

OpenCMISS-iron examples and tests used by OpenCMISS developers at University of Stuttgart, Germany

Christian Bleiler*, Andreas Hessenthaler*,
Thomas Klotz*, Aaron Krämer[†], Benjamin Maier[‡],
Sergio Morales*, Mylena Mordhorst*, Harry Saini*

July 3, 2017
13:15

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* Institute of Applied Mechanics (CE), University of Stuttgart, Pfaffenwaldring 7, 70569 Stuttgart, Germany

[†] Institute for Parallel and Distributed Systems, University of Stuttgart, Universitätsstraße 38, 70569 Stuttgart, Germany

[‡] Lehrstuhl Mathematische Methoden für komplexe Simulation der Naturwissenschaft und Technik, University of Stuttgart, Allmandring 5b, 70569 Stuttgart, Germany

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1 INTRODUCTION

This document contains information about examples used for testing *OpenCMISS-iron*. Read: How-to¹ and [1].

1.1 Cmgui files for cmgui-2.9

1.2 Variations to consider

- Geometry and topology
 - 1D, 2D, 3D
 - Length, width, height
 - Number of elements
 - Interpolation order
 - Generated or user meshes
 - quad/hex or tri/tet meshes
- Initial conditions
- Load cases
 - Dirichlet BC
 - Neumann BC
 - Volume force
 - Mix of previous items
- Sources, sinks
- Time dependence
 - Static
 - Quasi-static
 - Dynamic
- Material laws
 - Linear
 - Nonlinear (Mooney-Rivlin, Neo-Hookean, Ogden, etc.)
 - Active (Stress, strain)
- Material parameters, anisotropy
- Solver
 - Direct
 - Iterative
- Test cases
 - Numerical reference data
 - Analytical solution
- A mix of previous items

1.3 Folder structure

TBD..

¹ <https://bitbucket.org/hessenthaler/opencmisshowto>

2 HOW TO WORK ON THIS DOCUMENT

In the Google Doc at https://docs.google.com/spreadsheets/d/1RGKj8vVPqQ-PH0UwMX_e9TAzqaYavKi0z0D4pKY9RGI/edit#gid=0 please indicate what you are working on or if a given example was finished

- no mark: to be done
- x: currently working on it
- xx: done

Initials	Full name
CB	Christian Bleiler
AH	Andreas Hessenthaler
TK	Thomas Klotz
AK	Aaron Krämer
BM	Benjamin Maier
SM	Sergio Morales
MM	Mylena Mordhorst
HS	Harry Saini

Table 1: Initials of people working on examples, in alphabetical order (surnames).

3 DIFFUSION EQUATION

3.1 Equation in general form

The governing equation is,

$$\partial_t \mathbf{u} + \nabla \cdot [\boldsymbol{\sigma} \nabla \mathbf{u}] = \mathbf{f}, \quad (1)$$

with conductivity tensor $\boldsymbol{\sigma}$. The conductivity tensor is,

- defined in material coordinates (fibre direction),
- diagonal,
- defined per element.

3.2 Example-0001

Example uses generated regular meshes and solves a static problem, i.e., applies the boundary conditions in one step.

3.2.1 Mathematical model - 2D

We solve the following scalar equation,

$$\nabla \cdot \nabla u = 0 \quad \Omega = [0, 2] \times [0, 1], \quad (2)$$

with boundary conditions

$$u = 0 \quad x = y = 0, \quad (3)$$

$$u = 1 \quad x = 2, y = 1. \quad (4)$$

No material parameters to specify.

3.2.2 Mathematical model - 3D

We solve the following scalar equation,

$$\nabla \cdot \nabla u = 0 \quad \Omega = [0, 2] \times [0, 1] \times [0, 1], \quad (5)$$

with boundary conditions

$$u = 0 \quad x = y = z = 0, \quad (6)$$

$$u = 1 \quad x = 2, y = z = 1. \quad (7)$$

No material parameters to specify.

3.2.3 Computational model

- Commandline arguments are:

float: length along x-direction

float: length along y-direction

float: length along z-direction (set to zero for 2D)

integer: number of elements in x-direction

integer: number of elements in y-direction

integer: number of elements in z-direction (set to zero for 2D)

integer: interpolation order (1: linear; 2: quadratic)

integer: solver type (0: direct; 1: iterative)

- Commandline arguments for tests are:

2.0 1.0 0.0 2 1 0 1 0

2.0 1.0 0.0 4 2 0 1 0

2.0 1.0 0.0 8 4 0 1 0

2.0 1.0 0.0 2 1 0 2 0

2.0 1.0 0.0 4 2 0 2 0

2.0 1.0 0.0 8 4 0 2 0

2.0 1.0 0.0 2 1 0 1 1

2.0 1.0 0.0 4 2 0 1 1

2.0 1.0 0.0 8 4 0 1 1

2.0 1.0 0.0 2 1 0 2 1

```

2.0 1.0 0.0 4 2 0 2 1
2.0 1.0 0.0 8 4 0 2 1
2.0 1.0 1.0 2 1 1 1 0
2.0 1.0 1.0 4 2 2 1 0
2.0 1.0 1.0 8 4 4 1 0
2.0 1.0 1.0 2 1 1 2 0
2.0 1.0 1.0 4 2 2 2 0
2.0 1.0 1.0 8 4 4 2 0
2.0 1.0 1.0 2 1 1 1 1
2.0 1.0 1.0 4 2 2 1 1
2.0 1.0 1.0 8 4 4 1 1
2.0 1.0 1.0 2 1 1 2 1
2.0 1.0 1.0 4 2 2 2 1
2.0 1.0 1.0 8 4 4 2 1

```

3.2.4 Result summary

We use CHeart rev. 6292 to produce numerical reference solutions.

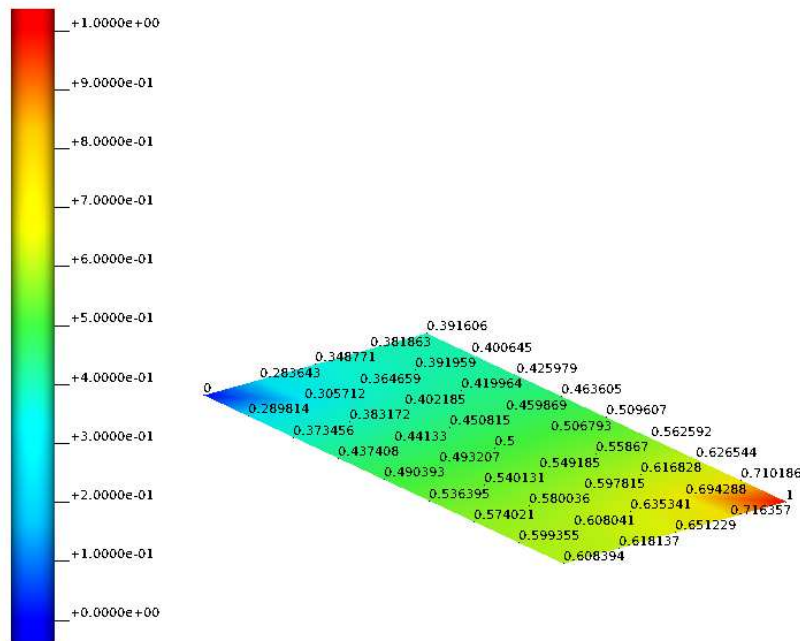


Figure 1: 2D results, iron reference w/ command line arguments [2.0 1.0 0.0 8 4 0 1 0].

