OpenCMISS-iron examples and tests used by *OpenCMISS* developers at University of Stuttgart, Germany

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1 INTRODUCTION

This document contains information about examples used for testing *OpenCMISS-iron*. Read: How-to[1] and [1].

1.1 Cmgui files for cmgui-2.9

1.2 Variations to consider

- Geometry and topology
  - 1D, 2D, 3D
  - Length, width, height
  - Number of elements
  - Interpolation order
  - Generated or user meshes
  - quad/hex or tri/tet meshes
- Initial conditions
- Load cases
  - Dirichlet BC
  - Neumann BC
  - Volume force
  - Mix of previous items
- Sources, sinks
- Time dependence
  - Static
  - Quasi-static
  - Dynamic
- Material laws
  - Linear
  - Nonlinear (Mooney-Rivlin, Neo-Hookean, Ogden, etc.)
  - Active (Stress, strain)
- Material parameters, anisotropy
- Solver
  - Direct
  - Iterative
- Test cases
  - Numerical reference data
  - Analytical solution
- A mix of previous items

1 https://bitbucket.org/hessenthaler/opencmiss-howto

1.3 Folder structure

TBD..

2 HOW TO WORK ON THIS DOCUMENT

In the Google Doc at https://docs.google.com/spreadsheets/d/1RGKj8vVPqQ-PH0UwMX_e9TAzqaYavKiOzOD4pKY9RGI/edit#gid=0 please indicate what you are working on or if a given example was finished

- no mark: to be done
- x: currently working on it
- xx: done

Initials	Full name
CB	Christian Bleiler
AH	Andreas Hessenthaler
TK	Thomas Klotz
AK	Aaron Krämer
BM	Benjamin Maier
SM	Sergio Morales
MM	Mylena Mordhorst
HS	Harry Saini

Table 1: Initials of people working on examples, in alphabetical order (surnames).

3 DIFFUSION EQUATION

3.1 Equation in general form

The governing equation is,

$$\partial_t u + \nabla \cdot [\boldsymbol{\sigma} \nabla u] = f, \tag{1}$$

with conductivity tensor $\boldsymbol{\sigma}$. The conductivity tensor is,

- defined in material coordinates (fibre direction),
- diagonal,
- defined per element.

4 LINEAR ELASTICITY

4.1 Equation in general form

$$\partial_{tt}\mathbf{u} + \nabla \cdot \boldsymbol{\sigma}(\mathbf{u}, t) = \mathbf{f}(\mathbf{u}, t) \tag{2}$$

5 FINITE ELASTICITY

6 NAVIER-STOKES FLOW

7 MONODOMAIN

7.1 Example-0401

7.1.1 *Mathematical model*

We solve the Monodomain Equation

$$\sigma\Delta V_m(t) = A_m\Big(C_m\frac{\partial V_m}{\partial t} + I_{ionic}(V_m)\Big) \quad \Omega = [0,1]\times[0,1], \quad t\in[0,3.0] \tag{3}$$

where $V_m(t)$ is given by the Hodgkin-Huxley system of ODEs
with boundary conditions

$$V_m = 0 \qquad x = y = 0, \tag{4}$$
$$V_m = 0 \qquad x = y = 1. \tag{5}$$

and initial values

$$V_m(t=0) = -75$$

Additionally a stimulation current I_{stim} is applied for $t_{stim} = [0, 0.1]$ at the center node of the domain (i.e. at $(x,y) = (\frac{1}{2}, \frac{1}{2},))$.
Material parameters:

$\sigma = 3.828$

$A_m = 500$

$C_m = 0.58$ for the slow-twitch case, $C_m = 1.0$ for the fast-twitch case

$I_{Stim} = 1200$ for the slow-twitch case, $I_{Stim} = 2000.0$ for the fast-twitch case

7.1.2 *Computational model*

- This example uses generated meshes
- Commandline arguments are:
  number elements X
  number elements Y
  interpolation order (1: linear; 2: quadratic)
  solver type (0: direct; 1: iterative)
  PDE step size
  stop time
  output frequency
  CellML Model URL
  slow-twitch
  ODE time-step
- Commands for tests are:

```
./folder/src/example 24 24 1 0 0.005 3.0 1 hodgkin_huxley_1952.cellml F 0.0001
./folder/src/example 24 24 1 0 0.005 3.0 1 hodgkin_huxley_1952.cellml F 0.005
./folder/src/example 10 10 1 0 0.005 3.0 1 hodgkin_huxley_1952.cellml F 0.0001
mpirun -n 2 ./folder/src/example 24 24 1 0 0.005 3.0 1 hodgkin_huxley_1952.cellml
```

```
mpirun -n 8 ./folder/src/example 24 24 1 0 0.005 3.0 1 hodgkin_huxley_1952.cellml
./folder/src/example 2 2 1 0 0.005 3.0 1 hodgkin_huxley_1952.cellml F 0.0001
mpirun -n 2 ./folder/src/example 2 2 1 0 0.005 3.0 1 hodgkin_huxley_1952.cellml F
```

- This is a dynamic problem.

