STAT 421/621 Spring 2023 - Project 2

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AR-GARCH Modeling

Background

Financial time series of daily returns may or may not exhibit autocorrelation in the returns themselves, but the variability in the returns as indicated by either the absolute or squared value of the daily returns will exhibit strong autocorrelation. In other words, the volatility or variance of the time series is also important to capture. The GARCH models provide one mechanism to model the volatility of the returns in addition to the returns themselves.

From an investors perspective, estimates and forecast of the volatility play an important role in pricing models, especially options written on the underlying stocks.

DATA: Use the `quantmod` command `quantmod::getSymbols("^DJI")` to obtain the Dow Jones Index. Note, the end date for the series is dependent on when you download (unless you set the parameters), but simply use what is automatically downloaded.

QUESTION 0 Provide a high-level executive summary with at MOST two plots that describes the volatiity patterns observed in the Dow Jones Index, and your ability to forecast the volatility one, two and twenty days ahead (3 distinct time points).

#This report analyzes and evaluates the previous and current adjusted closing price returns of the Dow Jones Index. First take the logarithm of the return. Because . The log of the price is often modeled as a random walk as there is very little measurableautocorrelation in the difference of the log price.The time series is intended to be modeled in a way that allows for the prediction of subsequent price returns. In order to avoid heteroskedasticity (variance volatility) in the time series from affecting the accuracy of the model, it was decided to build an ARMA-GARCH model. After building three models and comparing their AIC values, rma(1,0)+garch(1,1) is the best among these three models. According to the return series with the modeled conditional standard deviations' plot, this event sequence does have variance volatility.The variance volatility is successfully predicted by building this model.The log returns of using this model to predict the volatility one,two and twenty days ahead are 0.0008219478,0.0006498933 and 0.0006946032. The standard deviations of using this model to predict the volatility one,two and twenty days ahead are 0.006429471, 0.006588813 and 0.007653459.

```
## Loading required package: xts
```

```
## Loading required package: zoo
```

```
##
## Attaching package: 'zoo'
```

```
## The following objects are masked from 'package:base':
##
##     as.Date, as.Date.numeric
```

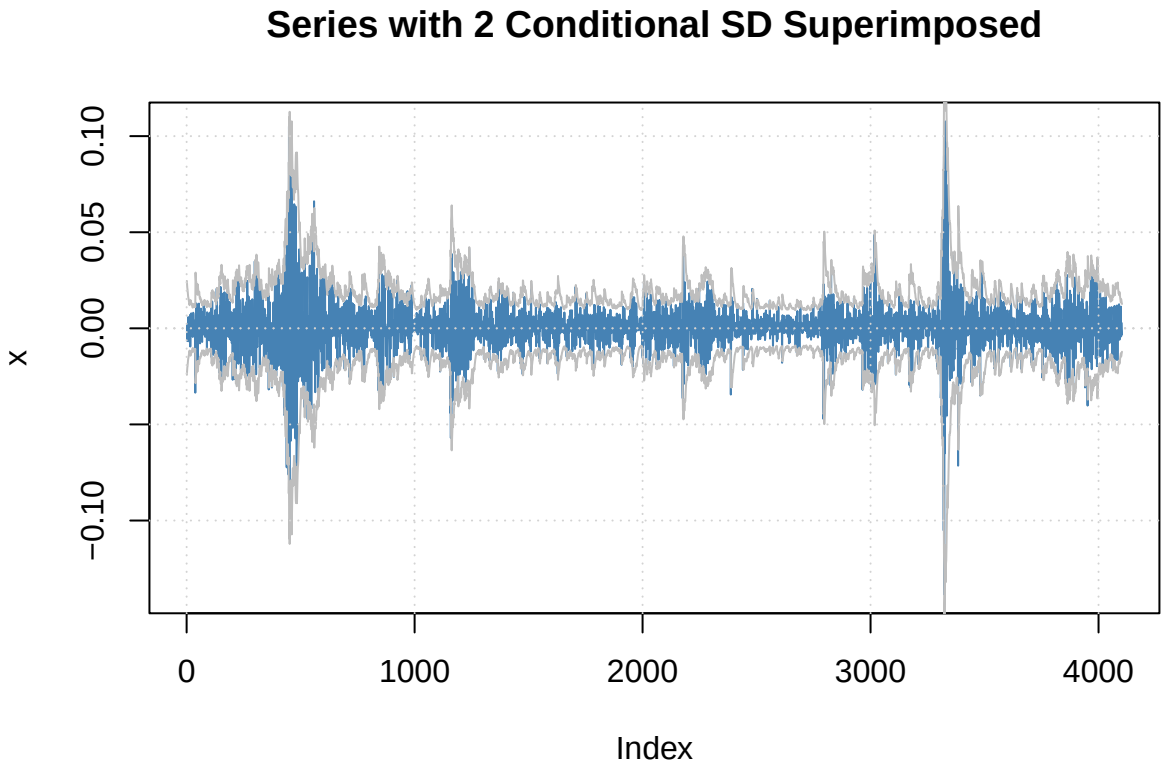
```
## Loading required package: TTR
```

```
## Registered S3 method overwritten by 'quantmod':
##   method            from
##   as.zoo.data.frame zoo

##
## Attaching package: 'tsibble'

## The following object is masked from 'package:zoo':
##
##     index

## The following objects are masked from 'package:base':
##
##     intersect, setdiff, union

##
## Attaching package: 'dplyr'

## The following objects are masked from 'package:xts':
##
##     first, last

## The following objects are masked from 'package:stats':
##
##     filter, lag

## The following objects are masked from 'package:base':
##
##     intersect, setdiff, setequal, union

## Loading required package: ggplot2

##
## Attaching package: 'TSA'

## The following objects are masked from 'package:stats':
##
##     acf, arima

## The following object is masked from 'package:utils':
##
##     tar

## Loading required package: fabletools

## [1] "^DJI"

## NOTE: Packages 'fBasics', 'timeDate', and 'timeSeries' are no longer
## attached to the search() path when 'fGarch' is attached.
##
## If needed attach them yourself in your R script by e.g.,
##         require("timeSeries")

##
## Attaching package: 'fGarch'

## The following object is masked from 'package:TTR':
##
##     volatility

##
## Series Initialization:
```

```
##  ARMA Model:                arma
##  Formula Mean:              ~ arma(1, 1)
##  GARCH Model:               garch
##  Formula Variance:          ~ garch(1, 1)
##  ARMA Order:                1 1
##  Max ARMA Order:            1
##  GARCH Order:               1 1
##  Max GARCH Order:           1
##  Maximum Order:             1
##  Conditional Dist:          norm
##  h.start:                   2
##  llh.start:                 1
##  Length of Series:          4103
##  Recursion Init:            mci
##  Series Scale:              0.01227742
##
## Parameter Initialization:
##  Initial Parameters:          $params
##  Limits of Transformations:   $U, $V
##  Which Parameters are Fixed?  $includes
##  Parameter Matrix:
##                      U           V      params includes
##     mu     -0.19779487   0.1977949  0.01976847     TRUE
##     ar1    -0.99999999   1.0000000 -0.17130733     TRUE
##     ma1    -0.99999999   1.0000000  0.03978818     TRUE
##     omega   0.00000100 100.0000000  0.10000000     TRUE
##     alpha1  0.00000001   1.0000000  0.10000000     TRUE
##     gamma1 -0.99999999   1.0000000  0.10000000    FALSE
##     beta1   0.00000001   1.0000000  0.80000000     TRUE
##     delta   0.00000000   2.0000000  2.00000000    FALSE
##     skew    0.10000000  10.0000000  1.00000000    FALSE
##     shape   1.00000000  10.0000000  4.00000000    FALSE
##  Index List of Parameters to be Optimized:
##     mu    ar1    ma1  omega alpha1  beta1
##      1      2      3      4      5      7
##  Persistence:                 0.9
##
##
## --- START OF TRACE ---
## Selected Algorithm: nlminb
##
## R coded nlminb Solver:
##
##   0:     4913.7639: 0.0197685 -0.171307 0.0397882 0.100000 0.100000 0.800000
##   1:     4786.8231: 0.0197689 -0.170006 0.0410653 0.0719670 0.0982503 0.784988
##   2:     4723.8401: 0.0197701 -0.167251 0.0437710 0.0446298 0.114219 0.784382
##   3:     4715.4337: 0.0197717 -0.164482 0.0464929 0.0525566 0.139285 0.802029
##   4:     4666.5846: 0.0197778 -0.155843 0.0549779 0.0258744 0.149463 0.794588
##   5:     4650.1915: 0.0197956 -0.135942 0.0744276 0.0313890 0.160331 0.804279
##   6:     4642.3268: 0.0198214 -0.127240 0.0824607 0.0166446 0.152725 0.828782
##   7:     4642.1306: 0.0198218 -0.127262 0.0824257 0.0207609 0.153278 0.829878
##   8:     4640.6183: 0.0198283 -0.126869 0.0826227 0.0191537 0.152692 0.828701
##   9:     4640.5283: 0.0198543 -0.126593 0.0821107 0.0189799 0.151190 0.829219
##  10:     4640.4645: 0.0199197 -0.126880 0.0798415 0.0194567 0.149970 0.829956
```

```
##  11:     4640.4037: 0.0199927 -0.126208 0.0783237 0.0188487 0.149030 0.830869
##  12:     4640.3667: 0.0200565 -0.123548 0.0790823 0.0192187 0.148615 0.830539
##  13:     4640.3629: 0.0200628 -0.123921 0.0785122 0.0195692 0.148103 0.830403
##  14:     4640.3503: 0.0200797 -0.123709 0.0782118 0.0192485 0.148063 0.830406
##  15:     4640.3341: 0.0200971 -0.123485 0.0779132 0.0193268 0.148050 0.830642
##  16:     4640.2230: 0.0204795 -0.118330 0.0715089 0.0183976 0.148034 0.832774
##  17:     4640.1151: 0.0208560 -0.113206 0.0651514 0.0194429 0.150922 0.829876
##  18:     4639.8988: 0.0212382 -0.107356 0.0591666 0.0190199 0.151974 0.828414
##  19:     4639.4959: 0.0221924 -0.0888389 0.0468081 0.0186098 0.143453 0.835227
##  20:     4639.4896: 0.0221927 -0.0888699 0.0467679 0.0187345 0.143512 0.835249
##  21:     4639.4851: 0.0221932 -0.0889338 0.0466850 0.0186740 0.143561 0.835181
##  22:     4639.4796: 0.0221977 -0.0889279 0.0465426 0.0187732 0.143646 0.835165
##  23:     4639.4713: 0.0222082 -0.0887953 0.0463326 0.0186825 0.143670 0.835092
##  24:     4634.9252: 0.0349744 0.125794 -0.155078 0.0209084 0.154116 0.821929
##  25:     4633.4445: 0.0374341 0.192022 -0.231883 0.0194019 0.154016 0.824481
##  26:     4633.0923: 0.0387706 0.250544 -0.292105 0.0207853 0.152842 0.823755
##  27:     4633.0653: 0.0388537 0.328654 -0.366266 0.0199129 0.154268 0.823557
##  28:     4632.8581: 0.0403701 0.255225 -0.300288 0.0190140 0.153116 0.826038
##  29:     4632.8463: 0.0406987 0.245303 -0.290144 0.0194681 0.153978 0.827190
##  30:     4632.7593: 0.0408973 0.241945 -0.287389 0.0191633 0.153645 0.826945
##  31:     4632.7276: 0.0412936 0.235120 -0.281805 0.0193213 0.153061 0.826649
##  32:     4632.5723: 0.0453163 0.150559 -0.202379 0.0193367 0.151951 0.827375
##  33:     4632.3381: 0.0612770 -0.193678 0.125547 0.0192013 0.148525 0.829706
##  34:     4632.1751: 0.0562785 -0.0649231 0.00478132 0.0194259 0.150833 0.828077
##  35:     4632.0012: 0.0599872 -0.121593 0.0621432 0.0193599 0.151460 0.827741
##  36:     4631.7048: 0.0768665 -0.392899 0.336609 0.0194072 0.152726 0.827005
##  37:     4631.6625: 0.0741273 -0.315394 0.264656 0.0190636 0.153106 0.826911
##  38:     4631.6389: 0.0752258 -0.339608 0.287356 0.0193309 0.153222 0.826663
##  39:     4631.6374: 0.0762670 -0.357425 0.304779 0.0192377 0.153164 0.826766
##  40:     4631.6367: 0.0760788 -0.352000 0.299593 0.0192679 0.153177 0.826736
##  41:     4631.6366: 0.0761361 -0.351592 0.299174 0.0192711 0.153177 0.826730
##  42:     4631.6366: 0.0762415 -0.351284 0.298795 0.0192712 0.153171 0.826730
##  43:     4631.6366: 0.0762476 -0.351362 0.298863 0.0192706 0.153170 0.826731
##
## Final Estimate of the Negative LLH:
##  LLH:  -13421.54    norm LLH:  -3.271152
##           mu           ar1           ma1         omega        alpha1
##  9.361238e-04 -3.513617e-01  2.988632e-01  2.904753e-06  1.531695e-01
##         beta1
##  8.267311e-01
##
## R-optimhess Difference Approximated Hessian Matrix:
##                   mu           ar1           ma1         omega        alpha1
## mu     -4.187594e+07  -36967.44450   -7710.71136 -9.999556e+08  3.152031e+04
## ar1    -3.696744e+04   -3854.32924   -3747.35285  2.469889e+06  1.103609e+01
## ma1    -7.710711e+03   -3747.35285   -3674.86674  2.818166e+06 -1.688442e+01
## omega  -9.999556e+08 2469889.19274 2818166.42313 -2.512409e+13 -7.501121e+08
## alpha1  3.152031e+04      11.03609     -16.88442 -7.501121e+08 -4.985448e+04
## beta1  -3.563300e+04      64.65094      72.70398 -1.219309e+09 -6.138775e+04
##                beta1
## mu     -3.563300e+04
## ar1     6.465094e+01
## ma1     7.270398e+01
## omega  -1.219309e+09
```

```
## alpha1 -6.138775e+04
## beta1  -8.851827e+04
## attr(,"time")
## Time difference of 0.07749701 secs
##
## --- END OF TRACE ---
##
##
## Time to Estimate Parameters:
##  Time difference of 0.3677099 secs
```