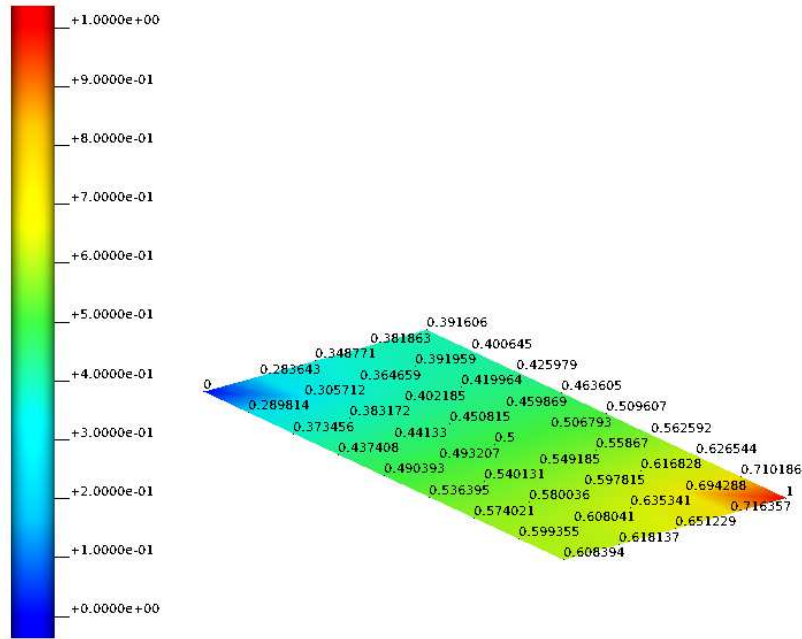


Figure 2: 2D results, current run w/ command line arguments [2.0 1.0 0.0 8 4 0 1 0].

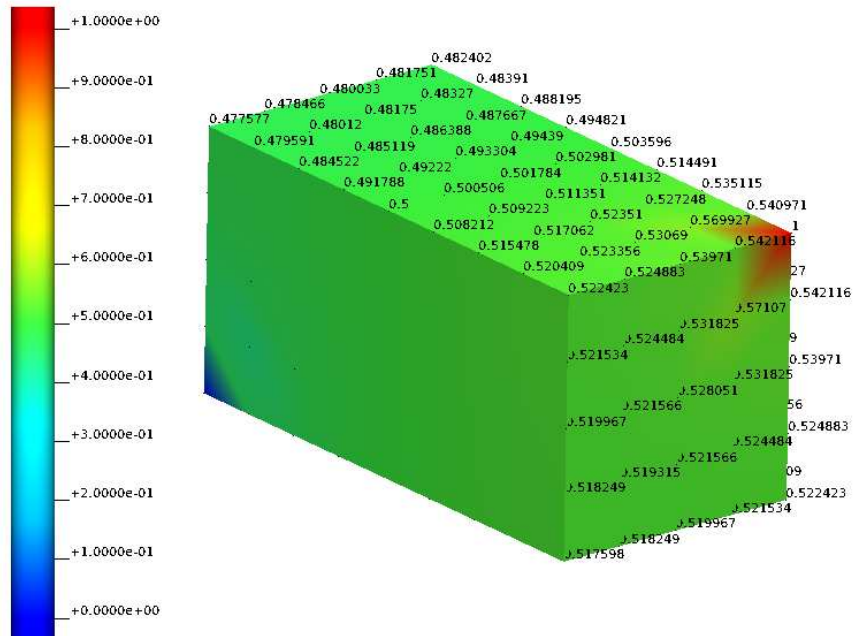


Figure 3: 3D results, iron reference w/ command line arguments [2.0 1.0 1.0 8 4 1 0].

3.3 Example-0001-u

Example uses user-defined regular meshes in CHeart mesh format and solves a static problem, i.e., applies the boundary conditions in one step.

3.3.1 Mathematical model - 2D

We solve the following scalar equation,

$$\nabla \cdot \nabla u = 0 \quad \Omega = [0, 2] \times [0, 1], \quad (8)$$

with boundary conditions

$$u = 0 \quad x = y = 0, \quad (9)$$

$$u = 1 \quad x = 2, y = 1. \quad (10)$$

No material parameters to specify.

3.3.2 Mathematical model - 3D

We solve the following scalar equation,

$$\nabla \cdot \nabla u = 0 \quad \Omega = [0, 2] \times [0, 1] \times [0, 1], \quad (11)$$

with boundary conditions

$$u = 0 \quad x = y = z = 0, \quad (12)$$

$$u = 1 \quad x = 2, y = z = 1. \quad (13)$$

No material parameters to specify.

3.3.3 Computational model

- Commandline arguments are:

float: length along x-direction

float: length along y-direction

float: length along z-direction (set to zero for 2D)

integer: number of elements in x-direction

integer: number of elements in y-direction

integer: number of elements in z-direction (set to zero for 2D)

integer: interpolation order (1: linear; 2: quadratic)

integer: solver type (0: direct; 1: iterative)

- Commandline arguments for tests are:

2.0 1.0 0.0 2 1 0 1 0

2.0 1.0 0.0 4 2 0 1 0

2.0 1.0 0.0 8 4 0 1 0

2.0 1.0 0.0 2 1 0 2 0

2.0 1.0 0.0 4 2 0 2 0

2.0 1.0 0.0 8 4 0 2 0

2.0 1.0 0.0 2 1 0 1 1

2.0 1.0 0.0 4 2 0 1 1

2.0 1.0 0.0 8 4 0 1 1

2.0 1.0 0.0 2 1 0 2 1

```

2.0 1.0 0.0 4 2 0 2 1
2.0 1.0 0.0 8 4 0 2 1
2.0 1.0 1.0 2 1 1 1 0
2.0 1.0 1.0 4 2 2 1 0
2.0 1.0 1.0 8 4 4 1 0
2.0 1.0 1.0 2 1 1 2 0
2.0 1.0 1.0 4 2 2 2 0
2.0 1.0 1.0 8 4 4 2 0
2.0 1.0 1.0 2 1 1 1 1
2.0 1.0 1.0 4 2 2 1 1
2.0 1.0 1.0 8 4 4 1 1
2.0 1.0 1.0 2 1 1 2 1
2.0 1.0 1.0 4 2 2 2 1
2.0 1.0 1.0 8 4 4 2 1

```

- Note: Binary uses command line arguments to search for the relevant mesh files.

3.3.4 Result summary

We use CHeart rev. 6292 to produce numerical reference solutions.

