

Classical Guitar Intonation and Compensation

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Abstract

TBD.

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1 Introduction and Background

Initial [1] and ongoing work by G. Byers[2].

Recent studies of steel-string guitars [3]

2 Simple Model of Guitar Intonation

Fundamental frequency of a string [4, 5]:

$$f_0 = \frac{1}{2L_0} \sqrt{\frac{T_0}{\mu_0}}, \quad (1)$$

where L_0 is the length of the free (unfretted) string from the saddle to the nut, T_0 is the tension in the free string, and $\mu_0 \equiv M/L_0$ is the linear mass density of a free string of mass M .

$$f_0 = \frac{1}{2L_0} \sqrt{\frac{T_0}{\mu_0}} \left[1 + B_0 + \left(1 + \frac{\pi^2}{8} \right) B_0^2 \right], \quad (2)$$

where B_0 is a “string stiffness parameter.” For a uniform string with a cylindrical cross section, B_0 given by [6]

$$B_0 \equiv \sqrt{\frac{\pi R^4 E}{T_0 L_0^2}}, \quad (3)$$

where R is the radius of the string and E is Young’s modulus (or the modulus of elasticity). For a typical nylon guitar string with $E \approx 2 - 4$ GPa, $T_0 \approx 50 - 70$ N, $R \approx 0.35 - 0.51$ mm, and $L_0 \approx 650$ mm, we have $B_0 \approx 0.007 - 0.026$.

Throughout this work, we will use *cents* to describe small differences in pitch [7]. One cent is one one-hundredth of a 12-TET half step, so that there are 1200 cents per octave. The difference in pitch between frequencies f_1 and f_2 is therefore defined as

$$\Delta v \equiv 1200 \log_2 \left(\frac{f_2}{f_1} \right). \quad (4)$$

We define $f \equiv (f_1 + f_2)/2$ and $\Delta f \equiv f_2 - f_1$. Then

$$\Delta v = 1200 \log_2 \left(\frac{f + \Delta f/2}{f - \Delta f/2} \right) \approx \frac{1200}{\ln 2} \frac{\Delta f}{f}, \quad (5)$$

where the last approximation applies when $\Delta f \ll f$. An experienced guitar player can distinguish beat notes with a difference frequency of $\Delta f \approx 1$ Hz, which corresponds to 8 cents at A_2 ($f = 220$ Hz) or 5 cents at E_4 ($f = 329.63$ Hz).

Our model begins with the simple form of the fundamental frequency of a string given by Eq. (1), and applies it to the frequency of a string pressed just behind the n^{th} fret:

$$f_n = \frac{1}{2L_n} \sqrt{\frac{T_n}{\mu_n}}, \quad (6)$$

where (as shown in Fig. TBD) L_n is the *resonant length* of the string from the saddle to fret n , and T_n and μ_n are respectively the corresponding tension and linear mass density of the fretted string. We note that T_n and μ_n depend on \mathcal{L}_n , the *total* length of the fretted string from the saddle to the nut. Ideally, in the 12-TET equal-temperament system [7],

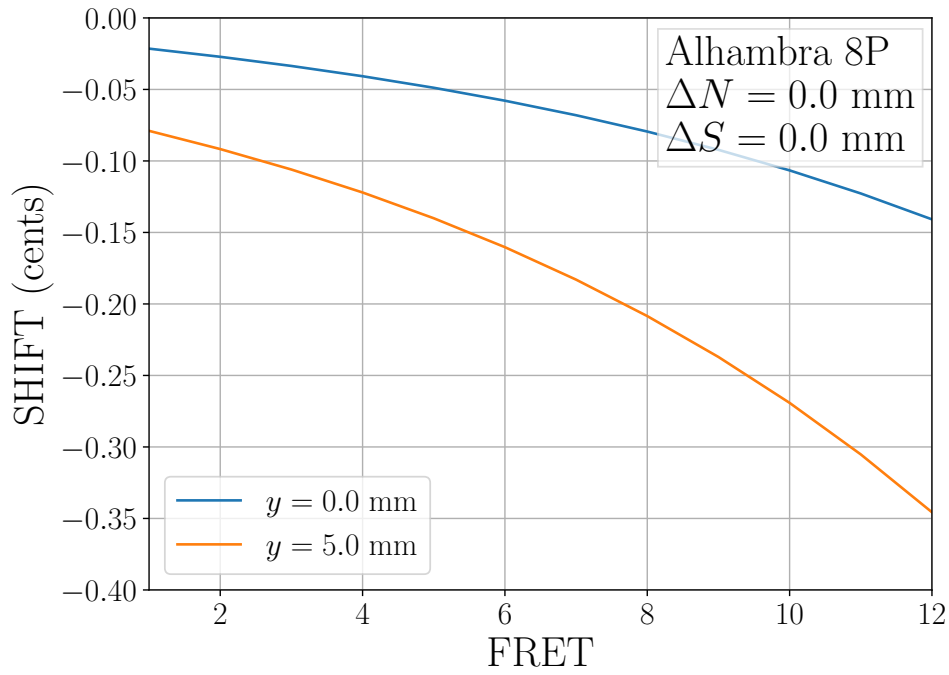
$$f_n = \gamma_n f_0, \quad (12\text{-TET ideal}) \quad (7)$$

