# The Beauty and Joy of Computing

## Lecture #24
## Limits of Computing

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## SEPARATION OF POWERS

Trouble brews as the CIA spies on Congressional committee computers as they investigate CIA interrogation techniques. Unfortunately, hacking is sometimes used as a powerful weapon between countries as well as within a country itself.

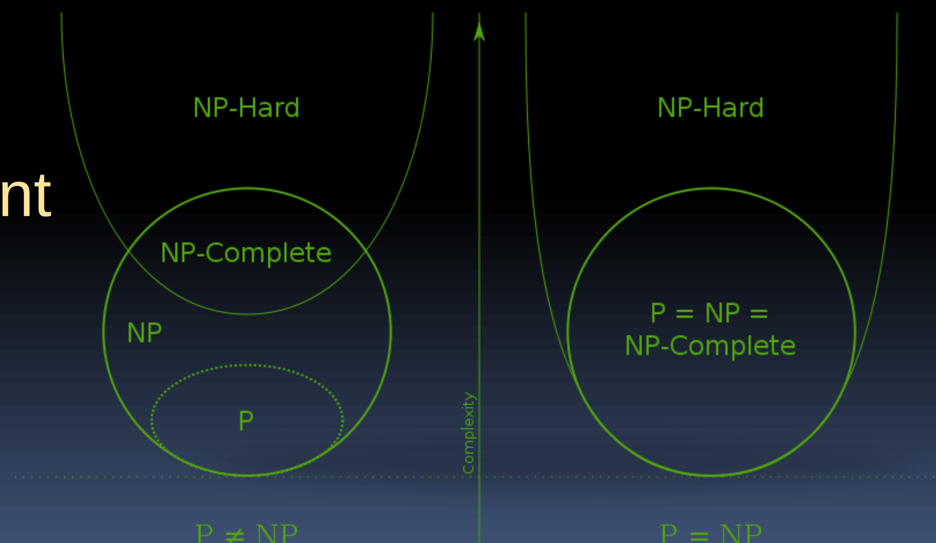
http://online.wsj.com/articles/investigation-faults-cia-employees-in-agency-feud-with-senate-panel-1406822353

# Computer Science … A UCB view

- CS research areas:
  - Artificial Intelligence
  - Biosystems & Computational Biology
  - Database Management Systems
  - Graphics
  - Human-Computer Interaction
  - Networking
  - Programming Systems
  - Scientific Computing
  - Security
  - Systems
  - Theory
    - Complexity theory
  - …

# Let's revisit algorithm complexity

- Problems that…
  - are tractable with efficient solutions in reasonable time
  - are intractable
  - are solvable approximately, not optimally
  - have no known efficient solution
  - are not solvable




http://en.wikipedia.org/wiki/NP-complete

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# Tractable with efficient sols in reas time

- Recall our algorithm complexity lecture, we've got several common orders of growth
  - Constant
  - Logarithmic
  - Linear
  - Quadratic
  - Cubic
  - Exponential
- Order of growth is polynomial in the size of the problem
- E.g.,
  - Searching for an item in a collection
  - Sorting a collection
  - Finding if two numbers in a collection are same
- These problems are called being "in P" (for polynomial)