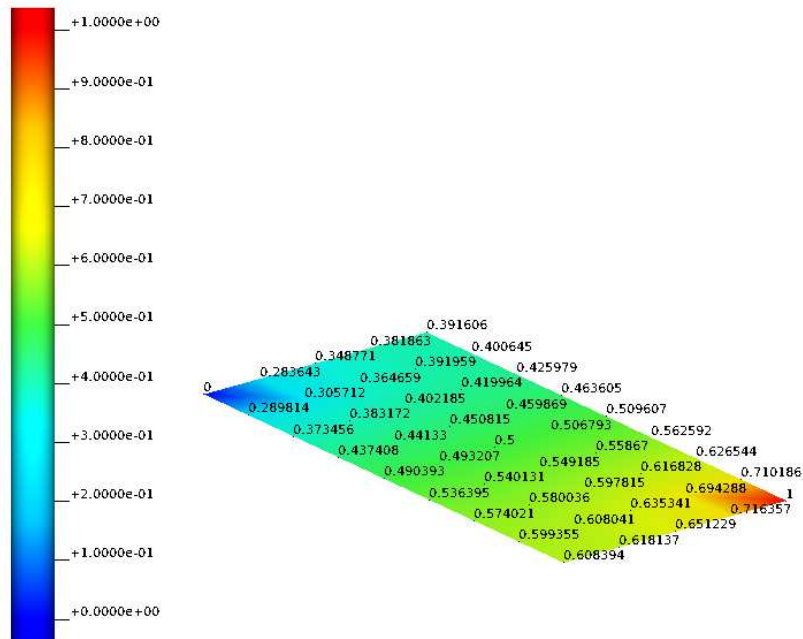


Figure 5: 2D results, iron reference w/ command line arguments [2.0 1.0 0.0 8 4 0 1 0].

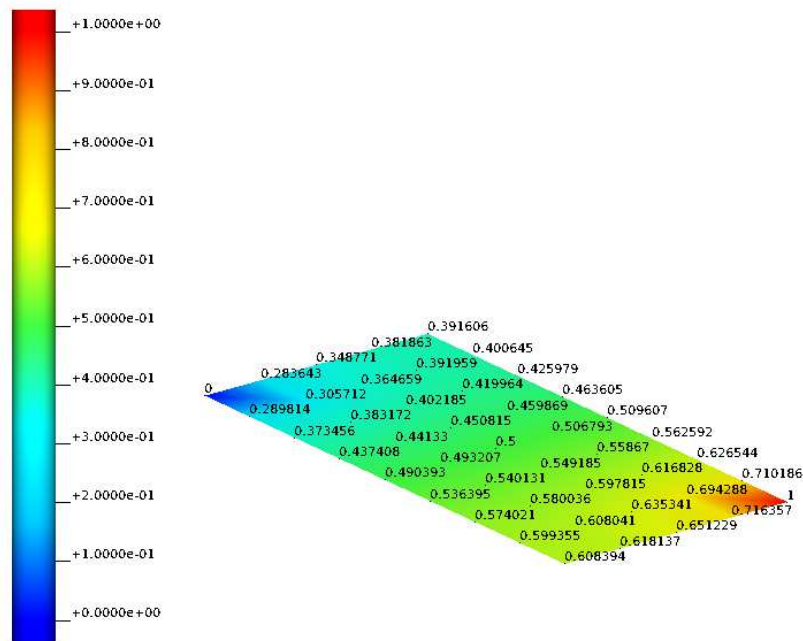


Figure 6: 2D results, current run w/ command line arguments [2.0 1.0 0.0 8 4 0 1 0].

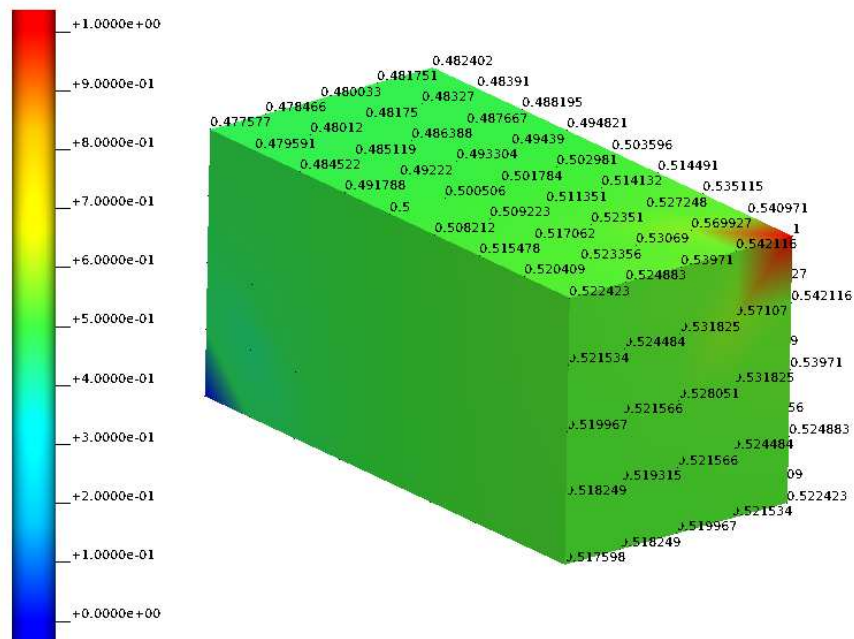


Figure 7: 3D results, iron reference w/ command line arguments [2.0 1.0 1.0 8 4 1 0].

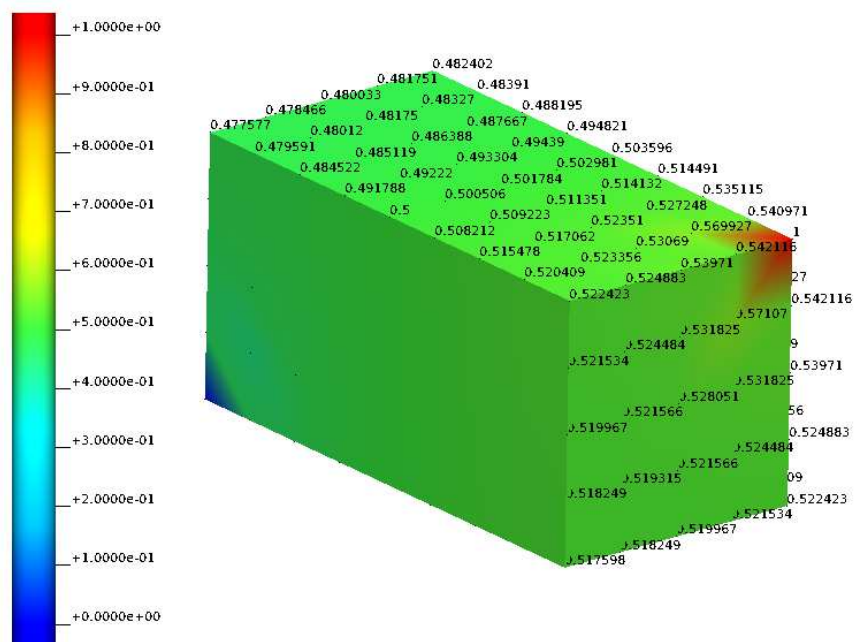


Figure 8: 3D results, current run w/ command line arguments [2.0 1.0 1.0 8 4 4 1 0].

