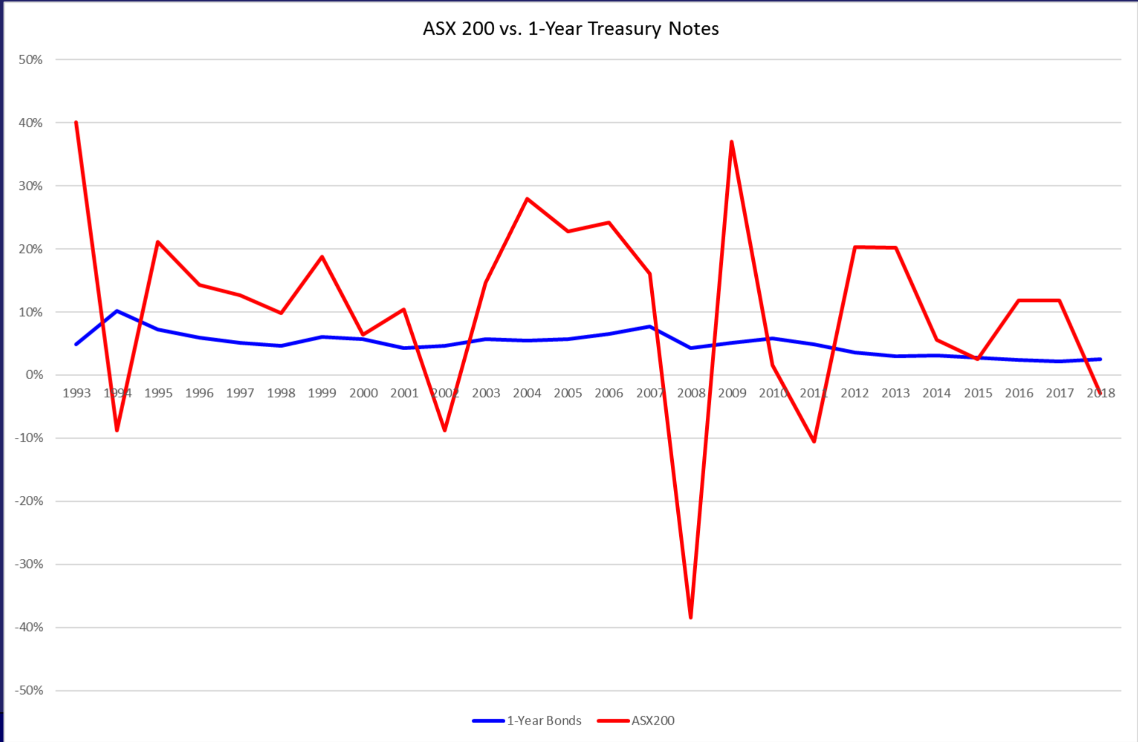
Investments

FNCE30001

Dr Patrick J Kelly

Suppose you have $1 million to invest.

What fraction would you put in the risk-free and in the risky asset?



Mean-Variance Criterion

- How do we increase the mean and reduce the variance?
- How can we improve returns?
- How can we control risk?

Plan for this lecture

- Controlling Risk through Capital Allocation
  - Using Complete Portfolios of a risky and a risk-free asset
  - Impact of preferences on the choice of complete portfolio and the choice of a portfolio of risky assets.

- Markowitz's Modern Portfolio Theory
  - Benefits of diversification and the implications for the choice of risky assets (separation property).
  - Calculating portfolio returns and covariances

- Asset Allocation in Practice

- Asset Pricing Implications
  - The role of firm-specific risk

Controlling Risk

Through Capital Allocation

How Can We Control Risk?

- Form Portfolios

- Mix less risky and more risky assets to get the desired return for a given amount of risk

- Split investment funds between safe and risky assets
  - Risk free asset: proxy; short-term government notes or bills
  - Risky asset: stock (or a portfolio)

Example

Treasury Note.

$r_f = 7\%$ $\sigma_f = 0\%$ ↓ risk free

$E[\tilde{r}_a] = 15\%$ $\sigma_a = 22\%$

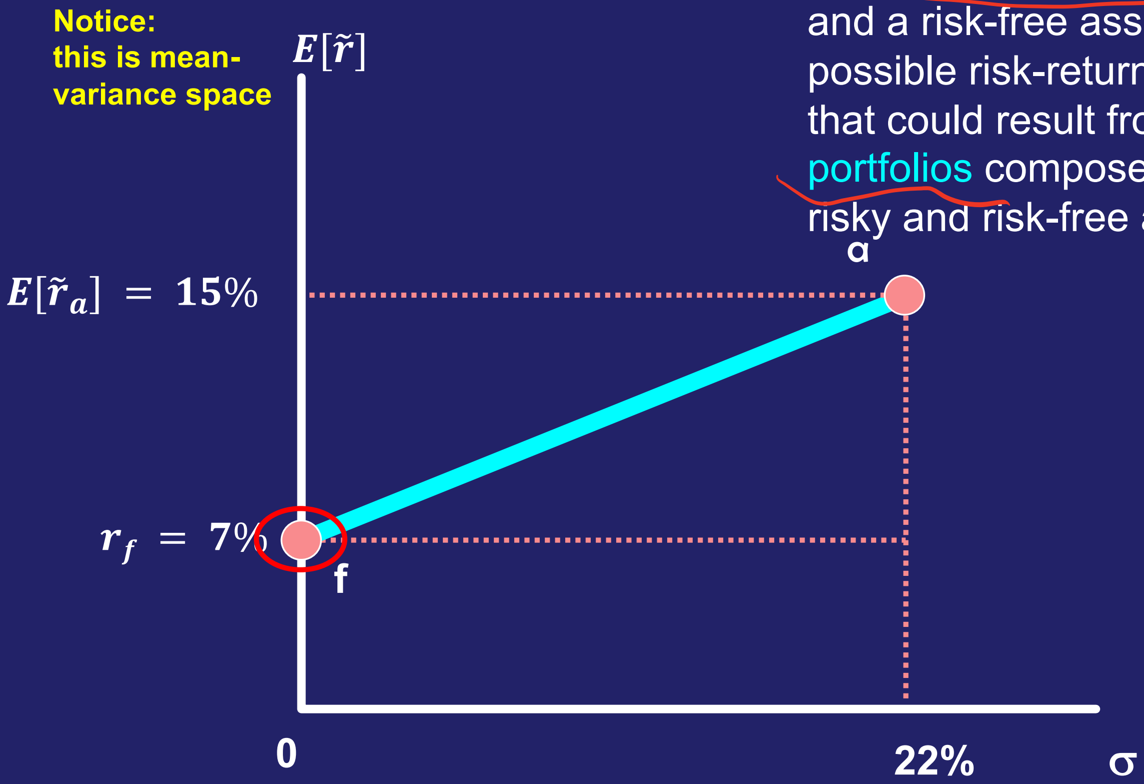
$w = \%\ in\ a$ $(1 - w) = \%\ in\ r_f$

weight of risky asset

- Let's plot the investment opportunity set
  - the set of all feasible portfolios of the assets, that come from using different weights in each asset.

What Return for What Risk?

CAL (Capital Allocation Line) is the investment opportunity set for a risky and a risk-free asset. It shows all possible risk-return combinations that could result from the complete portfolios composed of these two risky and risk-free assets.



Expected Returns for Portfolio of Risky and Risk Free

$$E\left[\tilde{r}_p\right] = w_a E[\tilde{r}_a] + (1 - w_a) r_f$$

- For example, w = .75

$$E\left[\tilde{r}_p\right] = .75(.15) + (.25)(.07) = 0.13 \; or \; 13\%$$

Portfolio risk when one asset is risky and one risk free

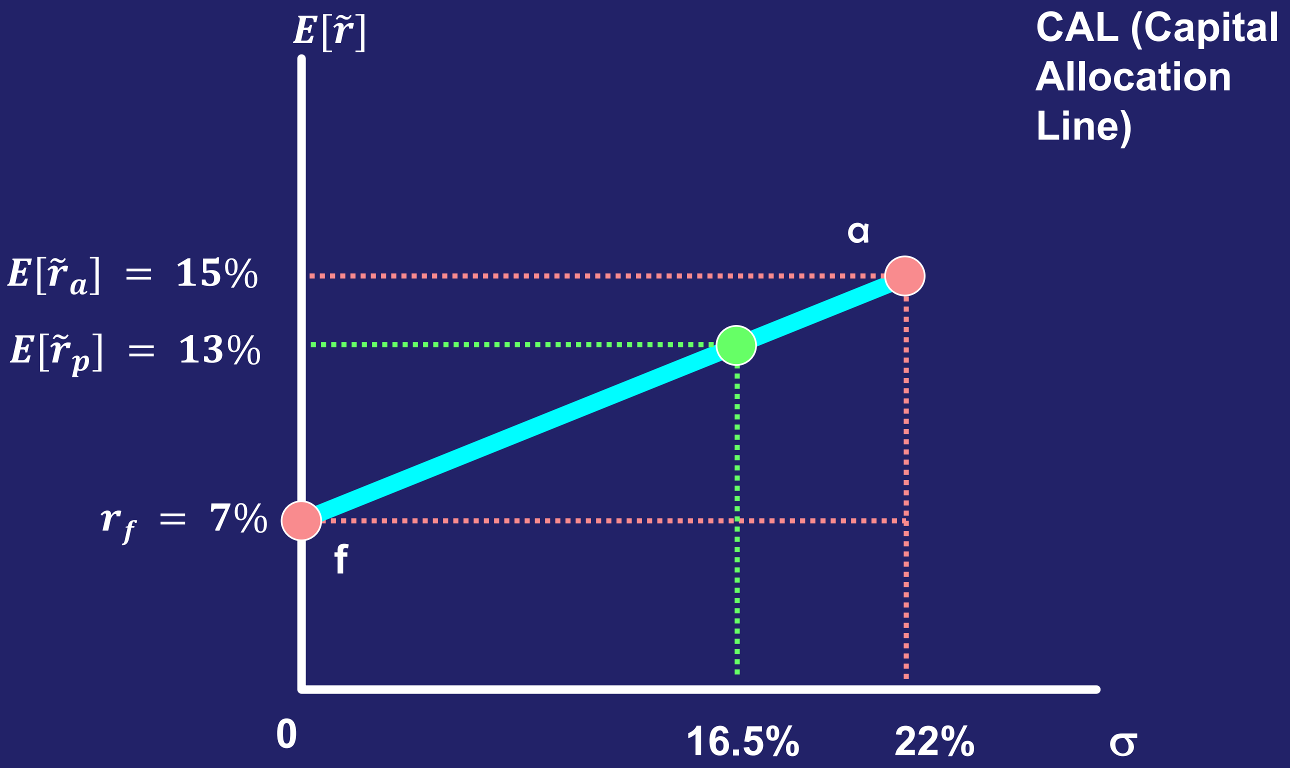
$$\sigma_p = w_a \sigma_a$$

CAUTION: Only works when one risky and one risk free

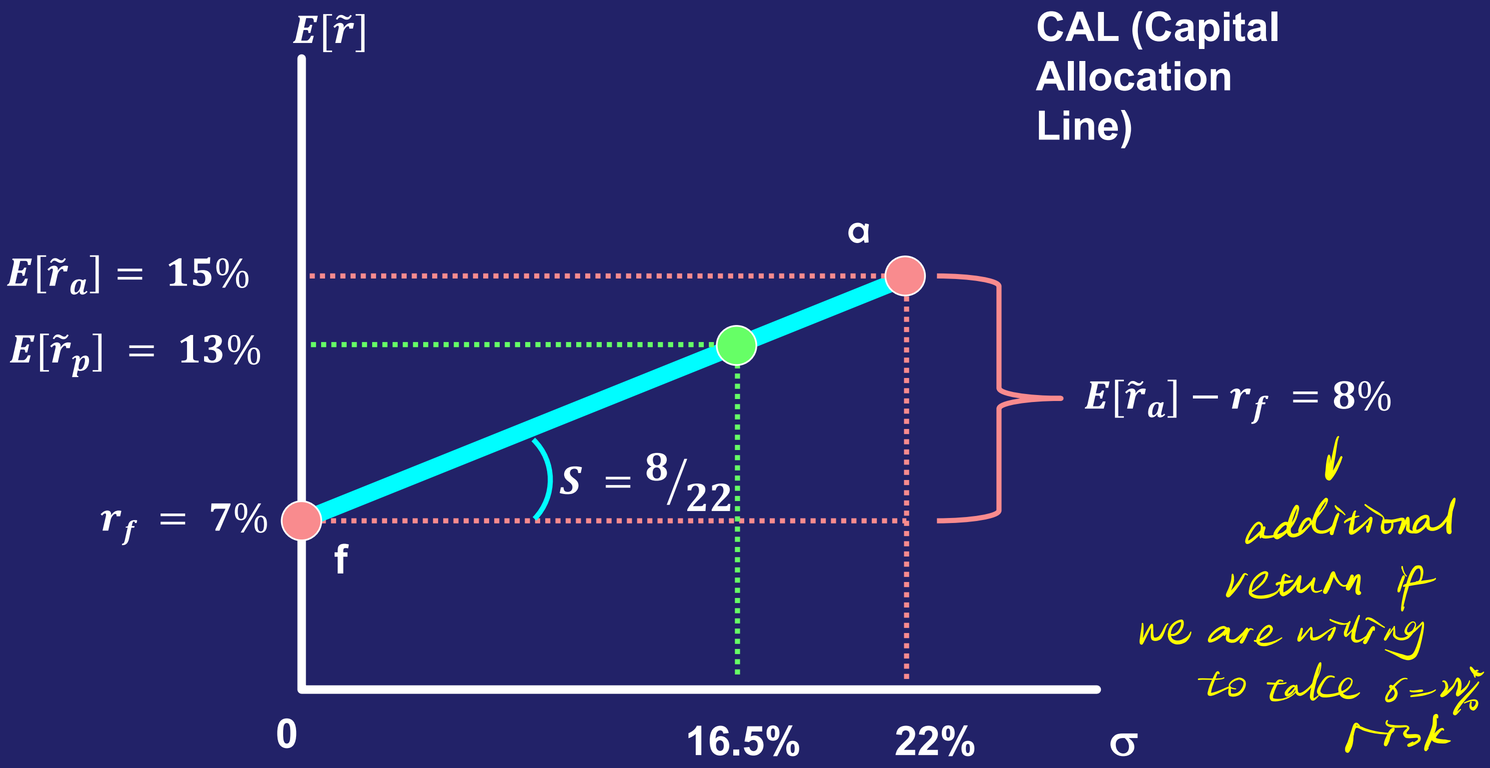
For example, w = .75

$$\sigma_p = .75 \times .22 = .165 \text{ or } 16.5\%$$

What Return for What Risk?