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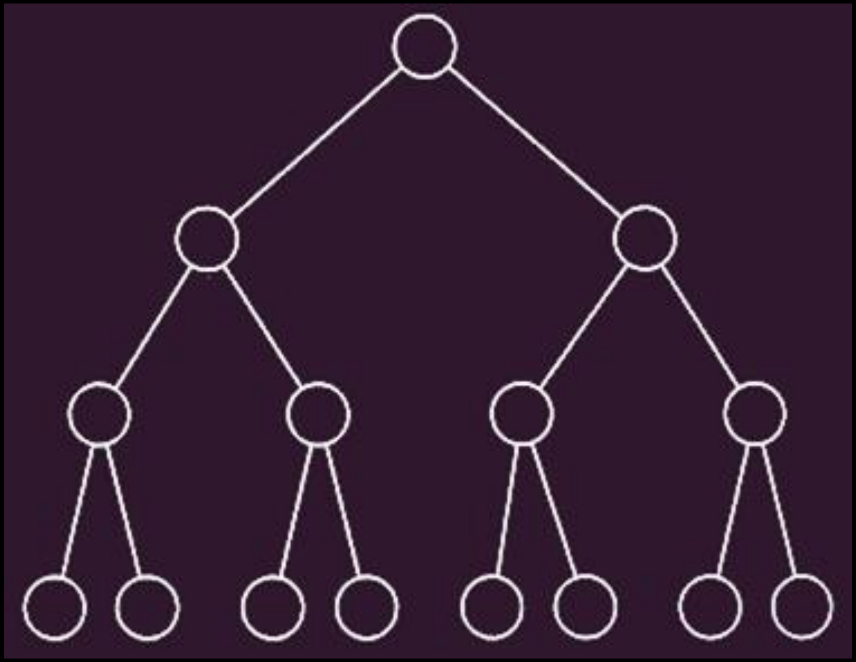
# Intractable problems

- Problems that can be solved, but not solved fast enough
- This includes exponential problems
  - E.g., $f(n) = 2^n$
    - as in the image to the right
- ~~This also includes poly-time algorithm with a huge exponent~~
  - ~~E.g, $f(n) = n^{10}$~~
- ~~Only solve for small n~~

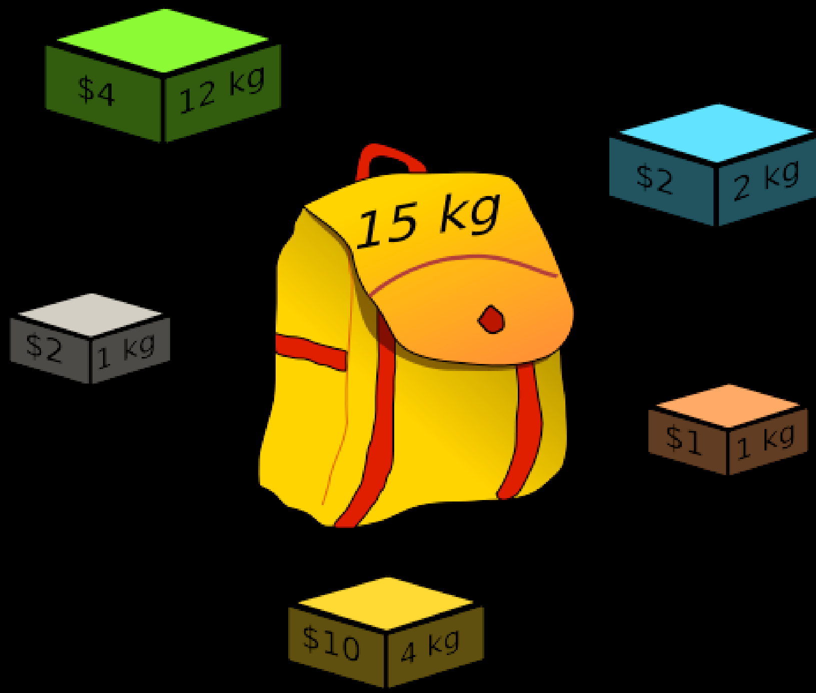


Imagine a program that calculated something important at each of the bottom circles. This tree has height n,
but there are $2^n$ bottom circles!

# Peer Instruction

What's the most you can put in your knapsack?

a) $10
b) $15
c) $33
d) $36
e) $40




Knapsack Problem

You have a backpack with a weight limit (here **15kg**), which boxes (with weights and values) should be taken to maximize value?