3.4 Example-0002

Example uses generated regular meshes and solves a static problem, i.e., applies the boundary conditions in one step.

3.4.1 Mathematical model - 2D

We solve the following scalar equation,

$$\nabla \cdot \nabla u = 0 \quad \Omega = [0, 2] \times [0, 1], \quad (14)$$

with boundary conditions

$$u = 15y \quad x = 0, \quad (15)$$

$$u = 25 - 18y \quad x = 2. \quad (16)$$

No material parameters to specify.

3.4.2 Mathematical model - 3D

We solve the following scalar equation,

$$\nabla \cdot \nabla u = 0 \quad \Omega = [0, 2] \times [0, 1] \times [0, 1], \quad (17)$$

with boundary conditions

$$u = 15y \quad x = 0, \quad (18)$$

$$u = 25 - 18y \quad x = 2. \quad (19)$$

No material parameters to specify.

3.4.3 Computational model

- Commandline arguments are:

float: length along x-direction

float: length along y-direction

float: length along z-direction (set to zero for 2D)

integer: number of elements in x-direction

integer: number of elements in y-direction

integer: number of elements in z-direction (set to zero for 2D)

integer: interpolation order (1: linear; 2: quadratic)

integer: solver type (0: direct; 1: iterative)

- Commandline arguments for tests are:

2.0 1.0 0.0 2 1 0 1 0

2.0 1.0 0.0 4 2 0 1 0

2.0 1.0 0.0 8 4 0 1 0

2.0 1.0 0.0 2 1 0 2 0

2.0 1.0 0.0 4 2 0 2 0

2.0 1.0 0.0 8 4 0 2 0

2.0 1.0 0.0 2 1 0 1 1

2.0 1.0 0.0 4 2 0 1 1

2.0 1.0 0.0 8 4 0 1 1

2.0 1.0 0.0 2 1 0 2 1

```

2.0 1.0 0.0 4 2 0 2 1
2.0 1.0 0.0 8 4 0 2 1
2.0 1.0 1.0 2 1 1 1 0
2.0 1.0 1.0 4 2 2 1 0
2.0 1.0 1.0 8 4 4 1 0
2.0 1.0 1.0 2 1 1 2 0
2.0 1.0 1.0 4 2 2 2 0
2.0 1.0 1.0 8 4 4 2 0
2.0 1.0 1.0 2 1 1 1 1
2.0 1.0 1.0 4 2 2 1 1
2.0 1.0 1.0 8 4 4 1 1
2.0 1.0 1.0 2 1 1 2 1
2.0 1.0 1.0 4 2 2 2 1
2.0 1.0 1.0 8 4 4 2 1

```

3.4.4 Result summary

We use CHeart rev. 6292 to produce numerical reference solutions.

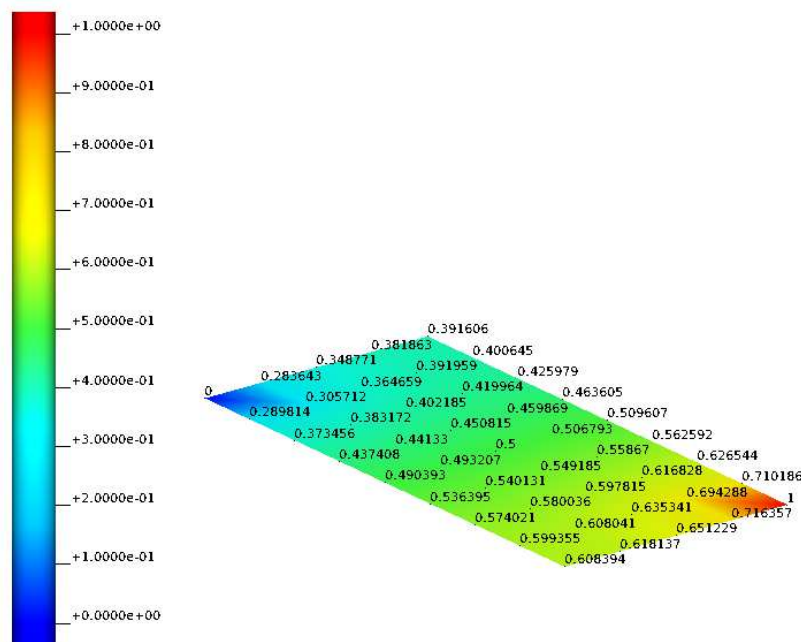


Figure 9: 2D results, iron reference w/ command line arguments [2.0 1.0 0.0 8 4 0 1 0].

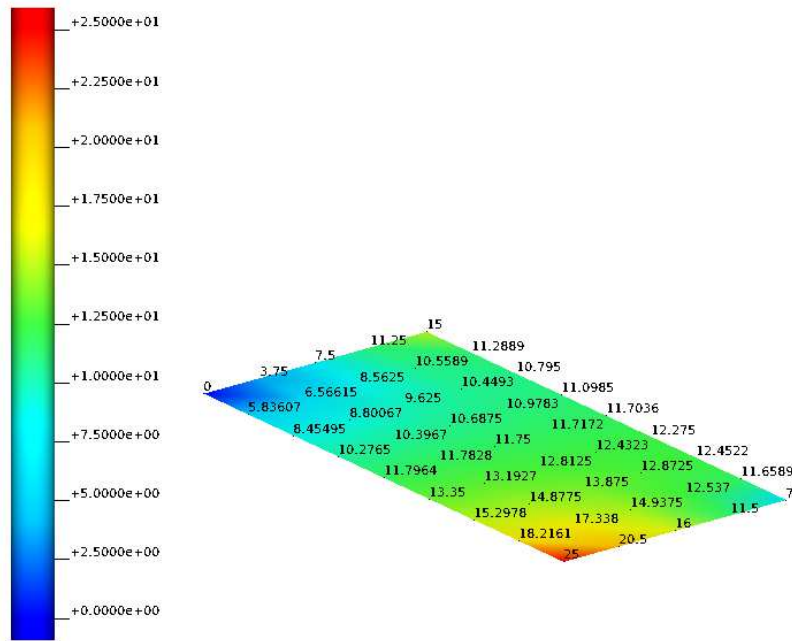


Figure 10: 2D results, current run w/ command line arguments [2.0 1.0 0.0 8 4 0 1 0].