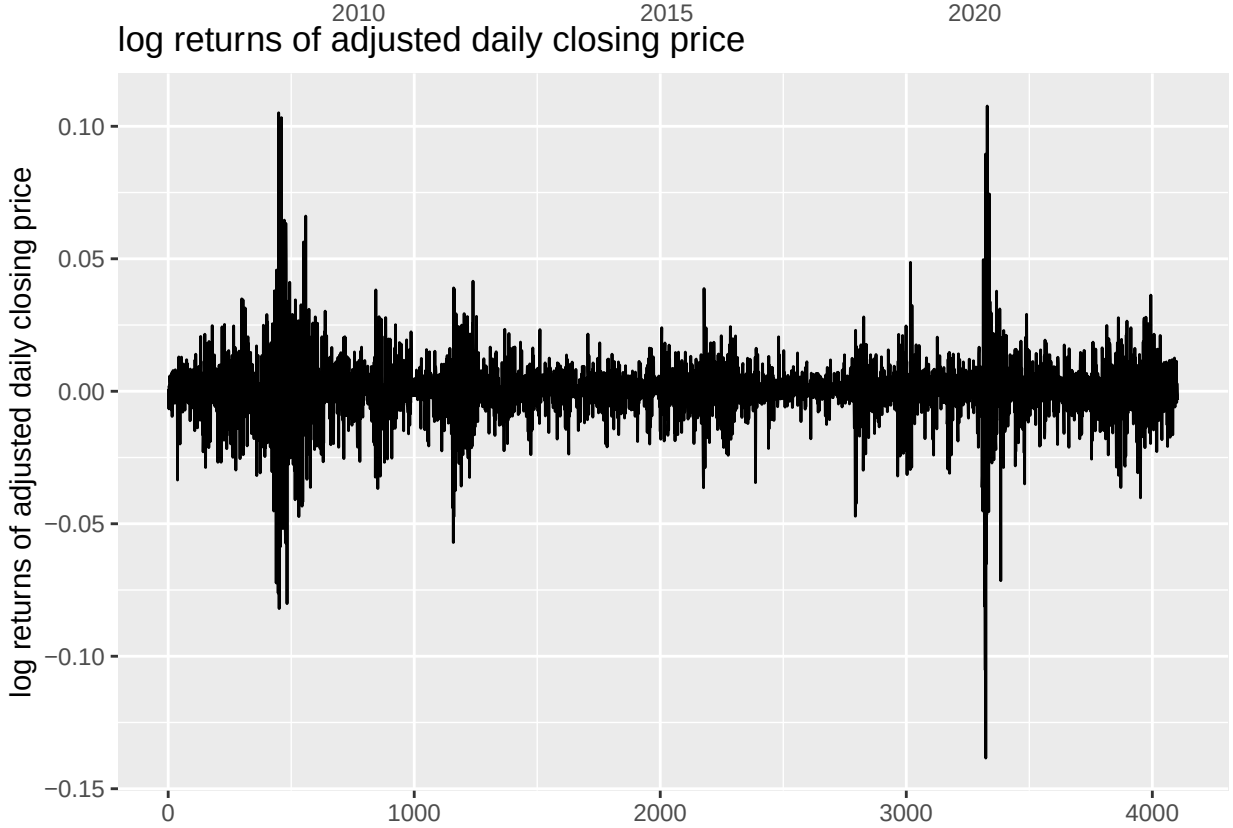


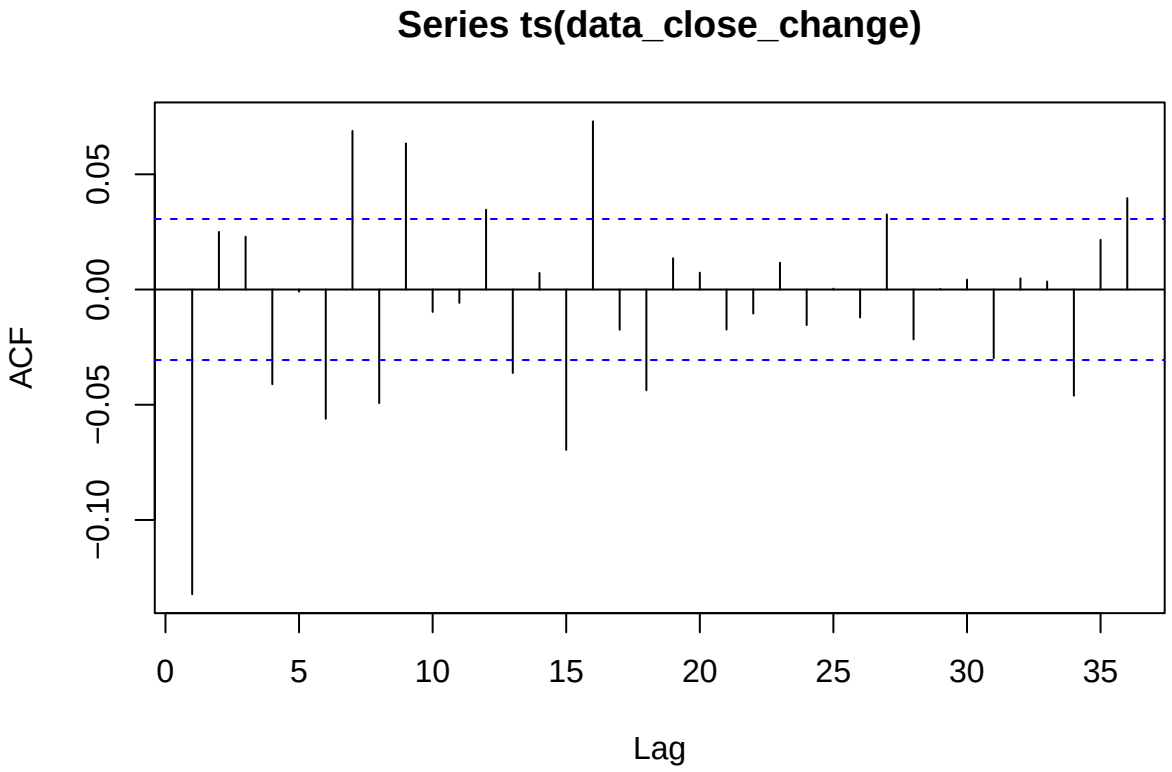
QUESTION 1 Produce the following plots

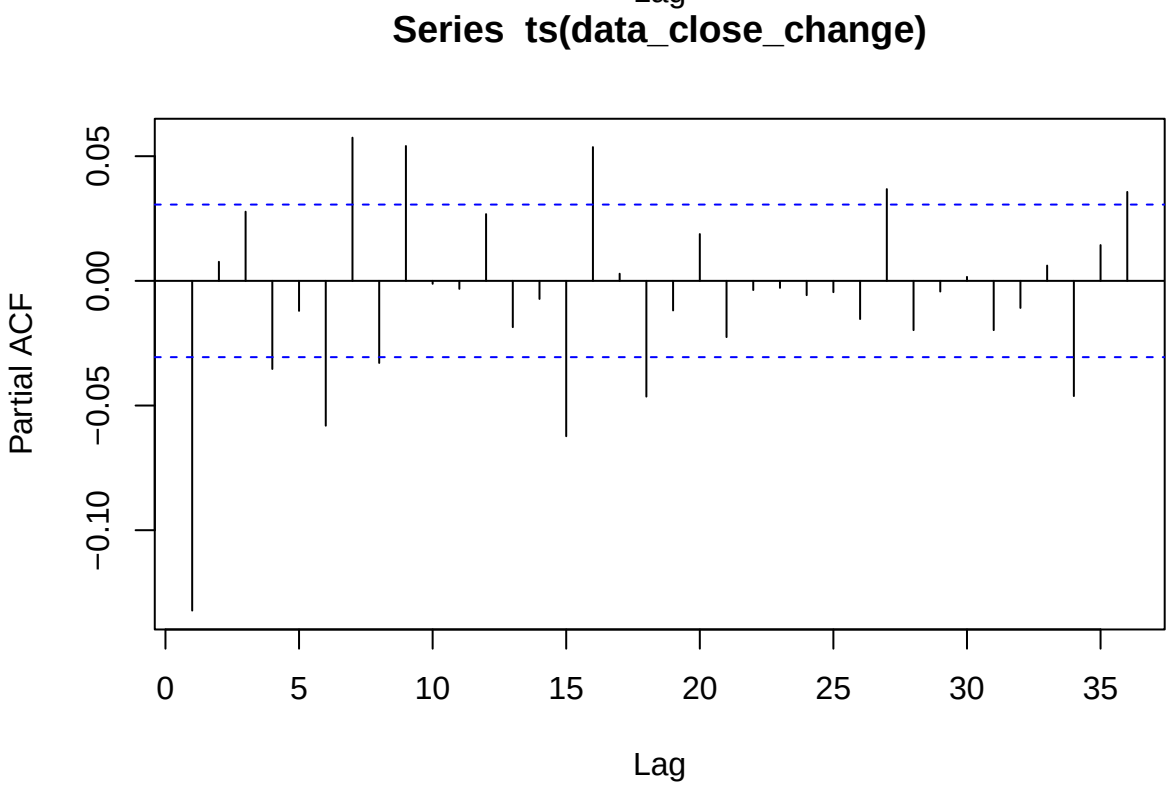
```
+ Plot of adjusted daily closing price
+ Plot of daily log returns based on the adjusted daily closing price
+ ACF and PACF of log returns
+ ACF and PACF of squared log returns
```

