Ex1. Técnica Primitivas

a) $\int x^2 = \frac{x^3}{3} + c$

b) $\int y^7 dy = \frac{y^8}{8} + c$

c) $\int (x+8)^5 = \frac{(x+8)^6}{6} + c$

d) $\int \frac{x+5}{x} dx = \int \left(1 + \frac{5}{x}\right) dx = \int 1\, dx + \int \frac{5}{x} dx$

$= x + 5 \int \frac{1}{x} dx = x + 5 \ln|x| + c$

e) $\int \frac{x^2 + 5x + 3}{x^{2/3}} dx = \int \left(x^{4/3} + 5x^{1/3} + 3x^{-2/3}\right) dx$

$= \frac{x^{\frac{4}{3}+1}}{\frac{4}{3}+1} + \frac{5x^{\frac{1}{3}+1}}{\frac{1}{3}+1} + 3\frac{x^{-\frac{2}{3}+1}}{-\frac{2}{3}+1} + c$

$= \frac{3}{7} x^{7/3} + \frac{15}{4} \times x^{4/3} + 9\, x^{1/3} + c$

f) $\int \left(\frac{x}{x^2} + \frac{4}{x\sqrt{x}} + 2\right) dx = \int \frac{1}{x} dx + 4 \int x^{-3/2} dx + \int 2\, dx$

$= \ln|x| + 4 \frac{x^{-3/2+1}}{-\frac{3}{2}+1} + 2x + C = \ln|x| + 4 \cdot \frac{1}{-\frac{1}{2} \times \sqrt{x}} + 2x + C$

$= \ln|x| - \frac{8}{\sqrt{x}} + 2x + C$

g) $\int y^6 \sqrt[10]{2y^7+3}\, dy = \frac{1}{14} \int 14 y^6 (2y^7+3)^{1/10} dy = \frac{1}{14} \cdot \frac{(2y^7+3)^{\frac{1}{10}+1}}{\frac{1}{10}+1} + c$

R2 $\quad f'\quad f$

$f = 2y^7 + 3 \to f' = 14 y^6$

$n = \frac{1}{10}$

$= \frac{1}{14} \frac{(2y^7+3)^{11/10}}{\frac{11}{10}} + c$

h) $\int \frac{x^3}{\sqrt{3x^4+5}} dx \underset{R2}{=} \frac{1}{12} \int 12x^3 \cdot (3x^4+5)^{-\frac{1}{2}} dx = \frac{1}{12} \frac{(3x^4+5)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + c$

$f = 3x^4 + 5 \to f' = 12 x^3$

$n = -2$

$= \frac{1}{12} \times \frac{(3x^4+5)^{\frac{1}{2}}}{\frac{1}{2}} + C$

$= \frac{1}{6} \sqrt{3x^4+5} + C$

i) $\int \frac{\sin(2x)}{\sqrt[4]{\cos^2 x + 9}} dx \underset{R3}{=} -\int - \sin(2x) (\cos^2 x + 9)^{-\frac{1}{4}} =$

$f = \cos^2 x + 9 \to f' = -2 \cos x \sin x = -\sin(2x)$

Ex2.

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a) $\int \frac{1}{x+3}\,dx \underset{R3}{=} \ln|x+3|+c$

$(f(x)=x+3 \to f'=1)$

b) $\int \frac{1}{3y+30}\,dy \underset{R3}{=} \frac{1}{3}\int \frac{1\times 3}{3y+30} = \frac{1}{3}\ln|3y+30|+c$

$(f(y)=3y+30 \to f'=3)$

c) $\int \frac{5}{8x-5}\,dx \underset{R3}{=} 5\times\frac{1}{8}\int \frac{1\times 8}{8x-5}\,dx = \frac{5}{8}\ln|8x-5|+c$

$(f(x)=8x-5 \to f'(x)=8)$

d) $\int \frac{5x}{2x^2+4}\,dx \underset{R2}{=} 5\times\frac{1}{4}\int \frac{4x}{2x^2+4} = \frac{5}{4}\ln|2x^2+4|+c$

$(f(x)=2x^2+4 \to f'=4x)$

Ex3.

a) $\int e^{5x+7}\,dx \underset{R4}{=} \frac{1}{5}\int 5\, e^{5x+7} = \frac{1}{5}e^{5x+7}$