What Return for What Risk?



Sharpe Ratio: Reward for Risk

- Slope S of CAL = $\frac{E[\tilde{r}_a]-r_f}{\sigma_{\tilde{r}_a}} = \frac{15-7}{22} = 0.364$
  - Measures the increase in expected return an investor obtains for taking on one additional unit of standard deviation risk

Also called the

- "reward-to-variability" or "reward-for-risk" ratio

  or

- **Sharpe Ratio**

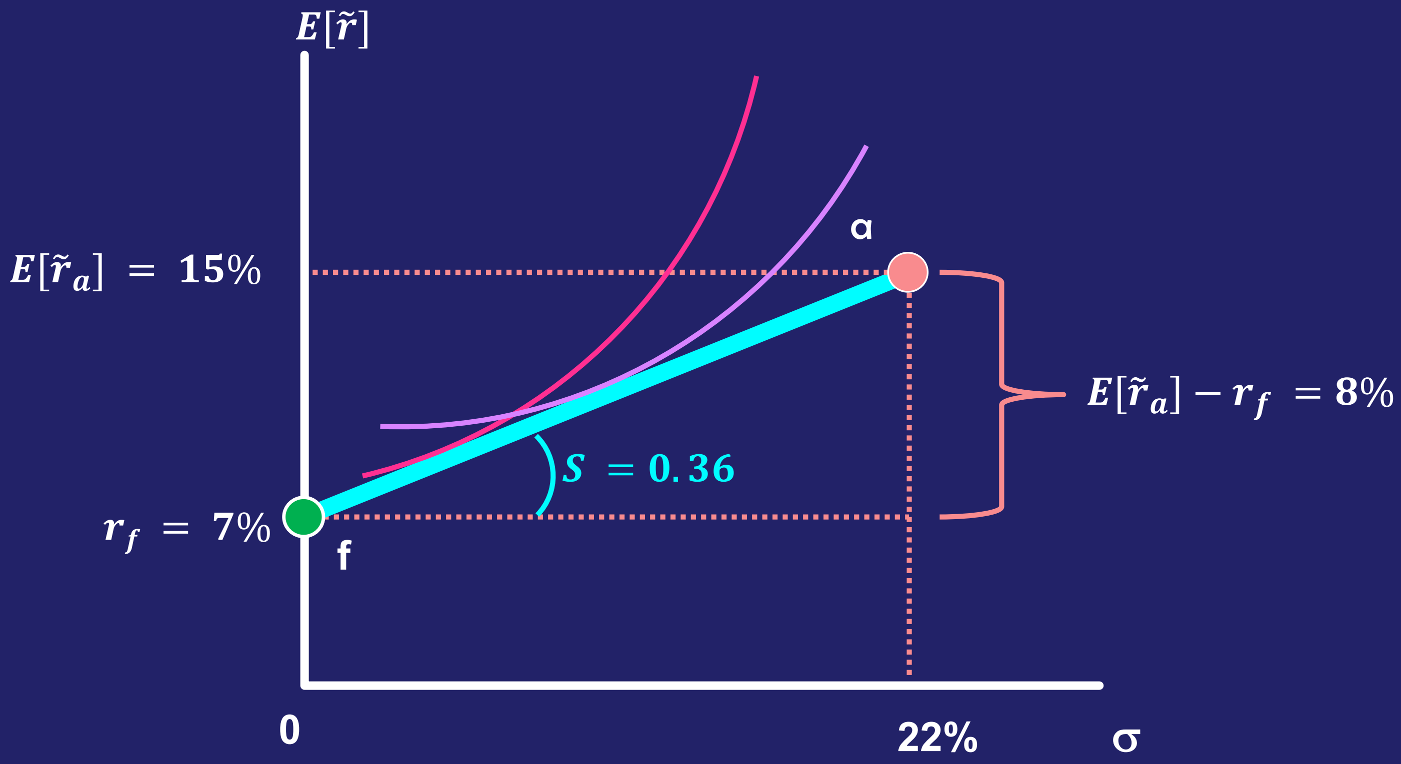
Optimal Portfolio Choice

with Mean-variance Maximising Preferences

The plan

- We'll see that the optimal mix of risky and risk-free asset depends on our preferences or risk aversion.
- We'll solve for the optimal portfolios weights for a person with mean-variance maximizing preferences.
- We will look at an example of how preferences (risk aversion) might change over the course of one's life.
- We'll see how we can take on more risk (if we desire) and how that impacts our Capital allocation line.
- We'll see that the Sharpe Ratio is an excellent way to evaluate which risky asset/portfolio is best to add to a complete portfolio.

Is there an optimal portfolio choice?



Solving for the optimal allocation

- Mean-Variance maximizing Utility:

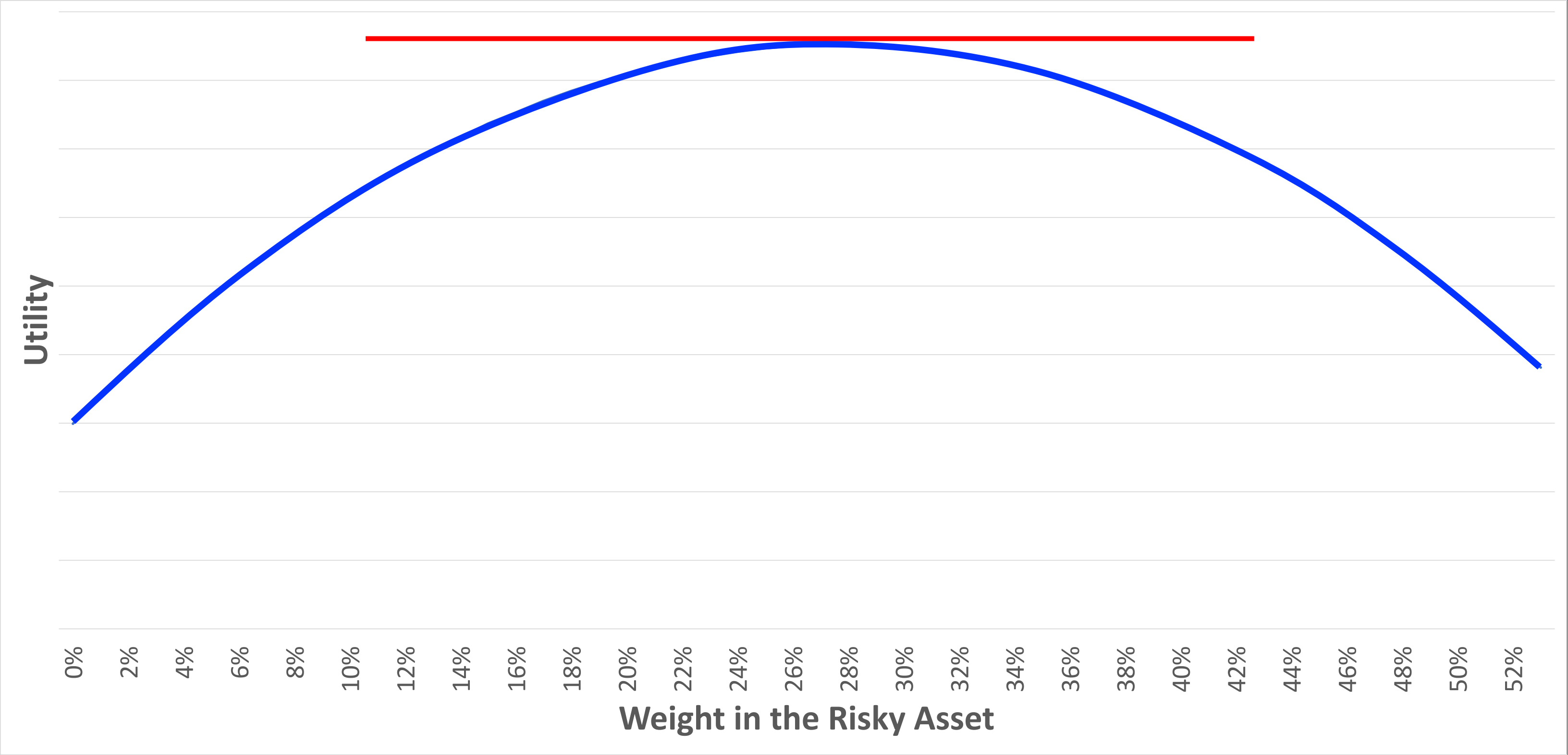
$$U = E[\tilde{r}] - \frac{1}{2}A\sigma^2$$

$$let \qquad E[\tilde{r}] = w_P E[\tilde{r}_P] + (1 - w_P)r_f$$
$$\sigma^2 = w_P^2 \sigma_P^2$$

$$\max U = w_P E[\tilde{r}_P] + (1 - w_P)r_f - \frac{1}{2}A w_P^2 \sigma_P^2$$

Use the First Order Conditions to find the maximum

Intuition of the First Order Condition



From the First Order Conditions

$$\max U = w_P E[\tilde{r}_P] + (1 - w_P)r_f - \frac{1}{2}Aw_P^2\sigma_P^2$$

$$\frac{\partial U}{\partial w_P} = E[\tilde{r}_P] - r_f - Aw_P\sigma_P^2 = 0$$

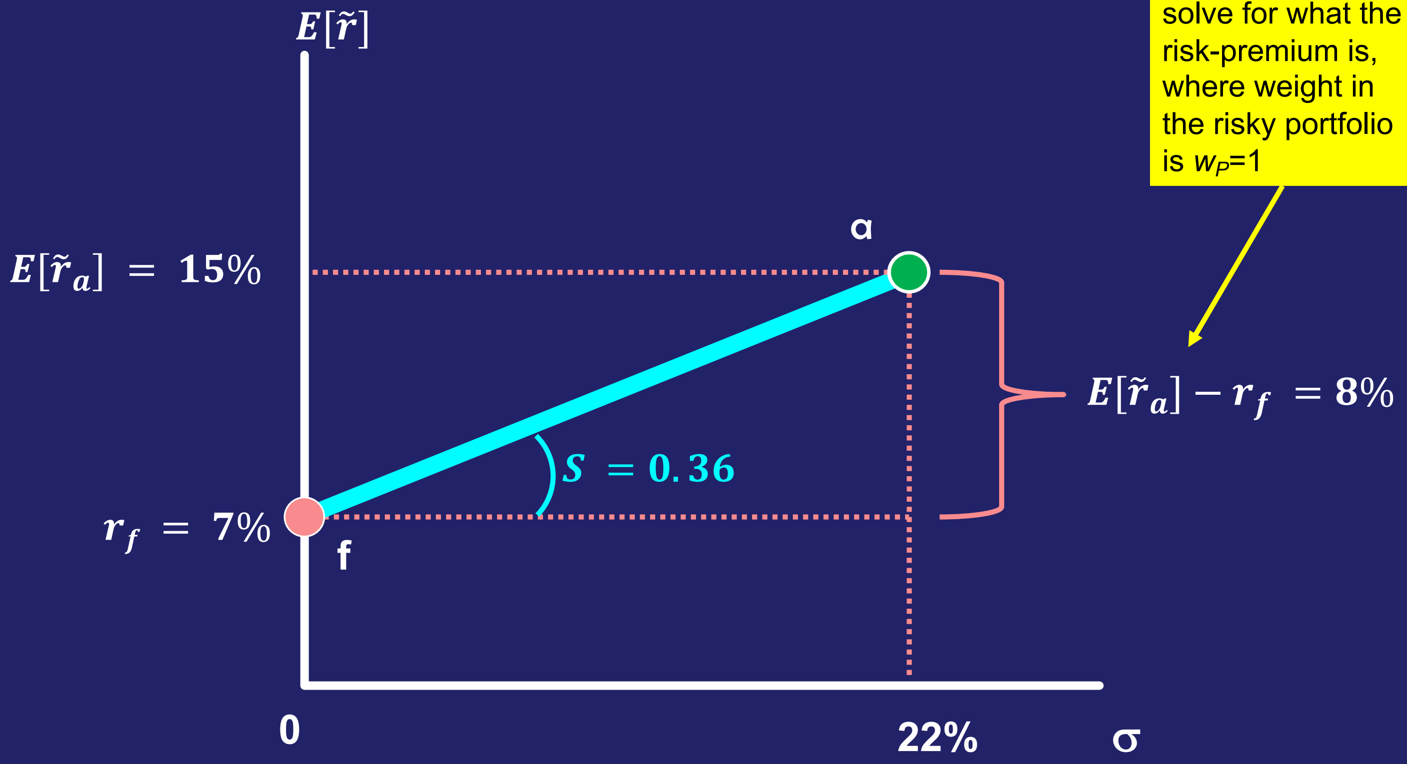
$$E[\tilde{r}_P] - r_f = Aw_P\sigma_P^2$$

$$w_P^* = \frac{E[\tilde{r}_P] - r_f}{A\sigma_P^2}$$

risk premium

$E[r] - r_f$

Now we can solve for what the risk-premium is, where weight in the risky portfolio is w_P=1



The Risk-Premium and Risk Aversion at Optimum

when put all
to risky asset

$$w_P^* = \frac{E[\tilde{r}_P] - r_f}{A\sigma_P^2} = 1$$

$$E[\tilde{r}_P] - r_f = A\sigma_P^2$$

Your textbook adds $^1/_2$ to make the math prettier.

