Delft University of Technology

Operations Optimisation

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Optimization model for the vehicle routing problem with drone delivery

Group 13

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1 Introduction

Environmental impact is a worldwide topic that in the past two decades has increased in popularity and also raised concerns, since the level of CO2 footprint has sky-rocketed to a peak. It is a concept that affects every industry, but the one that will be tackled in this paper is the transport industry, more specifically the delivery process.
The transport sector accounts for 20% of the total CO2 emissions [1] and is one of the most important systems that cannot be neglected when it comes to reduction of CO2 emissions. As the number of cars on the road has grown, the CO2 emissions have increased, and efforts to reduce the amount of CO2 emissions in countries around the world have intensified. Given that the delivery services are done mostly using trucks even for packages that are small in size, there is a big impact on the environment associated with it.
One solution can be the use of transportation vehicles that run on more sustainable type of fuel. For example, Amazon has already started to implement drone delivery services. It will reduce the impact on the environment by using those vehicles that run on electricity; but this solution is limited by the small battery capacity of the aircraft which means that they can only be used for short ranges.
In conclusion, the most effective way is to combine the already strong delivery system that uses trucks with the innovative and more sustainable drone usage. In this way, the trucks can focus on the delivery of the large packages while simultaneously delivering the small packages via drones. In order for this to work, the routing of the vehicles and drones needs to be optimised since the truck delivery process will be divided in two: ground and aerial vehicles. A good optimisation of the problem needs to be done in order to identify if the solution is beneficial and practical, and also if it would be a good investment or not.

2 Problem Statement

Drones have shown potential for becoming a tool in the industry of transportation since it also solves the problem with the CO2 footprint reduction simultaneously. In addition, it has developed an interest in researchers because of their high flight speed and low delivery costs [2].
To determine if this is true, a multi-objective optimization model for the vehicle routing problem with drone delivery (MOVRPDD)[2] has been proposed and solved.

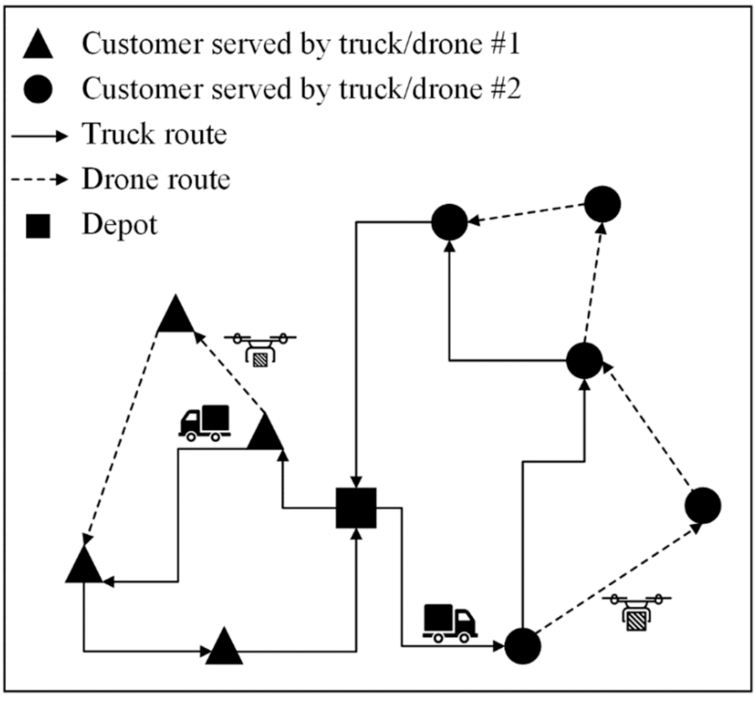

Figure 1: MOVRPDD model. [2]

The problem is intended to determine if the collaboration between trucks with high capacity of cargo and drones with small capacity of carrying products [2], can result in reducing costs and delivery times.

Figure 1 illustrates the model used for the problem. The truck fleet is leaving the depot with cargo and is also equipped with a drone. The truck has the choice of delivering the product or launch the drone to deliver it. If the second option is used, the aerial vehicle is going to return to the truck after its task is completed. Routes are determined in order to reach an optimal solution that minimizes total delivery costs and time, as well as the total distance travelled by trucks [2]. There can be multiple trucks and drones available. This number is defined by the set of vehicles, V.

The model is solved by taking into account the following assumptions:

1. Drones and trucks travel in a straight line between two nodes, and its route is measured using the Euclidean distance;
2. The drone cannot be re-launched until it is retrieved;
3. Drones cannot be launched from the trucks in the depot;
4. Every truck has to leave the depot;
5. After the drone has been launched, it has to return to the next customer the truck is going to serve;
6. Trucks cannot leave the nodes, until the corresponding drones have retrieved.

A similar problem was solved in [2] using a non-dominated sorting genetic algorithm in order to determine the optimum values. In this paper, the implementation of the problem was achieved by a combination of a programming language (Python) and MILP solver (Gurobi).