$(f(x)=5x+7 \to f'=5)$

b) $\int 3x\, e^{4x^2+5}\,dx \underset{R4}{=} 3\times\frac{1}{8}\int 8x\, e^{4x^2+5}\,dx = \frac{3}{8}e^{4x^2+5}+c$

$(f(x)=4x^2+5 \to f'=8x$

c) $\int 6^{3y-9}\,dy \underset{R5}{=} \frac{1}{3}\int 3\, 6^{3y-9}\,dy = \frac{1}{3}\frac{6^{3y-9}}{\ln 6}+c$

$a=6$

$f=3y-9 \to f'=3$

d) $\int 3\sin(2x)\, e^{\sin^2 x+8}\,dx \underset{R5}{=} 3\, e^{\sin^2 x+8}+c$

$f=\sin^2 x+8 \to f'=2\sin x\cos x = \sin(2x)$

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$\sec x = \frac{1}{\cos x}$

Slides Primitivas:

Ex4.

a) $\int \frac{1}{\sqrt{1-4x^2}}dx = \frac{1}{2}\int \frac{1\times 2}{\sqrt{1-(2x)^2}}dx = \frac{1}{2}\arcsin(2x)+C$

13.

$f=(2x) \to f'=2$

b) $\int \frac{1}{y\sqrt{1-\ln^2 y}}dy = \arcsin(\ln y)+C$

13.

$f=\ln y \to f'=\frac{1}{y}$

c) $\int \frac{x}{\sqrt{4-9x^2}}dx = -\frac{1}{18}\int -18x\cdot(4-9x^2)^{-1/2} = -\frac{1}{18}\frac{(4-9x^2)^{1/2}}{\frac{1}{2}}+C$

2. $\int f' f^{\alpha}$ $\qquad = -\frac{2}{18}\sqrt{4-9x^2}+C$

d) $\int \frac{\sec^2(\ln x)}{x}dx = \int \frac{1}{x}\cdot\frac{1}{\cos^2(\ln x)}dx = tg(\ln x)+C$

7. $\int \frac{f'}{\cos^2 f}$

e) $\int \frac{7x}{16+9x^4}dx = \int \frac{7x}{16(1+\frac{9}{16}x^4)}dx = \frac{7}{16}\int \frac{x}{(1+(\frac{3}{4}x^2)^2}dx$

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$= \frac{7}{16}\times\frac{2}{3}\int \frac{\frac{3}{2}x}{1+(\frac{3}{4}x^2)^2}dx = \frac{7}{24}\operatorname{arctg}(\frac{3}{2}x)+C$

$f=\frac{3}{4}x^2 \to f'=\frac{3}{4}\times 2x = \frac{3}{2}x$

f) $\int \frac{1}{x^2+2x+5}dx = \int \frac{1}{(x+1)^2+4}dx = \int \frac{1}{4(1+(x+1)^2\times\frac{1}{4})}$

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$= \frac{1}{4}\int \frac{1}{1+(\frac{x+1}{2})^2}dx = \frac{1}{4}\times 2\int \frac{\frac{1}{2}}{1+(\frac{x+1}{2})^2} = \frac{1}{2}\operatorname{arctg}\left(\frac{x+1}{2}\right)+C$

g) $\int \operatorname{sen}(3x)\,dx = \frac{1}{3}\int 3\operatorname{sen}(3x) = -\frac{1}{3}\times\cos(3x)+C$

Exercícios 5:

a) $\int \frac{1}{\sqrt{9+4x^2}}dx \underset{21}{=} \int \frac{1}{\sqrt{9(1+\frac{4}{9}x^2)}} = \frac{1}{2}\int \frac{1\times 2}{3\sqrt{1+(\frac{2}{3}x)^2}}dx$

$= \frac{1}{2}\operatorname{argsh}(\frac{2}{3}x) + C$

b) $\int \frac{1}{ch^2x(1-th^2x)}dx \underset{23}{=} \int \frac{\frac{1}{ch^2x}}{1-th^2x}dx \underset{23.}{=} \operatorname{argth}(thx)+C$

