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ECPE 121: Digital Signal Processing
Mini Exam 8 (25 pts total)

1. (10 pts) The signal $x(n) = 0.5^n u(n)$ is passed through the system $y(n) - 0.8y(n-1) = x(n)$. Obtain an expression for the DTFT of the output signal $y(n)$. Use a geometric approach to obtain a sketch of the magnitude spectrum $|Y(e^{j\theta})|$ and indicate the values at $\theta = 0$ and $\theta = \pi$ on the plot. Show all calculations.

2. (5 pts) Obtain the inverse DTFT of the signal $X(e^{j\theta}) = \text{j}\sin(2\theta)$.

3. (6 pts) The signal $x(n) = \delta(n) + 2\delta(n-1) + 3\delta(n-2)$ is zero padded to length 25 and a 25 point DFT $X(k)$ is computed. Evaluate $X(10)$ and put your result in polar form. Show all your work.

4. (4 pts) A signal $x(n)$ has length 30 samples and is obtained by sampling a signal at a sampling rate of 2000 samples / sec. $x(n)$ is zero padded to length 100 and a 100-point DFT is computed. What is the resolution of the DFT in Hz?

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2 $X[n] = \frac{1}{N}\sum_{n=0}^{N-1} x[k]\, e^{\left(j\frac{2\pi}{N}k\right)n}$

$X[e^{j\theta}] = \left[\frac{e^{j2\theta} - e^{-j2\theta}}{2j}\right]_{\theta} \Rightarrow \frac{e^{j2\theta} - e^{-j2\theta}}{2}$

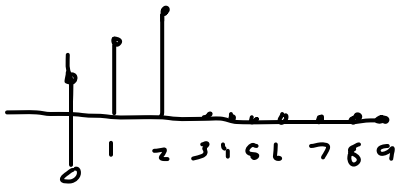
$X[z] = \frac{1}{2}\left[z^{2} - z^{-2}\right]$

$x(-2) = \frac{1}{2}$

$x(2) = \frac{1}{2}$

$$x(n) = \frac{1}{2}\left[\delta(n+2) + \delta(n-2)\right]$$

3.



$x = \{1, 2, 3\}$

25 $X(z) = 1 + 2z^{-1} + 3z^{-2}$

$= 1 + 2e^{-j\theta} + 3e^{-2j\theta}$

$\theta = \frac{2\pi k}{n}$

$n = 25$

$k = 10$

$X(10) = 1 + 2e^{-j\left(\frac{20\pi}{25}\right)} + 3e^{-j\left(\frac{40\pi}{25}\right)}$

$X(10) = 1.71e^{j1.389}$

4. $\Delta F = \frac{F_s}{N} \Rightarrow \frac{2000 \frac{\text{samples}}{\text{sec}}}{30 \text{ samples}}$

66.67 Hz

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5. (4 pts) The Fourier series coefficients of a periodic signal $x(n)$ of period 4 are $c_o = 1, c_1 = 3e^{j\pi/4}, c_2 = 2, c_3 = 3e^{-j\pi/4}$. Sketch the power density spectrum of $x(n)$ as a function of frequency θ in rad/sample. Also use your results to determine the average power of $x(n)$.