3 Mathematical Model

The mathematical model is based on the one presented in [2] but with small differences in the way the constraints are constructed. The sets, parameters and variables remain the same and they are the following:

Sets	
N:	The set of nodes representing all customers and also including the depot, N = {0,1, 2, ..., n}.
V:	The set of homogeneous trucks with drones, V = {1, 2, ..., v}.
Parameters	
q_i:	Demand for customer node i.
d_{ij}:	Euclidean distance for the truck or drone to travel from nodes i to j. If the positions of nodes i and j are (a_1, b_1) and (a_2, b_2), respectively, then $d_{ij} = \sqrt{(a_1 - a_2)^2 + (b_1 - b_2)^2}$
Q_T:	Load capacity of trucks.
Q_D:	Load capacity of drones.
C_T:	Travel cost of trucks per unit distance.
C_D:	Travel cost of drones per unit distance.
S_T:	Average travel speed of trucks.
S_D:	Average travel speed of drones.
M:	A large positive number.

Variables	
x_{ij}^v:	A binary number that equals 1 if truck v travels from node i to node j, otherwise 0; $x_{ij}^v \in \{0, 1\}$.
y_{ijk}^v:	A binary number that equals 1 if drone v (equipped on truck v) is launched at node i, serves customer at node j, and is retrieved at node k, otherwise 0; $y_{ijk}^v \in \{0, 1\}$.
t_{tvi}:	Arrival time of truck v at node i.
t_{dvi}:	Arrival time of drone v at node i.
$t_{wait,t_{vi}}$:	Interval of time the truck has to wait for the drone.
$t_{wait,d_{vi}}$:	interval of time the drone has to wait for the truck.
t_{total}:	Time each vehicle takes to finish the route.
$t_{totalmax}$:	The maximum route time of all vehicles.

The MOVRPPDD model that is evaluated in this paper has three optimization objectives, all three of them set for minimization. These are the total delivery time, total delivery cost, and distance travelled by trucks (proportional correspondence with energy consumption).

In the model, an hierarchical approach was used, as it was found to be the most suitable one. As a result, the main goal is to reduce the total delivery time the most, so that a good customer satisfaction is met. The other two goals, cost and distance (energy consumption) may be degraded. The latter objective only concerns trucks, since drones are running on electricity and their energy consumption does not have a value of interest. Its environmental impact is lower when compared to the trucks. The energy consumption of trucks can easily be calculated based on the travelled distance and the respective weight [2].

The problem formulation containing the objective functions and the constraints is presented below. Some were constructed with inspiration provided by [2]. Nonetheless, some have been originally created, changed or not included.

$$\text{minimize} \qquad f_1 = t_{totalmax}$$

$$\text{minimize} \qquad f_2 = \sum_{v \in V} \sum_{i \in N} \sum_{j \in N} x_{ij}^v d_{ij} C_T + \sum_{v \in V} \sum_{i \in N} \sum_{j \in N} \sum_{k \in N} C_D (d_{ij} + d_{jk}) y_{ijk}^v$$

$$\text{minimize} \quad f_3 = \sum_{v\in V}\sum_{i\in N}\sum_{j\in N} d_{ij}x_{ij}^v$$

subject to

$$\sum_{v\in V}\sum_{i\in N} x_{ij}^v + \sum_{v\in V}\sum_{i\in N}\sum_{k\in N} y_{ijk}^v = 1 \qquad \forall j \in (1,N), \tag{C1}$$

$$\sum_{v\in V} x_{jj}^v = 0 \qquad \forall j \in N, \tag{C2}$$

$$\sum_{v\in V}\sum_{k\in N} y_{jjk}^v = 0 \qquad \forall j \in N, \tag{C3}$$

$$\sum_{v\in V}\sum_{i\in N} y_{ijj}^v = 0 \qquad \forall j \in N, \tag{C4}$$

$$\sum_{v\in V}\sum_{i\in N} y_{iji}^v = 0 \qquad \forall j \in N, \tag{C5}$$

$$y_{0jk}^v = 0 \qquad \forall v \in V, j \in N, k \in N, \tag{C6}$$

$$\sum_{i\in N} x_{ij}^v = \sum_{i\in N} x_{ji}^v \qquad \forall v \in V, j \in N, \tag{C7}$$

$$\sum_{j\in N} x_{0j}^v = 1 \qquad \forall v \in V, \tag{C8}$$

$$\sum_{j\in N} x_{i0}^v = 1 \qquad \forall v \in V, \tag{C9}$$

$$\sum_{i\in N}\sum_{j\in N} x_{ij}^v q_j + \sum_{i\in N}\sum_{j\in N}\sum_{k\in N} y_{ijk}^v q_j \le Q_T \qquad \forall v \in V, \tag{C10}$$

$$\sum_{j\in N}\sum_{k\in N} y_{ijk}^v \le 1 \qquad \forall v \in V, i \in N, \tag{C11}$$

$$\sum_{i\in N}\sum_{j\in N} y_{ijk}^v \le 1 \qquad \forall v \in V, k \in N, \tag{C12}$$

$$y_{ijk}^v \le x_{ij}^v + M(1 - y_{ijk}^v) \qquad \forall v \in V, i \in N, j \in N, k \in N, \tag{C13}$$

$$\sum_{i\in N}\sum_{k\in N} y_{ijk}^v q_j \le Q_D \qquad \forall v \in V, j \in N, \tag{C14}$$

$$t_{tvi} \le t_{upper_i} \qquad \forall v \in V, i \in N, \tag{C15}$$

$$t_{tvi} \ge t_{lower_i} \qquad \forall v \in V, i \in N, \tag{C16}$$

$$t_{dvi} \le t_{upper_i} \qquad \forall v \in V, i \in N, \tag{C17}$$

$$t_{dvi} \ge t_{lower_i} \qquad \forall v \in V, i \in N, \tag{C18}$$

$$t_{dvi} \ge t_{tvi} - M(1 - \sum_{j\in N}\sum_{k\in N} y_{ijk}^v) \qquad \forall v \in V, i \in N, \tag{C19}$$