$(thx)' = \frac{1}{ch^2x}$

c) $\int sh(15x+3)\,dx \underset{18}{=} \frac{1}{15}\int 15\,sh(15x+3) = \frac{1}{15}ch(15x+3)+C$

d) $\int e^x \cot g(e^x)\,dx \underset{10.}{=} \ln|\operatorname{sen}(e^x)|+C$

e) $\int 3^x e^x dx = \int (3e)^x dx \underset{4.}{=} \frac{(3e)^x}{\ln(3e)}+C$

$\int a^f f'$

$f=x \rightarrow a=3e$

f) $\int \frac{sh(2x)}{3sh^2x+7}dx = \frac{1}{3}\int \frac{3sh(2x)}{3sh^2x+7} = \frac{1}{3}\ln|3sh^2x+7|+C$

$\int \frac{f'}{f}$

$f=3sh^2x+7 \rightarrow f' = 6shx.chx = 3\times\underbrace{2shx\,chx}=3sh(2x)$

g) $\int \frac{\sqrt[3]{th^2x}\,dx}{ch^2x} \underset{2}{=}$

$= \int \frac{1}{ch^2x}\cdot(thx)^{2/3}dx = \frac{th^{5/3}x}{\frac{5}{3}}+C$

h) $\int \frac{e^x}{\sqrt{e^{2x}-1}}dx \underset{22}{=} \int \frac{e^x}{\sqrt{(e^x)^2-1}}dx = \operatorname{argch}(e^x)+C$

$\int f \times g = F(x)\, g(x) - \int F(x)\, g'(x)\, dx$

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Ex6:

a) $\int \underbrace{x^3}_{f} \cdot \underbrace{\ln x}_{g}\, dx = \frac{x^4}{4} \cdot \ln x - \int \frac{x^{\cancel{4}\,3}}{4} \cdot \frac{1}{\cancel{x}}\, dx$

$= \frac{x^4}{4} \ln x - \frac{1}{4} \times \frac{x^4}{4} + C$

C.A.:
$F(x) = \int x^3\, dx = \frac{x^4}{4}$

b) $\int \underbrace{x^2}_{g} \cdot \underbrace{sh(x)}_{f} = chx \cdot x^2 - \int chx \cdot 2x\, dx$

$= x^2 \cdot chx - 2 \left(\int \underbrace{x}_{g} \cdot \underbrace{chx}_{f}\, dx \right)$

$= x^2 chx - 2 \left(shx \cdot x - \int shx \cdot 1\, dx \right)$

$= x^2 chx - 2x\, shx + 2\, chx + C$

c)

d) $\int \underbrace{(x^2+1)}_{g} \underbrace{e^x}_{f}\, dx = e^x (x^2+1) - \int e^x \cdot (2x)\, dx$

$= e^x (x^2+1) - 2 \left(\int \underbrace{e^x}_{f} \underbrace{x}_{g}\, dx \right)$

$= e^x (x^2+1) - 2 \left(e^x \cdot x - \int e^x\, dx \right)$

$= e^x (x^2+1) - 2x e^x + 2 e^x + C$

f) $\int \underbrace{1}_{f} \cdot \underbrace{\arcsin x}_{g}\, dx = x \cdot \arcsin x - \frac{}{2} \int 2x \cdot \frac{1}{\sqrt{1-x^2}}\, dx$

$= x \arcsin x - \frac{1}{2} \frac{(1-x^2)^{-\frac{1}{2}+1}}{\frac{1}{2}} + C$

? g) $\int \underbrace{x}_{f} \cdot \underbrace{\text{argcoth}\, x}_{g}\, dx = \frac{x^2}{2} \cdot \text{argcoth}\, x - \int \frac{x^2}{2\sqrt{x^2-1}}\, dx$

$= \frac{x^2}{2} \text{argcoth}\, x - \frac{1}{2} \int \underbrace{x}_{g}\, \underbrace{x(x^2-1)^{-\frac{1}{2}}}_{f}$

CA:

$F(x) = \int x = \frac{x^2}{2}$

$g(x) = \text{argcoth}\, x \to g'(x) = \frac{1}{\sqrt{x^2-1}}$

CA: $F(x) = \frac{1}{2} \int 2x(x^2-1)^{-1/2} = \frac{1}{2} \frac{(x^2-1)^{1/2}}{\frac{1}{2}}$

$= \frac{x^2}{2} \text{argcoth}\, x - \frac{1}{2} \left[\sqrt{x^2-1} \cdot x - \int 1 \cdot \sqrt{x^2-1}\, dx \right]$

$2x$

h) $\int 1 \cdot \ln^2 x\, dx = x \ln^2 x - \int \cancel{x} \cdot 2 \ln x \cdot \frac{1}{\cancel{x}}\, dx$

$= x \ln^2 x - 2 \int \ln x\, dx$

$= x \ln^2 x - 2 \left(x \ln x - \int x \cdot \frac{1}{x}\, dx \right)$

$= x \ln^2 x - 2x \ln x + x + C$

Ex 7.