***(any # of each box is available)***

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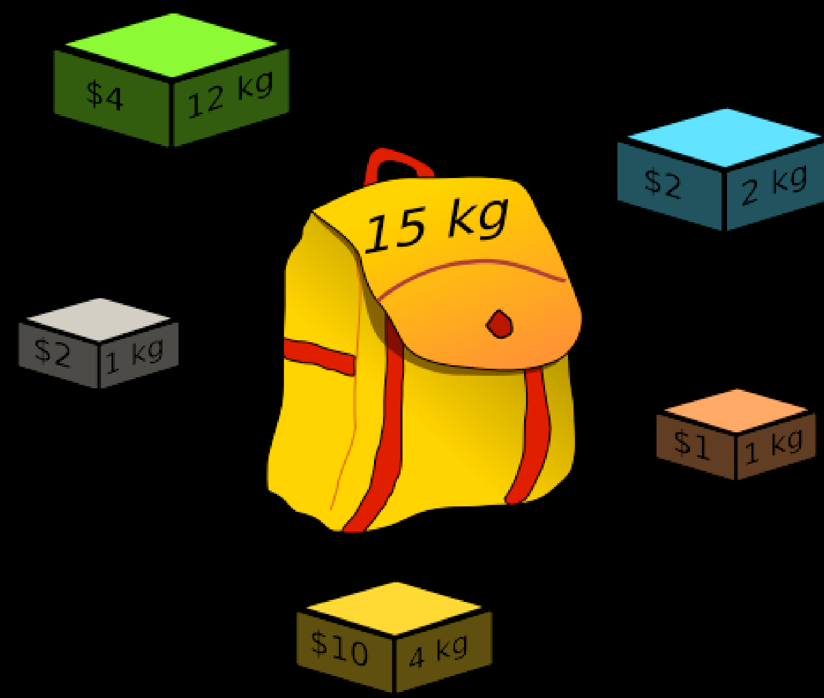
# Solvable approximately, not optimally in reas time

- A problem might have an optimal solution that cannot be solved in reasonable time
- BUT if you don't need to know the perfect solution, there might exist algorithms which could give pretty good answers in reasonable time




Knapsack Problem
You have a backpack with a weight limit (here 15kg), which boxes (with weights and values) should be taken to maximize value?

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# Have no known efficient solution

- Solving one of them would solve an entire class of them!
  - We can transform one to another, i.e., reduce
  - A problem P is “hard” for a class C if every element of C can be “reduced” to P
- If you’re in both “in NP” and “NP-hard”, then you’re “NP-complete”

| | | |
|---|---|---|
| -2 | -3 | 15 |
| 14 | 7 | -10 |

Subset Sum Problem
Are there a handful of these numbers (at least 1) that add together to get 0?

- If you guess an answer, can I verify it in polynomial time?
  - Called being “in NP”
  - Non-deterministic (the “guess” part) Polynomial

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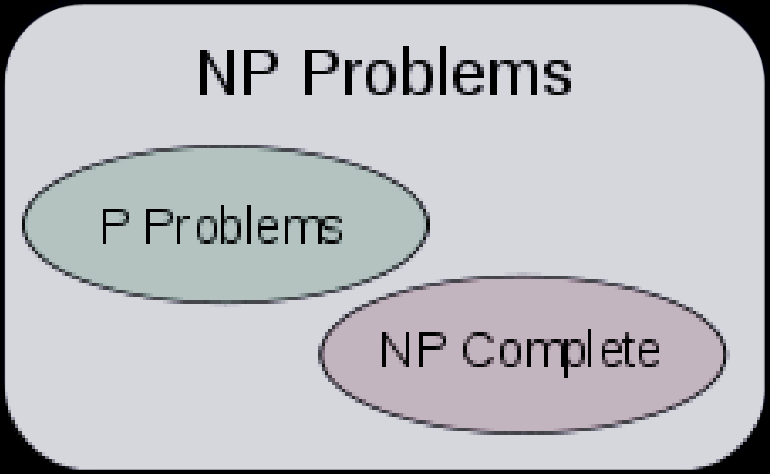
# The fundamental question. Is P = NP?

