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# Introductory Macroeconomics

In-Tutorial #8
Week Starting 3rd May 2021

**Questions.**

1. In the context of the Solow-Swan model, use a diagram to show the qualitative effect of an increase in the population growth rate, $n$, upon the steady state level of output per capita. Do these results seem sensible to you?

2. If we look at economic history we see that growth rates of output per capita were typically very low prior to the 1750s. In the post-1750 period, a set of countries has attained sustained growth in output per capita of about 2 per cent per year. In the context of the Solow-Swan model, what changes could account for a sustained increase in economic growth in the post-1750 period?

3. Suppose we consider a production function with the following functional form, $\frac{Y_t}{L_t} = A\frac{K_t}{L_t}$. In this case, $A$ is the level of technology and $\frac{K_t}{L_t}$ is the level of capital per worker. Capital per worker accumulates according to the following equation:

$$\frac{K_{t+1}}{L_{t+1}} = (1-d)\frac{K_t}{L_t} + \frac{I_t}{L_t}.$$

   (a) Maintain the standard assumption that investment is equal to a constant proportion of output. That is, $I_t = \theta Y_t$. Use the above information to derive an equation that describes the growth rate of capital per worker for this economy.

   (b) Does this economy have a steady state of capital per worker? If not, what will happen to this economy over time? What is the key difference between this environment and the Solow-Swan model that we discussed in lectures and the textbook?

**Solutions to In-Tutorial Work.**

1. The impact of an increase in population growth, $n$, is shown in Figure 1. The key points to note are that the $(d+n)K/L$ line shifts upwards in our Solow-Swan diagram. The intersection of the $\theta Y/L$ and $(n+d)K/L$ line moves from A to B. This leads to a lower steady state level of capital per worker. This reduces the level of output per worker. These results are generally regarded as quite intuitive and sensible.

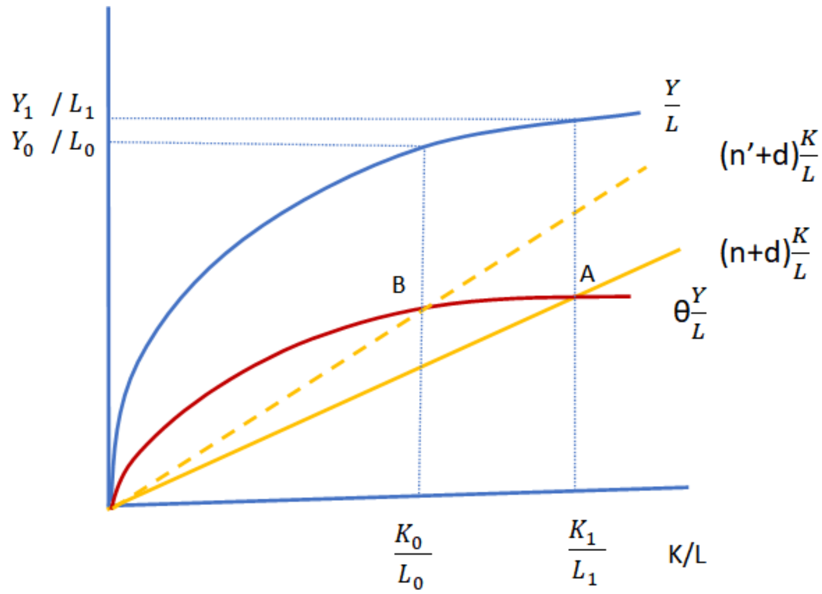


Figure 1: Impact of an increase in the rate of population growth

2. There are a few factors that could be considered:

   Capital accumulation does not lead to permanent growth in capital per worker or output per worker. The results from the Solow-Swan model show that, if the saving rate is constant, the level of capital per worker will eventually reach a constant level (that is, a steady state). In this steady state, output per worker will also be constant.

   If the saving rate increases, there can be a temporary increase in the growth of output per worker but this does not lead to permanent growth because the saving rate $\theta$ cannot grow permanently (its maximum value is 1). The population growth rate $n$ can grow over time, but this will reduce output per worker as we have seen in Question 1.

   A key implication of the Solow-Swan model is that changes in technology can drive changes in the output per capita. To explain the permanent growth in living standards, it would be necessary to allow permanent growth in total factor productivity. That is the $A$ in a production function $\frac{Y}{L} = A\left(\frac{K}{L}\right)^{\alpha}$. So one of the key questions associated with understanding economic history, is understanding what forces allowed for sustained growth in productivity in the post 1750 period.

3. (a) Substituting out investment in the capital accumulation equation implies

$$\begin{aligned}
\frac{K_{t+1}}{L_{t+1}} &= (1-d)\frac{K_t}{L_t} + \theta\frac{Y_t}{L_t} \\
&= (1-d)\frac{K_t}{L_t} + \theta A\frac{K_t}{L_t} \\
\to \frac{K_{t+1}}{L_{t+1}} - \frac{K_t}{L_t} &= -d\frac{K_t}{L_t} + \theta A\frac{K_t}{L_t} \\
\to \frac{\frac{K_{t+1}}{L_{t+1}} - \frac{K_t}{L_t}}{\frac{K_t}{L_t}} &= \theta A - d
\end{aligned}$$

This is an equation that describes the growth rate of capital per worker.

(b) The economy has a steady state of capital per worker only if $\theta A = d$. In this case, the level of capital per worker is not changing over time. The amount that is saved is exactly equal to depreciation in this case and the economy is neither growing nor shrinking. In this case, the existing level of capital per worker is what the level of capital per worker will be in the future. Therefore, any existing level of capital per worker is a steady state capital per worker.

In a second case, $\theta A > d$, the savings per worker by the economy exceed the amount by which capital per worker depreciates in each period. Therefore, there is no steady state capital per worker. The level of capital per worker in this economy will grow and, due to the production function, the level of output per worker in this economy will also grow over time.

In the third case, $\theta A < d$, the savings per worker are not enough to cover depreciation. Therefore, there is no steady state capital per worker. Capital per worker will decline over time until zero, and the economy will shrink.

The key difference between this question and what we considered in lectures is that this production function does not have *diminishing marginal returns* on capital. Hence, this economy can display continual economic growth (or decline).