where

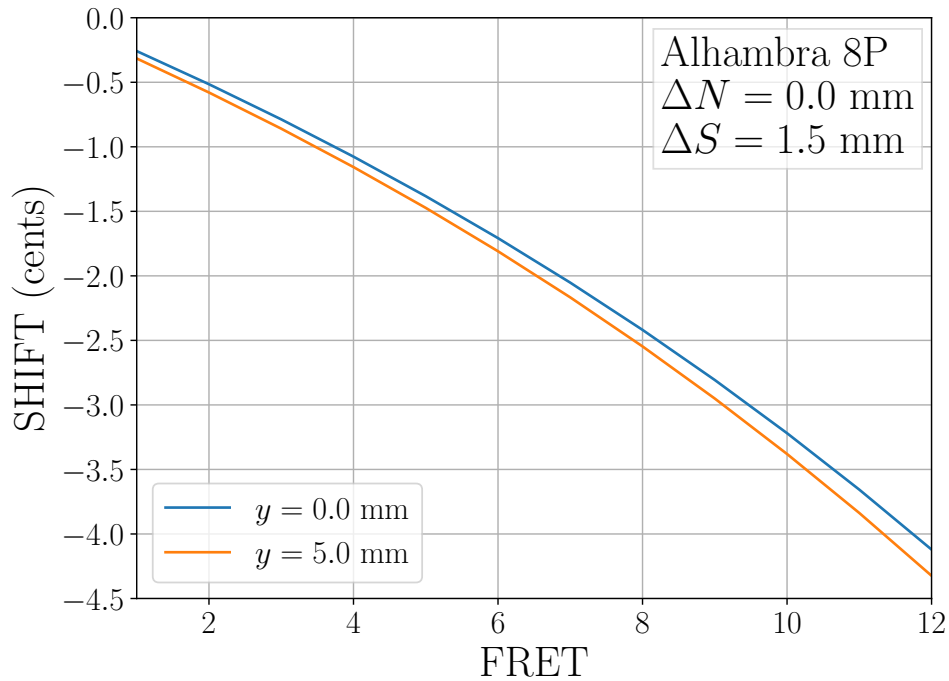
$$\gamma_n \equiv 2^{n/12}. \quad (8)$$

Therefore, the error interval expressed in cents is given by

$$\begin{aligned} \Delta v_n &= 1200 \log_2 \left(\frac{f_n}{\gamma_n f_0} \right) \\ &= 1200 \log_2 \left(\frac{L_0}{\gamma_n L_n} \sqrt{\frac{\mu_0}{\mu_n} \frac{T_n}{T_0}} \right). \end{aligned} \quad (9)$$