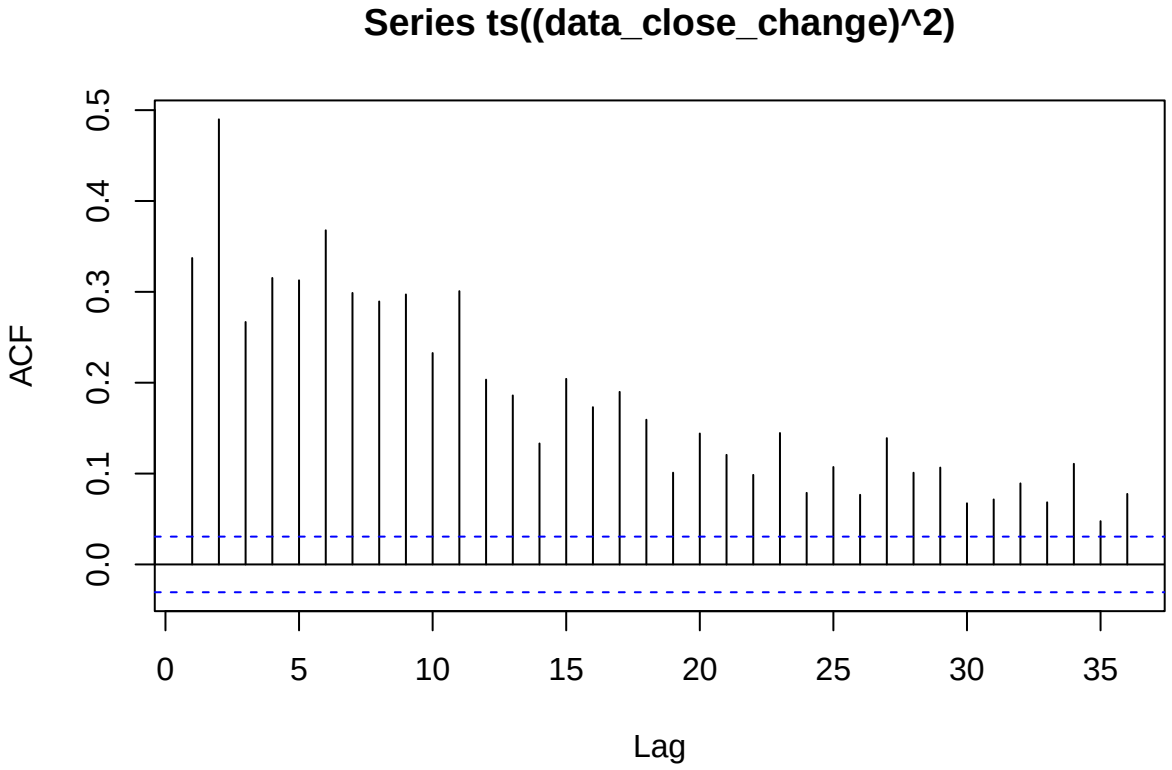
```
## [1] "^DJI"
```

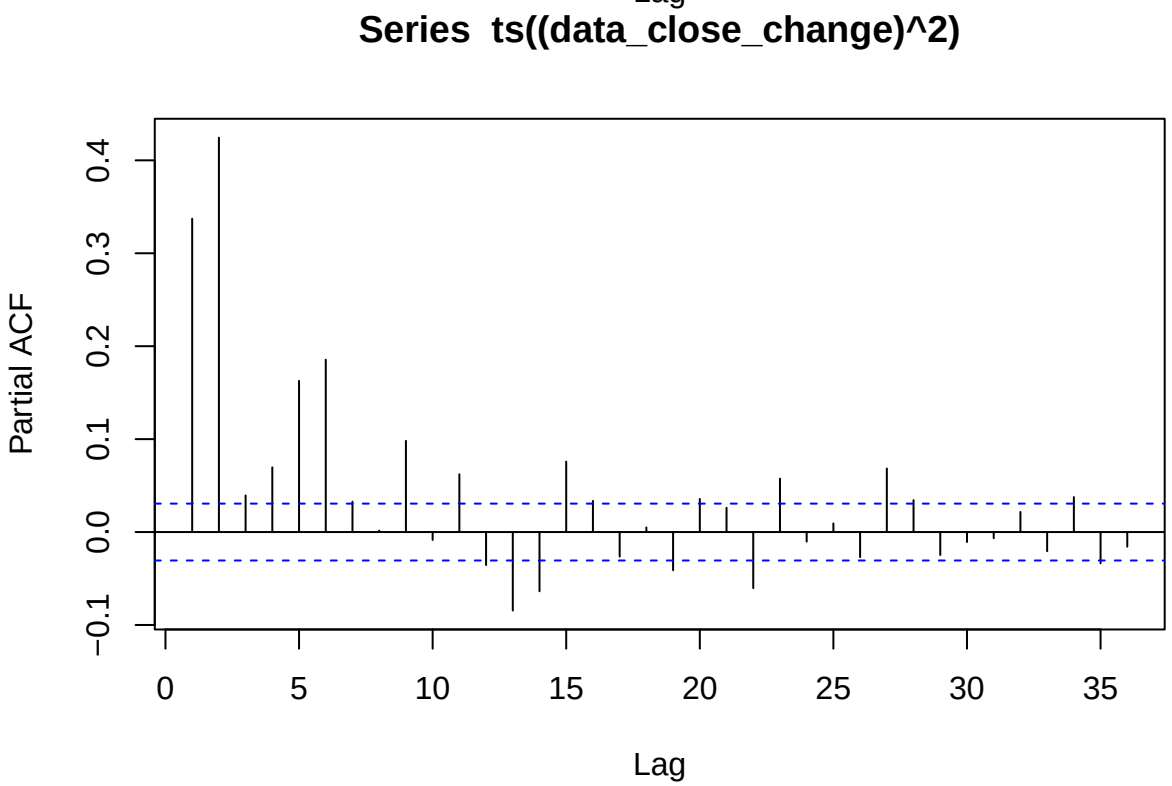

Dow Jone's adjusted daily closing price
adjusted daily closing price
30000
20000
10000
2010
2015
2020


log returns of adjusted daily closing price
log returns of adjusted daily closing price
0.10
0.05
0.00
−0.05
−0.10
−0.15
0
1000
2000
3000
4000


Series ts(data_close_change)
ACF
0.05
0.00
−0.05
−0.10
0
5
10
15
20
25
30
35
Lag


Series ts(data_close_change)
Partial ACF
0.05
0.00
−0.05
−0.10
0
5
10
15
20
25
30
35
Lag





QUESTION 2 Using the **fGarch** package, and the command **garchFit**, fit an ARMA(1,0)+GARCH(1,0) assuming a normal distribution.

```
##
## Series Initialization:
##  ARMA Model:                arma
```

```
##  Formula Mean:              ~ arma(1, 0)
##  GARCH Model:               garch
##  Formula Variance:          ~ garch(1, 0)
##  ARMA Order:                1 0
##  Max ARMA Order:            1
##  GARCH Order:               1 0
##  Max GARCH Order:           1
##  Maximum Order:             1
##  Conditional Dist:          norm
##  h.start:                   2
##  llh.start:                 1
##  Length of Series:          4103
##  Recursion Init:            mci
##  Series Scale:              0.01227742
##
## Parameter Initialization:
##  Initial Parameters:          $params
##  Limits of Transformations:   $U, $V
##  Which Parameters are Fixed?  $includes
##  Parameter Matrix:
##                       U           V      params includes
##     mu     -0.19779487   0.1977949  0.01978112     TRUE
##     ar1    -0.99999999   1.0000000 -0.13219769     TRUE
##     omega   0.00000100 100.0000000  0.10000000     TRUE
##     alpha1  0.00000001   1.0000000  0.10000000     TRUE
##     gamma1 -0.99999999   1.0000000  0.10000000    FALSE
##     delta   0.00000000   2.0000000  2.00000000    FALSE
##     skew    0.10000000  10.0000000  1.00000000    FALSE
##     shape   1.00000000  10.0000000  4.00000000    FALSE
##  Index List of Parameters to be Optimized:
##     mu    ar1  omega alpha1
##      1      2      3      4
##  Persistence:                 0.1
##
##
## --- START OF TRACE ---
## Selected Algorithm: nlminb
##
## R coded nlminb Solver:
##
##   0:     10652.888: 0.0197811 -0.132198 0.100000 0.100000
##   1:     5558.2579: 0.0197842 -0.158421  1.05312 0.401442
##   2:     5546.4891: 0.0201224 -0.237471 0.893496 0.868323
##   3:     5387.7818: 0.0203445 -0.0742760 0.462203  1.00000
##   4:     5333.6556: 0.0203741 -0.189913 0.654960 0.553379
##   5:     5305.0045: 0.0203871 -0.194495 0.576989 0.534242
##   6:     5301.5671: 0.0227845 -0.228694 0.498154 0.472660
##   7:     5297.0602: 0.0229789 -0.196303 0.525265 0.516059
##   8:     5296.5053: 0.0231742 -0.212164 0.511864 0.527743
##   9:     5296.3551: 0.0235950 -0.205323 0.512944 0.540570
##  10:     5296.3093: 0.0236206 -0.212950 0.515413 0.540538
##  11:     5296.2522: 0.0237671 -0.209852 0.514331 0.540018
##  12:     5296.2217: 0.0240767 -0.213941 0.512959 0.539534
##  13:     5296.1323: 0.0243794 -0.210346 0.516556 0.537626
```

```
##  14:     5296.1188: 0.0243991 -0.209862 0.514296 0.537677
##  15:     5296.1097: 0.0244393 -0.210644 0.515002 0.538728
##  16:     5296.0906: 0.0245330 -0.209748 0.513567 0.538646
##  17:     5295.8739: 0.0263176 -0.215259 0.513619 0.527140
##  18:     5294.4138: 0.0347279 -0.214195 0.520860 0.542149
##  19:     5292.6139: 0.0538727 -0.210824 0.506912 0.548629
##  20:     5292.4881: 0.0572433 -0.219888 0.509598 0.550445
##  21:     5292.4440: 0.0587730 -0.216073 0.510490 0.550188
##  22:     5292.4434: 0.0587467 -0.216318 0.509870 0.550553
##  23:     5292.4434: 0.0587254 -0.216317 0.509964 0.550504
##  24:     5292.4434: 0.0587268 -0.216316 0.509963 0.550504
##
## Final Estimate of the Negative LLH:
##  LLH:  -12760.73    norm LLH:  -3.110098
##           mu           ar1         omega        alpha1
##  0.0007210135 -0.2163159753  0.0000768693  0.5505042467
##
## R-optimhess Difference Approximated Hessian Matrix:
##                  mu          ar1         omega        alpha1
## mu     -42735136.71  -67836.6003     -38782103    10538.2207
## ar1       -67836.60   -5265.6031       3729600     -177.8215
## omega  -38782102.97 3729599.8340 -213825010731 -6118355.8155
## alpha1     10538.22    -177.8215      -6118356     -891.7238
## attr(,"time")
## Time difference of 0.02979994 secs
##
## --- END OF TRACE ---
##
##
## Time to Estimate Parameters:
##  Time difference of 0.144907 secs
```