7.1.3 *Results*

```
Passed tests: 6 / 6
```

```
No failed tests.
```

Figure 1: Results movie, 24×24 elements (only works in certain pdf viewers, e.g. Adobe Acrobat Reader)

7.1.4 *Validation*

We compare with a Matlab implementation.

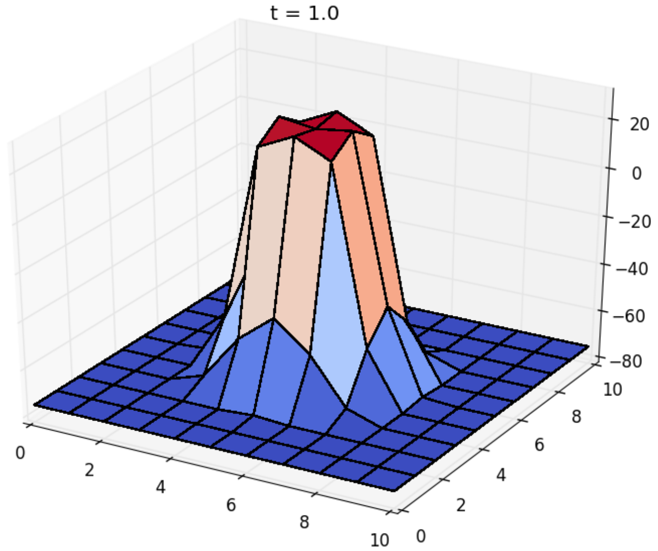

Figure 2: Results, 10×10 elements, $t = 200$

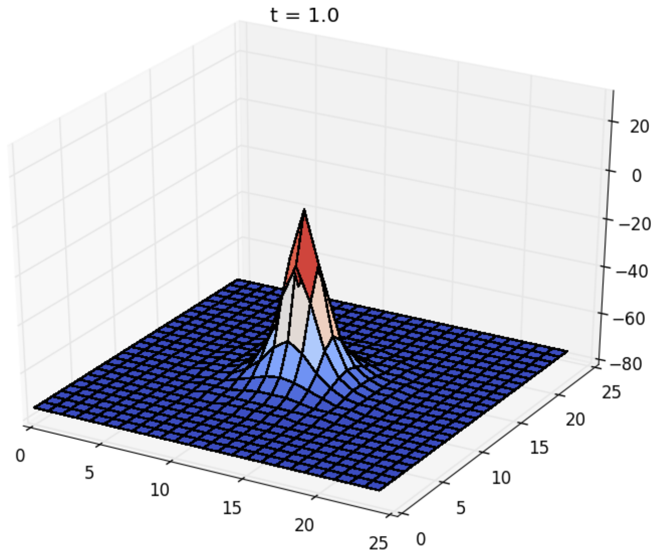

Figure 3: Results, 24×24 elements, $t = 200$

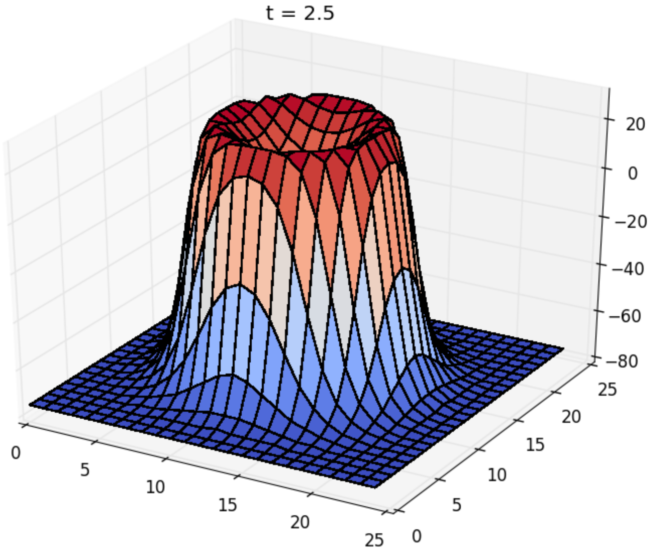

Figure 4: Results, 24×24 elements, $t = 500$

8 CELLML MODEL

REFERENCES

[1] Chris Bradley, Andy Bowery, Randall Britten, Vincent Budelmann, Oscar Camara, Richard Christie, Andrew Cookson, Alejandro F Frangi, Thiranja Babarenda Gamage, Thomas Heidlauf, et al. Opencmiss: a multi-physics & multi-scale computational infrastructure for the vph/-physiome project. *Progress in biophysics and molecular biology*, 107(1):32–47, 2011.