(a) $\int \cos^2 x \, dx = \int \frac{1+\cos 2x}{2} \, dx$

$\cos^2 x = \frac{1+\cos 2x}{2}$

$= \int \frac{1}{2} dx + \frac{1}{2 \cdot 2} \int 2\cos 2x \, dx = \frac{1}{2} x + \frac{1}{4} \sin(2x) + C$

(b) $\int ch^4 x \, dx = \int (ch^2 x)^2 \, dx =$

$= \int \left(\frac{ch 2x + 1}{2}\right)^2 dx = \frac{1}{4} \int (ch^2 (2x) + 2\, ch 2x + 1) \, dx$

$= \frac{1}{4} \left\{ \int \frac{ch(4x)+1}{2} dx + \int 2\, ch 2x \, dx + \int 1 \, dx \right\}$

$= \frac{1}{4} \left\{ \frac{1}{2} \times \frac{1}{4} \int 4\, ch 4x + \frac{1}{2} \int 1 \, dx + sh 2x + x \right\}$

$= \frac{1}{32} sh 4x + \frac{1}{8} x + \frac{1}{4} sh 2x + \frac{x}{4} + C$

(c) $\int \cos^3 x \, dx = \int \cos^2 x \cdot \cos x \, dx$

$= \int (1 - \sin^2 x) \cos x \, dx = \int \cos x \, dx - \int \cos x \cdot \sin^2 x \, dx$

$= \sin x - \frac{\sin^3 x}{3} + C$

(d) $\int \sin^3 x \cdot \cos^{12} x \, dx = \int \sin^2 x \cdot \sin x \cdot \cos^{12} x \, dx$

$= \int (1 - \cos^2 x) \cos^{12} x \cdot \sin x \, dx$

$= \int (\cos^{12} x - \cos^{14} x) \cdot \sin x \, dx$

$= (-)\int (-)\sin x \cos^{12} x \, dx - (-)\int(-)\sin x \cos^{14} x \, dx$

$= -\frac{\cos^{13} x}{13} + \frac{\cos^{15} x}{15} + C$

(e) $\int ch^5 x \, dx = \int (ch^2 x)^2 \cdot ch x \, dx$

$= \int (1 + sh^2 x)^2 \cdot ch x \, dx$

$ch^2 x = 1 + sh^2 x$

$= \int (1 + 2 sh^2 x + sh^4 x) \cdot ch x \, dx$

$= \int ch x \, dx + 2 \int ch x \cdot sh^2 x + \int ch x \cdot sh^4 x \, dx$

$= sh x + \frac{sh^3 x}{3} + \frac{sh^5 x}{5} + C$

(f) $\int th^3 x \, dx = \int th x \cdot \underbrace{th^2 x} \, dx$

$= \int th x \left(1 - \frac{1}{ch^2 x}\right) dx = \int th x \, dx - \int \overset{f^\alpha}{th x} \cdot \overset{f'}{\frac{1}{ch^2 x}} \, dx$

$th^2 x = 1 - \frac{1}{ch^2 x}$ $\qquad = \ln|ch x| - \frac{th^2 x}{2} + C$

(h) $\int tg^5 x \, dx = \int tg^3 x \cdot \underbrace{tg^2 x} \, dx$

$= \int tg^3 x \cdot \left(\frac{1}{\cos^2 x} - 1\right) dx$

$= \int \frac{1}{\cos^2 x} \cdot tg^3 x - \int tg x \cdot tg^2 x \, dx$

$= \frac{tg^4 x}{4} - \int tg x \left(\frac{1}{\cos^2 x} - 1\right) dx$

$= \frac{tg^4 x}{4} - \int tg x \cdot \frac{1}{\cos^2 x} \, dx + \int tg x \, dx$

$= \frac{tg^4 x}{4} - \frac{tg^2 x}{2} - \ln|\cos x| + C$

(g) $\int ch^4 x \, dx = \int (ch^2 x)^2 dx = \int \left(\frac{1 + ch 2x}{2}\right)^2 dx$