$$E[\tilde{r}_P] - r_f = \frac{1}{2}A\sigma_P^2$$

Average Risk Aversion in the Market

- Since the representative investor ($\approx$ *Average Investor*) must hold the market portfolio

$$w_P^* = \frac{E[\tilde{r}_P] - r_f}{A\sigma_P^2}$$

The market

$$w_M^* = \frac{E[\tilde{r}_M] - r_f}{A\sigma_M^2} = 1$$

$$A = \frac{E[\tilde{r}_M] - r_f}{\sigma_M^2}$$

Different Preferences (*A*) with age: Lifecycle Capital Allocation

- Reducing the risk of your portfolio as you age.
  - Because you might not have time to recover from negative shocks.

more risk averse with age

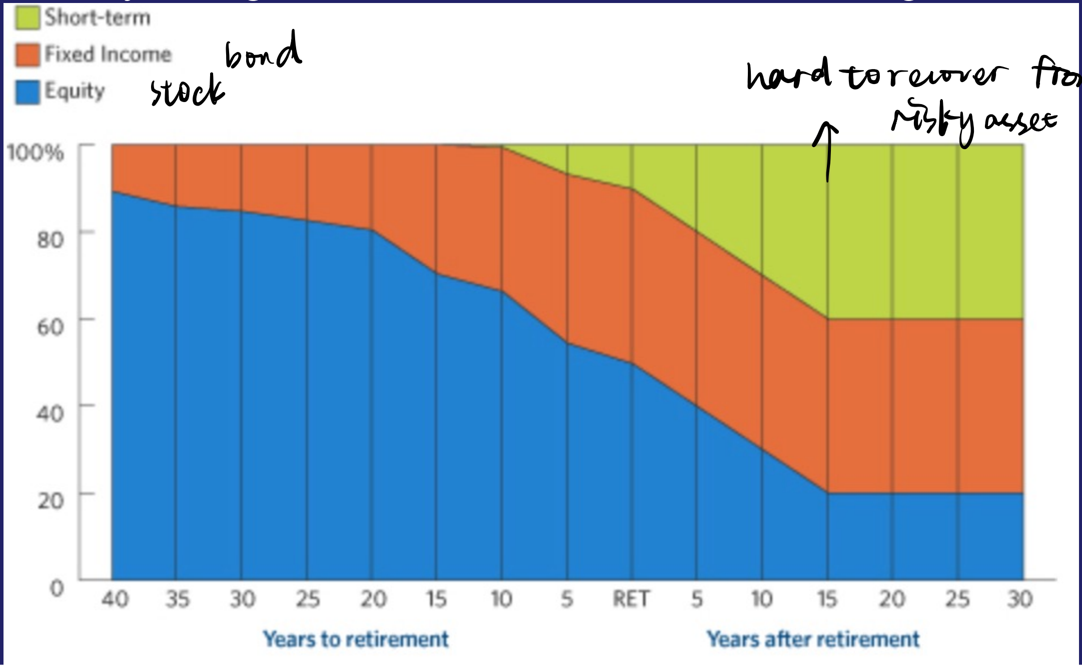
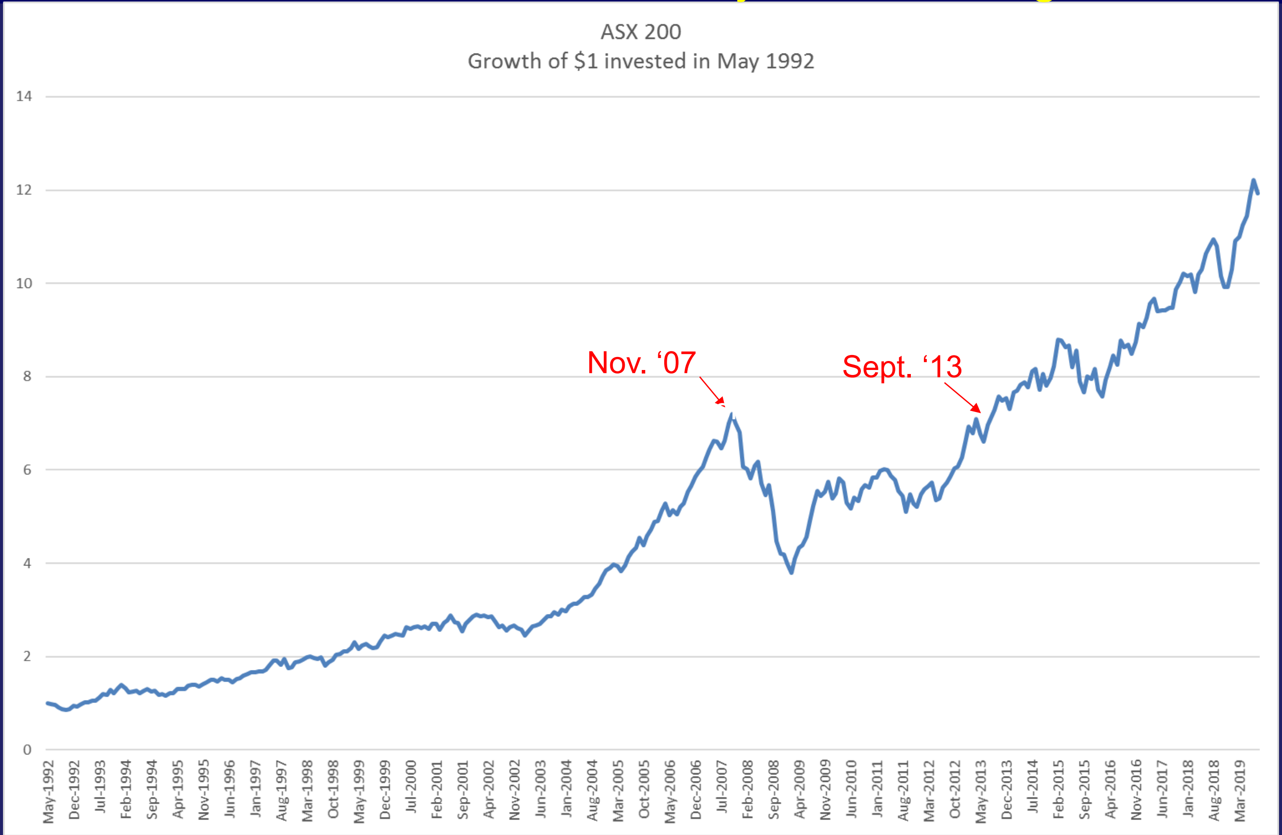


Illustration of the the benefits of life cycle investing

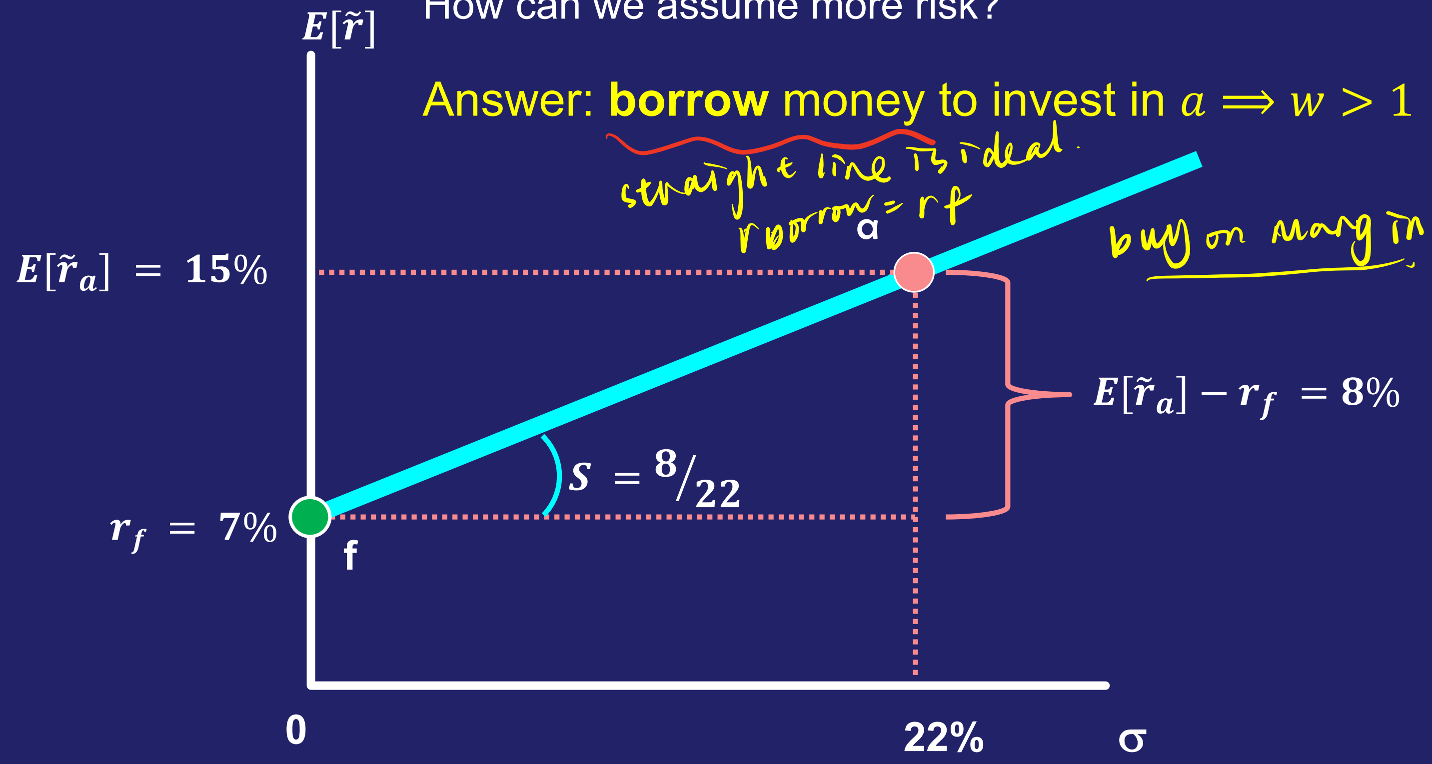


Extending the CAL

What if 15% Expected return and 22% risk is not enough?

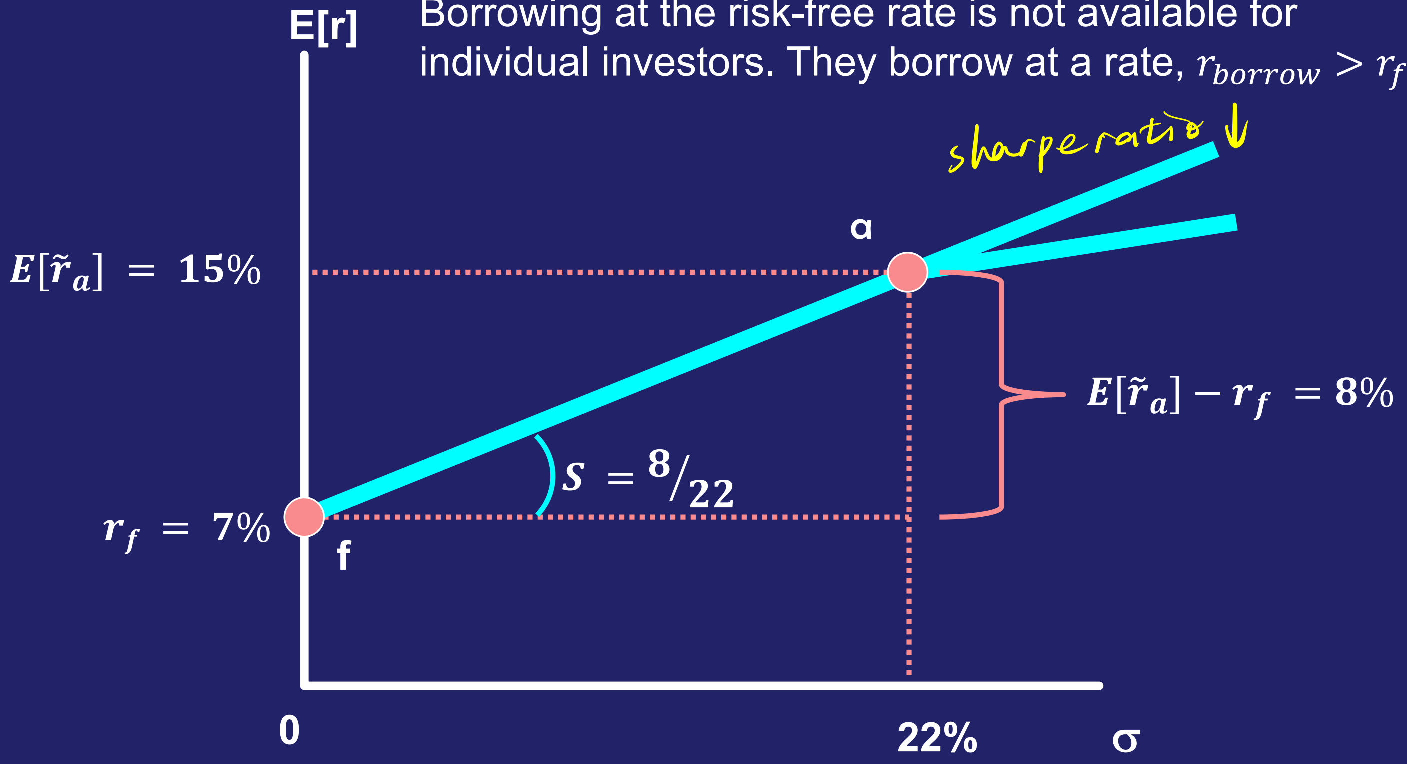
How can we assume more risk?

