THE UNIVERSITY OF MELBOURNE

FNCE10002 Principles of Finance

Semester 1, 2019

Introduction to Financial Mathematics II

Suggested Answers to Tutorial Questions for Week 2

Note that detailed answers to tutorial questions from Part II will only be provided in tutorials. The following abridged answers are intended as a guide to those detailed answers. This policy is in place to ensure that you attend your tutorial regularly and receive timely feedback from your tutor. If you are unsure of your answers you should check with your tutor, a pit stop tutor, online tutor or me.

While detailed answers to Part I appear below, if you are not sure of the answers to these questions please ask your tutor in the following week's tutorial.

Part I – Answers Submitted to Your Tutor

A. Problems and Case Studies

A1. We can calculate and compare the present values of each prize (rounded to the nearest dollar) as follows:

a) $$PV_0 = \frac{140000}{(1+0.08)^3} = \$111,137.$$

b) $$PV_0 = \left(\frac{28000}{0.08}\right)\left[1 - \frac{1}{(1+0.08)^5}\right] = \$111,796.$$

c) $$PV_0 = \left(\frac{9000}{0.08}\right) = \$112,500.$$

d) $$PV_3 = \frac{12000}{0.08} = \$150,000.$$

$$PV_0 = \frac{150000}{(1+0.08)^3} = \$119,075.$$

Based on the present values of the prizes you would prefer the fourth prize.

A2. We now have a growing annuity of \$80 per month where the growth rate is 0.5% per month. The future values over the two time horizons are as follows:

Over 10 years or 120 months

$$FV_{120}(GA)=\left(\frac{80}{0.01-0.005}\right)\left(1-\left(\frac{1+0.005}{1+0.01}\right)^{10\times 12}\right)\left(1+0.01\right)^{10\times 12}=\$23,696.$$

Over 50 years or 600 months

$$FV_{600}(GA)=\left(\frac{80}{0.01-0.005}\right)\left(1-\left(\frac{1+0.005}{1+0.01}\right)^{50\times 12}\right)\left(1+0.01\right)^{50\times 12}=\$5,946,359.$$

The present values of the growing annuity over the two time horizons are as follows:

Over 10 years or 120 months

$$PV_{120}(GA)=\left(\frac{80}{0.01-0.005}\right)\left(1-\left(\frac{1+0.005}{1+0.01}\right)^{10\times 12}\right)=\$7,180.$$

Over 50 years or 600 months

$$PV_{600}(GA)=\left(\frac{80}{0.01-0.005}\right)\left(1-\left(\frac{1+0.005}{1+0.01}\right)^{50\times 12}\right)=\$15,185.$$

A3. We now have a monthly repayment that is an unknown amount (that is, C) whose future value at the end of months 24 and 36 should be equal to the respective future values of \$58,126 at those points in time.

a) The future value of \$58,126 at the end of month 24:

$FV_{24} = 58126(1 + 0.015)^{24} = \$83,091.$

So, $FV_{24}=\dfrac{C}{0.015}\left[(1+0.015)^{24}-1\right]=83091.$

Solving for C, we get:

$$C=\frac{83091(0.015)}{\left[(1+0.015)^{24}-1\right]}=\$2,902.$$

b) Similarly, the future value of \$58,126 at the end of month 36:

$FV_{36} = 58126(1 + 0.015)^{36} = \$99,345.$

So, $FV_{36}=\dfrac{C}{0.015}\left[(1+0.015)^{36}-1\right]=99345.$

Solving for C, we get:

$$C = \frac{99345(0.015)}{\left[(1+0.015)^{36} - 1\right]} = \$2{,}101.$$

Part II – Submission of Answers Not Required

B. Multiple Choice Questions

B1. B is correct. This problem relates to the present value of a growing perpetuity where the growth rate (g) is unknown.

B2. B is correct. For these types of questions, I would suggest using a timeline to help you visualize the cash flows.

B3. B is correct. This is a three-year annuity which is deferred until the end of year 4. The present value at the end of year 3 is $12,434,260 and the present value today is $9,342,044.

B4. D is correct. Note that since the cash flows are received at the beginning of each year we have an annuity due.

B5. B is correct. Note that here we have a growing perpetuity where the growth rate is –2% per annum.

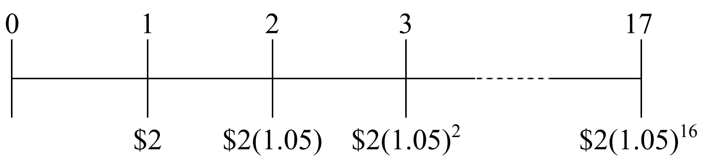
C. Problems and Case Studies

C1. The cash flows for the two alternatives look as follows:

End of Year	Alternative 1	Alternative 2
0	$1 million	--
1	$3 million	--
2	$3 million	$2 million
3	$3 million	$5 million
4	$3 million	$5 million
5	$3 million	$5 million
6	$5 million	$5 million
7	$5 million	$5 million
8	$5 million	$6 million
9	$5 million	$6 million
10	$5 million	$6 million
11	$5 million	$6 million
12	$5 million	$6 million

The total present value of entering the draft is $27,486,885 while the total present value of playing out his eligibility is $28,988,939. So, he should stay in university.

C2. a) The timeline for these cash flows (in millions) is as follows:



This is a growing annuity whose present value is: $21,861,456.

b) Using the present value of a growing perpetuity we can calculate the unknown profits in year 1 as: \$1,093,073.

C3. We now an unknown growing cash flow (C_1) with a known future value of \$3 million. The future value of this unknown amount must be equal to \$3 million at the end of 50 years. The unknown amount is \$1,416.

In the second case, the growth rate in the contribution is lower so the amount of contribution required would be higher. The unknown amount now is \$1,858.