$= \frac{1}{4} \left(\int (1 + 2 ch 2x + ch^2 2x) dx \right) = \frac{1}{4} x + \frac{1}{4} sh 2x + \frac{1}{4} \int \frac{1 + ch 4x}{2}$

$= \frac{1}{4} x + \frac{1}{4} sh 2x + \frac{1}{8} x + \frac{1}{8} x \frac{1}{4} \int 4 ch 4x \, dx = \frac{1}{4} x + \frac{1}{4} sh 2x + \frac{1}{8} x + \frac{1}{32} sh 4x + C$

Exercício 8

(a) $\int \sqrt{1-x^2}\, dx \qquad x = \underbrace{\cos t}_{g(t)}, \quad t \in [0,\pi]$

$dx = -\operatorname{sen} t \cdot dt$

Subst.

$$\int \sqrt{1-\cos^2 t}\,\cdot(-\operatorname{sen} t)\, dt = \int -\sqrt{\operatorname{sen}^2 t}\, \operatorname{sen} t\, dt$$

$\operatorname{sen}^2 t = 1-\cos^2 t$

$$= -\int \operatorname{sen}^2 t\, dt = -\int \frac{1-\cos 2t}{2}\, dt$$

$$= -\frac{1}{2}\left[\int 1\, dt - \frac{1}{2}\int 2\cos 2t\, dt\right]$$

$$= -\frac{1}{2}t + \frac{1}{4}\operatorname{sen} 2t + C = -\frac{1}{2}t + \frac{1}{4}\, 2\operatorname{sen} t \cos t$$

Desfazendo mud. var.

$x = \cos t \Rightarrow t = \arccos x$

$$\int \sqrt{1-x^2}\, dx = -\frac{1}{2}\arccos x + \frac{1}{2}\operatorname{sen} t \cdot x + C$$

$$= -\frac{1}{2}\arccos x + \frac{1}{2}\sqrt{1-x^2}\cdot x + C$$

C.A.:
$\operatorname{sen} t = \cancel{\pm}\sqrt{1-\cos^2 t}$
$\operatorname{sen} t = \sqrt{1-x^2}$

(b) $\int \frac{\arcsin\sqrt{x}}{\sqrt{x}}\, dx \quad ; \quad x = \underbrace{t^2}_{g(t)}, \quad t > 0$

Subst.

$g'(t) = 2t$

$dx = g'(t)\cdot dt$

$$\int \frac{\arcsin\sqrt{x}\, dx}{\sqrt{x}} = \int \frac{\arcsin\sqrt{t^2}}{\sqrt{t^2}}\cdot 2t\, dt$$

mud.

$$\int \frac{\arcsin t}{\cancel{t}}\cdot 2\cancel{t}\, dt = 2\int \arcsin t\, dt$$

R. P. PP

$$= 2\left[t\cdot\arcsin t - \int t\cdot\frac{1}{\sqrt{1-t^2}}\, dt\right]$$

$$= 2\left[t\arcsin t - \frac{1}{2}\int 2t\,(1-t^2)^{-\frac{1}{2}}\, dt\right]$$

$$= 2t\arcsin t - (1-t^2)^{1/2} + C$$

$= 2t \arcsin t - 2\sqrt{(1-t^2)} + C$

$= 2\sqrt{x} \arcsin\sqrt{x} - 2\sqrt{1-x} + C$

$x = t^2 \Rightarrow t = \cancel{\pm}\sqrt{x}$

Ex9

(a) $\int \frac{2x^2 - 3x + 7}{2 + x} dx = \int 2x - 7 + \frac{21}{2+x} dX =$

$= \frac{2x^2}{2} - 7x + 21 \int \frac{1}{2+x} dx$

$= x^2 - 7x + 21 + \ln|2+x| + C$

1º Dividir os polinómios.

$$\begin{array}{rl} 2x^2 - 3x + 7 & \underline{|2+x} \\ -2x^2 - 4x & 2x - 7 \\ \hline -7x + 7 & \\ +7x + 14 & \\ \hline 21 & \end{array}$$

(b) $\int \frac{x^2 + 5x + 4}{1 + x^2} = \int 1 + \frac{5x + 3}{1+x^2} dx$

$= \int 1 dx + \frac{5}{2} \int \frac{2x}{1+x^2} dx + \int \frac{3}{1+x^2} dX$

$= x + \frac{5}{2} \ln|1+x^2| + 3 \operatorname{arctg} x + C$

C.A.