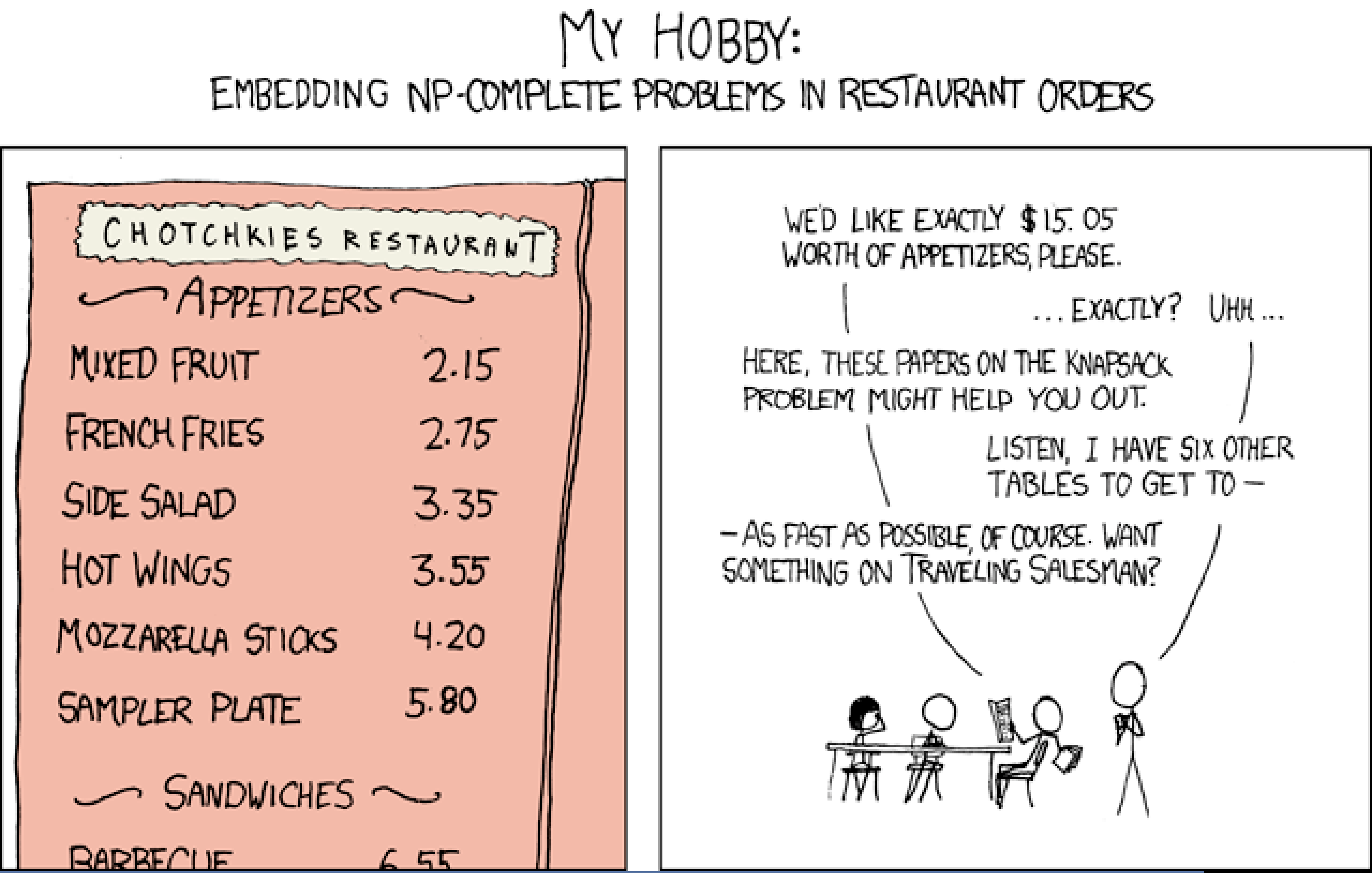
- This is THE major unsolved problem in Computer Science!
  - One of 7 “millennium prizes” w/a $1M reward
- All it would take is solving ONE problem in the NP-complete set in polynomial time!!
  - Huge ramifications for cryptography, others