Answer: **borrow** money to invest in $a \Rightarrow w > 1$



Extending the CAL when $r_{borrow} > r_{lend}$

Borrowing at the risk-free rate is not available for individual investors. They borrow at a rate, $r_{borrow} > r_f$

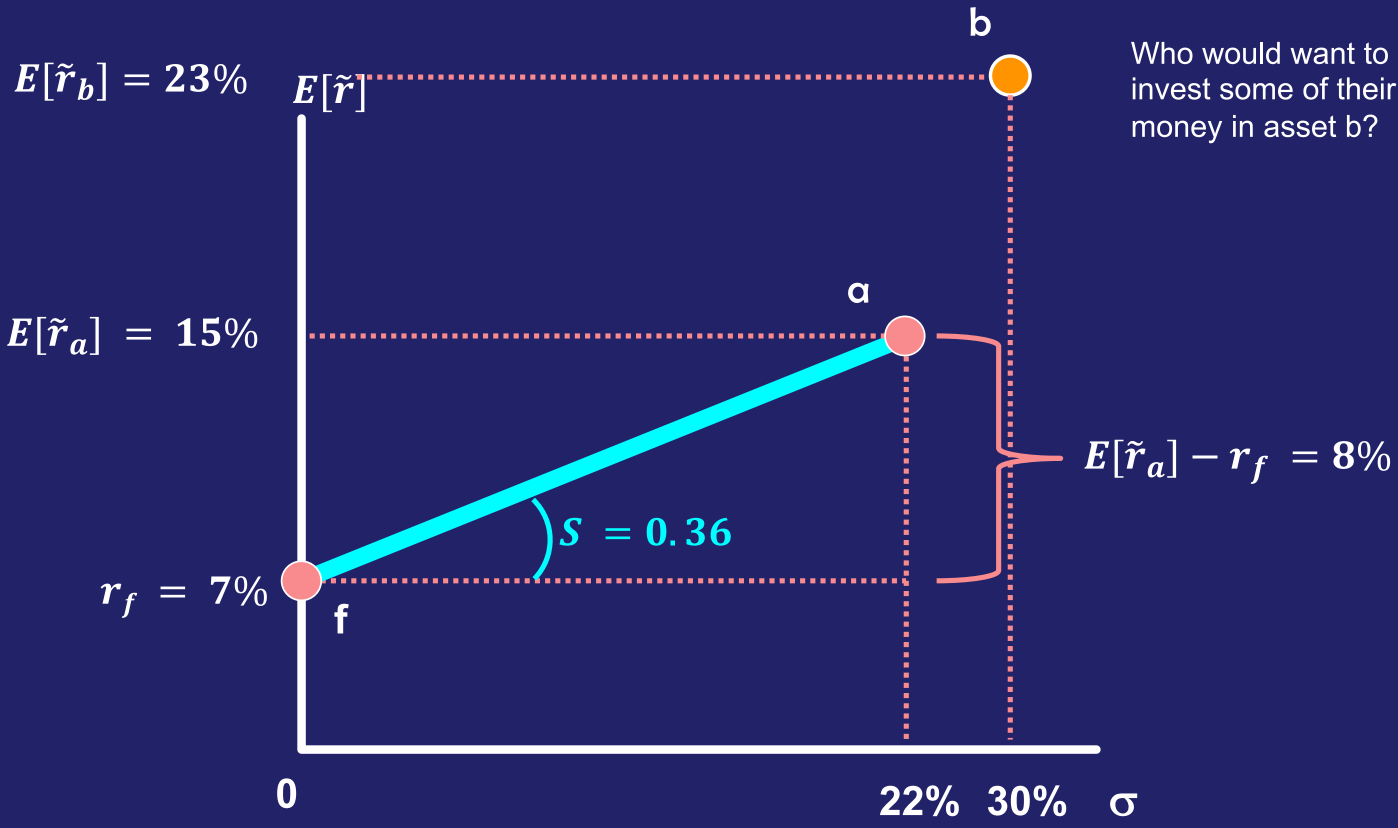


Consider this scenario

- Suppose you have $1 million to invest and two assets:
  - Asset a: E[r]=15% and Standard Deviation = 22%
  - Asset b: E[r]=23% and Standard Deviation = 30%

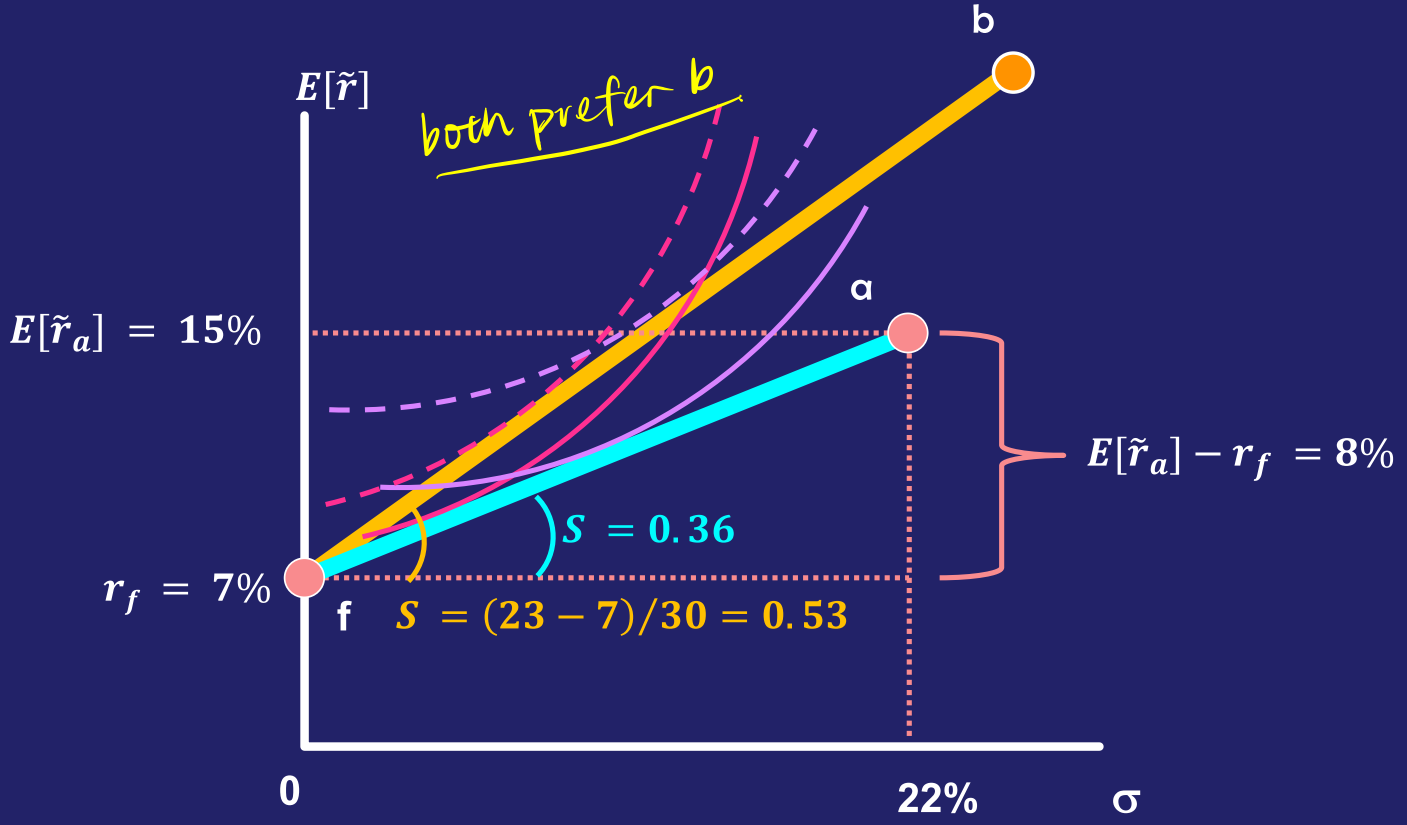
Suppose as a risky asset you can choose either a or b, but not both. Which is better? How does it depend on your risk preferences?

If we can only choose between 'a' and 'b', which is best?



Who would want to invest some of their money in asset b?

A Steeper CAL is always better



How Can We Raise the CAL?

$$S = \frac{E[\tilde{r}_P] - r_f}{\sigma_{\tilde{r}_P}} = \textit{Sharpe Ratio}$$

- Raise Return
  - If prices are correct, then the return will always be fair compensation for risk, and we cannot raise the return.
    - More on this when we get to asset pricing models.

- Lower risk

  ➔Diversify with Risky Assets

Portfolio Return and Variance

The plan

- Learn how to calculate returns of portfolios of assets

- Learn two common weighting schemes for weighting assets in a portfolio
  - Value weighted
  - Equally weighted

- Learn to calculate variance of portfolios of risky assets
  - We will see that how stock returns are correlated affects the portfolio variance (standard deviation).

Measuring Returns on Portfolio of Many Risky Assets

- Actual return on a portfolio is the weighted average of returns on *N* component securities:

$$r_p = \sum_{n=1}^{N} w_n r_n \xrightarrow{if\ N=2} w_A r_A + w_B r_B$$

- The ***expected*** return is also a weighted average

$$E[\tilde{r}_p] = \sum_{n=1}^{N} w_n E[\tilde{r}_n] \xrightarrow{if\ N=2} w_A E[\tilde{r}_A] + w_B E[\tilde{r}_B]$$

- Not any different from calculating the portfolio return of a risky asset and a risk free asset like with the CAL

Technical Aside: Returns for Portfolios of Many Assets

$$E[\tilde{r}_p] = \sum_{n=1}^{N} w_n E[\tilde{r}_n]$$

- Where w_n is the percentage of your investment in asset *n* and $\sum_{n=1}^{N} w_n = 1$

- Common Examples of weights:
  - ① Equally weighted portfolio: the same fraction of investment in all assets

$$w_n = \frac{1}{N}$$

  - ② Value weighted portfolio:

$$w_n = \frac{MV_n}{\sum_{n=1}^{N} MV_n}$$

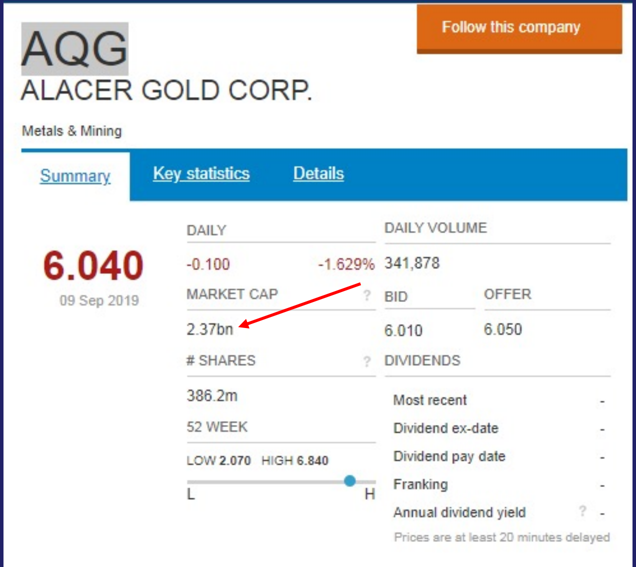
where MV_n is the market value of asset *n*. → # of shares outstanding × price of stock

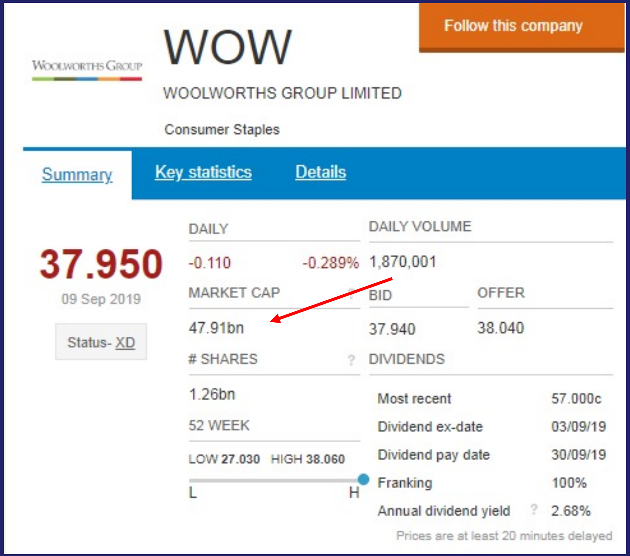
MV at start or at end?

start

- Not as simple for portfolio variance!

Example: Value and Equally weighted returns





https://www.asx.com.au/prices/company-information.htm

Technical Aside: example – equal weighting

$$E[\tilde{r}_p] = \sum_{n=1}^{N} w_n E[\tilde{r}_n], where\ w_n = \frac{1}{N}$$

- Two Stocks:

$$E[\tilde{r}_{WOW}] = 4\%$$
$$E[\tilde{r}_{AQG}] = 8\%$$

Equally weighted portfolio return

$$E[\tilde{r}_p] = \sum_{n=1}^{N} w_n E[\tilde{r}_n] = w_{WOW}E[\tilde{r}_{WOW}] + w_{AQG}E[\tilde{r}_{AQG}]$$

$$E[\tilde{r}_p] = \frac{1}{2}(4\%) + \frac{1}{2}(8\%) = 6\%$$

Technical Aside: example – value weighting

$$E[\tilde{r}_p] = \sum_{n=1}^{N} w_n E[\tilde{r}_n], where\ w_n = \frac{MV_n}{\sum_{n=1}^{N} MV_n}$$

- Two Stocks:

$$E[\tilde{r}_{WOW}] = 4\%\ and\ Market\ Capitalization = \$47.9\ billion$$
$$E[\tilde{r}_{AQG}] = 8\%\ and\ Market\ Capitalization = \$2.4\ billion$$

