MAST30027: Modern Applied Statistics

Assignment 2, 2021.

Due: 5pm Wednesday September 15th

- This assignment is worth 18% of your total mark.
- To get full marks, show your working including 1) R commands and outputs you use, 2) mathematics derivation, and 3) rigorous explanation why you reach conclusions or answers. If you just provide final answers, you will get zero mark.
- Your assignment must be submitted on Canvas LMS as a single PDF document only (no other formats allowed). Your answers must be clearly numbered and in the same order as the assignment questions.
- The LMS will not accept late submissions. It is your responsibility to ensure that your assignments are submitted correctly and on time, and problems with online submissions are not a valid excuse for submitting a late or incorrect version of an assignment.
- We will mark a selected set of problems. We will select problems worth $\geq 50\%$ of the full marks listed.
- Also, please read the "Assessments" section in "Subject Overview" page of the LMS.

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1. **(30 marks) Note: There is no unique answer for this problem. The report for this problem should be typed. Hand-written report or report including screen-captured R codes or figures won't be marked. An example report written by a student has been posted on LMS.**

   Experiment Design: Components are attached to an electronic circuit card assembly by a wave-soldering process. The soldering process involves baking and preheating the circuit card and then passing it through a solder wave by conveyor. Defects arise during the process, and an experiment was run to try and determine the effect on the number of defects of various aspects of the process.

   Data: The data is taken from Condra, Lloyd, Reliability Improvement with Design of Experiment. CRC Press, 2001. Full wavesolder data has 48 observations, each of which has the number of defects and seven predictor variables. In this assignment, we will consider only the number of defects (response variable), and four predictor variables, prebake, flux, cooling, temp. The data can be found in the file `assignment2_prob1_2021.txt`. The dataset has 48 rows representing 48 observations. Each row has entries for:

   - numDefects: number of defects
   - prebake: prebake condition - a factor with levels 1 2
   - flux: flux density - a factor with levels 1 2
   - cooling: cooling time - a factor with levels 1 2
   - temp: solder temperature - facctor with levels 1 2

   You can read the data using the following command.

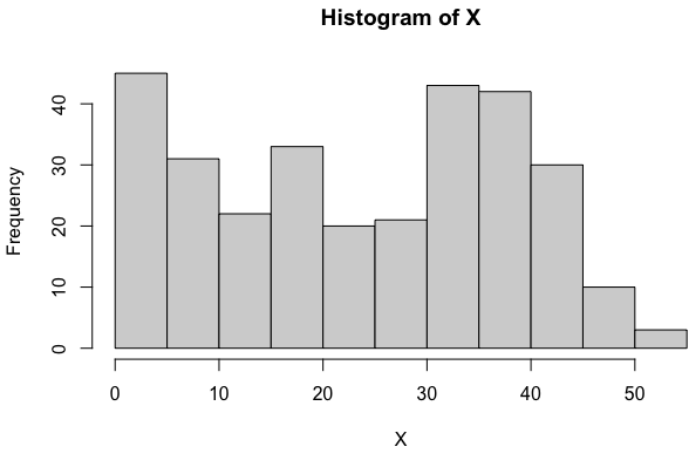
```
> data <- read.table(file ="assignment2_prob1_2021.txt", header=TRUE)
> dim(data)
[1] 48  5
> names(data)
[1] "numDefects" "prebake"    "flux"       "cooling"    "temp"
> wavesolder$prebake <- factor(wavesolder$prebake)
> wavesolder$flux <- factor(wavesolder$flux)
> wavesolder$cooling <- factor(wavesolder$cooling)
> wavesolder$temp <- factor(wavesolder$temp)
```

Problem: We want to determine which factors (prebake, flux, cooling, temp) and two-way interactions are related to the number of defects. Write a report on the analysis that should summarise the substantive conclusions and include the highlights of your analysis: for example, data visualisation, choice of model (e.g., Poisson, binomial, gamma, etc), model fitting and model selection (e.g., using AIC), diagnostic, check for overdispersion if necessary, and summary/interpretation of your final model.

At each step of you analysis, you should write why you do that and your interpretation/conclusion. For example, "I make an interaction plot to see whether there are interactions between X and Y", show a plot, and "It seems that there are some interaction between X and Y".

2. The file `assignment2_prob2_2021.txt` contains 300 observations. We can read the observations and make a histogram as follows.

```
> X = scan(file="assignment2_prob2_2021.txt", what=double())
Read 300 items
> length(X)
[1] 300
> hist(X)
```



We will model the observed data using a mixture of three Poisson distributions. Specifically, we assume the observations $X_1, \ldots, X_{300}$ are independent to each other, and each X_i follows this mixture model:

$$Z_i \sim \text{categorical } (\pi_1, \pi_2, 1-\pi_1-\pi_2),$$
$$X_i | Z_i = 1 \sim \text{Poisson}(\lambda_1),$$
$$X_i | Z_i = 2 \sim \text{Poisson}(\lambda_2),$$

$$X_i|Z_i = 3 \sim \text{Poisson}(\lambda_3).$$

The Poisson distribution has probability mass function

$$f(x;\lambda) = \frac{\lambda^x e^{-\lambda}}{x!}.$$

We aim to obtain MLE of parameters $\theta = (\pi_1, \pi_2, \lambda_1, \lambda_2, \lambda_3)$ using the EM algorithm.

(a) **(5 marks)** Let $X = (X_1, \ldots, X_{300})$ and $Z = (Z_1, \ldots, Z_{300})$. Derive the expectation of the complete log-likelihood, $Q(\theta, \theta^0) = E_{Z|X,\theta^0}[\log(P(X, Z|\theta))]$.

