Classical Guitar Intonation and Compensation

M. B. Anderson and R. G. Beausoleil
Rosewood Guitar
8402 Greenwood Ave. N, Seattle, WA 98103

September 6, 2020

Abstract

TBD.

Contents

1 Introduction and Background

Initial [1] and ongoing work by G. Byers[2].

Recent studies of steel-string guitars [3]

2 Simple Model of Guitar Intonation

Fundamental frequency of a string [4, 5]:

$$\nu_0 = \frac{1}{2\,L_0}\sqrt{\frac{T_0}{\mu_0}}\,, \tag{1}$$

where L_0 is the length of the free (unfretted) string from the saddle to the nut, T_0 is the tension in the free string, and $\mu_0 \equiv M/L_0$ is the linear mass density of a free string of mass M.

$$\nu_0 = \frac{1}{2\,L_0}\sqrt{\frac{T_0}{\mu_0}}\left[1 + B_0 + \left(1 + \frac{\pi^2}{8}\right) B_0^2\right] , \tag{2}$$

where B_0 is a "string stiffness parameter." For a uniform string with a cylindrical cross section, B_0 given by [6]

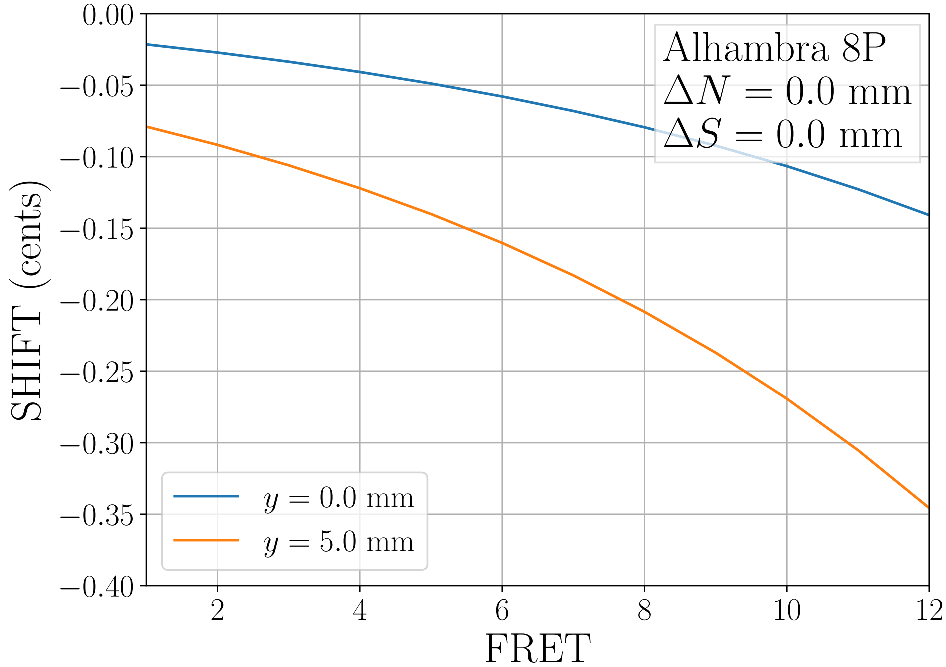
$$B_0 \equiv \sqrt{\frac{\pi\, R^4 E}{T_0\, L_0^2}} , \tag{3}$$

where R is the radius of the string and E is Young's modulus (or the modulus of elasticity). For a typical nylon guitar string with $E \approx 2 - 4$ GPa, $T_0 \approx 50 - 70$ N, $R \approx 0.35 - 0.51$ mm, and $L_0 \approx 650$ mm, we have $B_0 \approx 0.007 - 0.026$.