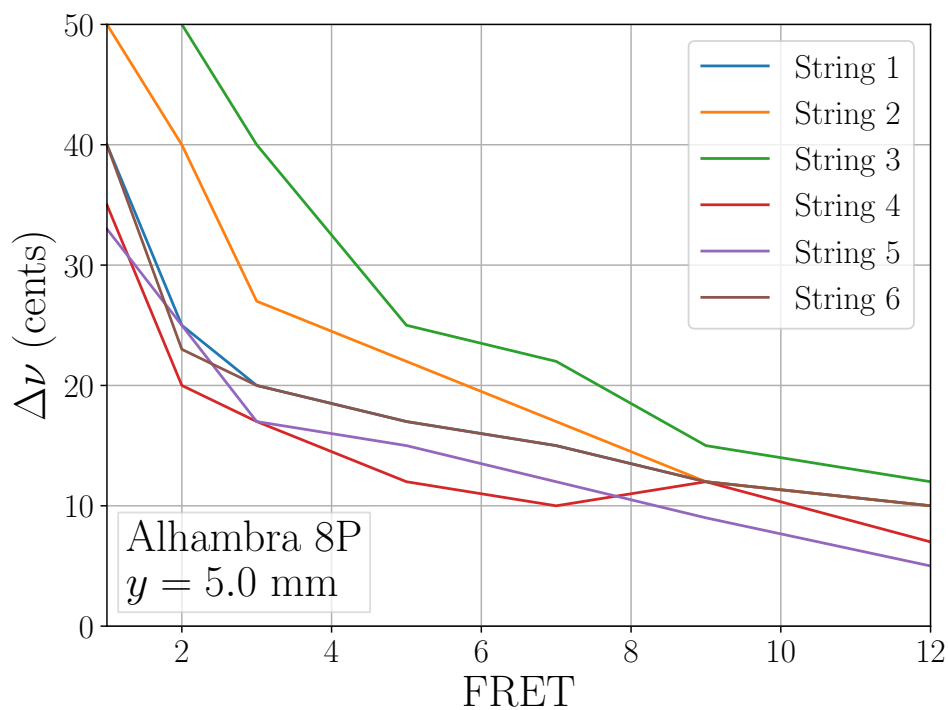


(a) Frequency shift for an uncompensated guitar



(b) Frequency shift for a factory guitar

Figure 1: Frequency shift (in cents) due to the fretted length L_n for an uncompensated (a) and factory (b) Alhambra 8P guitar, for both zero and nonzero lateral displacement y .



(a) Experimental data

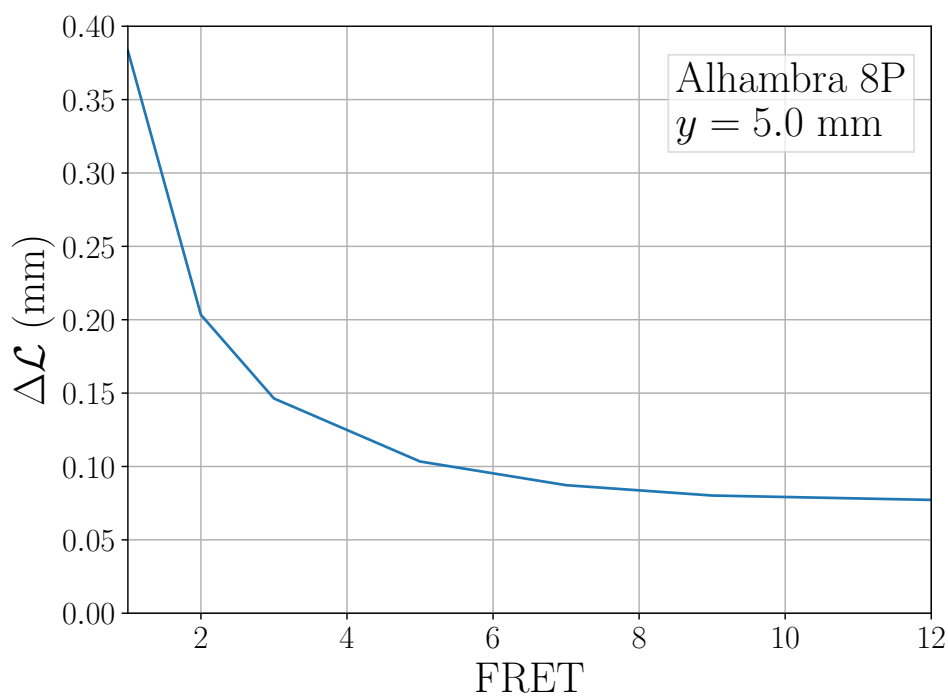
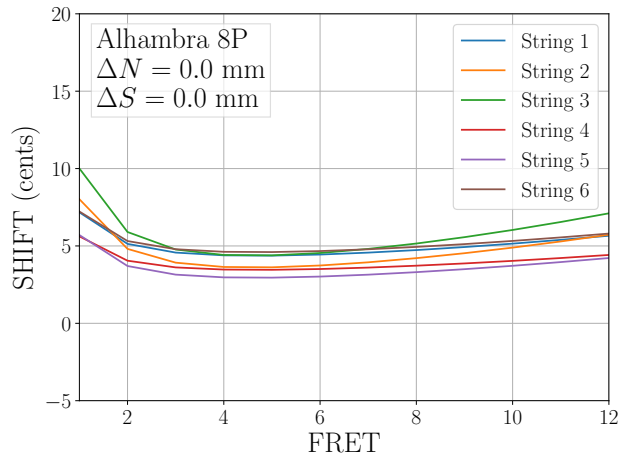
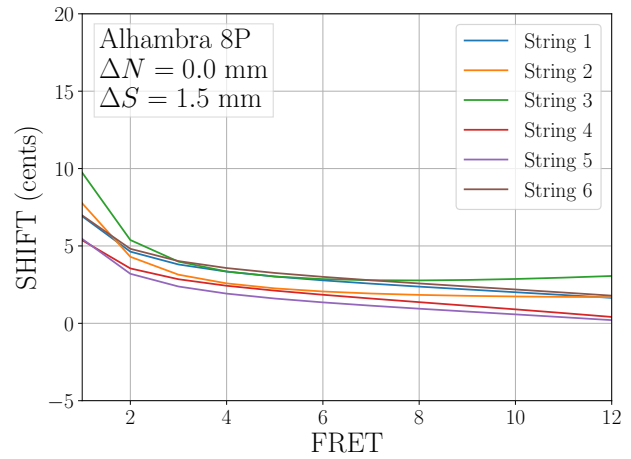
(b) Calculated change in total string length \mathcal{L}