If P ≠ NP, then

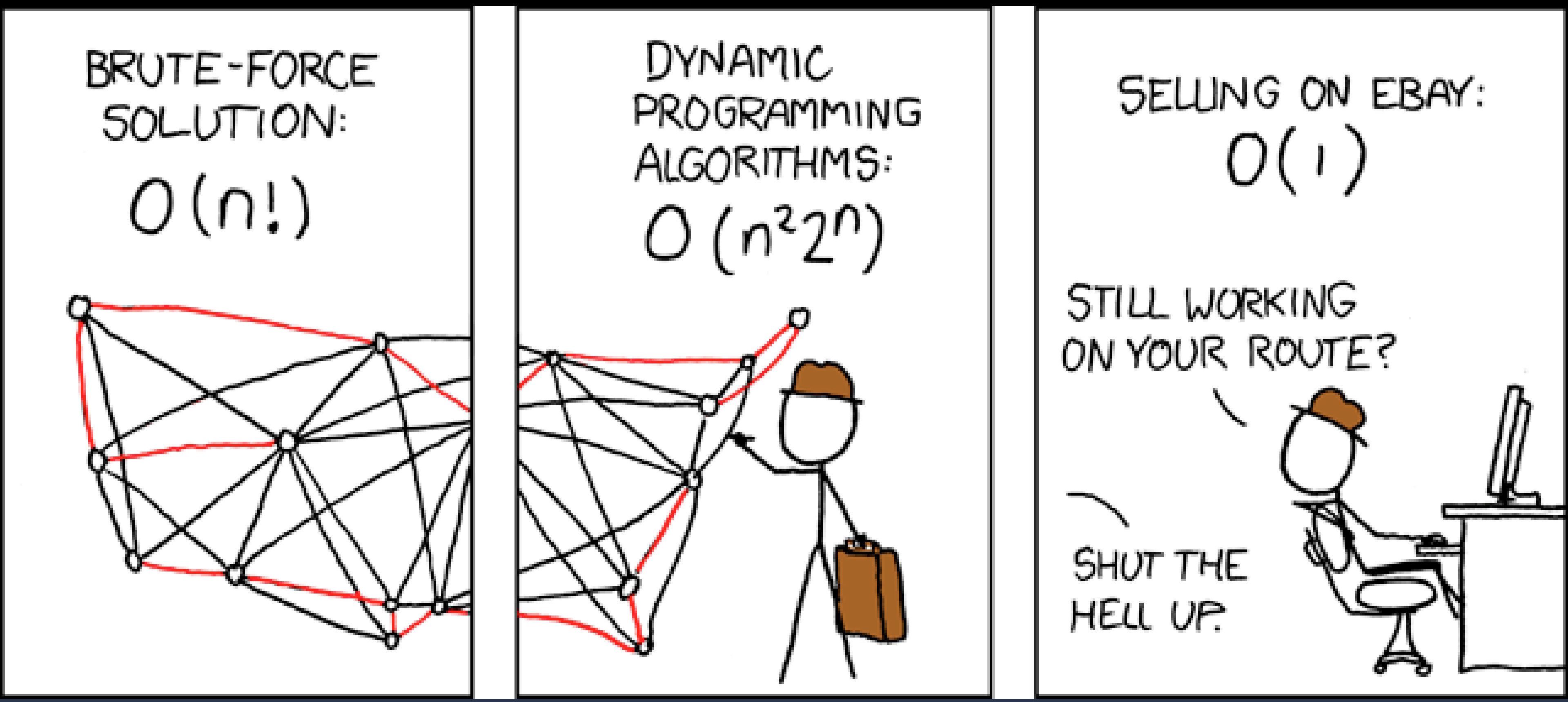



- Other NP-Complete
  - Traveling salesman who needs efficient route below a certain cost to visit all cities and return home

