Name Beau Pasquier

ECPE 121: Digital Signal Processing
Mini Exam 7 (25 pts total)

$$x(n) = \sum_{k=0}^{N-1} c_k e^{\frac{j2\pi kn}{N}} \qquad N=4 \qquad c_k = \frac{1}{N}\sum_{n=0}^{N-1} x(n)e^{\frac{-j2\pi kn}{N}}$$

1. (11 pts) A periodic signal $x(n)$ repeats every 4 samples. Its Fourier series coefficients are $c_o = 1, c_1 = 3e^{j\pi/4}, c_2 = 2, c_3 = 3e^{-j\pi/4}$. $x(n)$ is passed through a causal system whose transfer function is $H(z) = \frac{z}{z+0.8}$. Obtain an analytical expression for the resulting output signal $y(n)$. Your answer must contain only real-valued functions and must be simplified as much as possible.

$$c_k = \left\{1,\ 3e^{j\frac{\pi}{4}},\ 2,\ 3e^{-j\frac{\pi}{4}}\right\}$$

$$x(n) = c_0 + c_1 e^{\frac{j\pi n(1)}{2}} + c_2 e^{\frac{j\pi n(2)}{2}} + c_1^*$$

$$x(n) = \underbrace{1}_{c_0} + \underbrace{2\cos(\pi n)}_{c_2} + \underbrace{2\left[3\cos\left(\frac{\pi}{2}n + \frac{\pi}{4}\right)\right]}_{c_1 \,\&\, c_3}$$

$$H(z) = \frac{z}{z+0.8} \rightarrow \frac{e^{j\theta}}{e^{j\theta}+0.8}$$

θ	$\lvert H(e^{j\theta})\rvert$ Magnitude	$\angle H(e^{j\theta})$ angle
0	0.56	0
$\frac{\pi}{2}$	0.78	0.67
π	5	0

$$y(n) = 1(0.56) + (0.78)6\cos\left(\frac{\pi}{2}n + \frac{\pi}{4}\ (+0.67)\right) + 5(2\cos(\pi n))$$

$$\boxed{y(n) = 0.56 + 4.68\cos\left(\frac{\pi}{2}n + 1.46\right) + 10\cos(\pi n)}$$

$N = 6$

6FSC

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2. (10 pts) $x(n) = \{\ldots\ \check{1}, 2, 3, 4, 3, 2, 1, 2, 3, 4, 3, 2, 1 \cdots\}$ is a periodic signal (the symbol above the 1 indicates the position of the origin). Obtain the Fourier series coefficients of $x(n)$. Show all your work.

$$= \frac{1}{6}\sum_{n=0}^{5} x(n)\, e^{-j\frac{2\pi k n}{6}}$$

$$c_k = \frac{1}{6}\left[1\times 1 + 2e^{-j\frac{2\pi k}{6}} + 3e^{-j\frac{4\pi k}{6}} + 4e^{-j\pi k} + 3e^{-j\frac{8\pi k}{6}} + 2e^{-j\frac{10\pi k}{6}}\right]$$

$$c_0 = \frac{1}{6}\left[1+2+3+4+3+2\right] = \boxed{2.5 \quad c_0}$$

$$c_1 = \frac{1}{6}\left[1 + 2e^{-j\frac{\pi}{3}} + 3e^{-j\frac{2\pi}{3}} + 4e^{-j\pi} + 3e^{-j\frac{4\pi}{3}} + 2e^{-j\frac{5}{3}\pi}\right]$$

$$= \frac{1}{6}\left[4e^{-j\pi}\right] \Rightarrow \boxed{\frac{2}{3}e^{-j\pi} \quad c_1}$$

$$c_2 = \frac{1}{6}\left[1 + 2e^{-j\frac{4\pi}{6}} + 3e^{-j\frac{8\pi}{6}} + 4e^{-j2\pi k} + 3e^{-j\frac{16\pi}{6}} + 2e^{-j\frac{20\pi}{6}}\right]$$

$$c_2 = \frac{1}{6}\left[0\right] \Rightarrow \boxed{0 \quad c_2}$$

$$c_3 = \frac{1}{6}\left[1 + 2\pi^{-j\pi} + 3e^{-j2\pi} + 4e^{-3j\pi} + 3e^{-j4\pi} + 2^{-j5\pi}\right]$$

$$c_3 = \frac{1}{6}\left[e^{j\pi}\right] \Rightarrow \boxed{0.167e^{j\pi} \quad c_3}$$

$$c_4 = c_{4-2} = c_2^{*} \Rightarrow \boxed{0 \quad c_4}$$

$$c_5 = c_{5-1} = c_1^{*} \Rightarrow \boxed{0.67e^{j\pi} \quad c_5}$$

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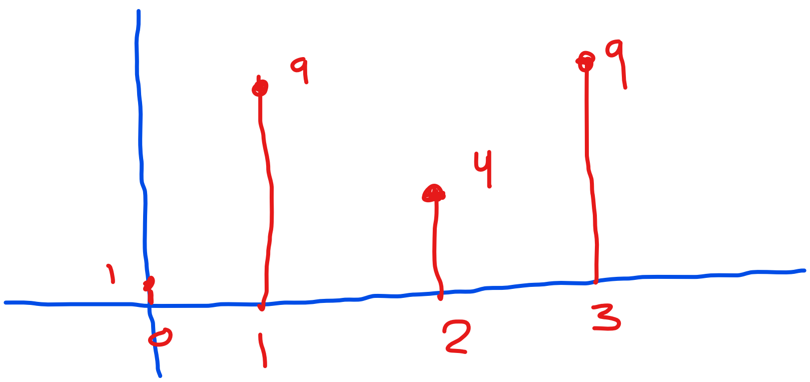
3. (4 pts) The Fourier series coefficients of a periodic signal $x(n)$ of period 4 are $c_o = 1, c_1 = 3e^{j\pi/4}, c_2 = 2, c_3 = 3e^{-j\pi/4}$. Sketch the power density spectrum of $x(n)$ as a function of frequency θ in rad/sample. Also use your results to determine the average power of $x(n)$.

$$c_k = \left\{ 1,\ 3e^{j\frac{\pi}{4}},\ 2,\ 3e^{-j\frac{\pi}{4}} \right\}$$

$|c_k|^2$ as function of $\theta = \frac{2\pi k}{N}$ $\qquad \theta = \frac{2\pi k}{4}$

$$|c_k|^2 = \{ 1,\ 9,\ 4,\ 9 \}$$



$\frac{2\pi(1)}{4}$ $\quad \frac{2\pi(2)}{4}$ $\quad \frac{2\pi(3)}{4}$

power density spectrum k
in rads/sample

Avg power

$$P = \sum_{n=0}^{N-1} |c_k^2|$$

$$P = 1 + 9 + 4 + 9$$

$$\boxed{P = 23}$$

Avg. Pwr