$$t_{dvi} \le t_{tvi} + M(1 - \sum_{j\in N}\sum_{k\in N} y_{ijk}^v) \qquad \forall v \in V, i \in N, \tag{C20}$$

$$t_{dvk} \ge t_{tvk} - M(1 - \sum_{i\in N}\sum_{j\in N} y_{ijk}^v) \qquad \forall v \in V, i \in N, k \in N, \tag{C21}$$

$$t_{dvk} \le t_{tvk} + M(1 - \sum_{i\in N}\sum_{j\in N} y_{ijk}^v) \qquad \forall v \in V, i \in N, k \in N, \tag{C22}$$

$$t_{tvj} \ge t_{tvi} + t_{wait,t_{vi}} + t_{travel,t_{ij}} - M(1 - x_{ij}^v) \qquad \forall v \in V, i \in N, j \in (1,N), \tag{C23}$$

$$t_{dvj} \ge t_{dvi} + t_{wait,d_{vi}} + t_{travel,d_{ij}} - M(1 - y_{ijk}^v) \qquad \forall v \in V, i \in N, j \in (1,N), k \in N, \tag{C24}$$

$$t_{dvk} \ge t_{dvj} + t_{wait,d_{vj}} + t_{travel,d_{jk}} - M(1 - y_{ijk}^v) \qquad \forall v \in V, i \in N, j \in N, k \in N, \tag{C25}$$

$$t_{total_v} \le t_{tvi} + t_{travel,t_{i0}} + M(1 - x_{i0}^v) \qquad \forall v \in V, i \in N, \tag{C26}$$

$$t_{total_v} \ge t_{tvi} + t_{travel,t_{i0}} - M(1 - x_{i0}^v) \qquad \forall v \in V, i \in N, \tag{C27}$$

$$t_{total_{max}} - t_{total_v} \ge 0 \qquad \forall v \in V \tag{C28}$$

Constraint C1 guarantees that all customers are either served by trucks or drones. The equations displayed in C2, C3, C4 and C5 ensure that the both trucks and drones cannot go to the same node that they departed from. Constraint C6 ensures that the drones cannot be launched from the trucks in the depot. C7 is a flow conservation equation that makes sure that the number of trucks that are arriving at a node has to be the same as the number of trucks leaving the respective node. Constraints C8 and C9 ensure that each trucks leaves the depot and arrives at the depot, respectively. Trucks cannot exceed their load capacity during the delivery as it is described in C10. Equations described by C11 and C12 ensure that the drone is launched or retrieved at most once at all customers and depot nodes. Constraint C13 describes that if the drone is launched at node i and retrieved at node k, the truck must pass through both nodes and it has to return to the node the truck is going to serve next. Each drone not being loaded beyond its load capacity during flight is ensured by C14. Constraints C15 and C16 are the time windows for trucks, and C17 and C18 are the time windows set for drones. The equations describing C19, C20, C21 and C22 guarantee that the arrival arrival times of each truck and its corresponding drone at the launch and retrieval nodes are synchronized. Constraints C23, C24 and C25 ensure that the arrival times of the trucks and drones are reasonable during movement. Waiting times were added in case the trucks had to wait for the drones to come back before resuming the route. The last three constraints: C26, C27, and C28 are computations for determining the total time taken by each set of vehicles to finish its route and the total time required for all vehicles to end the respective routes.

4 Verification

The model implemented is inspired by the previously mentioned paper [2]. Nevertheless, both the algorithm used and the constraints in the present work are slightly different. As a result, the verification procedure could not be conducted by comparing the results from the two processes.

Hence, in a first instance, only the main constraints regarding a simple vehicle routing problem were added, namely equations C1, C2, C3, C4, C5, C7, C8, and C9. In fact, several tests were carried out with reduced number of vehicles and nodes. In almost all iterations, the trucks were following an appropriate path. This could be tested by doing simple drawings in the paper and using simple maths. In some cases, there were sub-tours. The drones had to be more constrained to have the desired behaviour.

To eliminate the sub-tours, time windows were implemented (constraints C15, C16, C17, and C18). It was notorious that these intervals of time were fundamental to solve the problem, after running the algorithm with and without this constraint.

Moreover, after adding the constraints C6, C11, C12, and C13 on the drones, these were already making more adequate routes. Nonetheless, the times at which the drones left the nodes were not synchronized with the drones. This was a main issue for the objective of the model. As so, time constraints were implemented (C19, C20, C21, C22, C23, C24, and C25). These took into consideration the travelling times between nodes of each truck and drone and the time each truck had to wait for its respective drone to come back from another delivery. These constraints were verified by printing the routes performed (which were already verified), the times at which each truck and drone arrived at a certain node and the time they took to travel from one to another. After simple calculations done by a pencil and a paper, it was verified that these were working properly.

Ultimately, since every drone and truck have a load capacity and each customer have a certain demand. Constraints for this were applied (C10 and C14). In the excel file, the demand values were altered in a way that the previous optimal route could not be performed because of the load capacity of the trucks. It was, in fact, verified, that the route had to be changed, so that the constraint was properly enforced.

5 Sensitivity Analysis

In this section, an analysis of the sensitivity of the model is conducted, by changing some of the main parameters. These values can be observed in table 1. The coordinates of the nodes were generated randomly in MATLAB in a 100km by 100km square, with the depot placed in the origin.