QUESTION 3 Repeat question 2 but include a GARCH(2,0) component.

```
##
## Series Initialization:
##  ARMA Model:                arma
##  Formula Mean:              ~ arma(1, 0)
##  GARCH Model:               garch
##  Formula Variance:          ~ garch(2, 0)
##  ARMA Order:                1 0
##  Max ARMA Order:            1
##  GARCH Order:               2 0
##  Max GARCH Order:           2
##  Maximum Order:             2
##  Conditional Dist:          norm
##  h.start:                   3
##  llh.start:                 1
##  Length of Series:          4103
##  Recursion Init:            mci
##  Series Scale:              0.01227742
##
## Parameter Initialization:
##  Initial Parameters:          $params
##  Limits of Transformations:   $U, $V
```

```
##  Which Parameters are Fixed?  $includes
##  Parameter Matrix:
##                      U           V      params includes
##     mu     -0.19779487   0.1977949  0.01978112     TRUE
##     ar1    -0.99999999   1.0000000 -0.13219769     TRUE
##     omega   0.00000100 100.0000000  0.10000000     TRUE
##     alpha1  0.00000001   1.0000000  0.05000000     TRUE
##     alpha2  0.00000001   1.0000000  0.05000000     TRUE
##     gamma1 -0.99999999   1.0000000  0.10000000    FALSE
##     gamma2 -0.99999999   1.0000000  0.10000000    FALSE
##     delta   0.00000000   2.0000000  2.00000000    FALSE
##     skew    0.10000000  10.0000000  1.00000000    FALSE
##     shape   1.00000000  10.0000000  4.00000000    FALSE
##  Index List of Parameters to be Optimized:
##     mu    ar1  omega alpha1 alpha2
##      1      2      3      4      5
##  Persistence:                  0.1
##
##
## --- START OF TRACE ---
## Selected Algorithm: nlminb
##
## R coded nlminb Solver:
##
##   0:     8933.6959: 0.0197811 -0.132198 0.100000 0.0500000 0.0500000
##   1:     5468.3662: 0.0197872 -0.113429 0.901282 0.439753 0.503527
##   2:     5424.4468: 0.0199332 0.212866 0.668924 0.530733 0.788499
##   3:     5019.5340: 0.0199564 -0.0173333 0.266894 0.391218 0.662347
##   4:     5015.5279: 0.0200724 -0.00108155 0.387471 0.314681 0.561042
##   5:     4987.2897: 0.0201073 -0.0425345 0.312673 0.233189 0.430655
##   6:     4982.9505: 0.0202202 -0.0330623 0.361268 0.274495 0.394385
##   7:     4981.1972: 0.0204497 -0.0212338 0.331927 0.289184 0.396236
##   8:     4980.8200: 0.0207424 -0.0147554 0.340747 0.265954 0.418061
##   9:     4980.7379: 0.0207600 -0.0153330 0.336982 0.264825 0.415419
##  10:     4980.6952: 0.0208011 -0.0159266 0.339766 0.265574 0.412167
##  11:     4980.6549: 0.0209548 -0.0165776 0.336453 0.264104 0.407665
##  12:     4980.5603: 0.0211319 -0.0163513 0.339730 0.265910 0.407598
##  13:     4979.6683: 0.0262817 -0.0106174 0.327894 0.293429 0.411863
##  14:     4977.9029: 0.0314511 -0.0201226 0.338707 0.279345 0.406672
##  15:     4976.5590: 0.0418176 -0.0183298 0.324215 0.272448 0.404516
##  16:     4976.4731: 0.0521196 -0.0170738 0.330064 0.261930 0.463888
##  17:     4974.8923: 0.0572087 -0.0248068 0.334114 0.276411 0.416444
##  18:     4974.8807: 0.0572090 -0.0249974 0.331997 0.275246 0.414552
##  19:     4974.8684: 0.0572575 -0.0249336 0.333654 0.275370 0.413722
##  20:     4974.8622: 0.0573690 -0.0250505 0.333597 0.274778 0.411343
##  21:     4974.8600: 0.0575764 -0.0253441 0.334295 0.275064 0.409917
##  22:     4974.8600: 0.0574716 -0.0253784 0.334326 0.274913 0.409942
##  23:     4974.8600: 0.0575154 -0.0253131 0.334306 0.274936 0.409974
##  24:     4974.8600: 0.0575211 -0.0253448 0.334300 0.274963 0.409968
##  25:     4974.8600: 0.0575169 -0.0253354 0.334303 0.274952 0.409968
##
## Final Estimate of the Negative LLH:
##  LLH:  -13078.31    norm LLH:  -3.1875
##             mu           ar1         omega        alpha1        alpha2
```

```
##  7.061589e-04 -2.533542e-02  5.039121e-05  2.749523e-01  4.099676e-01
##
## R-optimhess Difference Approximated Hessian Matrix:
##                   mu           ar1         omega        alpha1        alpha2
## mu      -59322098.49  -62952.98191 -1.559702e+08  1.605590e+04  1.189657e+04
## ar1        -62952.98   -2897.85840 -9.960179e+05 -1.246045e+02  8.782543e+01
## omega  -155970242.48 -996017.93836 -3.488508e+11 -1.200862e+07 -8.998554e+06
## alpha1      16055.90    -124.60454 -1.200862e+07 -1.739989e+03 -4.849619e+02
## alpha2      11896.57      87.82543 -8.998554e+06 -4.849619e+02 -1.310658e+03
## attr(,"time")
## Time difference of 0.05847311 secs
##
## --- END OF TRACE ---
##
##
## Time to Estimate Parameters:
##  Time difference of 0.2099741 secs
```