(b) **(3 marks)** Derive E-step of the EM algorithm.

(c) **(5 marks)** Derive M-step of the EM algorithm.

(d) **(5 marks) Note: Your answer for this problem should be typed. Answers including screen-captured R codes or figures won't be marked.**

Implement the EM algorithm and obtain MLE of the parameters by applying the implemented algorithm to the observed data, $X_1, \ldots, X_{300}$. Set EM iterations to stop when either the number of EM-iterations reaches 100 (max.iter = 100) or the incomplete log-likelihood has changed by less than 0.00001 ($\epsilon = 0.00001$). Run the EM algorithm two times with the following two different initial values and report estimators with the highest incomplete log-likelihood.

	π_1	π_2	λ_1	λ_2	λ_3
1st initial values	0.3	0.3	3	20	35
2nd initial values	0.1	0.2	5	25	40

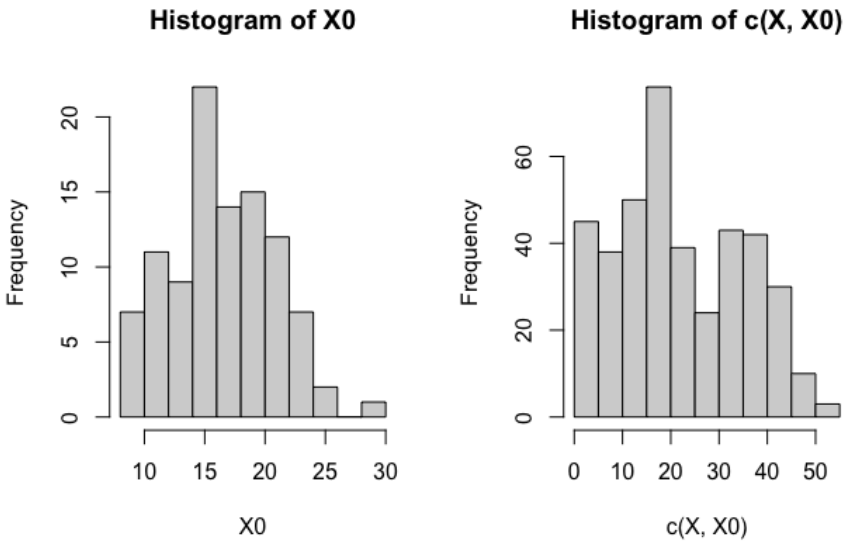
For each EM run, check that the incomplete log-likelihoods increase at each EM-step by plotting them.

3. The file `assignment2_prob3_2021.txt` contains 100 observations. We can read the 300 observations from the problem 2 and the new 100 observations and make histograms as follows.

```
> X = scan(file="assignment2_prob2_2021.txt", what=double())
Read 300 items
> X0 = scan(file="assignment2_prob3_2021.txt", what=double())
Read 100 items
> length(X)
[1] 300
> length(X0)
[1] 100
> par(mfrow=c(1,2))
> hist(X0)
> hist(c(X,X0))
```

Let $X_1, \ldots, X_{300}$ and $X_{301}, \ldots, X_{400}$ denote the 300 observations from `assignment2_prob2_2021.txt` and the 100 observations from `assignment2_prob3_2021.txt`, respectively. We assume the observations $X_1, \ldots, X_{400}$ are independent to each other. We model $X_1, \ldots, X_{300}$ (from `assignment2_prob2_2021.txt`) using the mixture of three Poisson distributions (as we did in the problem 3), but we model $X_{301}, \ldots, X_{400}$ (from `assignment2_prob3_2021.txt`) using one of the three Poisson distributions. Specifically, for $i = 1, \ldots, 300$, X_i follows this mixture model:

$$Z_i \sim \text{categorical } (\pi_1, \pi_2, 1 - \pi_1 - \pi_2),$$



$$X_i|Z_i = 1 \sim \text{Poisson}(\lambda_1),$$
$$X_i|Z_i = 2 \sim \text{Poisson}(\lambda_2),$$
$$X_i|Z_i = 3 \sim \text{Poisson}(\lambda_3),$$

and for $i = 301, \ldots, 400$,

$$X_i \sim \text{Poisson}(\lambda_2).$$

We aim to obtain MLE of parameters $\theta = (\pi_1, \pi_2, \lambda_1, \lambda_2, \lambda_3)$ using the EM algorithm.

(a) **(5 marks)** Let $X = (X_1, \ldots, X_{400})$ and $Z = (Z_1, \ldots, Z_{300})$. Derive the expectation of the complete log-likelihood, $Q(\theta, \theta^0) = E_{Z|X,\theta^0}[\log(P(X, Z|\theta))]$.

(b) **(5 marks)** Derive E-step and M-step of the EM algorithm.

(c) **(5 marks) Note: Your answer for this problem should be typed. Answers including screen-captured R codes or figures won't be marked.**

Implement the EM algorithm and obtain MLE of the parameters by applying the implemented algorithm to the observed data, $X_1, \ldots, X_{400}$. Set EM iterations to stop when either the number of EM-iterations reaches 100 (max.iter = 100) or the incomplete log-likelihood has changed by less than 0.00001 ($\epsilon = 0.00001$). Run the EM algorithm two times with the following two different initial values and report estimators with the highest incomplete log-likelihood.

	π_1	π_2	λ_1	λ_2	λ_3
1st initial values	0.3	0.3	3	20	35
2nd initial values	0.1	0.2	5	25	40

For each EM run, check that the incomplete log-likelihoods increase at each EM-step by plotting them.

4. **(3 marks)** Read the "Assessments" section of "Subject Overview" page of the LMS. Provide the requested information in the first page of your assignment.