Trucks			**Drones**		
Cost/Km	**Load Capacity [Kg]**	**Speed [Km/h]**	**Cost/Km**	**Load Capacity [Kg]**	**Speed [Km/h]**
25	500	60	1	3	80

Table 1: Main parameters of the model.

It is important to highlight that the time windows are completely relaxed in the original parameters, with lower and upper bounds of 0 and 90 hours, respectively. Furthermore, the demand of each customer is defined as in Table 2.

Node	Demand	Node	Demand
1	0.4	6	2
2	0.6	7	4
3	1	8	0.6
4	1.2	9	0.6
5	0.8	10	1

Table 2: Demand of each customer.

Moreover, along the section, the number of vehicles, the cost per km for drones and trucks, its speed, the time windows, and the hierarchical position of the objective functions are changed. All the solutions shown below are optimal, with a gap of 0%.

5.1 Number of Vehicles

For 10 nodes, the model is going to be simulated for 1, 2, 3, and 4 sets of vehicles. Each set has 1 truck and 1 drone integrated. The objectives are evaluated in the light of each scenario. Moreover, this subsection is crucial to, then, compare with the other changes made in subsections 5.2, 5.3, 5.4, and 5.5.

The optimized routes for the 4 scenarios are represented in figures 2, 3, 4, and 5. More information on these routes are made available in Tables 3 and 4.

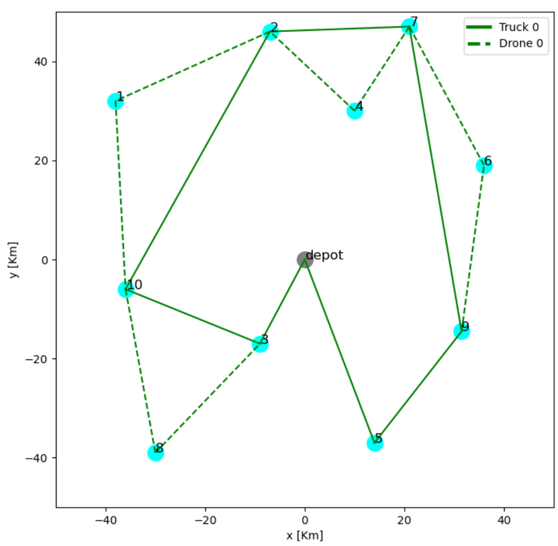

Figure 2: 10 customers served by 1 set of vehicles.

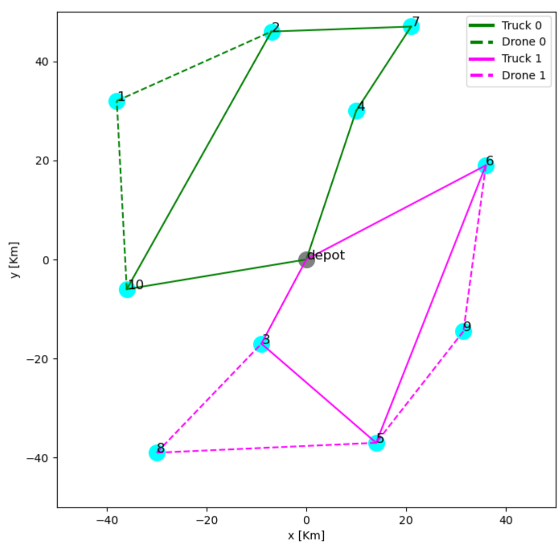

Figure 3: 10 customers served by 2 set of vehicles.

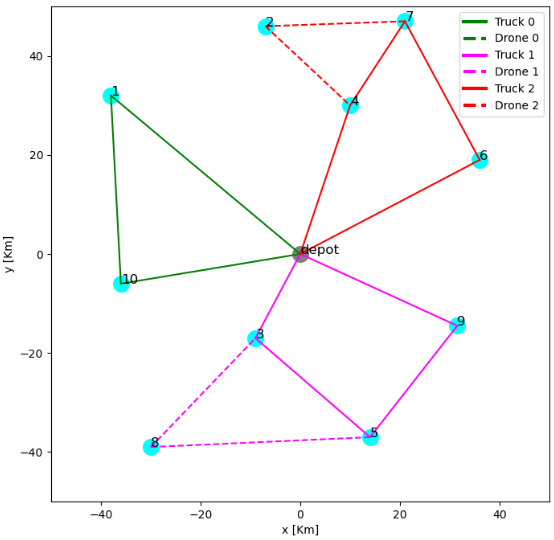

Figure 4: 10 customers served by 3 set of vehicles.

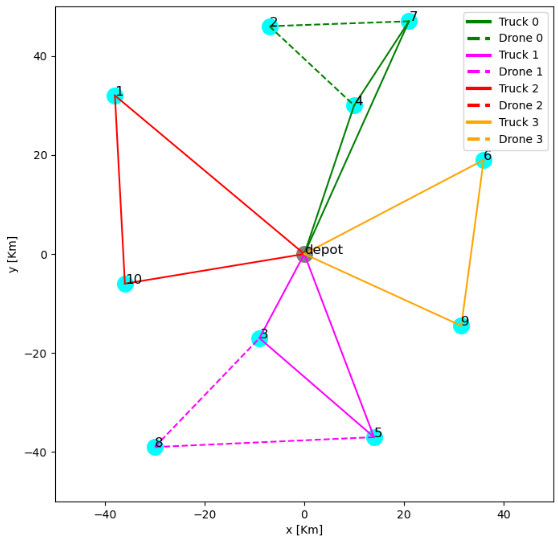

Figure 5: 10 customers served by 4 set of vehicles.