Value weighted portfolio return

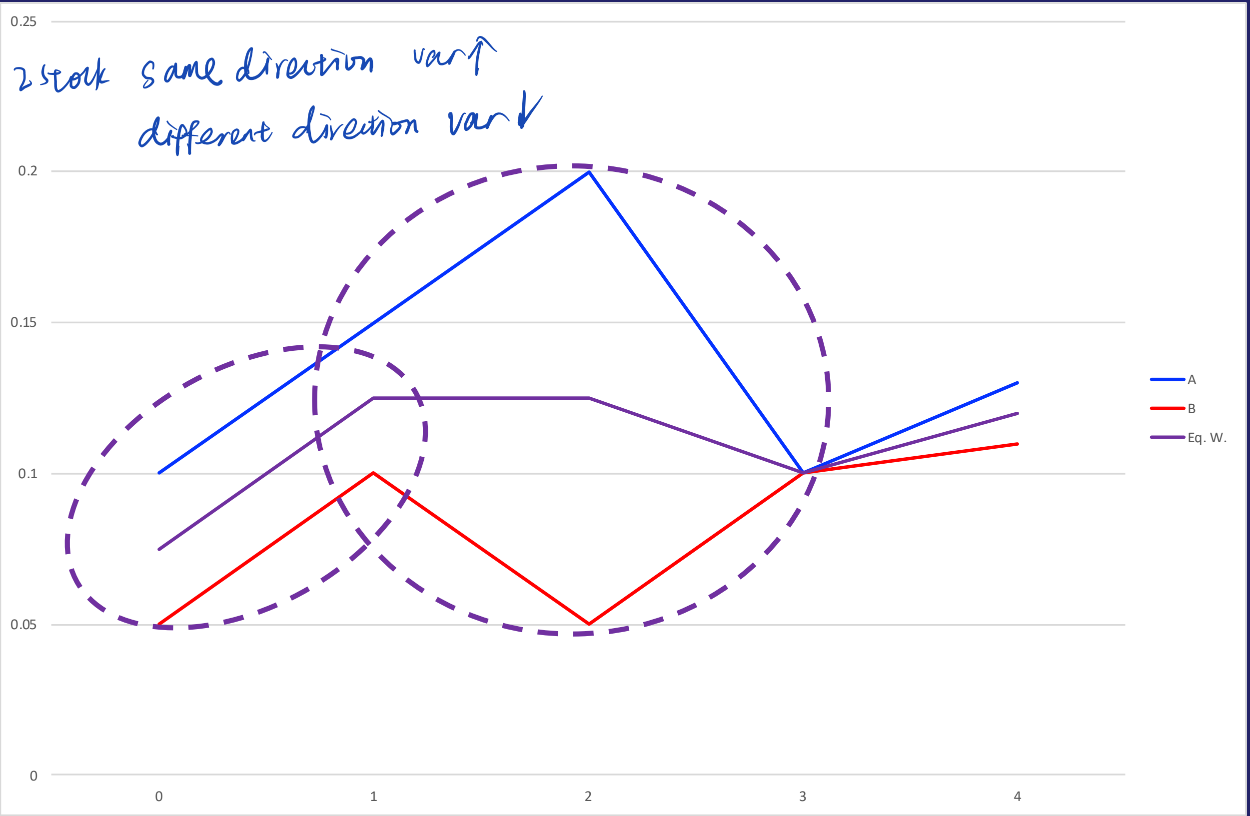
$$E[\tilde{r}_p] = \sum_{n=1}^{N} w_n E[\tilde{r}_n] = w_{WOW} E[\tilde{r}_{WOW}] + w_{AQG} E[\tilde{r}_{AQG}]$$

$$E[\tilde{r}_p] = \frac{47.9}{2.4 + 47.9}(4\%) + \frac{2.4}{2.4 + 47.9}(8\%) = 4.2\%$$

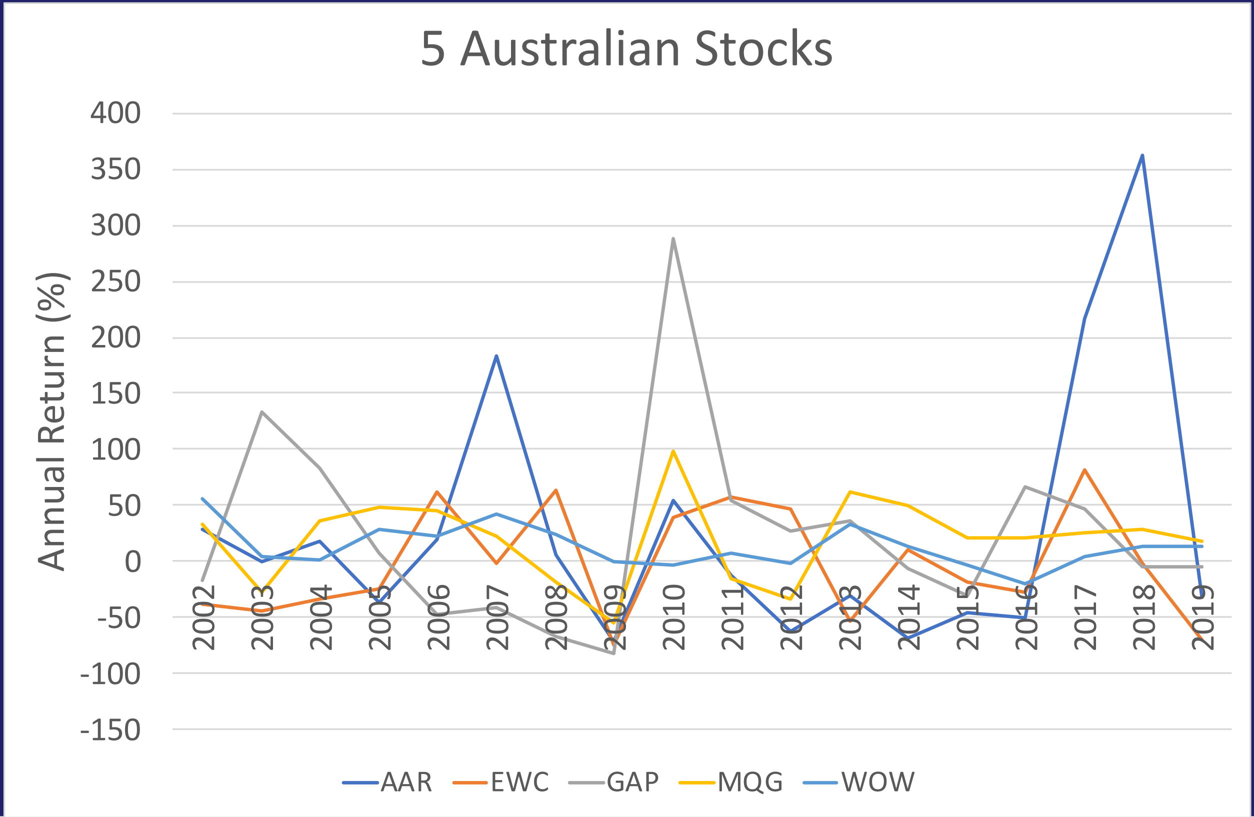
Portfolio Variance – The data underlying next slide

T	A	B	Equally Weighted Return
0	0.10	0.05	0.075
1	0.15	0.10	0.125
2	0.20	0.05	0.125
3	0.10	0.10	0.100
4	0.13	0.11	0.120

Portfolio Variance

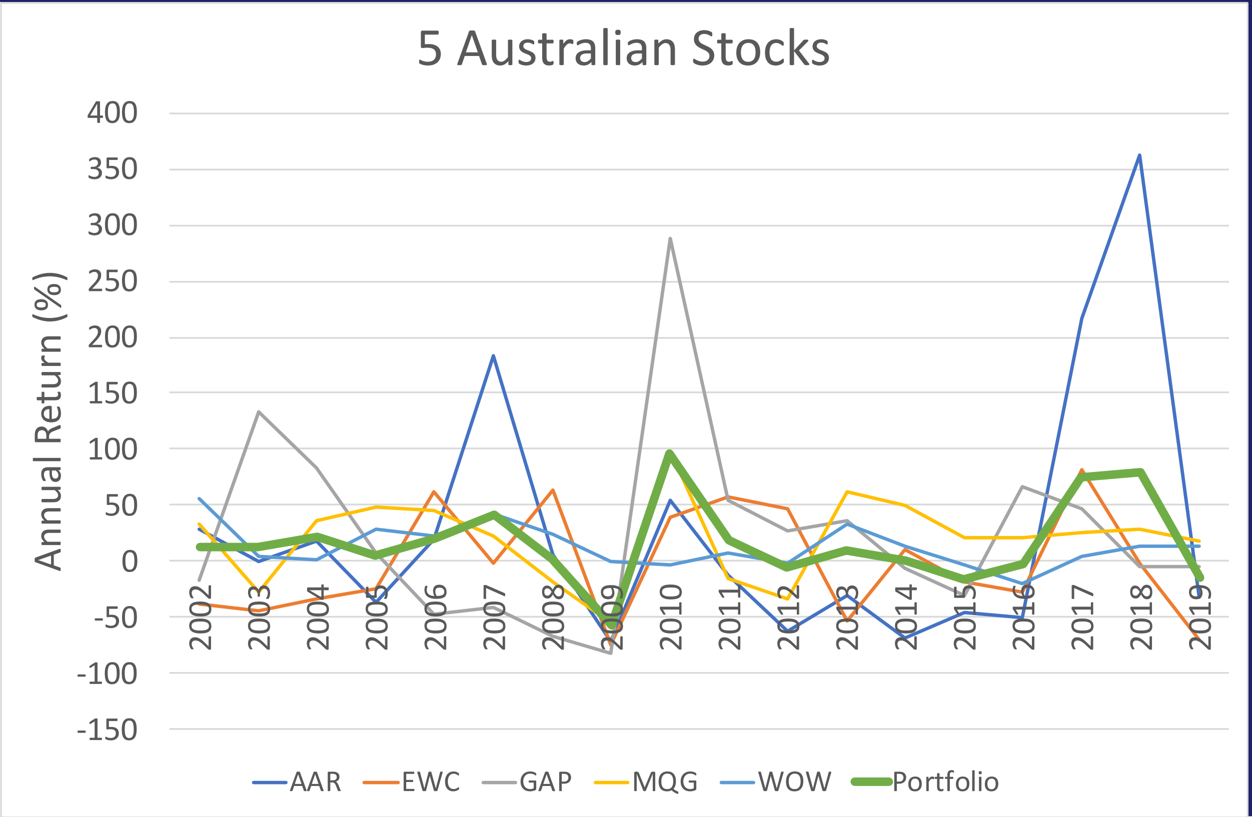


The Difference with Variance



Source: finance.yahoo.com

The Difference with Variance



Portfolios and Risk

- The variance of a group has to take into account how that group moves together

- A 2-asset portfolio, assets A and B:

$$\sigma_p^2 = w_A^2\ \sigma_A^2 + w_B^2\ \sigma_B^2 + 2w_A w_B \sigma_{AB}$$

-or-

$$\sigma_p^2 = w_A^2\ \sigma_A^2 + w_B^2\ \sigma_B^2 + 2w_A w_B \sigma_A \sigma_B \rho_{AB}$$

Technical Aside: How to Calculate Covariance

- Historic Covariance:

$$\sigma_{AB} = \frac{1}{T-1}\sum_{t=1}^{T}(r_{A,t} - \bar{r}_A)(r_{B,t} - \bar{r}_B)$$

$r_{AB}\uparrow$ A & B deviate at the same direction

- Scenario Covariance:

$r_{AB}\downarrow$ A & B deviate in different directions

$$\sigma_{AB} = \sum_{s=1}^{\mathbb{S}} p(s)(r_{A,s} - E[\tilde{r}_A])(r_{B,s} - E[\tilde{r}_B])$$

Example: Calculating Scenario Covariance

$$\sigma_{AB} = \sum_{s=1}^{\mathbb{S}} p(s)\left(r_{A,s} - E[\tilde{r}_A]\right)\left(r_{B,s} - E[\tilde{r}_B]\right)$$

	Pr	Stock A	Stock B
Boom	.20	.30	.08
Normal	.65	.08	.07
Bust	.15	-.05	.05
E[r]		**.1045**	**.069**

Example: Calculating Scenario Covariance

	Pr	Stock A	Stock B
Boom	.20	.30	.08
Normal	.65	.08	.07
Bust	.15	-.05	.05
E[r]		**.1045**	**.069**

- $\sigma_{AB} = \sum_{s=1}^{\mathbb{S}} p(s)\left(r_{A,s} - E[\tilde{r}_A]\right)\left(r_{B,s} - E[\tilde{r}_B]\right)$
- $\sigma_{AB} = .20(.30 - .1045)(.08 - .069) + .65(.08 - .1045)(.07 - .069) + .15(-.05 - .1045)(.05 - .069)$
- $\sigma_{AB} = .20(.1955)(.011) + .65(-.0245)(.001) + .15(-.1545)(-.019)$
- $\sigma_{AB} = .0004301 - .00001592 + .00044032$
- $\sigma_{AB} = .0008545$

Covariance and Correlation

- Covariance has inconvenient units (like variance) -- Correlation removes all units:

$$Corr(\tilde{r}_A, \tilde{r}_B) = \frac{Cov(\tilde{r}_A, \tilde{r}_B)}{StDev(\tilde{r}_A)StDev(\tilde{r}_B)}$$

$$\rho_{AB} = \frac{\sigma_{AB}}{\sigma_A \sigma_B}$$

perfect negative correlation

perfect positive correlative

- Correlation is bounded: $-1 \leq \rho_{AB} \leq 1$

Covariance/Correlation Implications for the Choice of Assets

The point

- To maximize the benefits of diversification you want assets with low correlation.
  - But with low transaction costs, anything with less-than-perfect correlation is good enough to create reductions in risk.

- For any number of risky assets plus a risk-free bond, there will be only one, combination of these risky assets that is optimal.
  - We will show this with 2 assets and move to many assets later.

