SÉRIES TEMPORAIS/TIME SERIES

Worksheet 1: Autocorrelation function (ACF) and Sample ACF
Academic year 2022/23

1. Consider the process $Y = (Y_t, t \in Z)$ such as

$$Y_t = \epsilon_t - \theta\epsilon_{t-1}, \text{ with } \theta \in [-1, 1]$$

where $\varepsilon = (\epsilon_t, t \in \mathbf{Z})$ is a gaussian white noise with variance $\sigma^2 > 0$.

   (a) Determine the mean value of the model and the autocorrelation function (ACF) of Y.
   (b) What can be said concerning the stationarity of the process?

2. Consider the gaussian white noise process w_t with mean value 0 and variance σ^2.

   (a) Obtain the autocorrelation function.
   (b) Simulate a series with 500 observations under the previous conditions, make the cronogram, calculate $\hat{\rho}(20)$ and compare with $\rho(20)$. Remark: Consider $\sigma^2 = 1$.
   (c) Redo the previous item with $n = 50$ to analyze how the value of n affects the results.

3. Consider the smoothed white noise process defined by $x_t = \frac{1}{3}(w_{t-1} + w_t + w_{t+1}), t = 2, 3, \ldots$ where $w_t \sim N(0, \sigma^2)$ is Gaussian white noise.

   (a) Determine μ_t, σ_t^2, the autocovariance function and the autocorrelation function of the smoothed process (moving averages).
   (b) Simulate a trajectory of this model with dimension $n = 500$, plot it and plot the Sample ACF. Note: Consider $\sigma^2 = 1$.
   (c) Compare $\rho(20)$ with $\hat{\rho}(20)$.
   (d) Change the sample size and compare the values of $\rho(20)$ and $\hat{\rho}(20)$ to each other.

4. Consider the autocorrelations model defined by the second order difference equation::

$$x_t = x_{t-1} - 0.9x_{t-2} + w_t, t = 1, 2, \ldots, 500, w_t \sim N(0, 1)$$

   (a) Simulate a trajectory of this autoregressive process, make the respective cronogram, the graph of sample acf. To what extent does the sample autocorrelation function indicate the cyclical behavior of the data? Hint: simulate 50 additional values and consider the last 500 observations.
   (b) Answer the question about the cyclical behavior of the series considering

$$x_t = 0.2x_{t-1} - 0.4x_{t-2} + w_t$$

5. Varying the sample size, we intend to compare the sample autocorrelation function with the theoretical ACF. The model is built based on the following experience: consider an artificial dataset generated by tossing a coin with heads and tails, where $x_t = 1$ represents heads and $x_t = -1$ corresponds to the tails output; the model is defined by

$$y_t = 5 + x_t - 0.7x_{t-1}$$

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   (a) Show that ACF is given by $\rho(h) = 0, |h| > 1$ and $\rho(1) = -0.47$.
   (b) Simulate 10 observations according to the model y_t and plot the cronogram.
   (c) Get sample ACF values until lag $h = 6$ and compare with theoretical values.
   (d) Increase the sample size to 100, perform the requested comparison and verify that the variability increases as sample size decreases.