1º Dividir os polinómios

$$\begin{array}{rl} x^2 + 5x + 4 & \underline{|1+x^2} \\ -x^2 \quad\quad - 1 & 1 \\ \hline 0 \quad 5x + 3 & \end{array}$$

(3) $\int \frac{4y^4 + y + 1}{4y^4 + 1} dy = \int 1 + \frac{y}{4y^4 + 1} dy$

$= \int 1 dy + \frac{1}{4} \int \frac{4y}{(2y^2)^2 + 1} dy = y + \frac{1}{4} \operatorname{arctg}(2y^2) + C$

CA:

1º Dividir os polinómios

$$\begin{array}{rl} 4y^4 + 0y^2 + 1y + 1 & \underline{|4y^4 + 1} \\ -4y^4 \quad\quad\quad - 1 & 1 \\ \hline y & \end{array}$$

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Slides

Ex9

a) $\int \frac{18}{x(x^2-9)} dx = \int \frac{18}{x(x-3)(x+3)} dx = \int \frac{-2}{x} \frac{-2}{x-3} + \frac{4}{x+3} d$

$= -2 \int \frac{1}{x} dx - 2 \int \frac{1}{x-3} dx + 4 \int \frac{1}{x+3} dx$

$= -2 \ln|x| - 2 \ln|x-3| + 4 \ln|x+3| + C$

C.A:

1º Não é preciso dividir os pols.

2º Calcular os zeros do pol. denominador:

$x(x^2-9)=0 \Leftrightarrow x=0 \vee x^2=9 \Leftrightarrow x=0 \vee x=\pm 3$ → raízes reais

3º Decompor ~~Função~~ função racional em fracções simples:

$\cancel{\frac{18}{x(x^2-9)}}$ $\boxed{\frac{18}{x(x-3)(x+3)} = \frac{A^{=-2}}{x} + \frac{B^{=-2}}{x-3} + \frac{C^{=4}}{x+3}}$

$(x^2-9) \quad x(x+3) \quad x(x-3)$

4º Método dos Coef. Indeterminados:

$\frac{18}{x(x^2-9)} = \frac{A(x^2-9)}{x(x^2-9)} + \frac{B\,x(x+3)}{x(x^2-9)} + \frac{C\,x(x-3)}{x(x^2-9)}$

~~Ent~~.

$18 = A(x^2-9) + B(x^2+3x) + C(x^2-3x)$

sist. a resol.

$\begin{cases} A+B+C=0 \\ 3A-3B=0 \\ -9A=18 \end{cases} \quad \begin{cases} — \\ -6-3B=0 \\ A=\frac{18}{-9}=-2 \end{cases} \quad \begin{cases} C=-A-B=4 \\ B=\frac{6}{-3}=-2 \\ A=-2 \end{cases}$

e) $\displaystyle\int \frac{t^2+1}{t^3+t^4}\,dt = \int \frac{t^2+1}{t^3(1+t)}\,dt =$

$\displaystyle = \int \frac{2}{t} - \frac{1}{t^2} + \frac{1}{t^3} - \frac{2}{1+t}\,dt = 2\int \frac{1}{t}\,dt - \int t^{-2}\,dt + \int t^{-3}\,dt$

$\displaystyle - 2\int \frac{1}{1+t}\,dt = 2\ln|t| - \frac{t^{-1}}{-1} + \frac{t^{-2}}{-2} - 2\ln|1+t| + c$

$\displaystyle = 2\ln|t| + \frac{1}{t} - \frac{1}{2t^2} - 2\ln|1+t| + C$

C.A.:

1) Dividir polinómios X

2) Decompor o pól. denominador nas suas raízes ✓

$t^3 + t^4 = 0 \Leftrightarrow t^3(1+t) = 0 \Leftrightarrow t^3 = 0 \lor t = -1$

$\Leftrightarrow \underbrace{t=0}_{\text{ap. 3 vezes}} \lor t = -1$

3) Decompor a F.R. em fracções simples:

$$\boxed{\frac{t^2+1}{t^3(1+t)}} = \frac{\overset{=2}{A}}{t} + \frac{\overset{=-1}{B}}{t^2} + \frac{\overset{=1}{C}}{t^3} + \frac{\overset{=-2}{D}}{1+t}$$

4) Calcular A, B, C, D $\to$ Método C. Ind.

