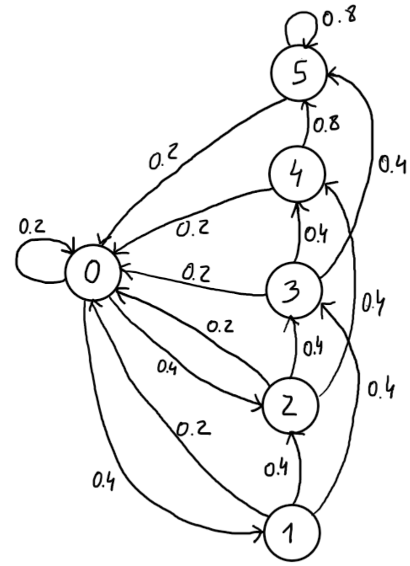
Homework 1 – Group 30

a) States = {0, 1, 2, 3, 4, 5}



$P(0,1) = 0.8 \times 0.5 = 0.4$

$$P = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0.2 & 0.4 & 0.4 & 0 & 0 & 0 \\ 0.2 & 0 & 0.4 & 0.4 & 0 & 0 \\ 0.2 & 0 & 0 & 0.4 & 0.4 & 0 \\ 0.2 & 0 & 0 & 0 & 0.4 & 0.4 \\ 0.2 & 0 & 0 & 0 & 0 & 0.8 \\ 0.2 & 0 & 0 & 0 & 0 & 0.8 \end{bmatrix} \end{array}$$

b) $0 \xrightarrow[2]{} 2 \xrightarrow[2]{} 4 \xrightarrow[2]{} 5$ Fastest Path is in 3 steps

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$$3 \text{ movements} = P^3 = \begin{bmatrix} 0,2 & 0,08 & 0,112 & 0,128 & 0,224 & 0,256 \\ 0,2 & 0,08 & 0,112 & 0,064 & 0,096 & 0,448 \\ 0,2 & 0,08 & 0,112 & 0.064 & 0,032 & 0,512 \\ 0,2 & 0,08 & 0,112 & 0,064 & 0,032 & 0,512 \\ 0,2 & 0,08 & 0,112 & 0,064 & 0,032 & 0,512 \\ 0,2 & 0,08 & 0,112 & 0,064 & 0,032 & 0,512 \end{bmatrix}$$

$P(0,5) = 0,256 = 25,6\%$

c) We want to know the stationary distribution μ

$\mu = [\mu(0) \quad \mu(1) \quad \mu(2) \quad \mu(3) \quad \mu(4) \quad \mu(5)]$

$\sum_{x \in X} \mu(x) = 1$

$$\mu(0) = \sum_{y \in X} \mu(y) P(0|y) = 0.2\mu(0) + 0.2\mu(1) + 0.2\mu(2) + 0.2\mu(3) + 0.2\mu(4) + 0.2\mu(5) =$$
$$= 0.2\,\underbrace{(\mu(0) + \mu(1) + \mu(2) + \mu(3) + \mu(4) + \mu(5))}_{=1} = 0.2 \times 1 = 0.2$$

$\mu(1) = 0.4 \times \mu(0) = 0.08$

$\mu(2) = 0.4 \times \mu(0) + 0.4 \times \mu(1) = 0.112$

$\mu(3) = 0.4 \times \mu(1) + 0.4 \times \mu(2) = 0.0768$

$\mu(4) = 0.4 \times \mu(2) + 0.4 \times \mu(3) = 0.07552$

$\mu(5) = 1 - \mu(0) - \mu(1) - \mu(2) - \mu(3) - \mu(4) = 0.45568$

d) To be an ergodic chain it must be:
- irreducible
- aperiodic
- stationary distribution

Our chain is irreducible, as proven in b) the P^3 matrix has no probability as 0, thus every state can reach every other, in at least 3 steps.

It is also aperiodic since every state has probability greater than 0 to go back to state 0, and since 0 can also transition to itself with prob>0 then state 0 is aperiodic. Since every state transitions to an aperiodic state, the chain is also aperiodic.

in c) we calculated μ^* and we can affirm that $\lim_{t\to\infty} \mu_0 P^t = \mu^*, \forall \mu_0$.

stationary distribution

-checking all this 3 properties, we can justify that our chain is ergodic.