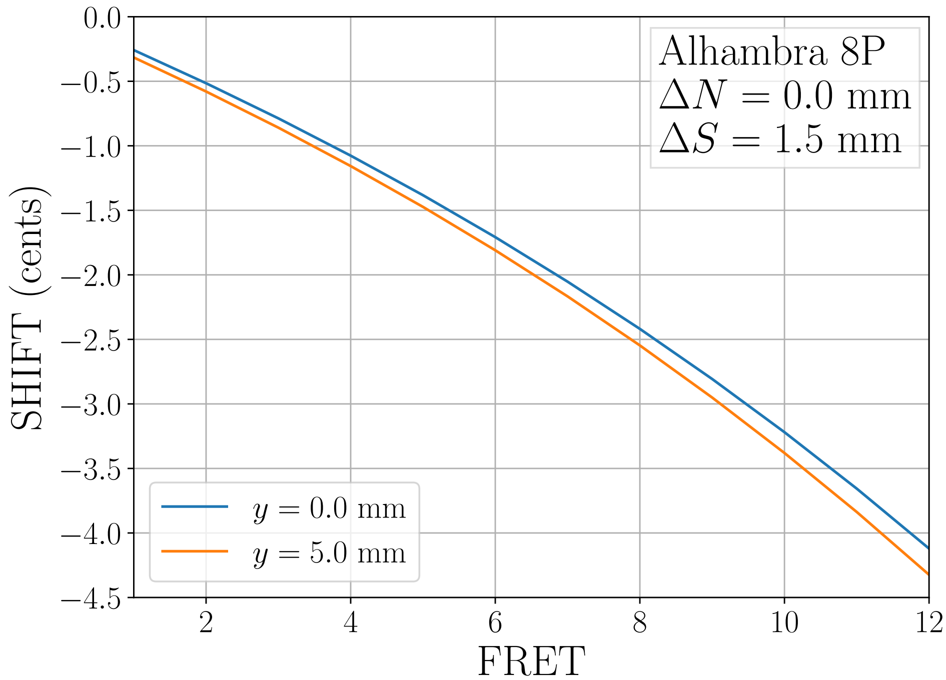
3 Experimental Estimate of the String Constant

