School of Mathematics and Statistics
MAST20009 Vector Calculus, Semester 1 2020
Assignment 4 and Cover Sheet

Student Name	*Student Number*
Tutor's Name	*Tutorial Day/Time*

Submit your assignment via the MAST20009 website before 11am on Monday 25th May.

Note:

- This assignment is worth 5% of your final MAST20009 mark.
- Assignments must be neatly handwritten in blue or black pen on A4 paper. Diagrams can be drawn in pencil.
- Full working must be shown in your solutions.
- Marks will be deducted for incomplete working, insufficient justification of steps, incorrect mathematical notation and for messy presentation of solutions.

1. Consider a thin wire lying along a smooth curve W with mass per unit length $\mu(x, y, z)$. The centre of mass (x_c, y_c, z_c) of the wire is given by

$$x_c = \frac{\int_W x\mu\, ds}{\text{mass W}}, \quad y_c = \frac{\int_W y\mu\, ds}{\text{mass W}}, \quad z_c = \frac{\int_W z\mu\, ds}{\text{mass W}}.$$

Find the centre of mass of a thin wire parametrised by

$$\mathbf{c}(t) = t\mathbf{i} + 2t\mathbf{j} + \frac{2}{3}t^{\frac{3}{2}}\,\mathbf{k}, \qquad 0 \le t \le 2,$$

if the mass per unit length of the wire is $\mu(x, y, z) = 3\sqrt{5 + x}$.

2. Let the path C traverse part of the ellipse $16x^2 + y^2 = 16$ from $(0, -4)$ to $(0, 4)$, in a clockwise direction.

   (a) Write down a parametrisation for C in terms of an increasing parameter t.

   (b) Using part (a), determine the work done by the force

$$\mathbf{F}(x, y) = 2y\mathbf{i} + 3x\mathbf{j}$$

   to move a particle along C.

3. Let R be the region cut from the plane $x + 2y + 2z = 5$ by the cylinder whose walls are $x = y^2$ and $x = 2 - y^2$.

   (a) Sketch R, clearly labelling any intercepts.

   (b) Using an appropriate surface integral, find the area of R.

4. A greenhouse has a glass dome in the shape of the paraboloid $z = 8 - 2x^2 - 2y^2$ and a flat dirt floor at $z = -10$. Let S be the closed surface formed by the dome and the floor, oriented with outward unit normal.

   Suppose that the temperature in the greenhouse is given by

   $$T(x, y, z) = x^2 + y^2 + 3(z - 2)^2.$$

   The temperature gives rise to a heat flux density field

   $$\mathbf{H}(x, y, z) = -k\boldsymbol{\nabla}T$$

   where k is a positive constant that depends on the insulating properties of the medium. Assume that $k = 1$ on the glass dome and $k = 3$ on the dirt floor of the greenhouse.

   (a) Sketch S, clearly labelling any intercepts and the direction of the normal vector.

   (b) *Without using any integral theorems*, find the total heat flux

   $$\iint_S \mathbf{H} \cdot d\mathbf{S}$$

   across S in the direction of the outward unit normal.