QUESTION 4 Repeat question 2 but include any other model features you think might be important. You can use your judgement and do not need to go through an extensive model selection process.

```
##
## Series Initialization:
##  ARMA Model:                arma
##  Formula Mean:              ~ arma(1, 1)
##  GARCH Model:               garch
##  Formula Variance:          ~ garch(1, 1)
##  ARMA Order:                1 1
##  Max ARMA Order:            1
##  GARCH Order:               1 1
##  Max GARCH Order:           1
##  Maximum Order:             1
##  Conditional Dist:          norm
##  h.start:                   2
##  llh.start:                 1
##  Length of Series:          4103
##  Recursion Init:            mci
##  Series Scale:              0.01227742
##
## Parameter Initialization:
##  Initial Parameters:          $params
##  Limits of Transformations:   $U, $V
##  Which Parameters are Fixed?  $includes
##  Parameter Matrix:
##                      U           V      params includes
##     mu     -0.19779487   0.1977949  0.01976847     TRUE
##     ar1    -0.99999999   1.0000000 -0.17130733     TRUE
##     ma1    -0.99999999   1.0000000  0.03978818     TRUE
##     omega   0.00000100 100.0000000  0.10000000     TRUE
##     alpha1  0.00000001   1.0000000  0.10000000     TRUE
##     gamma1 -0.99999999   1.0000000  0.10000000    FALSE
##     beta1   0.00000001   1.0000000  0.80000000     TRUE
##     delta   0.00000000   2.0000000  2.00000000    FALSE
##     skew    0.10000000  10.0000000  1.00000000    FALSE
##     shape   1.00000000  10.0000000  4.00000000    FALSE
```

```
##  Index List of Parameters to be Optimized:
##     mu    ar1    ma1  omega alpha1  beta1
##      1      2      3      4      5      7
##  Persistence:                  0.9
##
##
## --- START OF TRACE ---
## Selected Algorithm: nlminb
##
## R coded nlminb Solver:
##
##   0:     4913.7639: 0.0197685 -0.171307 0.0397882 0.100000 0.100000 0.800000
##   1:     4786.8231: 0.0197689 -0.170006 0.0410653 0.0719670 0.0982503 0.784988
##   2:     4723.8401: 0.0197701 -0.167251 0.0437710 0.0446298 0.114219 0.784382
##   3:     4715.4337: 0.0197717 -0.164482 0.0464929 0.0525566 0.139285 0.802029
##   4:     4666.5846: 0.0197778 -0.155843 0.0549779 0.0258744 0.149463 0.794588
##   5:     4650.1915: 0.0197956 -0.135942 0.0744276 0.0313890 0.160331 0.804279
##   6:     4642.3268: 0.0198214 -0.127240 0.0824607 0.0166446 0.152725 0.828782
##   7:     4642.1306: 0.0198218 -0.127262 0.0824257 0.0207609 0.153278 0.829878
##   8:     4640.6183: 0.0198283 -0.126869 0.0826227 0.0191537 0.152692 0.828701
##   9:     4640.5283: 0.0198543 -0.126593 0.0821107 0.0189799 0.151190 0.829219
##  10:     4640.4645: 0.0199197 -0.126880 0.0798415 0.0194567 0.149970 0.829956
##  11:     4640.4037: 0.0199927 -0.126208 0.0783237 0.0188487 0.149030 0.830869
##  12:     4640.3667: 0.0200565 -0.123548 0.0790823 0.0192187 0.148615 0.830539
##  13:     4640.3629: 0.0200628 -0.123921 0.0785122 0.0195692 0.148103 0.830403
##  14:     4640.3503: 0.0200797 -0.123709 0.0782118 0.0192485 0.148063 0.830406
##  15:     4640.3341: 0.0200971 -0.123485 0.0779132 0.0193268 0.148050 0.830642
##  16:     4640.2230: 0.0204795 -0.118330 0.0715089 0.0183976 0.148034 0.832774
##  17:     4640.1151: 0.0208560 -0.113206 0.0651514 0.0194429 0.150922 0.829876
##  18:     4639.8988: 0.0212382 -0.107356 0.0591666 0.0190199 0.151974 0.828414
##  19:     4639.4959: 0.0221924 -0.0888389 0.0468081 0.0186098 0.143453 0.835227
##  20:     4639.4896: 0.0221927 -0.0888699 0.0467679 0.0187345 0.143512 0.835249
##  21:     4639.4851: 0.0221932 -0.0889338 0.0466850 0.0186740 0.143561 0.835181
##  22:     4639.4796: 0.0221977 -0.0889279 0.0465426 0.0187732 0.143646 0.835165
##  23:     4639.4713: 0.0222082 -0.0887953 0.0463326 0.0186825 0.143670 0.835092
##  24:     4634.9252: 0.0349744 0.125794 -0.155078 0.0209084 0.154116 0.821929
##  25:     4633.4445: 0.0374341 0.192022 -0.231883 0.0194019 0.154016 0.824481
##  26:     4633.0923: 0.0387706 0.250544 -0.292105 0.0207853 0.152842 0.823755
##  27:     4633.0653: 0.0388537 0.328654 -0.366266 0.0199129 0.154268 0.823557
##  28:     4632.8581: 0.0403701 0.255225 -0.300288 0.0190140 0.153116 0.826038
##  29:     4632.8463: 0.0406987 0.245303 -0.290144 0.0194681 0.153978 0.827190
##  30:     4632.7593: 0.0408973 0.241945 -0.287389 0.0191633 0.153645 0.826945
##  31:     4632.7276: 0.0412936 0.235120 -0.281805 0.0193213 0.153061 0.826649
##  32:     4632.5723: 0.0453163 0.150559 -0.202379 0.0193367 0.151951 0.827375
##  33:     4632.3381: 0.0612770 -0.193678 0.125547 0.0192013 0.148525 0.829706
##  34:     4632.1751: 0.0562785 -0.0649231 0.00478132 0.0194259 0.150833 0.828077
##  35:     4632.0012: 0.0599872 -0.121593 0.0621432 0.0193599 0.151460 0.827741
##  36:     4631.7048: 0.0768665 -0.392899 0.336609 0.0194072 0.152726 0.827005
##  37:     4631.6625: 0.0741273 -0.315394 0.264656 0.0190636 0.153106 0.826911
##  38:     4631.6389: 0.0752258 -0.339608 0.287356 0.0193309 0.153222 0.826663
##  39:     4631.6374: 0.0762670 -0.357425 0.304779 0.0192377 0.153164 0.826766
##  40:     4631.6367: 0.0760788 -0.352000 0.299593 0.0192679 0.153177 0.826736
##  41:     4631.6366: 0.0761361 -0.351592 0.299174 0.0192711 0.153177 0.826730
##  42:     4631.6366: 0.0762415 -0.351284 0.298795 0.0192712 0.153171 0.826730
```

```
##  43:     4631.6366: 0.0762476 -0.351362 0.298863 0.0192706 0.153170 0.826731
##
## Final Estimate of the Negative LLH:
##  LLH:  -13421.54    norm LLH:  -3.271152
##           mu           ar1           ma1         omega        alpha1
##  9.361238e-04 -3.513617e-01  2.988632e-01  2.904753e-06  1.531695e-01
##         beta1
##  8.267311e-01
##
## R-optimhess Difference Approximated Hessian Matrix:
##                    mu           ar1           ma1         omega        alpha1
## mu     -4.187594e+07  -36967.44450   -7710.71136 -9.999556e+08  3.152031e+04
## ar1    -3.696744e+04   -3854.32924   -3747.35285  2.469889e+06  1.103609e+01
## ma1    -7.710711e+03   -3747.35285   -3674.86674  2.818166e+06 -1.688442e+01
## omega  -9.999556e+08 2469889.19274 2818166.42313 -2.512409e+13 -7.501121e+08
## alpha1  3.152031e+04      11.03609     -16.88442 -7.501121e+08 -4.985448e+04
## beta1  -3.563300e+04      64.65094      72.70398 -1.219309e+09 -6.138775e+04
##                beta1
## mu     -3.563300e+04
## ar1     6.465094e+01
## ma1     7.270398e+01
## omega  -1.219309e+09
## alpha1 -6.138775e+04
## beta1  -8.851827e+04
## attr(,"time")
## Time difference of 0.07926011 secs
##
## --- END OF TRACE ---
##
##
## Time to Estimate Parameters:
##  Time difference of 0.3301978 secs
```

According to the ACF and PACF, they are not cutting the tail, so I want to fit the model with ARMA(1,1)

QUESTION 5 Of your 3 models, choose the one that is the best representation and justify your choice with model diagnostics. Use this model for the remaining questions.

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~arma(1, 0) + garch(1, 0), data = return,
##     cond.dist = "norm")
##
## Mean and Variance Equation:
##  data ~ arma(1, 0) + garch(1, 0)
## <environment: 0x7fdff0e4bcd0>
##  [data = return]
##
## Conditional Distribution:
```

```
##  norm
##
## Coefficient(s):
##          mu          ar1        omega       alpha1
##  7.2101e-04  -2.1632e-01   7.6869e-05   5.5050e-01
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##          Estimate  Std. Error  t value Pr(>|t|)
## mu      7.210e-04   1.553e-04    4.644 3.42e-06 ***
## ar1    -2.163e-01   1.420e-02  -15.237  < 2e-16 ***
## omega   7.687e-05   2.451e-06   31.357  < 2e-16 ***
## alpha1  5.505e-01   3.792e-02   14.517  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  12760.73    normalized:  3.110098
##
## Description:
##  Fri Apr 21 13:31:17 2023 by user:
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  2726.108  0
##  Shapiro-Wilk Test  R    W      0.9458824 0
##  Ljung-Box Test     R    Q(10)  51.58608  1.361139e-07
##  Ljung-Box Test     R    Q(15)  60.23441  2.298481e-07
##  Ljung-Box Test     R    Q(20)  65.40448  1.005975e-06
##  Ljung-Box Test     R^2  Q(10)  898.8016  0
##  Ljung-Box Test     R^2  Q(15)  1248.814  0
##  Ljung-Box Test     R^2  Q(20)  1572.377  0
##  LM Arch Test       R    TR^2   615.0744  0
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -6.218245 -6.212085 -6.218247 -6.216064
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~arma(1, 0) + garch(2, 0), data = return,
##     cond.dist = "norm")
##
## Mean and Variance Equation:
##  data ~ arma(1, 0) + garch(2, 0)
## <environment: 0x7fdfd10bd660>
##  [data = return]
##
```

```
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##         mu          ar1        omega       alpha1       alpha2
##  7.0616e-04  -2.5335e-02   5.0391e-05   2.7495e-01   4.0997e-01
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      7.062e-04   1.320e-04    5.350 8.82e-08 ***
## ar1    -2.534e-02   1.887e-02   -1.342    0.179
## omega   5.039e-05   2.053e-06   24.542  < 2e-16 ***
## alpha1  2.750e-01   2.788e-02    9.861  < 2e-16 ***
## alpha2  4.100e-01   3.089e-02   13.272  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  13078.31    normalized:  3.1875
##
## Description:
##  Fri Apr 21 13:31:17 2023 by user:
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  1467.689  0
##  Shapiro-Wilk Test  R    W      0.9626674 0
##  Ljung-Box Test     R    Q(10)  9.975723  0.4426257
##  Ljung-Box Test     R    Q(15)  18.61099  0.2319062
##  Ljung-Box Test     R    Q(20)  20.09198  0.4521894
##  Ljung-Box Test     R^2  Q(10)  210.491   0
##  Ljung-Box Test     R^2  Q(15)  323.0707  0
##  Ljung-Box Test     R^2  Q(20)  403.5083  0
##  LM Arch Test       R    TR^2   275.8051  0
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -6.372563 -6.364862 -6.372566 -6.369837
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~arma(1, 1) + garch(1, 1), data = return,
##     cond.dist = "norm")
##
## Mean and Variance Equation:
##  data ~ arma(1, 1) + garch(1, 1)
## <environment: 0x7fdfc6703a78>
```