# imgs.xkcd.com/comics/np_complete.png



# imgs.xkcd.com/comics/travelling_salesman_problem.png



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# Problems NOT solvable

- Decision problems answer YES or NO for an infinite # of inputs
  - E.g., is N prime?
  - E.g., is sentence S grammatically correct?
- An algorithm is a solution if it correctly answers YES/NO in a finite amount of time
- A problem is decidable if it has a solution

**June 23, 2012 was his 100th birthday celebration!!**

Alan Turing
He asked:
"Are all problems decidable?"
(people used to believe this was true)
**Turing proved it wasn't for CS!**

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# Review: Proof by Contradiction

- Infinitely Many Primes?
- Assume the contrary, then prove that it's impossible
  - Only a finite set of primes, numbered $p_1, p_2, \ldots, p_n$
  - Consider $q=(p_1 \cdot p_2 \cdot \ldots \cdot p_n)+1$
  - Dividing $q$ by $p_i$ has remainder 1
  - $q$ either prime or composite
    - If prime, $q$ is not in the set
    - If composite, since no $p_i$ divides $q$, there must be another p that does that is not in the set.
  - So there's infinitely many primes

Euclid
www.hisschemoller.com/wp-content/uploads/2011/01/euclides.jpg

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# Turing's proof : The Halting Problem

- Given a program and some input, will that program eventually stop? (or will it loop)
- Assume we could write it, then let's prove a contradiction
  - 1. write Stops on Self?
  - 2. Write Weird
  - 3. Call Weird on itself



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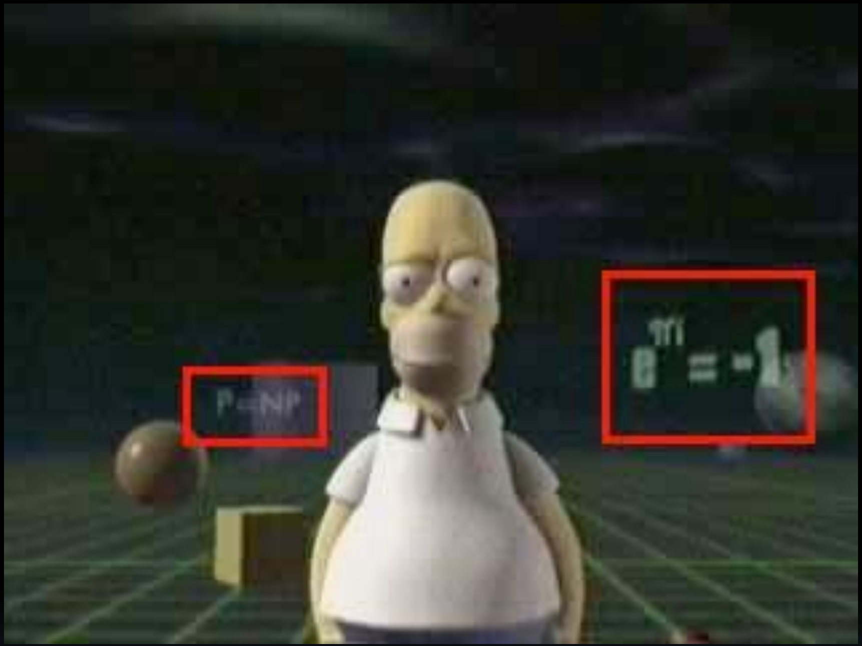
# Conclusion

- Complexity theory important part of CS
- If given a hard problem, rather than try to solve it yourself, see if others have tried similar problems
- If you don't need an exact solution, many approximation algorithms help
- Some not solvable!




P=NP question even made its way into popular culture, here shown in the Simpsons 3D episode!

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