Number of the set of vehicles (V)	**V = 1**	**V = 2**
Route T: Truck D: Drone	T0 : 0 ->5 ->9 ->7 ->2 ->10 ->3 ->0 D0 : 9->6->7 // 7->4->2 // 2->1->10 // 10->8->3	T0 : 0 ->10 ->2 ->7 ->4 ->0 D0 : 10 ->1 ->2 T1: 0 ->6 ->5 ->3 ->0 D1: 6 ->9 ->5 // 5->8->3
Objective 1 (Total Time)	4.83 hours	2.93 hours
Objective 2 (Total Cost)	6905.17	8372.07
Objective 3 (Truck Distance)	266.40 Km	326.53 Km

Table 3: Routes and optimal solutions for 10 nodes served by 1 and 2 sets of vehicles, with the original parameters.

By analysing Tables 3 and 4, it is noticeable that for a greater number of vehicles the total time decreases, whilst the other two objective function values increase. This outcome is expected. By having more vehicles performing the routes, each one has to visit less customers, which reduces the delivery period. Nonetheless, with more vehicles, there are some extra kilometers that have to be travelled, since there are more trucks that need to leave and return to the depot. With fewer trucks, these could go directly visit the remaining nodes, which could be in the surroundings. As a result, since the distance rises, the cost also follows this tendency. This is due to the fact that the cost is based on the distance travelled.

Number of the set of vehicles (V)	**V = 3**	**V = 4**
Route T: Truck D: Drone	T0 : 0 ->10 ->1 ->0 T1: 0 ->9 ->5 ->3 ->0 D1: 5 ->8 ->3 T2: 0 ->6 ->7 ->4 ->0 D2: 7 ->2 ->4	T0 : 0 ->7 ->4 ->0 D0: 7 ->2 ->4 T1: 0 ->5 ->3 ->0 D1: 5 ->8 ->3 T2: 0 ->10 ->1 ->0 T3: 0 ->9 ->6 ->0
Ojective 1 (Total Time)	2.38 hours	2.07 hours
Objective 2 (Total Cost)	9162.58	10777.1
Objective 3 (Truck Distance)	361.4 Km	426.05 Km

Table 4: Routes and optimal solutions for 10 nodes served by 3 and 4 sets of vehicles, with the original parameters.

5.2 Cost per Km for Drones and Trucks

In this part of the paper, the different outcomes for changing the travel cost for trucks (C_T) and drones (C_D) shall be analyzed. Also, those scenarios were conducted for 1 and 2 vehicles.

The drone costs were changed to 25 units of cost per km and the trucks cost remained the same, to see how the routes would be changed my penalizing more the drone trips.

Figures 6 and 7 illustrate the respective constructed routes.

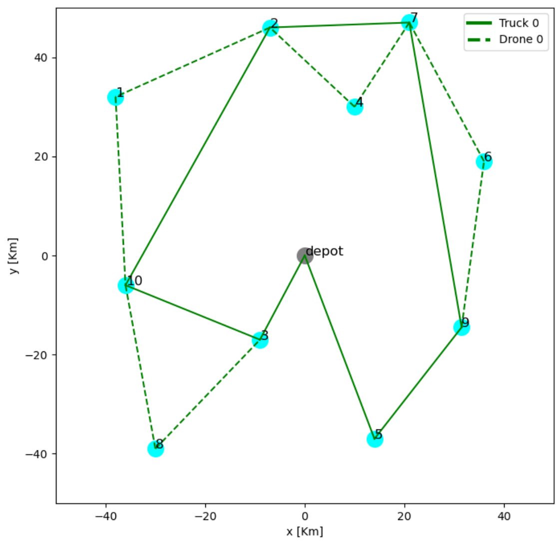

Figure 6: 10 customers served by 1 vehicle, for C_T=C_D=25.

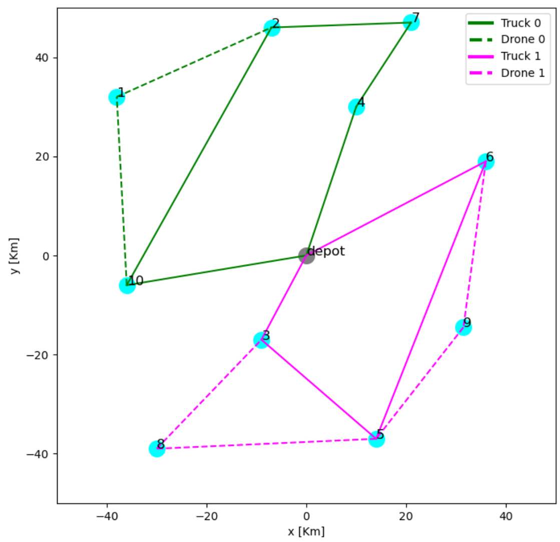

Figure 7: 10 customers served by 2 vehicles, for C_T=C_D=25.

For this scenario, the routes and optimal solutions are summarized in Table 5.