```
##  [data = return]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##          mu          ar1          ma1        omega       alpha1        beta1
##  9.3612e-04  -3.5136e-01   2.9886e-01   2.9048e-06   1.5317e-01   8.2673e-01
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##          Estimate  Std. Error  t value Pr(>|t|)
## mu      9.361e-04   2.489e-04    3.761  0.00017 ***
## ar1    -3.514e-01   2.793e-01   -1.258  0.20839
## ma1     2.989e-01   2.849e-01    1.049  0.29413
## omega   2.905e-06   3.760e-07    7.726 1.11e-14 ***
## alpha1  1.532e-01   1.278e-02   11.987  < 2e-16 ***
## beta1   8.267e-01   1.234e-02   66.992  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  13421.54    normalized:  3.271152
##
## Description:
##  Fri Apr 21 13:31:17 2023 by user:
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  746.8043  0
##  Shapiro-Wilk Test  R    W      0.9772689 0
##  Ljung-Box Test     R    Q(10)  11.84163  0.2957956
##  Ljung-Box Test     R    Q(15)  22.04793  0.1065532
##  Ljung-Box Test     R    Q(20)  23.24832  0.2767664
##  Ljung-Box Test     R^2  Q(10)  9.889716  0.4502216
##  Ljung-Box Test     R^2  Q(15)  10.5638   0.782859
##  Ljung-Box Test     R^2  Q(20)  11.86986  0.9204824
##  LM Arch Test       R    TR^2   10.21484  0.5971193
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -6.539380 -6.530138 -6.539384 -6.536108
```

#According to the AIC, arma(1,0)+garch(1,1) is the best among these three models.And it also passes the Ljung-Box Test.

QUESTION 6 Write down the model in final form.

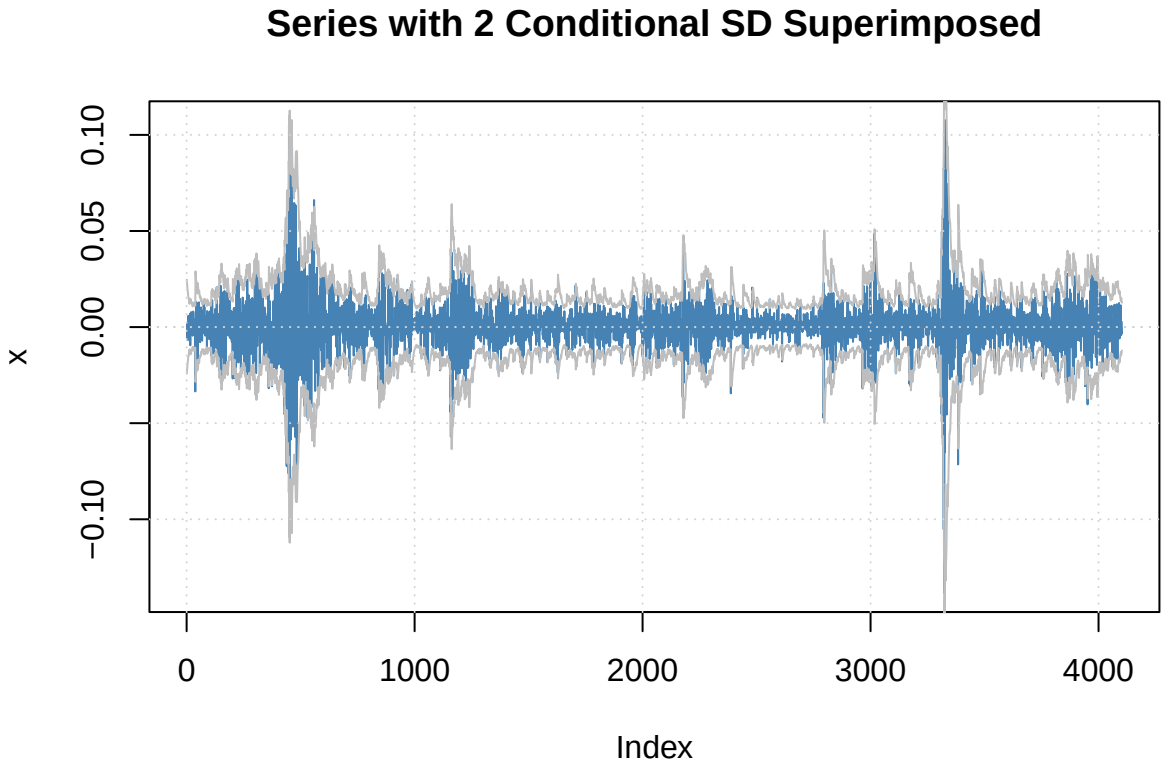
$r_t = 9.3856*(1/10)^4 - 3.512*(1/10)^2*r_t - 1 + \sigma_t w_t + 2.9865*(1/10)\sigma_t - 1 w_t - 1\sigma_t^2 = 2.9*(1/10)^6 + 1.53*(1/10)^1*(r_t - 1)^2 + 8.27*\sigma_t -$

QUESTION 7 Plot the return series with the modeled conditional standard deviations times 2 indicated. This is one of the default plots in fGarch.