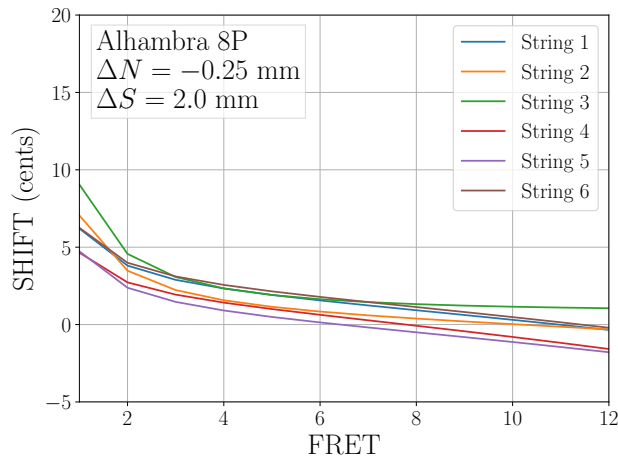
Figure 2: Frequency shift (in cents) (a) and change in total string length \mathcal{L} (b) due to lateral displacement y .



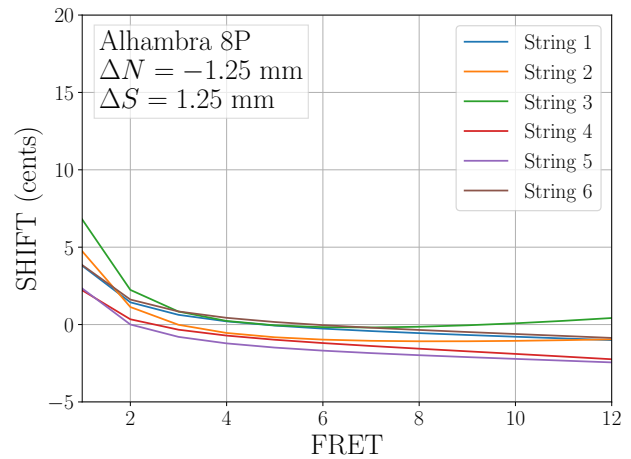
(a) Uncompensated



(b) Factory guitar



(c) Saddle and nut



(d) Preserved scale length

Figure 3: Frequency shift (in cents) for four different strategies of saddle and nut compensation.

3 Experimental Estimate of the String Constant

4 Classical Guitar Compensation

5 Conclusion

References

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- [7] D. S. Durfee and J. S. Colton, “The physics of musical scales: Theory and experiment,” *Amer. J. Phys.* **835–842**, 368 (2015).