Number of the set of vehicles (V)	**V = 1**	**V = 2**
Route T: Truck D: Drone	T0 : 0 ->5 ->9 ->7 ->2 ->10 ->3 ->0 D0: 9 ->6 ->7 //7 ->4 ->2 // 2 ->1 ->10 // 10 ->8 ->3	T0 : 0 ->10 ->2 ->7 ->4 ->0 D0 : 10 ->1 ->2 T1: 0 ->6 ->5 ->3 ->0 D1: 6 ->9 ->5 // 5 ->8 ->3
Objective 1 (Total Time)	4.83 hours	2.93 hours
Objective 2 (Total Cost)	12789.24	13383.75
Objective 3 (Truck Distance)	266,40 Km	326,53 Km

Table 5: Routes and optimal solutions when C_T=C_D=25.

By analysing Tables 3 and 5, it can be observed that the routes remained the same. As so the first and third objective values remained the same. In fact, only the second one was sharply increased, as it was expected. The main reason for this outcome is based on the fact that the first minimization function has a very high importance when compared to the other two. As a consequence, with the original hierarchical order, changing the relative cost of trucks and drones would not make a marked difference.

5.3 Speed of Trucks and Drones

The purpose of this section is to determine what would be the consequences of the road being congested with traffic, which would, thus force the trucks to reduce their speed. The original parameters for the speed of trucks and drones, used in the previous sections, were 60 km/h and 80 km/h, respectively. It is assumed that the speed of the trucks and drones is constant.

Thus, the speed of the trucks was decreased from 60 km/h to 30 km/h and the outcomes, for 1 and 2 vehicles, are portrayed in Figures 8 and 9.

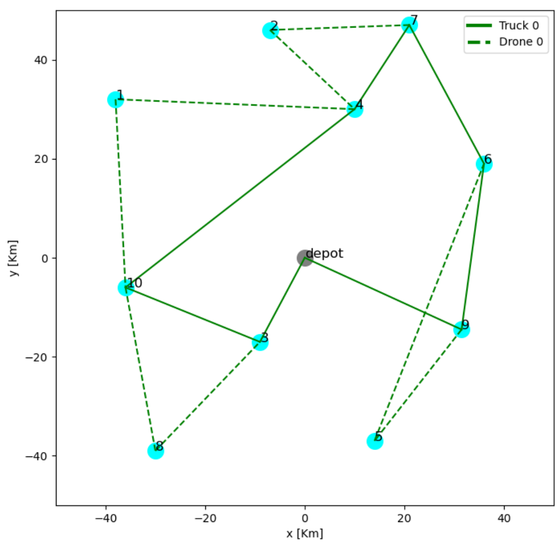

Figure 8: 10 customers served by 1 vehicle, for $S_T = 30Km/h$ and for $S_D = 80Km/h$.

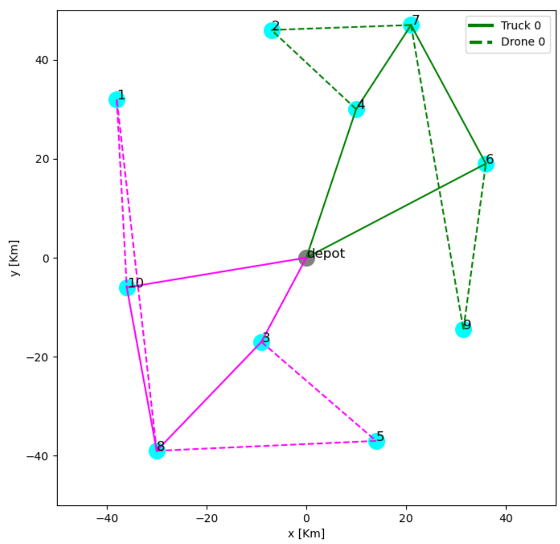

Figure 9: 10 customers served by 2 vehicles, for $S_T = 30Km/h$ and for $S_D = 80Km/h$.

The optimal solution values are displayed in Table 6.

Number of the set of vehicles (V)	**V = 1**	**V = 2**
Route T: Truck D: Drone	T0 : 0 ->9 ->6 ->7 ->4 ->10 ->3 ->0 D0: 4 ->1 ->10 //7 ->2 ->4 // 9 ->5 ->6 // 10 ->8 ->3	T0 : 0 ->4 ->7 ->6 ->0 D0 : 4 ->2 ->7// 7 ->9 ->6 T1: 0 ->3 ->8 ->10 ->0 D1: 3 ->5 ->8 // 8 ->1 ->10
Objective 1 (Total Time)	7.57 hours	4.28 hours
Objective 2 (Total Cost)	5.972,33	6.432,34
Objective 3 (Truck Distance)	227,29 Km	244,03 Km

Table 6: Routes and optimal solutions when speed for the trucks is 30 km/h and for drones is 80 km/h.

By comparing the results from Tables 6 and Table 3, it is immediately noticeable that the total time has increased considerably, since the trucks are operating at a lower speed. This obviously forces the trucks to take a longer time to deliver the products. As a result, in this scenario, the routes took approximately more 3 hours and more 1 hour and a half, for 1 and 2 vehicles, than they would take with the original parameters. Moreover, in the model proposed, the drones have to return to the next customer is going to serve. If this was not the case and if the drones had more freedom of movement, the results would, probably, be better and more efficient.

What is interesting and also expected is that the distance travelled by trucks has decreased. Since the primary minimization objective is based on the total time, the optimization tool verified that it would be better for the trucks to travel the shortest path possible and for the drones to cover more ground, since its speed is very high in comparison. As so, the distance travelled by trucks diminished around 40 and 80 km, respectively for 1 and 2 vehicles, when compared to the original parameters.