The KEY to Understanding Risk

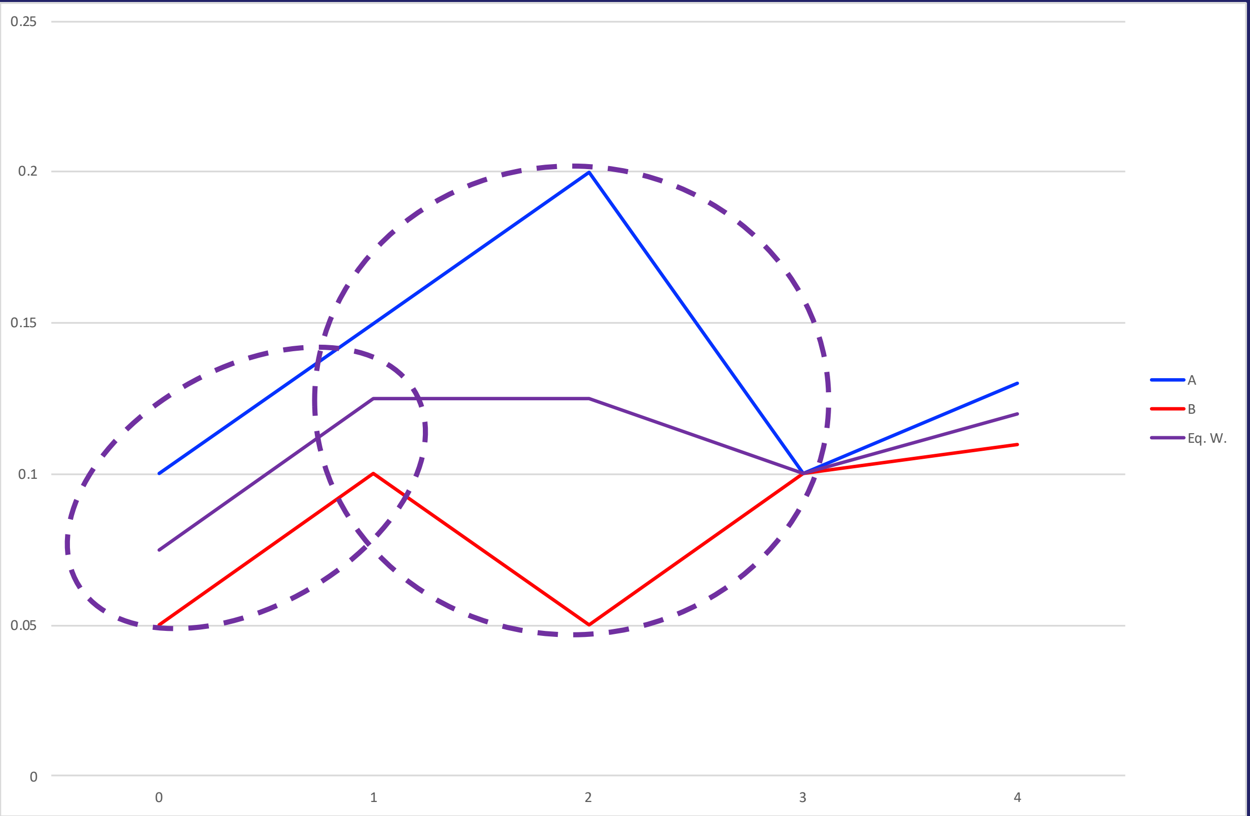
- Variance of portfolio of TWO securities:

$$\sigma_p^2 = w_A^2\ \sigma_A^2 + w_B^2\ \sigma_B^2 + 2w_A w_B \sigma_A \sigma_B \rho_{AB}$$

- As correlation *decreases*, so does ***portfolio's*** variance.
  – Why? As members move "out of synch", their fluctuations tend to cancel each other out more often.

- How will this affect portfolio selection?

Portfolio Variance



Portfolio Example

- Consider an example where we can invest into risky assets (stocks, funds) 1 and 2.

- Asset 1: $E[\tilde{r}_1] = 10\%$ $\sigma_1 = 12\%$
- Asset 2: $E[\tilde{r}_2] = 17\%$ $\sigma_2 = 25\%$

- What is the expected portfolio return and standard deviation?

Benefits from Diversification

Asset 1: $E[\tilde{r}_1]$ = 10% σ_1 = 12%
Asset 2: $E[\tilde{r}_2]$ = 17% σ_2 = 25%

$$E[\tilde{r}_p] = w_1 E[\tilde{r}_1] + w_2 E[\tilde{r}_2]$$

$$\sigma_p = \sqrt{w_1^2\ \sigma_1^2 + w_2^2\ \sigma_2^2 + 2w_1 w_2 \sigma_1 \sigma_2 \rho_{1,2}}$$

		Portfolio Standard Deviation (%) for Given Correlation				
Weight in 1	E(r) portf.	$\rho_{1,2}$ = - 1	$\rho_{1,2}$ = 0	$\rho_{1,2}$ = 0.2	$\rho_{1,2}$ = 0.5	$\rho_{1,2}$ = 1
0	17.0					
0.2	15.6					
0.4	14.2					
0.6	12.8					
0.8	11.4					
1	10.0					

Even though the expected return is the same

The lower the correlation the lower the portfolio variance

Benefits from Diversification

Asset 1: $E[\tilde{r}_1]$ = 10% σ_1 = 12%
Asset 2: $E[\tilde{r}_2]$ = 17% σ_2 = 25%

$$E[\tilde{r}_p] = w_1 E[\tilde{r}_1] + w_2 E[\tilde{r}_2]$$

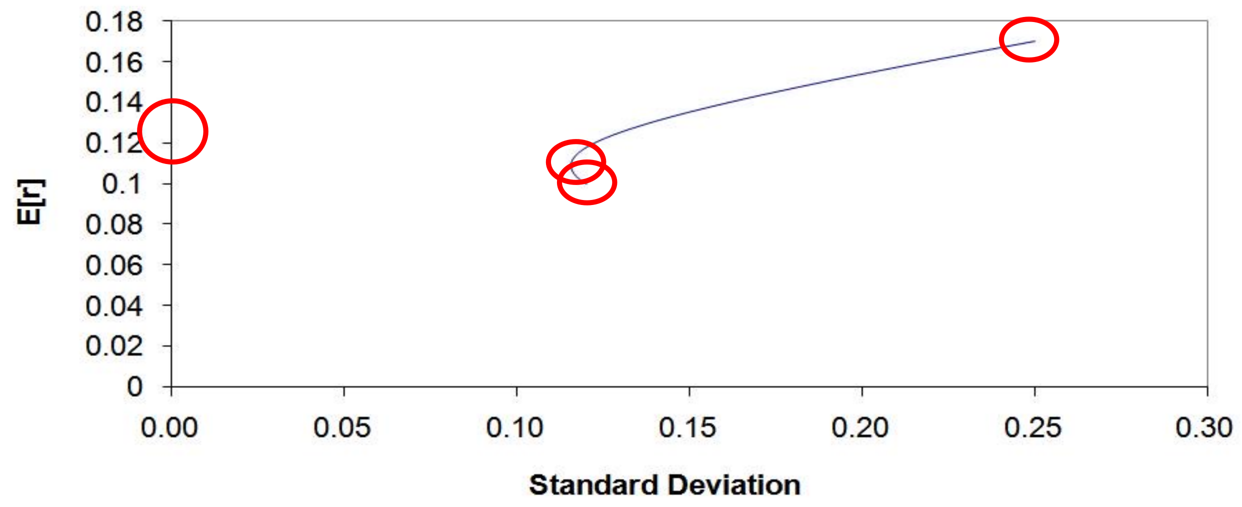
$$\sigma_p = \sqrt{w_1^2\,\sigma_1^2 + w_2^2\,\sigma_2^2 + 2w_1 w_2 \sigma_1 \sigma_2 \rho_{1,2}}$$

		Portfolio Standard Deviation (%) for Given Correlation				
Weight in 1	E(r) portf.	$\rho_{1,2} = -1$	$\rho_{1,2} = 0$	$\rho_{1,2} = 0.2$	$\rho_{1,2} = 0.5$	$\rho_{1,2} = 1$
0	17.0	25.0	25.0	25.0	25.0	25.0
0.2	15.6	17.6	20.1	20.6	21.3	22.4
0.4	14.2	10.2	15.7	16.6	17.9	19.8
0.6	12.8	2.8	12.3	13.4	15.0	17.2
0.8	11.4	4.6	10.8	11.7	12.9	14.6
1	10.0	12.0	12.0	12.0	12.0	12.0

$\sigma \downarrow$

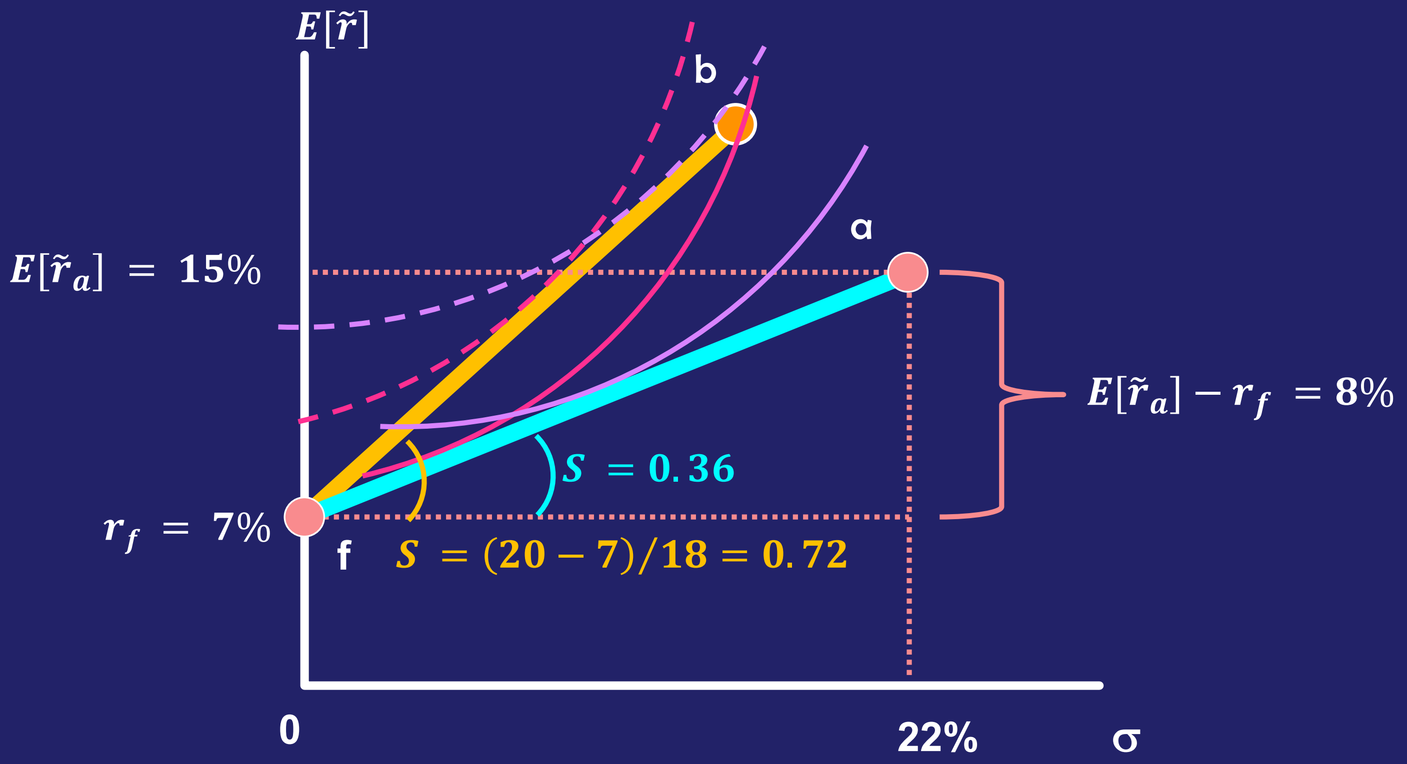
How does risk reduction depend on ρ?

Asset 1: $E(r_1) = 10\%$ $\sigma_1 = 12\%$
Asset 2: $E(r_2) = 17\%$ $\sigma_2 = 25\%$



corr= 0.2

Trying to find a steeper CAL: It's always better



Adding the Risk-Free Rate and Making CAL Steeper

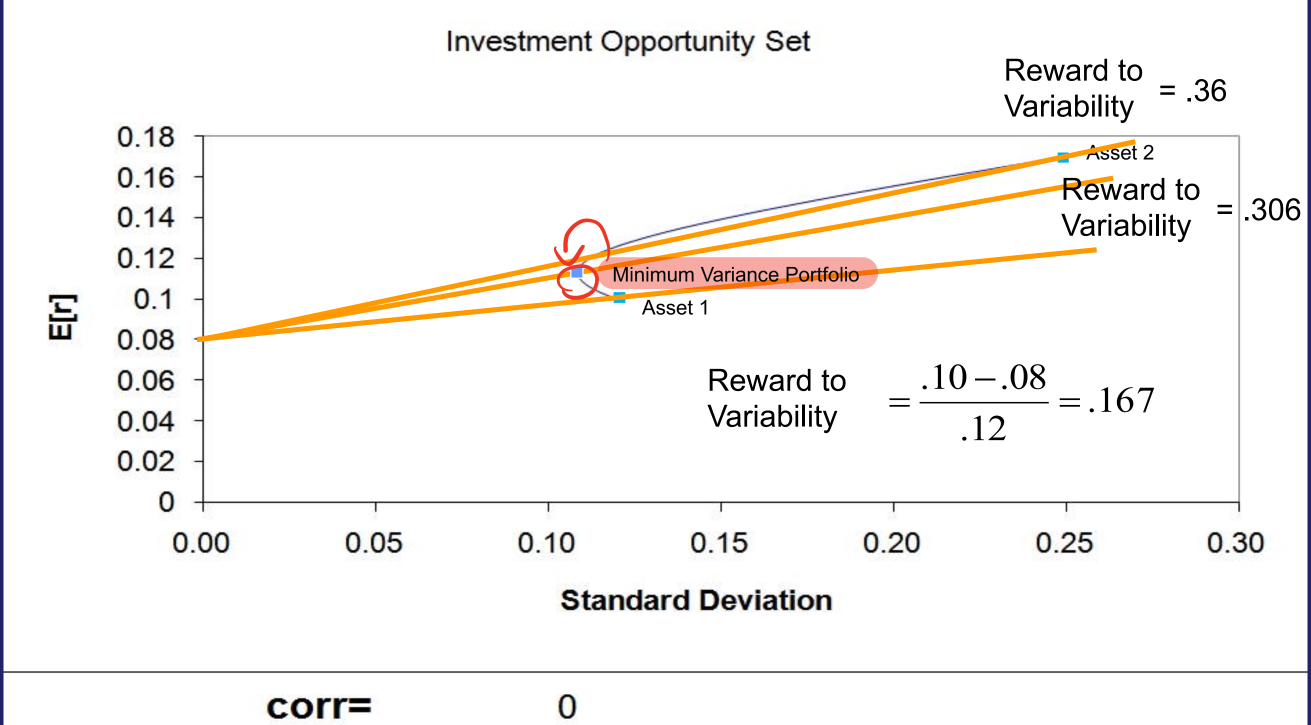
- We now know how to choose the risky assets that go into our portfolios
- Return to our initial problem of having one risky portfolio and one risk-less asset