4 Classical Guitar Compensation

5 Conclusion

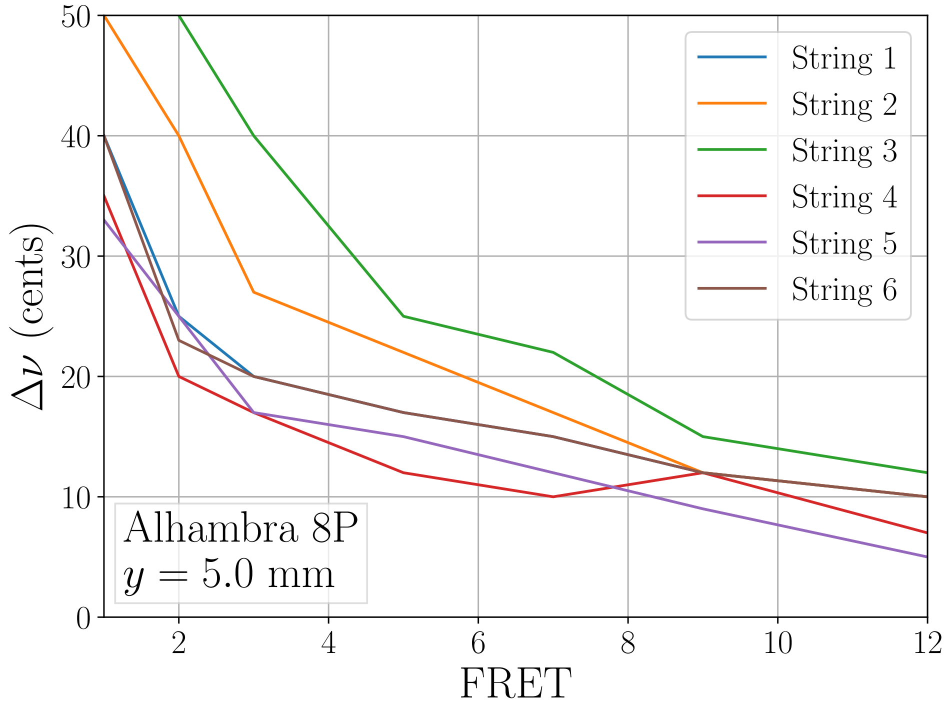


(a) Frequency shift for an uncompensated guitar

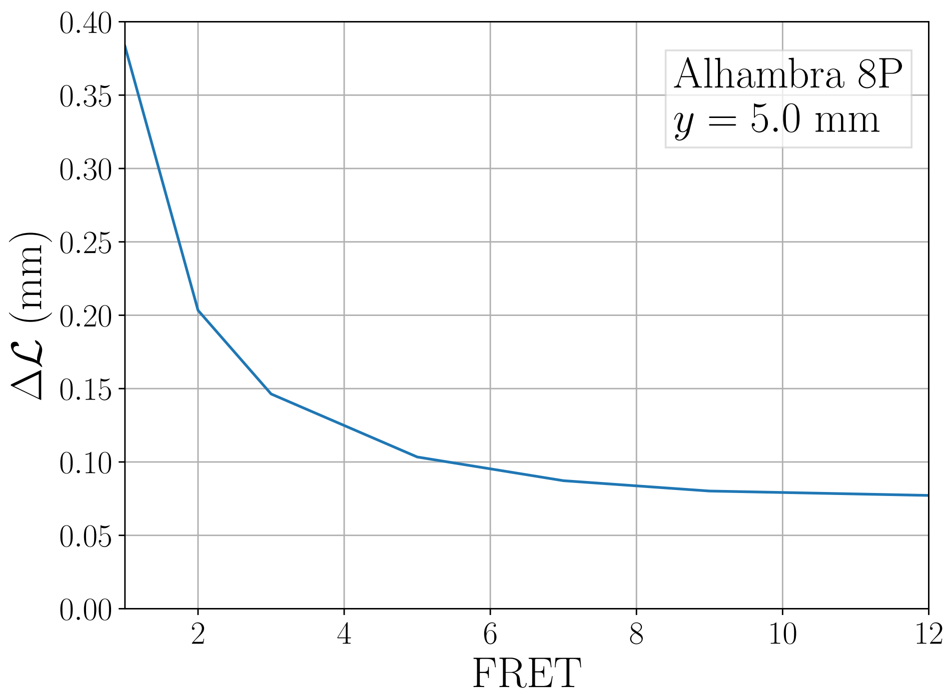


(b) Frequency shift for a factory guitar

Figure 1: Frequency shift (in cents) due to the fretted length L_n for an uncompensated (a) and factory (b) Alhambra 8P guitar, for both zero and nonzero lateral displacement y.