5.4 Time Windows

Time windows in the original problem were completely relaxed. As so, in order to assess the sensitivity of the model to this parameter, time windows were redefined to the values that are illustrated in table 7a. The times at which each drone and truck arrive at a certain node are summarized in table 7b.

Nodes	**Time Windows**	
	Lower Limit	**Upper Limit**
1	0	2
2	0	2
3	3	5
4	1	5
5	3	6
6	1	3
7	0	10
8	0	13
9	0	11
10	1	5

(a) Interval of time each customer can be served.

Node	**Arrival Times [hours]**		
	V = 1	**V = 2**	
	Vehicle 0	**Vehicle 0**	**Vehicle 1**
0	0	0	0
1	0.828	-	2.00 (D)
2	1.39	-	1.40
3	3.72	-	3.34
4	1.69 (D)	-	1.11 (D)
5	3.33 (D)	3.0	-
6	2.33 (D)	1.0	-
7	1.93	-	0.86
8	4.10 (D)	-	2.95 (D)
9	2.98	0.58	-
10	4.51	-	2.54

(b) Arrival times in the customers, which were served either by trucks or only by drones (D).

Table 7: Time windows and the actual times each customer was served.

As it can be observed in Table 7, all the actual times are in between the two values defined as upper and lower limits.

Moreover, for 1 and 2 sets of vehicles, the optimization resulted in the routes illustrated in figures 10 and 11.

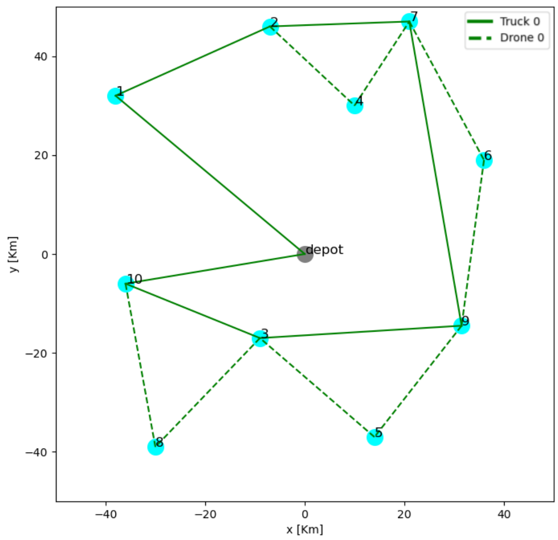

Figure 10: 10 customers served by 1 set of vehicles with time windows.

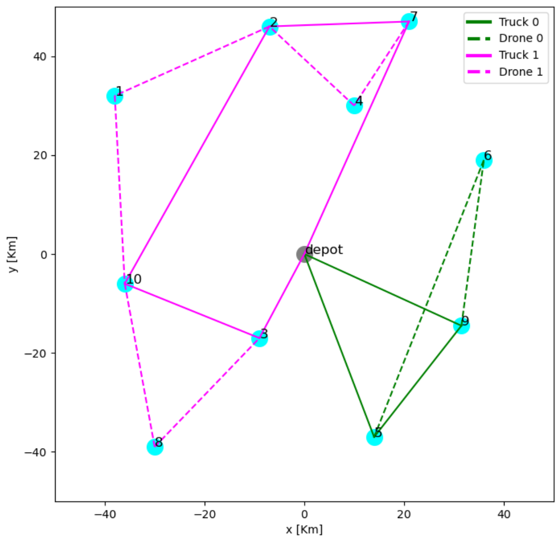

Figure 11: 10 customers served by 2 set of vehicles with time windows.

The routes and optimal solutions of each objective are portrayed in Table 8.

Number of the set of vehicles (V)	**V = 1**	**V = 2**
Route T: Truck D: Drone	T0 : 0 ->1 ->2 ->7 ->9 ->3 ->10 ->0 D0: 2 ->4 ->7 // 3 ->8 ->10 // 7 ->6 ->9 // 9 ->5 ->3	T0 : 0 ->9 ->5 ->0 D0 : 9 ->6 ->5 T1: 0 ->7 ->2 ->10 ->3 ->0 D1: 2->1->10 // 7 ->4 ->2 // 10 ->8 ->3
Objective 1 (Total Time)	5.12 hours	3.66 hours
Objective 2 (Total Cost)	7240.34	7527.83
Objective 3 (Truck Distance)	280.33 Km	290.17 Km

Table 8: Routes and optimal solutions when strict time windows are imposed.

By comparing Tables 3 and 8, it is noticeable that the primary optimization (total time), when time windows were implemented, degraded. For 1 vehicle, the total time rose approximately 17 minutes and, for 2 vehicles, it increased by around 44 minutes. This is an expected result, since the model has more constraints on the time the vehicles arrive at a certain node.

In terms of cost, for 1 vehicle, it rose by 335,17. For 2 vehicles, it decreased by 844,24 units of cost, which is what was expected since, in comparison with the case in Section 5.1, there were one extra node being served by drones instead of by trucks. And the cost of drones is rather low. Furthermore, regarding the third objective, the distance travelled by trucks decreased around 50 km for both cases.

In fact, these two last objectives have a very low importance in the model, so these fluctuations do not come unexpectedly.

5.5 Hierarchical Positions of Objective Functions

In fact, as mentioned earlier, the hierarchical order of the objective functions was prioritizing the total time the routes were taking to be completed.

