Faculty of Business and Economics
Department of Economics

THE UNIVERSITY OF MELBOURNE

ECON10003
Pre-Tutorial #7

Introductory Macroeconomics

Pre-Tutorial #7
Week Starting 26th April 2021

The Tutorial. This week's tutorial looks at saving, investment, and production function.

Note that your tutor is under no obligation to go through the answers to the pre-tutorial work in detail. The focus in the tutorial will be on the tutorial work itself – the questions here are preparatory.

Reading Guide. You should look carefully over lecture 13 and 14. You may also find Chapter 4, 13, and 14 of BOFAH useful.

Key Concepts. Stock and flow. Saving and investment. Cobb-Douglas production function.

Problems.

1. Distinguish between a "stock" and "flow" as concepts. Describe the following economic variables as describing either a "stock" or a "flow":

   - GDP
   - Savings
   - Wealth
   - Investment

2. Demonstrate that, in a closed economy with no government, the level of national saving is equal to the level of investment.

3. In the lecture, we assumed that firms maximise profits by selecting a level of capital to use in the production process. Suppose $Y = 2K^{1/2}$. What is the level of profits as a function of K in this setting? What is the optimal value of K as a function of the interest rate, the rate of depreciation and the price of output?

4. To a large extent, the level of output per person determines living standards. Why do economists consider growth in average labour productivity to be the key factor in determining long-run living standards?

5. The Cobb-Douglas production function is given by $Y = AK^{\alpha}L^{(1-\alpha)}$. Here α is a given parameter that satisfies $0 < \alpha < 1$.

   (a) Find the marginal product of capital. Show that there are diminishing marginal product of capital.

   (b) What do we mean by the phrase, "constant returns to scale"? How is this concept different to the concept of "diminishing marginal product"?

   (c) Show that the Cobb-Douglas production function displays constant returns to scale and that the level of output per worker can be written as a function of the level of capital per worker.

Solutions to Pre-Tutorial Work.

1. The following are stocks - wealth. At a given point in time there is a stock of wealth available. The rest of the concepts are flow concepts. Note that if GDP is a flow concept, then its components (C, I, G, X, M) must also be flow concepts.

2. In a closed economy without government sector, the National Accounting identity states that

$$Y = C + I$$

In this economy, national saving equals private saving, which is defined as output less consumption:

$$S = Y - C$$

Combining these two equations implies that $S = I$.

3. In the lecture, we saw that the profit of a firm using K units of capital is

$$2pK^{1/2} - (r + \delta)K,$$

where p is the price of output, r is the real interest rate, and δ is the depreciation rate. We can think of this as a firm that hires K units of capital produces $2K^{1/2}$ units of output and is able to sell output at a price of p. Hence $2pK^{1/2}$ determines the revenue produced. The cost of hiring capital depends upon the real interest rate and the rate at which capital depreciates. The total cost is $(r + \delta)K$. In our setting this equation becomes after substituting in values.

The first order condition for this problem becomes

$$pK^{-1/2} - (r + \delta) = 0$$

The first term is the marginal revenue product of capital, or marginal benefit of increasing one unit of capital. The second term is the cost of raising one unit of capital, the interest (and cover depreciation) for the capital rented. We can re-arrange this first order condition to find,

$$K^* = \left(\frac{p}{r + \delta}\right)^2.$$

where K^* is the optimal level of capital. This optimal level of capital will be increasing in p the price of output and decreasing in r and δ, the cost of hiring capital.

4. Real GDP per person (a basic determinant of living standards) equals average labour productivity times the share of the population that is employed.The share of the population that is employed can only rise so far; it can never exceed 100 percent. In fact, for a long period of time, this ratio has remained roughly constant. Thus, large long-term gains in output per person (and hence living standards) generally must come from increases in average labour productivity.

5. The Cobb-Douglas production function is $Y = AK^{\alpha}L^{1-\alpha}$.

(a) The marginal product of capital is given by

$$\begin{aligned}\frac{\partial Y}{\partial K} &= \alpha A K^{\alpha-1} L^{1-\alpha} \\ &= \alpha \frac{A K^{\alpha} L^{1-\alpha}}{K} \\ &= \alpha \frac{Y}{K}\end{aligned}$$

The first derivative of the marginal product of capital is

$$\frac{\partial \left(\frac{\partial Y}{\partial K}\right)}{\partial K} = \alpha \underbrace{(\alpha - 1)}_{<0} A K^{\alpha-2} L^{1-\alpha}$$

Each of the terms in this derivative will be positive except for the component $\alpha - 1$ which is labelled as less than zero in the equation above. This implies that the first derivative of the marginal product of capital is negative. The economic interpretation that we can take away from this is that the marginal product of capital is positive but it is decreasing as K increases.

(b) The phrase constant returns to scale implies that if we increase the inputs used in the production process by a certain multiple, say X, then the overall level of output produced in the economy will increase by the same multiple, X. This is different from the concept of diminishing marginal product. A production function that displays diminishing marginal products implies that the amount produced increases but at a decreasing rate when we consider changes to *one* factor of production. The concept of constant returns to scale applies when we consider changing *all* inputs into the production process by the same proportional amount.

(c) If we start with an arbitrary level of capital K and labour L we can produce an amount of output denoted Y_0. Then we can increase both capital by a factor of X and labour by a factor of X. Then total output will be

$$\begin{aligned}Y &= A(XK)^{\alpha}(XL)^{1-\alpha} \\ &= A X^{\alpha} K^{\alpha} X^{1-\alpha} L^{1-\alpha} \\ &= A X K^{\alpha} L^{1-\alpha} \\ &= X Y_0\end{aligned}$$

So increasing inputs by a factor of X leads to an increase in output by a factor of X. Finally, for any constant returns to scale production function, the level of output per worker can be written as a function of capital per worker the production function implies,

$$\begin{aligned}Y &= A K^{\alpha} L^{1-\alpha} \\ \rightarrow \frac{Y}{L} &= A K^{\alpha} L^{-\alpha} \\ &= A \left(\frac{K}{L}\right)^{\alpha}\end{aligned}$$

which implies that we can denote the level of output per worker as a function of the level of capital per worker.