$$\frac{t^2+1}{t^3(1+t)} = \frac{A}{t}_{t^2(1+t)} + \frac{B}{t^2}_{t(1+t)} + \frac{C}{t^3}_{(1+t)} + \frac{D}{1+t}_{t^3}$$

$\Rightarrow t^2+1 = At^2(1+t) + Bt(1+t) + C(1+t) + Dt^3$

$\Leftrightarrow t^2+1 = A(t^2+t^3) + B(t+t^2) + C(1+t) + Dt^3$

$t^2+1 = (A+D)t^3 + (A+B)t^2 + (B+C)t + C$

Sist. a resol.

$$\begin{cases} \overset{=2}{A} + D = 0 \\ A + \overset{=-1}{B} = 1 \\ B + \overset{=1}{C} = 0 \\ C = 1 \end{cases} \qquad \begin{cases} D = -2 \\ A = 2 \\ B = -1 \\ C = 1 \end{cases}$$

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f) $\int \frac{2x^3+3x}{(x^2+1)^2}\,dx = \int \frac{2x}{x^2+1} + \frac{x}{(x^2+1)^2}\,dx$

$= \int \frac{2x}{x^2+1}\,dx + \frac{1}{2}\int 2x\,(x^2+1)^{-2}\,dx$

$= \ln|x^2+1| + \frac{1}{2}\,\frac{(x^2+1)^{-1}}{-1} + C$

C.A.

1) Dividir os polinómios X

2) Decompor o pol. denominador nas suas raízes ✓

$(x^2+1)^2=0 \Leftrightarrow (x^2+1)\cdot(x^2+1)=0 \Leftrightarrow$

$x^2+1=0 \vee x^2+1=0 \Leftrightarrow x^2=-1 \vee x^2=-1$

raízes complexas ✓

3) Decompor F.R. em fracções simples.

$\left[\frac{2x^3+3x}{(x^2+1)^2} = \frac{\overset{2}{A}x+B^{=0}}{x^2+1} + \frac{\overset{1}{C}x+D^{=0}}{(x^2+1)^2}\right]$

4) Calcular A, B, C, D → Método C. Indet.

$\frac{2x^3+3x}{(x^2+1)^2} = \frac{Ax+B}{\underset{(x^2+1)}{x^2+1}} + \frac{Cx+D}{(x^2+1)^2}$

$\Rightarrow 2x^3+3x = (Ax+B)(x^2+1) + Cx+D$

$\Leftrightarrow 2x^3+3x = (Ax^3+Bx^2+Ax+B) + Cx+D$

$\Leftrightarrow 2x^3+3x = Ax^3+Bx^2+(A+C)x+B+D$

S.A. a resolver

$\begin{cases} A=2 \\ B=0 \\ \overset{=2}{A}+C=3 \\ \underset{0}{B}+D=0 \end{cases} \quad \begin{cases} A=2 \\ B=0 \\ C=1 \\ D=0 \end{cases}$

$\boxed{\text{Ex 10.}}$

$$F(x) = \int f'(x)\,dx = \int \frac{1}{(x-2)(x-1)}\,dx$$

$$= \int \frac{1}{x-2} - \frac{1}{x-1}\,dx = \ln|x-2| - \ln|x-1| + C$$

Agora: Como $F(0) = 0$, vem $\ln 2 - \underbrace{\ln 1}_{=0} + C = 0$

$$C = -\ln 2$$

Portanto

$$F(x) = \ln|x-2| - \ln|x-1| - \ln 2$$

C.A.:

1) $$\boxed{\frac{1}{(x-2)(x-1)} = \frac{\overset{=1}{A}}{x-2} + \frac{\overset{=-1}{B}}{x-1}}$$

2) Calcular A, B → Mé todo C. Ind.

$$\frac{1}{(x-2)(x-1)} = \frac{A}{x-2}_{(x-1)} + \frac{B}{x-1}_{(x-2)}$$

$$\Rightarrow 1 = A(x-1) + B(x-2)$$

$$\Leftrightarrow 1 = (A+B)x + (-A-2B)$$

Sist. a resolver:

$$\begin{cases} A+B=0 \\ -A-2B = 1 \end{cases} \Leftrightarrow \begin{cases} A=-B \\ B-2B=1 \end{cases} \quad \begin{cases} A=1 \\ -B=1 \end{cases} \Leftrightarrow \begin{cases} A=1 \\ B=-1 \end{cases}$$