Thus, this order was reversed. The distance travelled by trucks became the most important minimization and the total time became the least prioritized. The model was tested for 10 nodes and for 1 and 2 sets of vehicles.

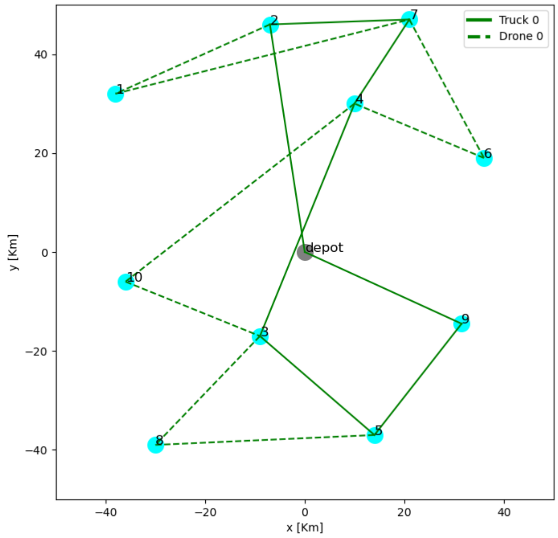

Figure 12: 10 customers served by 1 set of vehicles with reversed hierarchy of objective functions.

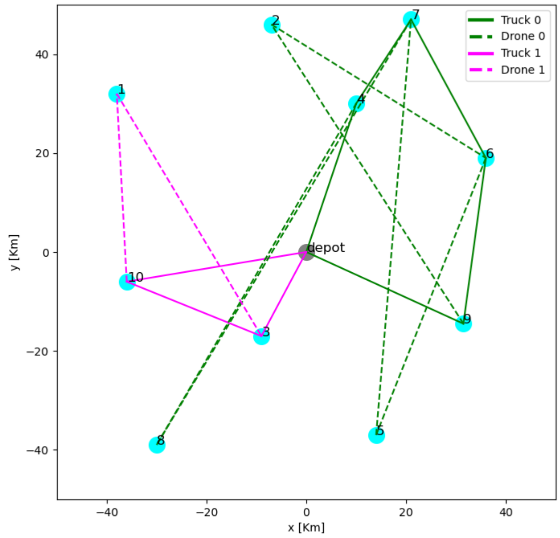

Figure 13: 10 customers served by 2 set of vehicles with reversed hierarchy of objective functions.

As it can be observed in Figures 12 and 13, the results correspond to the expectations, since the trucks perform the shortest path and the routes performed by drones are longer and can go up to the farthest distances. This occurs, specially, because, in the two primary minimization functions, the drones are almost not constrained. They have a very small cost and no importance at all regarding the kilometers travelled. In Figure 13, this is noticed the most.

Hence, it is clear that the total time minimization plays a crucial role in the original set of parameters. It is particularly important for a good customer service.

The routes and optimal objective functions of each scenario are illustrated in table 9.

Number of the set of vehicles (V)	**V = 1**	**V = 2**
Route T: Truck D: Drone	T0 : 0 ->2 ->7 ->4 ->3 ->5 ->9 ->0 D0: 2 ->1 ->7 // 3 ->8 ->5 // 4 ->10 ->3 // 7 ->6 ->4	T0 : 0 ->9 ->6 ->7 ->4 ->0 D0 : 9 ->2 ->6 // 6 ->5 ->7 // 7 ->8 ->4 T1: 0 ->10 ->3 ->0 D1: 10->1->3
Ojective 1 (Truck Distance)	239.16 Km	237 Km
Objective 2 (Total Cost)	6295.9	6466.7
Objective 3 (Total Time)	5.79 hours	6.83 hours

Table 9: Routes and optimal solutions.

By comparing tables 3 and 9, the results have the expected outcome. In fact, with the changes made, the total time to finish the routes have a significant rise, of approximately 1 and 5 hours, for 1 and 2 sets of vehicles, respectively. On the contrary, the distance travelled by trucks is considerably decreased for both cases.

6 Conclusions and Final Remarks

Drone delivery is an industry that is set to grow exponentially in the upcoming years. There has been some active research in the field and enterprises are considering this method as a great candidate for the transportation of their products to the respective customers. Additionally, it is a way of the businesses having a delivery alternative that is faster, cheaper, and more environmentally friendly.

Hence, the model proposed is of great relevance and is applicable to real life scenarios. In fact, all the objectives of the model were fulfilled. Therefore, in order to use this in a real situation, only the parameters have to be adjusted for the intended purpose. Furthermore, there are some additional constraints that can be added to approximate the model to a more realistic one, namely adding some restrictions on the flight endurance of the drones or computing the cost of the travels depending on the weight each truck is carrying along the route. Nevertheless, these are out of the scope of the current project.

Moreover, the major limitation of the model is the computational effort that is required. By increasing the number of nodes, the run time would easily go from a few minutes to a great number of hours. However, if these simulations are made in a more powerful computer, this might no longer be an issue.

Ultimately, taking everything in consideration, this novel multi-objective optimization model for the vehicle routing problem, with drone delivery, is step forward for ameliorated future deliveries. It takes into account total delivery time, total delivery costs, and total energy consumption of trucks. Furthermore, to solve this complex model, the optimization tool *Gurobi* was used and the optimal routes for both drones and trucks were outputted. Optimal routes were also generated by changing some of the main parameters of the model and by analysing its sensitivity to it.

7 References

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