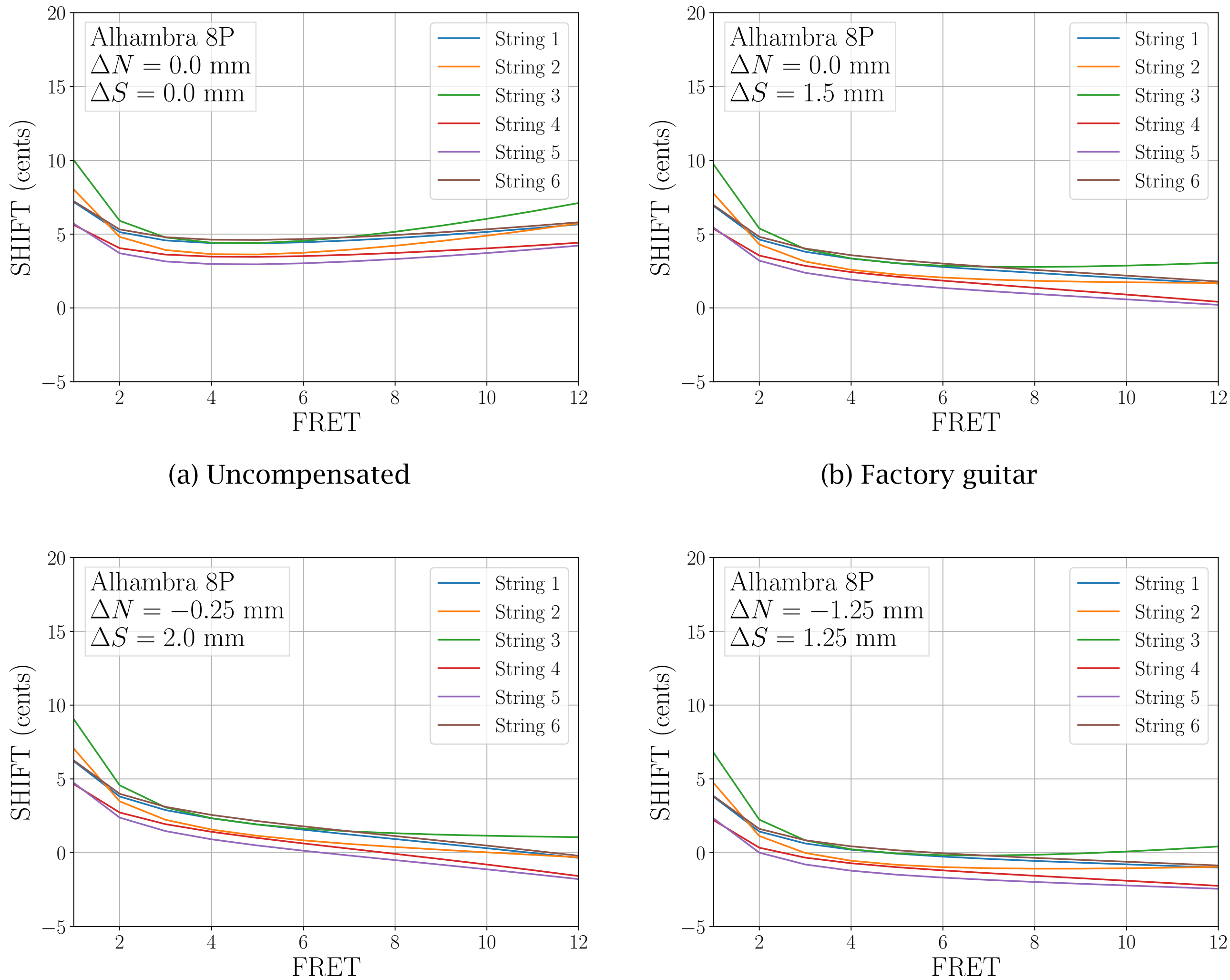


(a) Experimental data



(b) Calculated change in total string length $\mathcal{L}$

Figure 2: Frequency shift (in cents) (a) and change in total string length $\mathcal{L}$ (b) due to lateral displacement y.



(a) Uncompensated

(b) Factory guitar

(c) Saddle and nut

(d) Preserved scale length

Figure 3: Frequency shift (in cents) for four different strategies of saddle and nut compensation.

References

[1] G. Byers, Guitars of Gregory Byers: Intonation (2020). See http://byersguitars.com/intonation and http://www.byersguitars.com/Research/Research.html.

[2] G. Byers, "Classical Guitar Intonation," American Lutherie **47**, 368 (1996).

[3] G. U. Varieschi and C. M. Gower, "Intonation and Compensation of Fretted String Instruments," Amer. J. Phys. **78**, 47 (2010).

[4] P. M. Morse, *Vibration and Sound* (Acoustical Society of America, New York, 1981).

[5] P. M. Morse, *Vibration and Sound*, pp. 84–85, in [4] (1981).

[6] P. M. Morse, *Vibration and Sound*, pp. 166–170, in [4] (1981).