- Our earlier example with two risky assets:
  - A: $E[\tilde{r}_1] = 10\%$ $\sigma_1 = 12\%$
  - B: $E[\tilde{r}_2] = 17\%$ $\sigma_2 = 25\%$
  - $\rho_{1,2} = 0$

- Add T-bill which returns $r_f = 8\%$
- Plot possible CALs and compare reward-to-variability ratios

Possible CALs

Asset 1: $E(r_1) = 10\%$ $\sigma_1 = 12\%$
Asset 2: $E(r_2) = 17\%$ $\sigma_2 = 25\%$



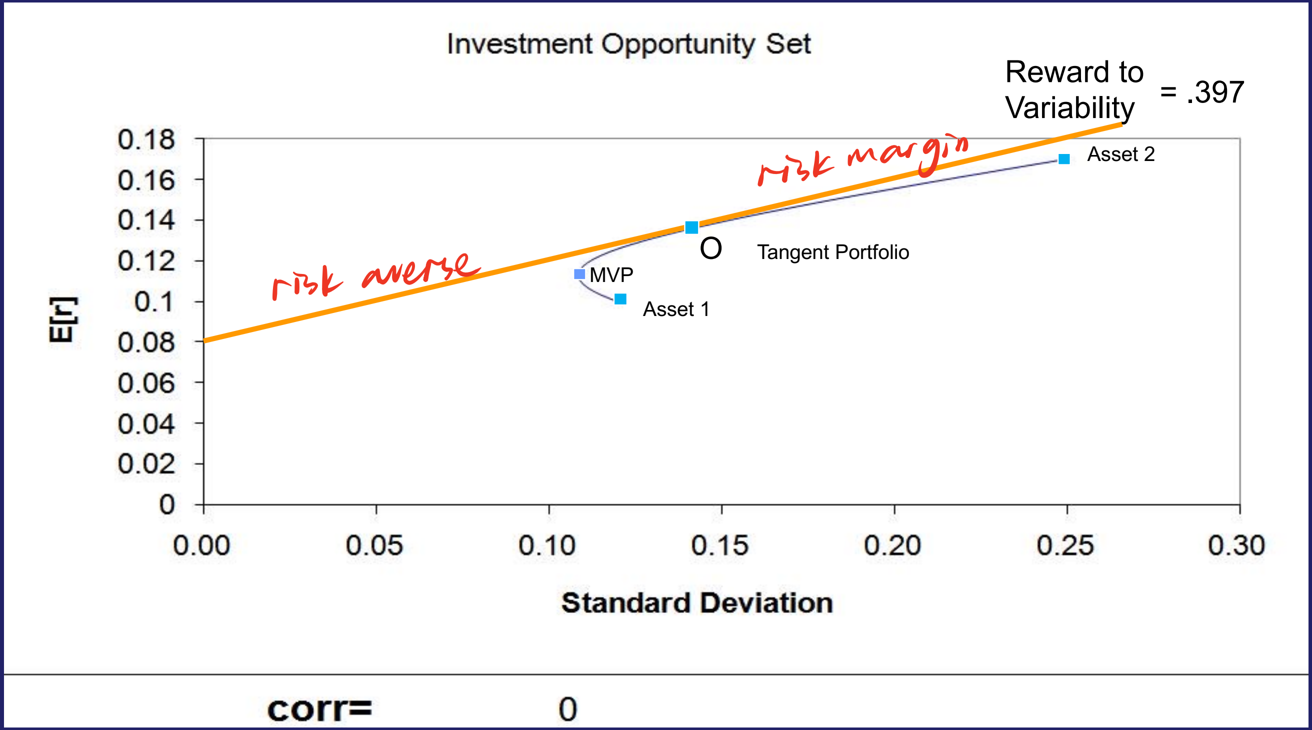
Maximum Reward-to-Variability

- CAL has to intersect with the investment opportunity set, but only just!

- Tangent point, where CAL touches the investment opportunity set will be optimal
  - It gives us the highest reward to variability ratio

Possible CALs

Asset 1: $E(r_1) = 10\%$ $\sigma_1 = 12\%$
Asset 2: $E(r_2) = 17\%$ $\sigma_2 = 25\%$



Which Portfolio is Best?

- It **doesn't** depend on the level of risk aversion (as long investors are mean-variance maximizers).
- Only one portfolio is best!!!
- Which one?
- The Tangent Portfolio
  - It makes for the steepest CAL with the highest Sharpe Ratio

Optimal CAL (con't)

- The slope of the optimal CAL is:

$$S = \frac{E[\tilde{r}_P] - r_f}{\sigma_{\tilde{r}_P}} \qquad S = \frac{0.1356 - 0.08}{0.1402} = 0.397$$

- For each additional unit of risk, we obtain 0.397% additional expected return

- The equation for the optimal CAL is:

$$E[\tilde{r}_P] = .08 + 0.397\sigma_{\tilde{r}_P}$$

Question for you:

- Suppose you work for an investment company and your boss gives you $1 million to invest in:
  - A risk-free savings account
  - Any stock or stocks listed on the ASX

- Alternatives:
  - Invest in one stock?
    - Which ones?
  - Invest in a few of them?
    - Which ones? What weights?
  - Invest in all of them?
    - What weights?

Diversification with Many Assets

Markowitz (1952) Portfolio Theory

The point

- If diversifying with 2 assets is good, well, diversifying with many (ALL!) is even better!
  - The more assets you add the more firm-specific risk cancels out.

care about portfolio risk

- The key insight of Markowitz (1952) Portfolio Theory: Only portfolio risk is important to investors, because firm-specific risk can be completely eliminated through diversification.

- Tobin's Separation Property follows from this. All investing collapses to 2 decisions:
  - Figure out what the optimal portfolio is
  - Decide how much you want in the risk-free asset to balance risk.

Raising the CAL even Further

- What if we add more assets? What happens to the ***Investment Opportunity Set***?

- Instead of looking at all possible weights in different stocks (*the Investment Opportunity Set*),

- let's only look at **Efficient Portfolios** of Stocks – those that minimize portfolio variance for a given return.

lowest risk given level of return

Which Portfolios are Efficient?

- We are looking for the lowest variance portfolio for a given return

$$MIN\ \sigma_p^2 = \sum_{i=1}^{N} w_i^2 \sigma_i^2 + \sum_{i=1}^{N} \sum_{\substack{j=1 \\ j \neq i}}^{N} w_i w_j \sigma_{i,j}$$

Subject to:

constraint ①

$$E[\tilde{r}_p] = a\ given\ return$$

and

constraint ②

$$\sum_{i=1}^{N} w_i = 1$$

assumption: (negative weight)
no constraint on short sale.
no constraint on buy on margin

Which Portfolios are Efficient? (not on any exam)

- We are looking for the lowest variance portfolio for a given return

$$MIN\ \sigma_p^2 = \vec{w}'\vec{\Omega}\vec{w}$$

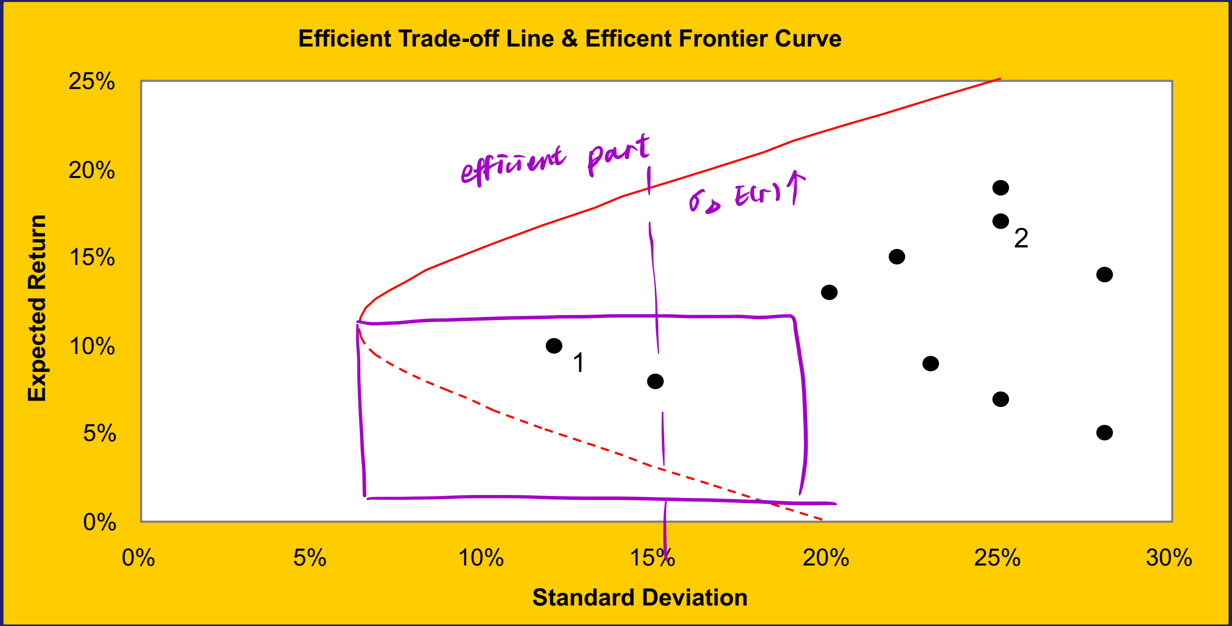
s.t.

$$\vec{w}'\vec{r} = \bar{r}$$

and

$$\vec{w}'\vec{\iota} = 1$$

What Happens When We Add More Assets?



Why is the efficient frontier shifting out?

- Through diversification firm-specific risk cancels out
- This lowers the risk of the portfolio of risky assets.

Diversification and Portfolio Variance: Simulation

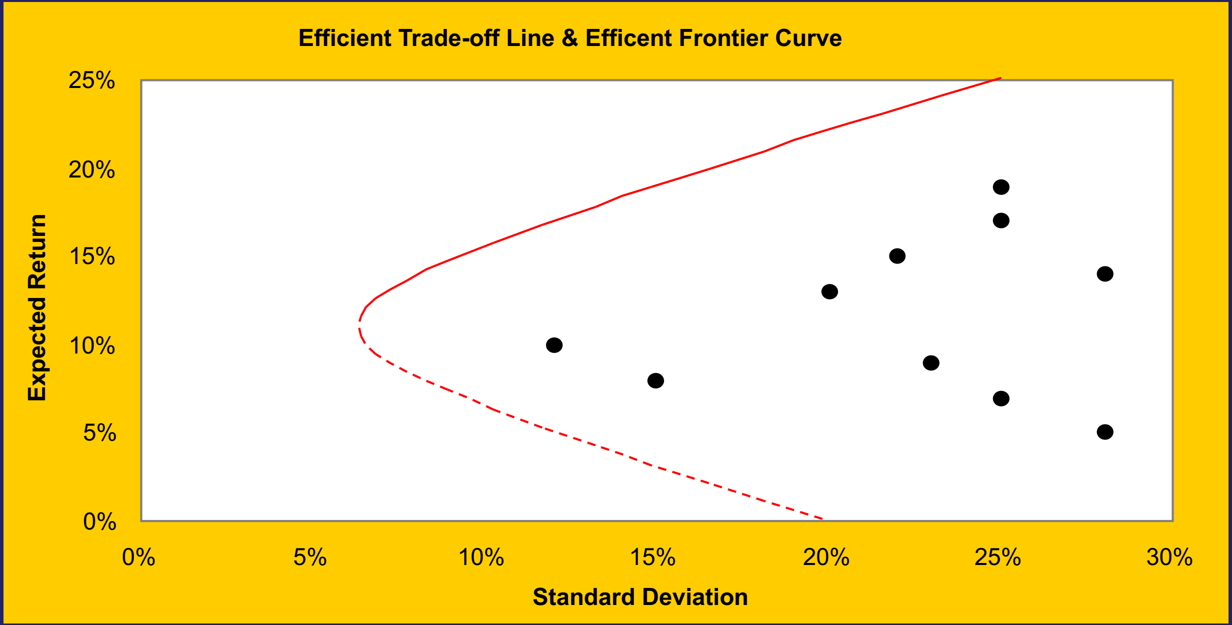
Avg. Std. Dev.	30%
Avg. Correlation	0.2

firm risk.

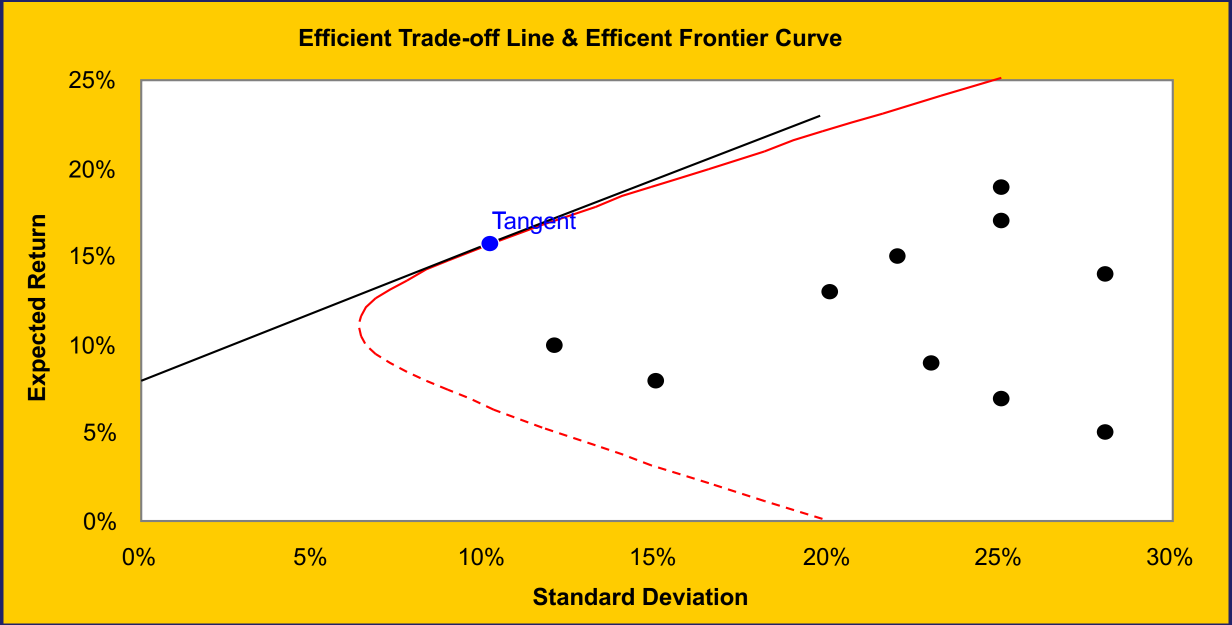
# of Assets	$\boxed{\text{Portfolio Std. Dev.}}$	Due to Variances	Due to Covariances
2	23.24%	83.33%	16.67%
3	20.49%	71.43%	28.57%
4	18.97%	62.50%	37.50%
100	13.68%	4.81%	95.19%
1000	13.44%	0.50%	99.50%
10000	13.42%	0.05%	99.95%

the more
risk ↓.