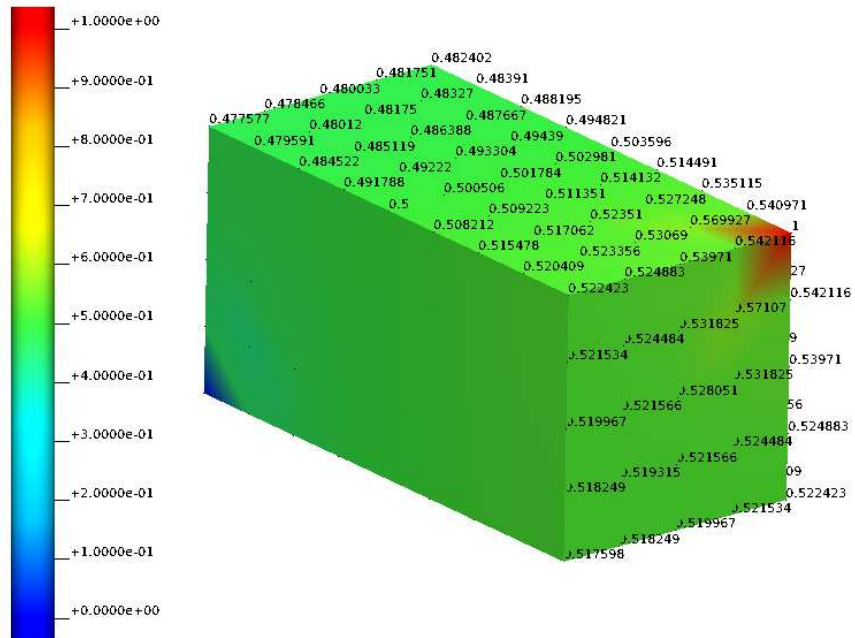


Figure 11: 3D results, iron reference w/ command line arguments [2.0 1.0 1.0 8 4 4 1 0].

3.5 Example-0003

Example uses generated regular meshes and solves a static problem, i.e., applies the boundary conditions in one step.

3.5.1 Mathematical model - 2D

We solve the following scalar equation,

$$\nabla \cdot \nabla u = 0 \quad \Omega = [0, 2] \times [0, 1], \quad (20)$$

with boundary conditions

$$u = 15y \quad x = 0, \quad (21)$$

$$\partial_n u = 25 - 18y \quad x = 2. \quad (22)$$

No material parameters to specify.

3.5.2 Mathematical model - 3D

We solve the following scalar equation,

$$\nabla \cdot \nabla u = 0 \quad \Omega = [0, 2] \times [0, 1] \times [0, 1], \quad (23)$$

with boundary conditions

$$u = 15y \quad x = 0, \quad (24)$$

$$\partial_n u = 25 - 18y \quad x = 2. \quad (25)$$

No material parameters to specify.

3.5.3 Computational model

- Commandline arguments are:

float: length along x-direction

float: length along y-direction

float: length along z-direction (set to zero for 2D)

integer: number of elements in x-direction

integer: number of elements in y-direction

integer: number of elements in z-direction (set to zero for 2D)

integer: interpolation order (1: linear; 2: quadratic)

integer: solver type (0: direct; 1: iterative)

- Commandline arguments for tests are:

2.0 1.0 0.0 2 1 0 1 0

2.0 1.0 0.0 4 2 0 1 0

2.0 1.0 0.0 8 4 0 1 0

2.0 1.0 0.0 2 1 0 2 0

2.0 1.0 0.0 4 2 0 2 0

2.0 1.0 0.0 8 4 0 2 0

2.0 1.0 0.0 2 1 0 1 1

2.0 1.0 0.0 4 2 0 1 1

2.0 1.0 0.0 8 4 0 1 1

2.0 1.0 0.0 2 1 0 2 1

```

2.0 1.0 0.0 4 2 0 2 1
2.0 1.0 0.0 8 4 0 2 1
2.0 1.0 1.0 2 1 1 1 0
2.0 1.0 1.0 4 2 2 1 0
2.0 1.0 1.0 8 4 4 1 0
2.0 1.0 1.0 2 1 1 2 0
2.0 1.0 1.0 4 2 2 2 0
2.0 1.0 1.0 8 4 4 2 0
2.0 1.0 1.0 2 1 1 1 1
2.0 1.0 1.0 4 2 2 1 1
2.0 1.0 1.0 8 4 4 1 1
2.0 1.0 1.0 2 1 1 2 1
2.0 1.0 1.0 4 2 2 2 1
2.0 1.0 1.0 8 4 4 2 1

```

3.5.4 *Result summary*

We use CHeart rev. 6292 to produce numerical reference solutions.

Figure 13: 2D results, iron reference w/ command line arguments [2.0 1.0 0.0 8 4 0 1 0].

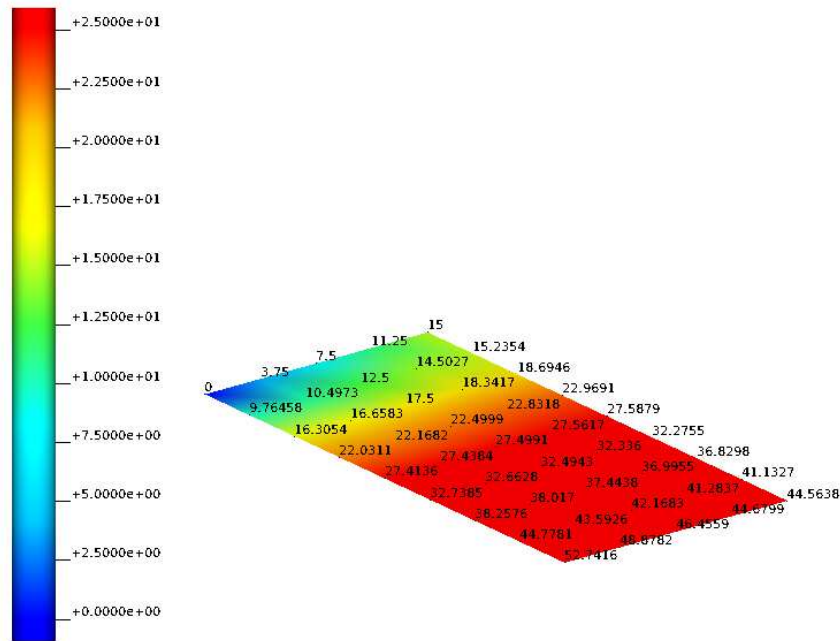


Figure 14: 2D results, current run w/ command line arguments [2.0 1.0 0.0 8 4 0 1 0].

Figure 15: 3D results, iron reference w/ command line arguments [2.0 1.0 1.0 8 4 4 1 0].

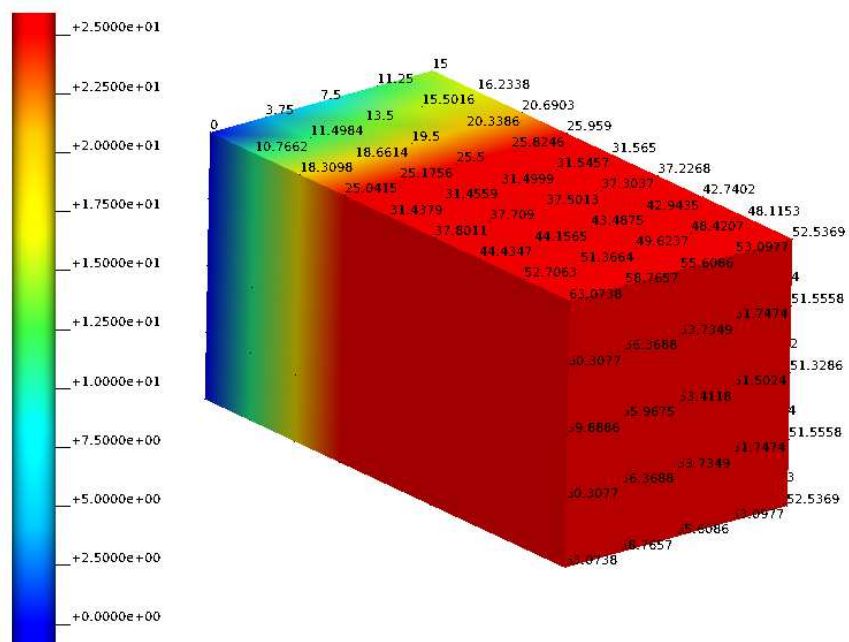


Figure 16: 3D results, current run w/ command line arguments [2.0 1.0 1.0 8 4 1 0].

