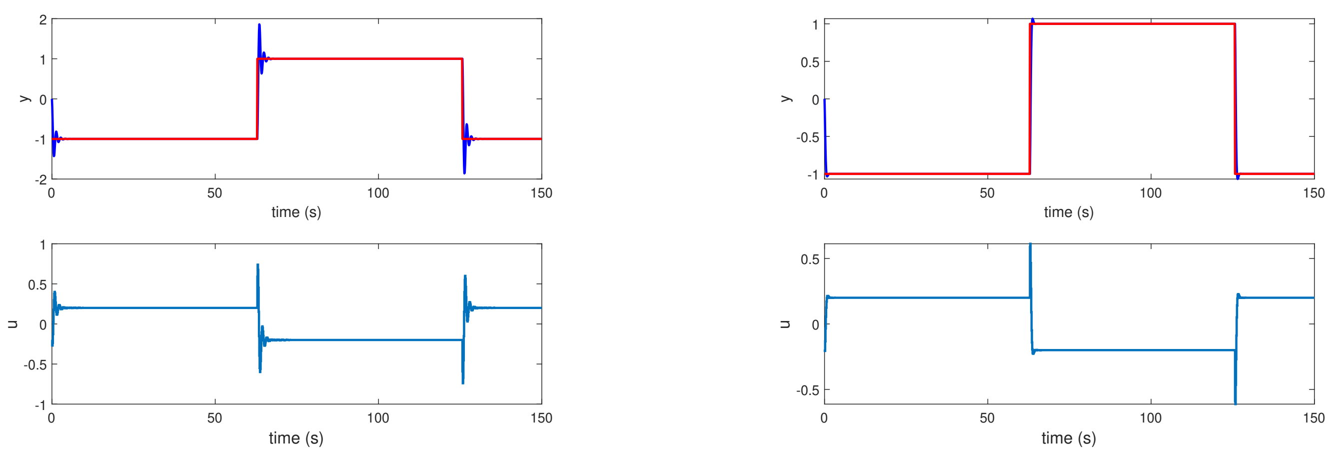
Figure 12: Effect of the variation of cost weights Q and R.

For $R < Q$, the output will follow the reference as precisely as possible. In addition, the control input amplitude is higher, as expected, since, in order to minimize the function, for lower R, the value of u has to be greater. For the situation in which $R > Q$, the aggressiveness of the control is lower, following the opposite tendency of what was previously considered. Nevertheless, the output shows oscillations and overshoot, which is not desirable.

3.1.5 Effect of the horizon

In order to evaluate the effect of the horizon in the control and output two different situations will be analyzed.

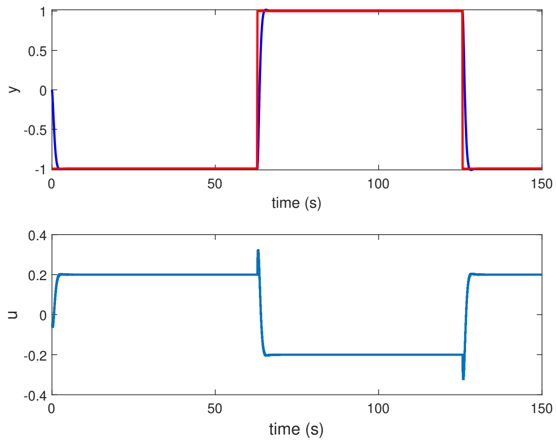
Firstly, for a $H_u = H_p$, that is the same prediction and control horizon, it can be seen in the plots 13a and 13b that enlarging the horizon allows a better reference tracking but has the advantage of costing a lot more computationally speaking. From $H_p > 10$ the variation is not notorious but with the increase of H_p, the overshoot becomes lower and has a better response.



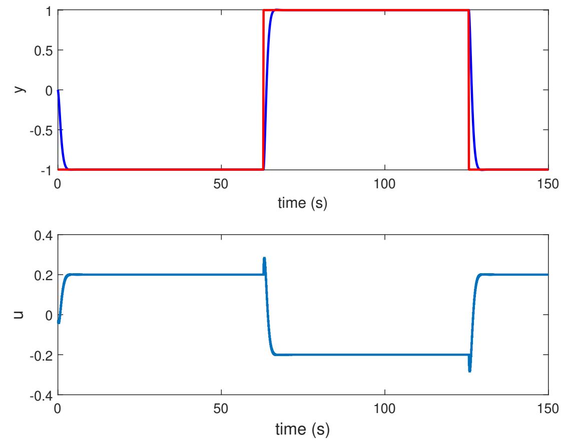
(a) $H_p = H_u = 2$. (b) $H_p = H_u = 50$.

Figure 13: Effect of the horizon for $H_p = H_u$.

Secondly, for a $H_u < H_p$, it can be seen that for a larger H_u the reference tracking gets better with almost no overshoot and oscillations for $H_u = 25$ in figure 14b.



(a) $H_p = 50$ and $H_u = 10$.



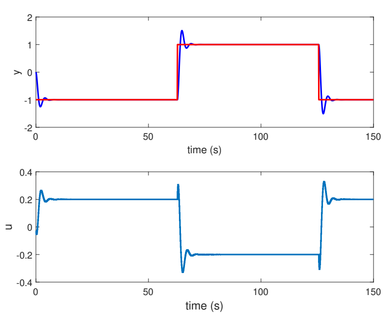
(b) $H_p = 50$ and $H_u = 25$.

Figure 14: Effect of the horizon for $H_u < H_p$.

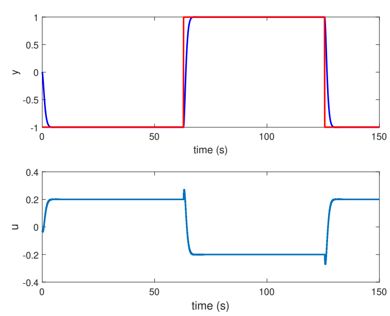
3.2 Question 2

On this question, considering a large R and moderated constraints on the control amplitude ($u_{min} = -0.4$ and $u_{max} = 0.4$), it will be analysed the influence of enlarging the prediction horizon in the performance. There are many definitions of performance but the group considered that a good performance is a consequence of a good tracking reference using a low computational load.

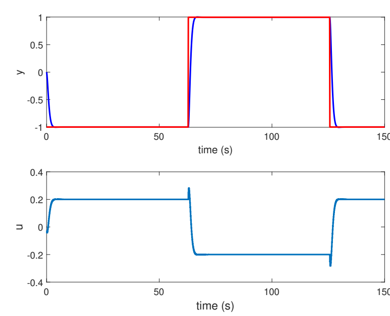
It can be observed from plots 15a, 15b and 15c that increasing the H_p allowed a better reference tracking but from a certain value of the prediction horizon ($H_p = 13$) there was not a observable difference between the oscillations. Therefore, for $H_p > 13$ the results do not diverge considerably, with the advantage of using a low computational load.



(a) $H_p = 8$.



(b) $H_p = 13$.



(c) $H_p = 50$.

Figure 15: Effect of the prediction horizon for a fixed R.