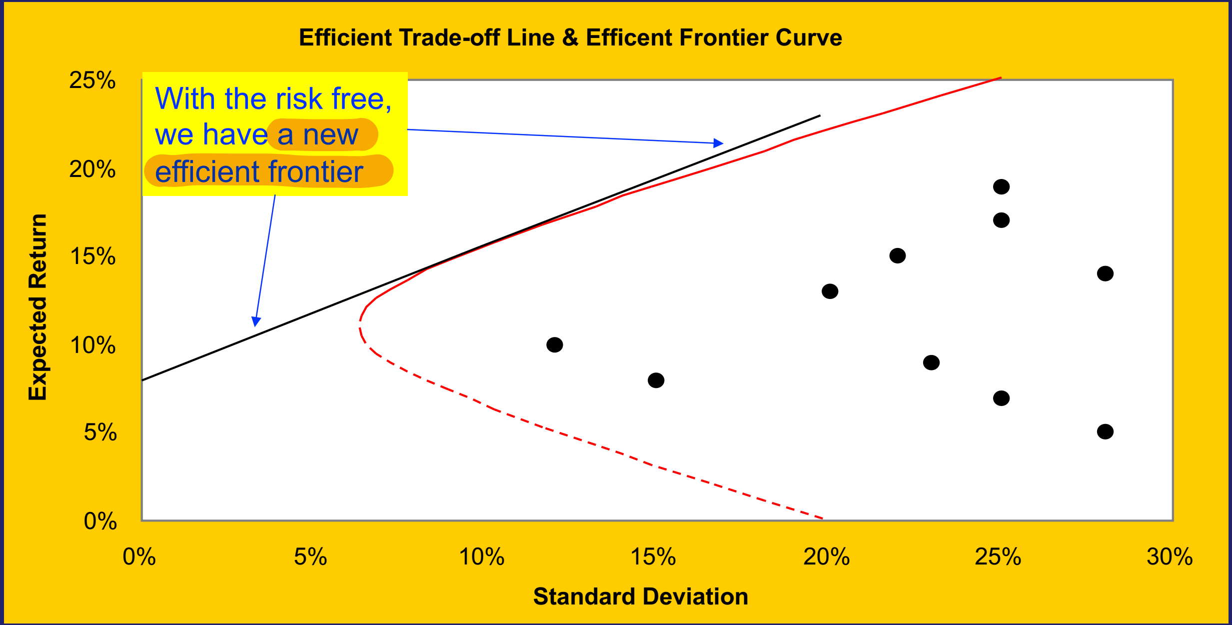
What if we add in the risk-free rate?



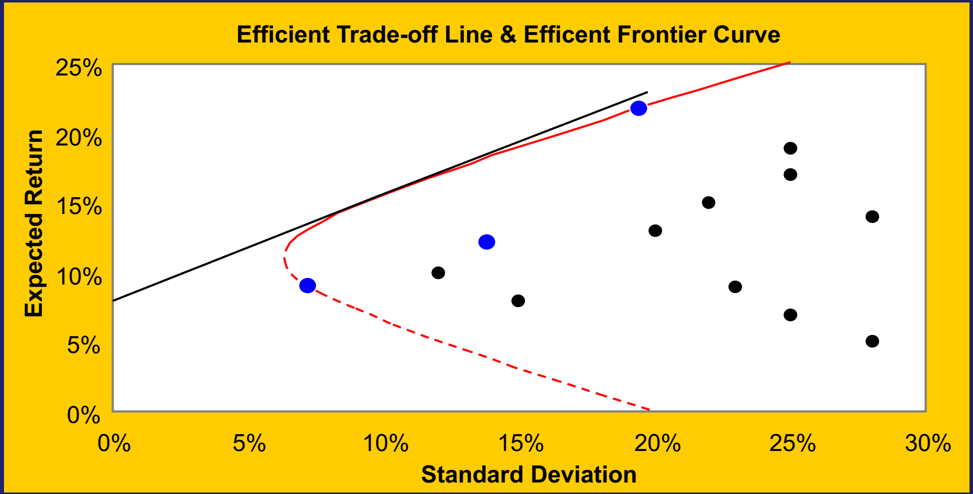
A new efficient frontier



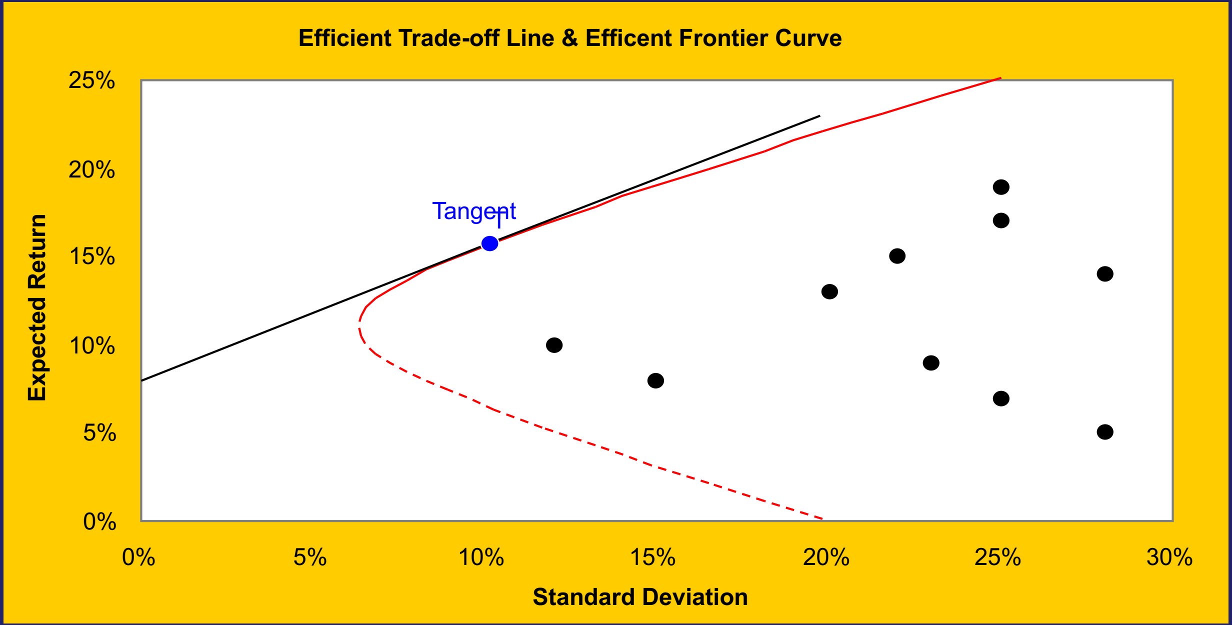
What if we add in the risk-free rate?



Which portfolio would any rational investor choose?



There is only one best risky portfolio



Optimal Decision Rule

If everyone prefers more return to more risk and
everyone sees the same assets, and
believes the same about returns, variance and covariance,
Then there is only one best risky portfolio.

Meaning:

- Optimal Portfolio Selection takes 2 steps:
  - 1. Choose Optimal ***Risky*** Portfolio
  - 2. Optimal ***Allocation*** between Risky and Riskless
- This is called the **Separation Property**
  - – Tobin (1958)

Implication of the Separation Property

Notice this means:

- All rational risk-averse investors
  - passively index holdings to some risky fund and
    - 75% of all institutional funds are passively indexed
  - account for risk aversion by keeping some money totally safe in the risk free

Asset Allocation and Portfolio Optimization: Example

Example with real data

- https://www.portfoliovisualizer.com/efficient-frontier

- Thank you Silicon Cloud Technologies LLC !!

- What do we need for portfolio optimization?
  1. Some risky assets
  2. Their expected returns, $E[\tilde{r}]$
  3. Their variance, σ^2
  4. Correlations or covariances among assets

① why they use geometric mean ⇒ pull average down ⇒ more conservative
geometric < arithmetic

Asset Allocation In Practice

The point

- In practice many investment managers diversify across asset classes, that is, across portfolios of stock grouped by some common characteristic.

- Ultimately, there are limits to the benefits of diversification. You cannot get rid of all risk.
  - What remains is systematic risk.

A top-down approach

- Many investing institutions diversify using a top-down approach
  1. Asset Allocation: how much should be invested in different asset classes
     - Foreign/domestic
     - Sectors (Natural Resources, Health, Manufacturing, etc.)

  2. Security Selection
     - The choice of individual stocks or bonds within each asset class
       – We will touch on a simple version of security selection in the next topic.

- Broadly speaking, these two are the same, we need to estimate:
  – Expected Returns
  – Variances
  – Covariances

Estimation Challenges

- It is common to use:
  - Historical average returns to approximate $E[\tilde{r}]$
  - Historical variance and covariance to estimate the variances and covariances of returns in the future.

- Using historical data
  - Assumes the future is like the past

Drawback

  - Tends to overweight extremes
    - If $observed\ r = E[\tilde{r}] + \epsilon$
    - If a stock has high observed returns it may be due to high ϵ and not high $E[\tilde{r}]$

Estimation Challenges

- Estimating
  - expected returns,
  - variance and
  - covariance

- for a large numbers of assets introduces
  - Estimation error.
  - Difficult and time consuming

Partial Solutions

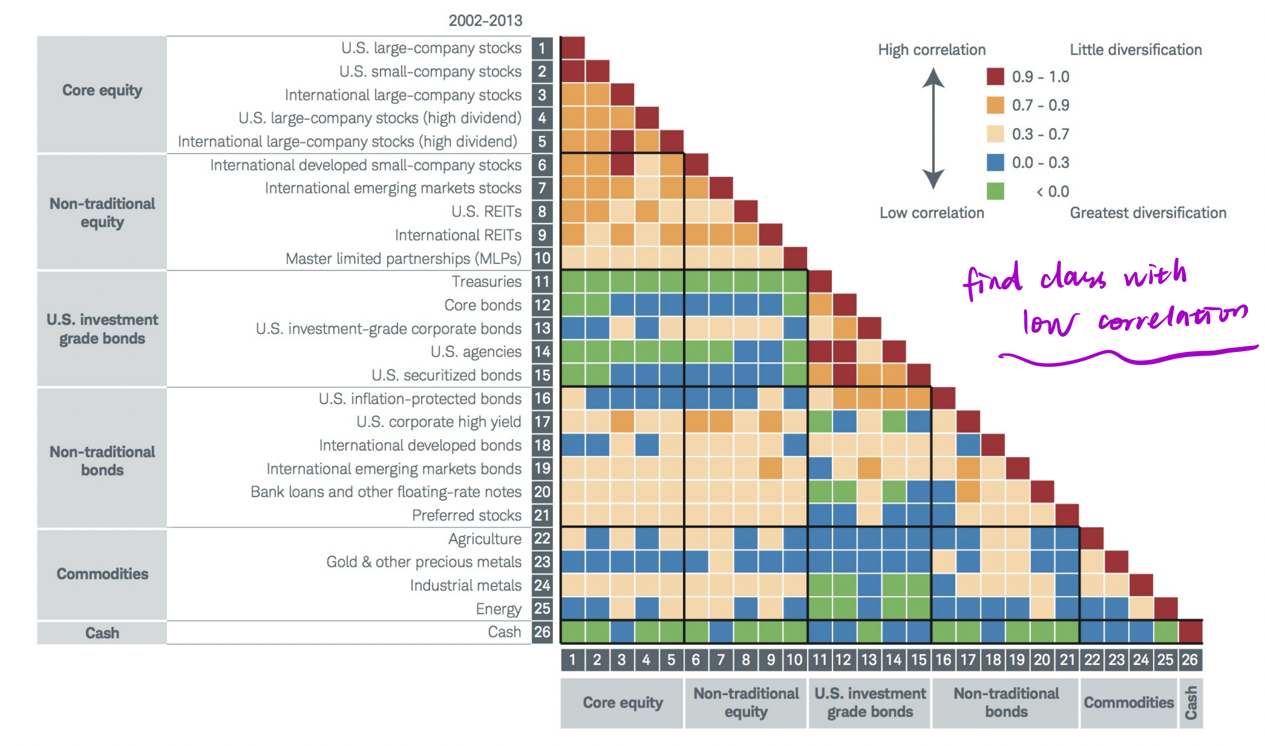
- Reducing dimensionality through asset pricing or index models (we'll cover this in a future lecture)
- Focusing on asset classes, instead of individual assts.

Asset Classes

Growth	U.S. large-company stocks	U.S. small-company stocks	International developed large-company stocks	International developed small-company stocks	International emerging markets stocks	
Growth and income	U.S. large-company stocks (high dividend)		International developed large-company stocks (high dividend)		Master limited partnerships (MLPs)	
Income	U.S. investment grade corporate bonds	U.S. corporate high-yield bonds	U.S. securitized bonds	International emerging markets bonds	Preferred stocks	Bank loans & other floating-rate notes
Inflation	U.S. inflation-protected bonds	U.S. REITs	International REITs	Energy	Industrial metals	Agriculture
Defensive assets	Cash	Treasuries		Gold & other precious metals	International developed bonds	U.S. agencies

Source: Charles Schwab Investment Advisory, Inc.

Asset Classes and Their Correlations



find class with low correlation

Limits to Diversification: Simulation

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limit in diversification due to systematic risk

Limits to Diversification: with US stock

- Randomly draw N stocks from the NYSE.
- Calculate the equally weighted average return for the portfolio.
- Repeat 1000 times. The portfolio standard deviation is in black