3.6 Example-0004

Example uses generated regular meshes and solves a static problem, i.e., applies the boundary conditions in one step.

3.6.1 *Mathematical model - 2D*

We solve the following scalar equation,

$$\nabla \cdot \nabla u = 0 \quad \Omega = [0, 2] \times [0, 1], \quad (26)$$

with boundary conditions

$$u = 2.0e^x \cdot \cos(y) \quad \text{on } \partial\Omega. \quad (27)$$

No material parameters to specify.

3.6.2 *Computational model*

- Commandline arguments are:
 - integer: number of elements in x-direction
 - integer: number of elements in y-direction
 - integer: number of elements in z-direction (set to zero for 2D)
 - integer: interpolation order (1: linear; 2: quadratic)
 - integer: solver type (0: direct; 1: iterative)
- Commandline arguments for tests are:
 - 4 2 0 1 0
 - 8 4 0 1 0
 - 2 1 0 2 0
 - 4 2 0 2 0
 - 8 4 0 2 0
 - 4 2 0 1 1
 - 8 4 0 1 1
 - 2 1 0 2 1
 - 4 2 0 2 1
 - 8 4 0 2 1
 - 100 50 0 1 0 (not tested yet..)
 - 100 50 0 2 0 (not tested yet..)
 - 100 50 0 1 1 (not tested yet..)
 - 100 50 0 2 1 (not tested yet..)

3.6.3 *Result summary*

We use CHeart rev. 6292 to produce numerical reference solutions.

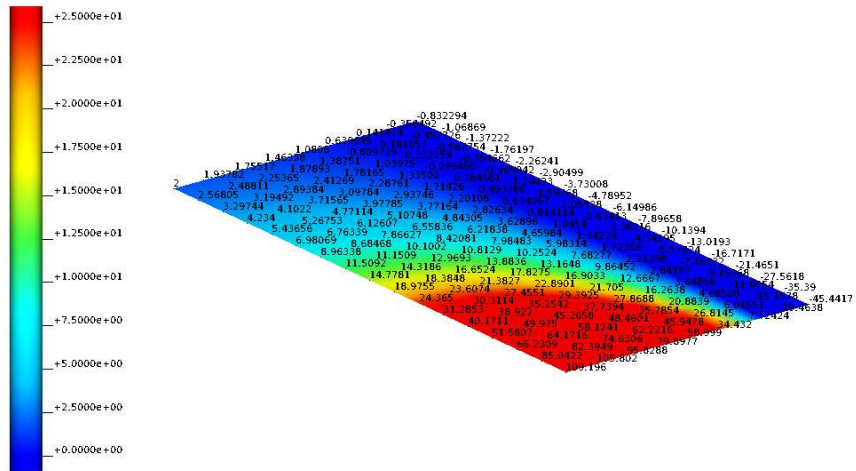


Figure 17: 2D results, iron reference w/ command line arguments [8 4 0 2 0].

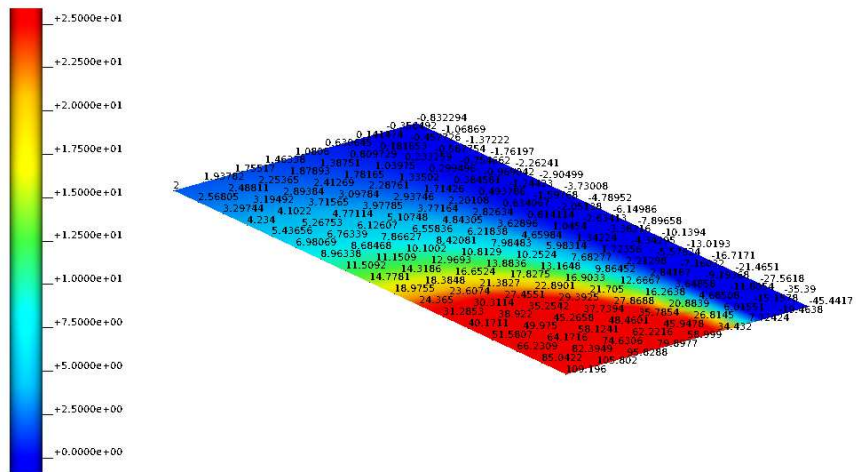


Figure 18: 2D results, current run w/ command line arguments [8 4 0 2 0].

3.7 Example-0011

Example uses generated regular meshes and solves a static problem, i.e., applies the boundary conditions in one step.

3.7.1 Mathematical model - 2D

We solve the following scalar equation,

$$\nabla \cdot [\sigma \nabla u] = 0 \quad \Omega = [0, 2] \times [0, 1], \quad (28)$$

with boundary conditions

$$u = 0 \quad x = y = 0, \quad (29)$$

$$u = 1 \quad x = 2, y = 1. \quad (30)$$

The conductivity tensor is defined as,

$$\sigma(x, t) = \sigma = \mathbf{I}. \quad (31)$$

3.7.2 Mathematical model - 3D

We solve the following scalar equation,

$$\nabla \cdot [\sigma \nabla u] = 0 \quad \Omega = [0, 2] \times [0, 1] \times [0, 1], \quad (32)$$

with boundary conditions

$$u = 0 \quad x = y = z = 0, \quad (33)$$

$$u = 1 \quad x = 2, y = z = 1. \quad (34)$$

The conductivity tensor is defined as,

$$\sigma(x, t) = \sigma = \mathbf{I}. \quad (35)$$

3.7.3 Computational model

- Commandline arguments are:

float: length along x-direction

float: length along y-direction

float: length along z-direction (set to zero for 2D)

integer: number of elements in x-direction

integer: number of elements in y-direction

integer: number of elements in z-direction (set to zero for 2D)

integer: interpolation order (1: linear; 2: quadratic)

integer: solver type (0: direct; 1: iterative)

float: σ_{11}

float: σ_{22}

float: σ_{33} (ignored for 2D)

- Commandline arguments for tests are:

2.0 1.0 0.0 2 1 0 1 0 1 1

2.0 1.0 0.0 4 2 0 1 0 1 1

2.0 1.0 0.0 8 4 0 1 0 1 1

2.0 1.0 0.0 2 1 0 2 0 1 1

```

2.0 1.0 0.0 4 2 0 2 0 1 1
2.0 1.0 0.0 8 4 0 2 0 1 1
2.0 1.0 0.0 2 1 0 1 1 1 1
2.0 1.0 0.0 4 2 0 1 1 1 1
2.0 1.0 0.0 8 4 0 1 1 1 1
2.0 1.0 0.0 2 1 0 2 1 1 1
2.0 1.0 0.0 4 2 0 2 1 1 1
2.0 1.0 0.0 8 4 0 2 1 1 1
2.0 1.0 1.0 2 1 1 1 0 1 1 1
2.0 1.0 1.0 4 2 2 1 0 1 1 1
2.0 1.0 1.0 8 4 4 1 0 1 1 1
2.0 1.0 1.0 2 1 1 2 0 1 1 1
2.0 1.0 1.0 4 2 2 2 0 1 1 1
2.0 1.0 1.0 8 4 4 2 0 1 1 1
2.0 1.0 1.0 2 1 1 1 1 1 1 1
2.0 1.0 1.0 4 2 2 1 1 1 1 1
2.0 1.0 1.0 8 4 4 1 1 1 1 1
2.0 1.0 1.0 2 1 1 2 1 1 1 1
2.0 1.0 1.0 4 2 2 2 1 1 1 1
2.0 1.0 1.0 8 4 4 2 1 1 1 1

```

3.7.4 Result summary

We use CHeart rev. 6292 to produce numerical reference solutions.

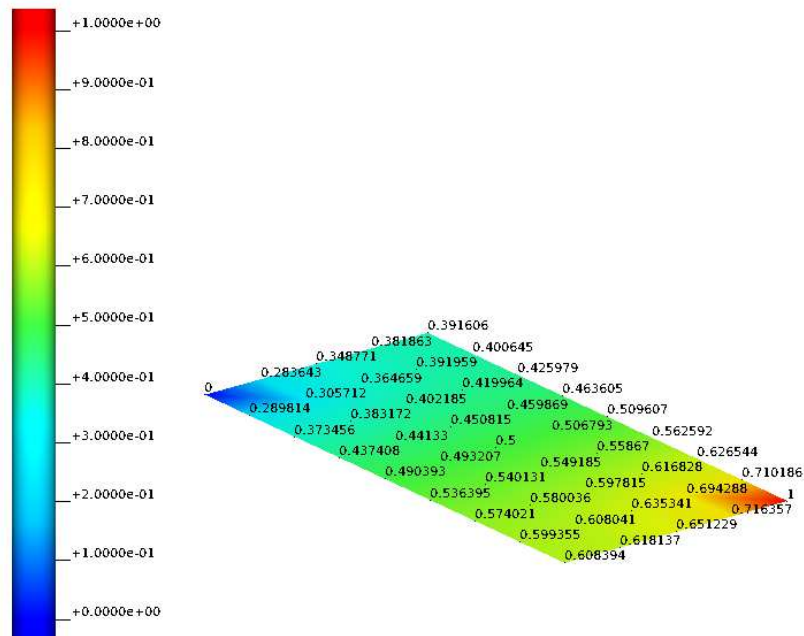


Figure 19: 2D results, iron reference w/ command line arguments [2.0 1.0 0.0 8 4 0 1 0 1 1].

4 LINEAR ELASTICITY

4.1 Equation in general form

$$\partial_{tt}\mathbf{u} + \nabla \cdot \boldsymbol{\sigma}(\mathbf{u}, t) = \mathbf{f}(\mathbf{u}, t) \quad (36)$$

4.2 Example-0101

4.2.1 Mathematical model

We solve the following equation (both 2D and 3D domains are considered),

$$\nabla \cdot \sigma(\mathbf{u}, t) = 0 \quad \Omega = [0, 160] \times [0, 120] \times [0, 120], t \in [0, 5], \quad (37)$$

with time step size $\Delta_t = 1$ and $\mathbf{u} = [u_x, u_y]$ in 2D $\mathbf{u} = [u_x, u_y, u_z]$ in 3D. The boundary conditions in 2D are given by

$$u_x = u_y = 0 \quad x = y = 0, \quad (38)$$

$$u_x = 16 \quad x = 160, \quad (39)$$

and in 3D by

$$u_x = u_y = u_z = 0 \quad x = y = z = 0, \quad (40)$$

$$u_x = 16 \quad x = 160. \quad (41)$$

The material parameters are

$$E = 10000 \text{MPa}, \quad (42)$$

$$\nu = 0.3, \quad (43)$$

$$\rho = 5 \times 10^{-9} \text{tonne} \cdot \text{mm}^3. \quad (44)$$

4.2.2 Computational model

- Commandline arguments are:
 - float: length along x-direction
 - float: length along y-direction
 - float: length along z-direction (set to zero for 2D)
 - integer: number of elements in x-direction
 - integer: number of elements in y-direction
 - integer: number of elements in z-direction (set to zero for 2D)
 - integer: interpolation order (1: linear; 2: quadratic)
 - integer: solver type (0: direct; 1: iterative)
 - float: elastic modulus
 - float: Poisson ratio
 - float: displacement percentage load
- Commandline arguments for tests are:
 - ...

4.2.3 Results

4.2.4 Validation

CHeart rev. 6328, Abaqus 2017, analytical reference solution, whatever...

Figure 23: Results, analytical solution.

Figure 24: Results, Abaqus reference.

Figure 25: Results, iron reference.