```
##
## Series Initialization:
##  ARMA Model:                arma
##  Formula Mean:              ~ arma(1, 1)
##  GARCH Model:               garch
##  Formula Variance:          ~ garch(1, 1)
##  ARMA Order:                1 1
##  Max ARMA Order:            1
##  GARCH Order:               1 1
##  Max GARCH Order:           1
##  Maximum Order:             1
##  Conditional Dist:          norm
##  h.start:                   2
##  llh.start:                 1
##  Length of Series:          4103
##  Recursion Init:            mci
##  Series Scale:              0.01227742
##
## Parameter Initialization:
##  Initial Parameters:          $params
##  Limits of Transformations:   $U, $V
##  Which Parameters are Fixed?  $includes
##  Parameter Matrix:
##                      U           V      params includes
##     mu     -0.19779487   0.1977949  0.01976847     TRUE
##     ar1    -0.99999999   1.0000000 -0.17130733     TRUE
##     ma1    -0.99999999   1.0000000  0.03978818     TRUE
##     omega   0.00000100 100.0000000  0.10000000     TRUE
##     alpha1  0.00000001   1.0000000  0.10000000     TRUE
##     gamma1 -0.99999999   1.0000000  0.10000000    FALSE
##     beta1   0.00000001   1.0000000  0.80000000     TRUE
##     delta   0.00000000   2.0000000  2.00000000    FALSE
##     skew    0.10000000  10.0000000  1.00000000    FALSE
##     shape   1.00000000  10.0000000  4.00000000    FALSE
##  Index List of Parameters to be Optimized:
##     mu    ar1    ma1  omega alpha1  beta1
##      1      2      3      4      5      7
##  Persistence:                 0.9
##
##
## --- START OF TRACE ---
## Selected Algorithm: nlminb
##
## R coded nlminb Solver:
##
##   0:     4913.7639: 0.0197685 -0.171307 0.0397882 0.100000 0.100000 0.800000
##   1:     4786.8231: 0.0197689 -0.170006 0.0410653 0.0719670 0.0982503 0.784988
##   2:     4723.8401: 0.0197701 -0.167251 0.0437710 0.0446298 0.114219 0.784382
##   3:     4715.4337: 0.0197717 -0.164482 0.0464929 0.0525566 0.139285 0.802029
##   4:     4666.5846: 0.0197778 -0.155843 0.0549779 0.0258744 0.149463 0.794588
##   5:     4650.1915: 0.0197956 -0.135942 0.0744276 0.0313890 0.160331 0.804279
##   6:     4642.3268: 0.0198214 -0.127240 0.0824607 0.0166446 0.152725 0.828782
##   7:     4642.1306: 0.0198218 -0.127262 0.0824257 0.0207609 0.153278 0.829878
##   8:     4640.6183: 0.0198283 -0.126869 0.0826227 0.0191537 0.152692 0.828701
```

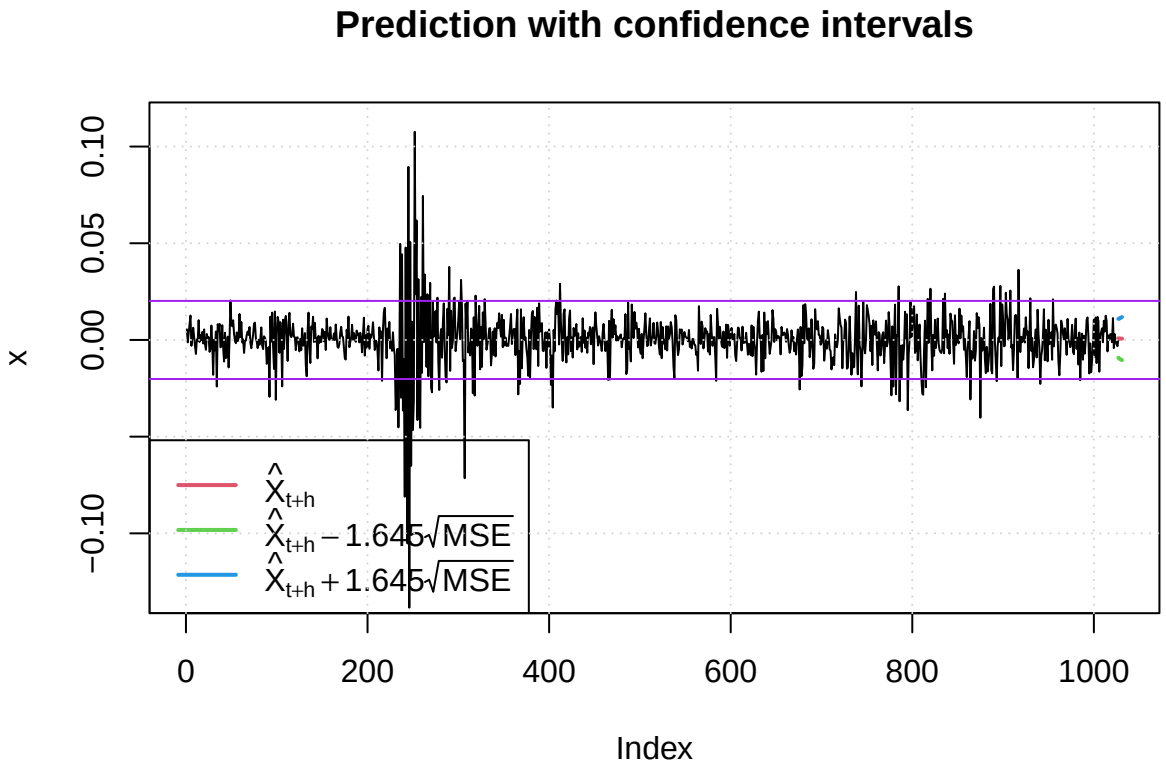
```
##   9:     4640.5283: 0.0198543 -0.126593 0.0821107 0.0189799 0.151190 0.829219
##  10:     4640.4645: 0.0199197 -0.126880 0.0798415 0.0194567 0.149970 0.829956
##  11:     4640.4037: 0.0199927 -0.126208 0.0783237 0.0188487 0.149030 0.830869
##  12:     4640.3667: 0.0200565 -0.123548 0.0790823 0.0192187 0.148615 0.830539
##  13:     4640.3629: 0.0200628 -0.123921 0.0785122 0.0195692 0.148103 0.830403
##  14:     4640.3503: 0.0200797 -0.123709 0.0782118 0.0192485 0.148063 0.830406
##  15:     4640.3341: 0.0200971 -0.123485 0.0779132 0.0193268 0.148050 0.830642
##  16:     4640.2230: 0.0204795 -0.118330 0.0715089 0.0183976 0.148034 0.832774
##  17:     4640.1151: 0.0208560 -0.113206 0.0651514 0.0194429 0.150922 0.829876
##  18:     4639.8988: 0.0212382 -0.107356 0.0591666 0.0190199 0.151974 0.828414
##  19:     4639.4959: 0.0221924 -0.0888389 0.0468081 0.0186098 0.143453 0.835227
##  20:     4639.4896: 0.0221927 -0.0888699 0.0467679 0.0187345 0.143512 0.835249
##  21:     4639.4851: 0.0221932 -0.0889338 0.0466850 0.0186740 0.143561 0.835181
##  22:     4639.4796: 0.0221977 -0.0889279 0.0465426 0.0187732 0.143646 0.835165
##  23:     4639.4713: 0.0222082 -0.0887953 0.0463326 0.0186825 0.143670 0.835092
##  24:     4634.9252: 0.0349744 0.125794 -0.155078 0.0209084 0.154116 0.821929
##  25:     4633.4445: 0.0374341 0.192022 -0.231883 0.0194019 0.154016 0.824481
##  26:     4633.0923: 0.0387706 0.250544 -0.292105 0.0207853 0.152842 0.823755
##  27:     4633.0653: 0.0388537 0.328654 -0.366266 0.0199129 0.154268 0.823557
##  28:     4632.8581: 0.0403701 0.255225 -0.300288 0.0190140 0.153116 0.826038
##  29:     4632.8463: 0.0406987 0.245303 -0.290144 0.0194681 0.153978 0.827190
##  30:     4632.7593: 0.0408973 0.241945 -0.287389 0.0191633 0.153645 0.826945
##  31:     4632.7276: 0.0412936 0.235120 -0.281805 0.0193213 0.153061 0.826649
##  32:     4632.5723: 0.0453163 0.150559 -0.202379 0.0193367 0.151951 0.827375
##  33:     4632.3381: 0.0612770 -0.193678 0.125547 0.0192013 0.148525 0.829706
##  34:     4632.1751: 0.0562785 -0.0649231 0.00478132 0.0194259 0.150833 0.828077
##  35:     4632.0012: 0.0599872 -0.121593 0.0621432 0.0193599 0.151460 0.827741
##  36:     4631.7048: 0.0768665 -0.392899 0.336609 0.0194072 0.152726 0.827005
##  37:     4631.6625: 0.0741273 -0.315394 0.264656 0.0190636 0.153106 0.826911
##  38:     4631.6389: 0.0752258 -0.339608 0.287356 0.0193309 0.153222 0.826663
##  39:     4631.6374: 0.0762670 -0.357425 0.304779 0.0192377 0.153164 0.826766
##  40:     4631.6367: 0.0760788 -0.352000 0.299593 0.0192679 0.153177 0.826736
##  41:     4631.6366: 0.0761361 -0.351592 0.299174 0.0192711 0.153177 0.826730
##  42:     4631.6366: 0.0762415 -0.351284 0.298795 0.0192712 0.153171 0.826730
##  43:     4631.6366: 0.0762476 -0.351362 0.298863 0.0192706 0.153170 0.826731
##
## Final Estimate of the Negative LLH:
##  LLH:  -13421.54    norm LLH:  -3.271152
##           mu           ar1           ma1         omega        alpha1
##  9.361238e-04 -3.513617e-01  2.988632e-01  2.904753e-06  1.531695e-01
##         beta1
##  8.267311e-01
##
## R-optimhess Difference Approximated Hessian Matrix:
##                   mu           ar1           ma1         omega        alpha1
## mu     -4.187594e+07  -36967.44450   -7710.71136 -9.999556e+08  3.152031e+04
## ar1    -3.696744e+04   -3854.32924   -3747.35285  2.469889e+06  1.103609e+01
## ma1    -7.710711e+03   -3747.35285   -3674.86674  2.818166e+06 -1.688442e+01
## omega  -9.999556e+08 2469889.19274 2818166.42313 -2.512409e+13 -7.501121e+08
## alpha1  3.152031e+04      11.03609     -16.88442 -7.501121e+08 -4.985448e+04
## beta1  -3.563300e+04      64.65094      72.70398 -1.219309e+09 -6.138775e+04
##                beta1
## mu     -3.563300e+04
## ar1     6.465094e+01
```

```
## ma1     7.270398e+01
## omega  -1.219309e+09
## alpha1 -6.138775e+04
## beta1  -8.851827e+04
## attr(,"time")
## Time difference of 0.07851887 secs
##
## --- END OF TRACE ---
##
##
## Time to Estimate Parameters:
##  Time difference of 0.32461 secs
```



QUESTION 8 Produce forecasts for the next 5 days (one trading week) of both the return level and the standard deviation.

```
##   meanForecast   meanError standardDeviation lowerInterval upperInterval
## 1 0.0008640269 0.006111787       0.006111787  -0.009188969    0.01091702
## 2 0.0006325379 0.006293716       0.006285532  -0.009719704    0.01098478
## 3 0.0007138742 0.006460664       0.006451246  -0.009912972    0.01134072
## 4 0.0006852957 0.006619407       0.006609601  -0.010202659    0.01157325
## 5 0.0006953371 0.006771257       0.006761176  -0.010442390    0.01183306
```



QUESTION 9 Summarize your findings assuming model is PERFECT. Comment on what the model tells you and the forecasts obtained.

This model tells us that this event sequence does have variance volatility.The variance volatility is successfully predicted by building this model.

According to the plot of forecast, the mean error and standard deviation, the model is a good choice for estimators.Besides, this model tells us the returns and variance in next 5 days keep comparatively flat.