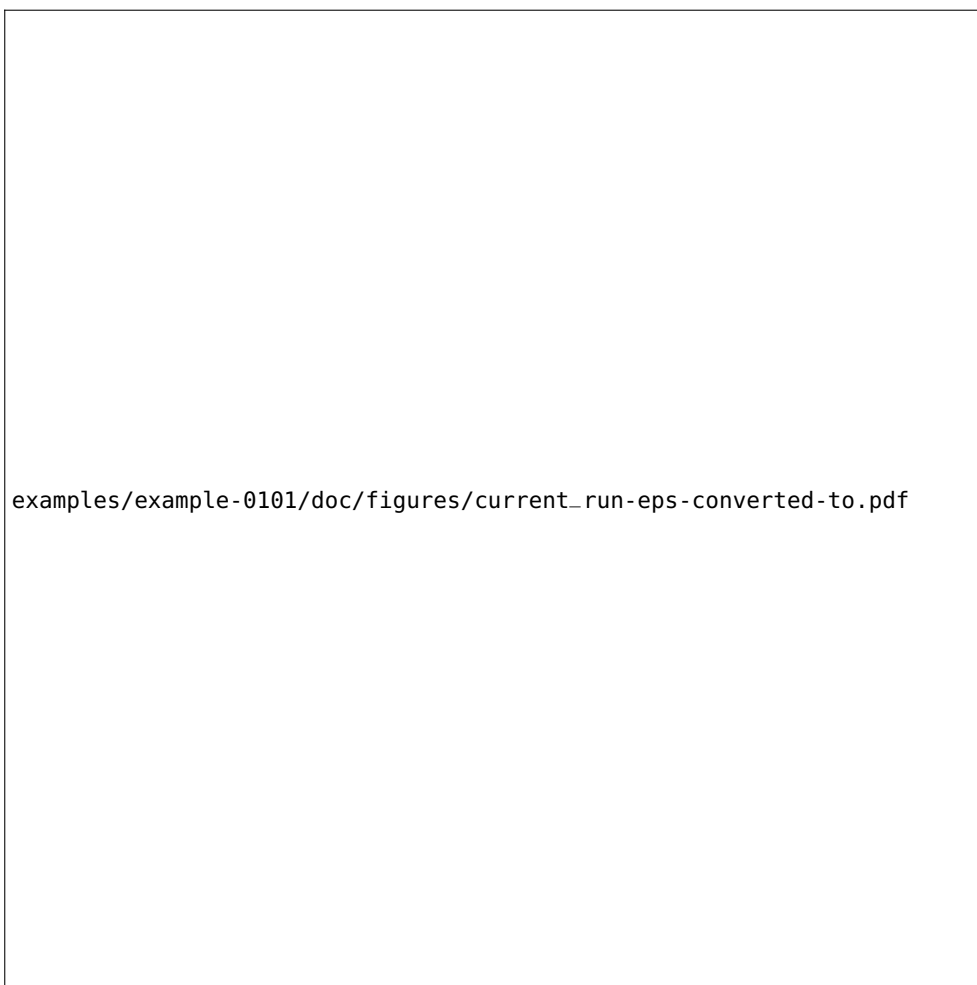


Figure 26: Results, current run.

4.3 Example-0102

4.3.1 Mathematical model

We solve the following equation (both 2D and 3D domains are considered),

$$\nabla \cdot \sigma(\mathbf{u}, t) = 0 \quad \Omega = [0, 160] \times [0, 120] \times [0, 120], t \in [0, 5], \quad (45)$$

with time step size $\Delta_t = 1$ and $\mathbf{u} = [u_x, u_y]$ in 2D $\mathbf{u} = [u_x, u_y, u_z]$ in 3D. The boundary conditions in 2D are given by

$$u_x = u_y = 0 \quad y = 0, \quad (46)$$

$$u_y = 8 \quad x = 160, \quad (47)$$

and in 3D by

$$u_x = u_z = 0 \quad x = 0, \quad (48)$$

$$u_y = 0 \quad y = 0, \quad (49)$$

$$u_x = 160 \quad x = 160, \quad (50)$$

$$u_y = 8 \quad x = 160. \quad (51)$$

The material parameters are

$$E = 10000 \text{ MPa}, \quad (52)$$

$$\nu = 0.3, \quad (53)$$

$$\rho = 5 \times 10^{-9} \text{ tonne} \cdot \text{mm}^3. \quad (54)$$

4.3.2 Computational model

- Commandline arguments are:
 - float: length along x-direction
 - float: length along y-direction
 - float: length along z-direction (set to zero for 2D)
 - integer: number of elements in x-direction
 - integer: number of elements in y-direction
 - integer: number of elements in z-direction (set to zero for 2D)
 - integer: interpolation order (1: linear; 2: quadratic)
 - integer: solver type (0: direct; 1: iterative)
 - float: elastic modulus
 - float: Poisson ratio
 - float: displacement percentage load
- Commandline arguments for tests are:
 - ...

4.3.3 Results

4.3.4 Validation

CHeart rev. 6328, Abaqus 2017, analytical reference solution, whatever...

Figure 27: Results, analytical solution.

Figure 28: Results, Abaqus reference.

Figure 29: Results, iron reference.

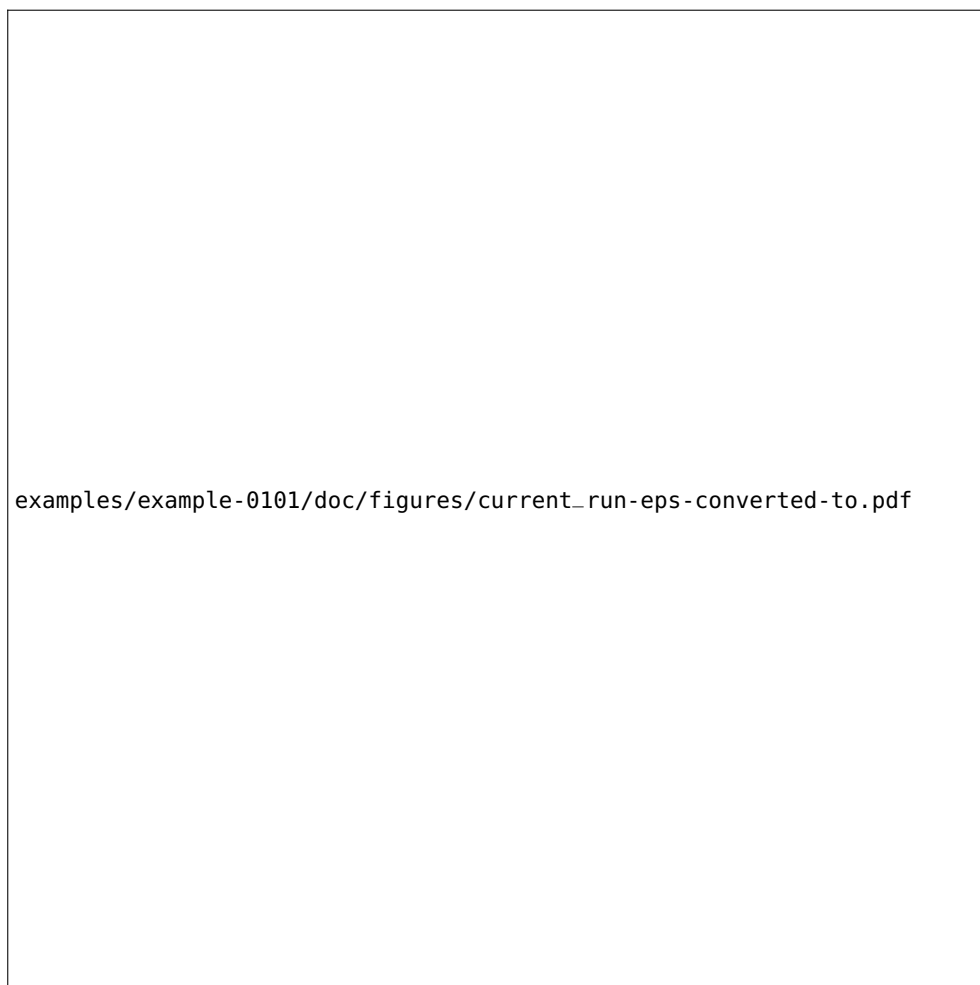


Figure 30: Results, current run.

5 FINITE ELASTICITY

6 NAVIER-STOKES FLOW

7 MONODOMAIN

8 CELLML MODEL

REFERENCES

- [1] Chris Bradley, Andy Bowery, Randall Britten, Vincent Budelmann, Oscar Camara, Richard Christie, Andrew Cookson, Alejandro F Frangi, Thiranj Babarenda Gamage, Thomas Heidlauf, et al. Opencmiss: a multi-physics & multi-scale computational infrastructure for the vph/physiome project. *Progress in biophysics and molecular biology*, 107(1):32–47, 2011.