PHYC10003 Physics 1

Lecture 2: Motion

Frames, position, speed, acceleration

Last lecture

- Galileo's principle, inertial frames of reference, universal time
- Units: base units, manufactured and fundamental standards
- Mass
- Length
- Time

Kinematics

- **Kinematics** is the classification and comparison of motions
- In this Lecture, we restrict motion in three ways:
  1. We consider motion along a straight line only
  2. We discuss only the motion itself, not the forces that cause it
  3. We consider the moving object to be a **particle**
- A **particle** is either:
  - A point-like object (such as an electron)
  - Or a rigid body: an object that moves such that each part travels in the same direction at the same rate (no rotation or stretching)

2.1 Position

"In an infinite universe, the one thing sentient life cannot afford to have is a sense of proportion",

…Douglas Adams, (The Hitchhiker's Guide to the Galaxy, 1978)

2.1 Position

- Position is measured relative to a reference point:
  - The **origin**, or zero point, of an axis
- Position has a sign:
  - **Positive direction** is in the direction of increasing numbers
  - **Negative direction** is opposite the positive

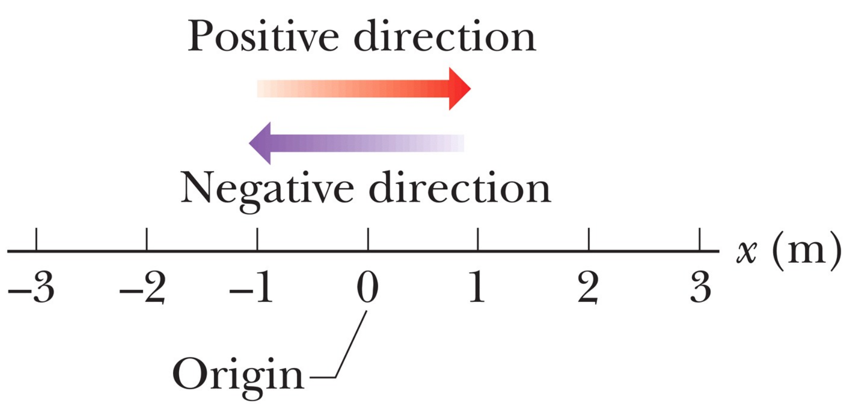


Figure 2-1

2.1 Displacement

- A change in position is called **displacement**
  - Δx is the change in x, (*final position*) – (*initial position*)

$$\Delta x = x_2 - x_1$$

Eq. (2-1)

Examples A particle moves . . .

- From x = 5 m to x = 12 m: Δx = 7 m (positive direction)
- From x = 5 m to x = 1 m: Δx = -4 m (negative direction)
- From x = 5 m to x = 200 m to x = 5 m: Δx = 0 m

2.1 Displacement

- Displacement is therefore a **vector quantity**
  - Direction: along a single axis, given by sign (+ or -)
  - Magnitude: length or distance, in this case metres
- Ignoring sign, we get its **magnitude** (absolute value)
  - The magnitude of Δx = -4 m is 4 m.

Checkpoint 1

Here are three pairs of initial and final positions, respectively, along an x axis. Which pairs give a negative displacement: (a) -3 m, $+5$ m; (b) -3 m, -7 m; (c) 7 m, -3 m?

- Answer: pairs (b) and (c)

(b) -7 m – -3 m = -4 m   (c) -3 m – 7 m = -10 m

2.1 Velocity

- **Average velocity** is the ratio of:
  - A displacement, Δx
  - To the time interval in which the displacement occurred, Δt

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$$

Eq. (2-2)

- Average velocity has units of (*distance*) / (*time*)
  - Metres per second, m/s

2.1 Velocity

Spoiler alert!

Mechanics is primarily about rates of change of position, velocity or related quantities.

The natural language for discussing mechanics is *calculus*.

The equations that determine the relationship between these quantities are *differential equations*.

2.1 Velocity

- On a graph of x vs. t, the average velocity is the **slope** of the straight line that connects two points
- Average velocity is therefore a vector quantity
  - Positive slope means positive average velocity
  - Negative slope means negative average velocity

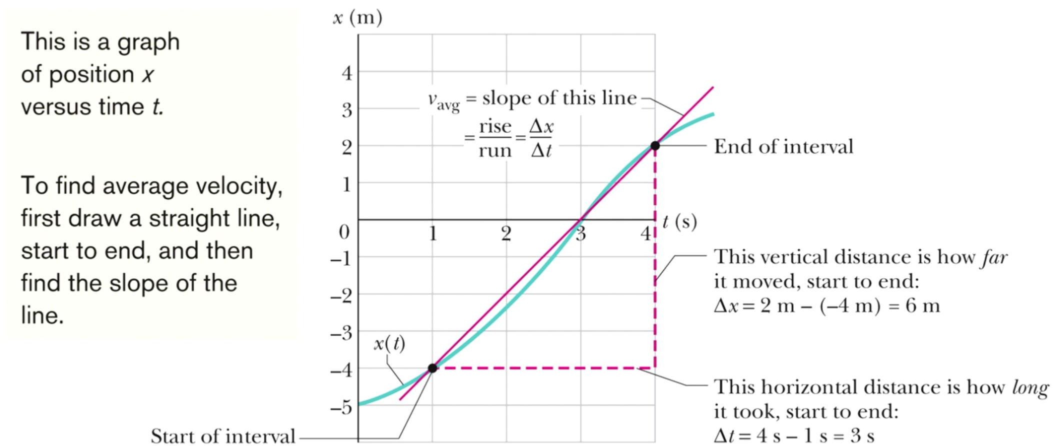


Figure 2-4

2.1 Velocity

- **Average speed** is the ratio of:
  - The total distance covered
  - To the time interval in which the distance was covered, Δt

$$s_{\text{avg}} = \frac{\text{total distance}}{\Delta t}$$

Eq. (2-3)

- Average speed is always positive (no direction)

Example A particle moves from x = 3 m to x = -3 m in 2 seconds.

  - Average velocity = -3 m/s; average speed = 3 m/s

2.2 Instantaneous velocity

- **Instantaneous velocity**, or just **velocity**, *v*, is:
  - At a single moment in time
  - Obtained from average velocity by shrinking Δt
  - The slope of the position-time curve for a particle at an instant (the derivative of position)
  - A vector quantity with units (*distance*) / (*time*)
    - The sign of the velocity represents its direction

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

Eq. (2-4)

2.2 Velocity

- **Speed** is the magnitude of (instantaneous) velocity

Example A velocity of 5 m/s and -5 m/s both have an associated speed of 5 m/s.

Checkpoint 2

The following equations give the position $x(t)$ of a particle in four situations (in each equation, x is in meters, t is in seconds, and $t > 0$): (1) $x = 3t - 2$; (2) $x = -4t^2 - 2$; (3) $x = 2/t^2$; and (4) $x = -2$. (a) In which situation is the velocity v of the particle constant? (b) In which is v in the negative x direction?

Answers:

(a) Situations 1 and 4 (zero)

(b) Situations 2 and 3

2.2 Velocity

Example

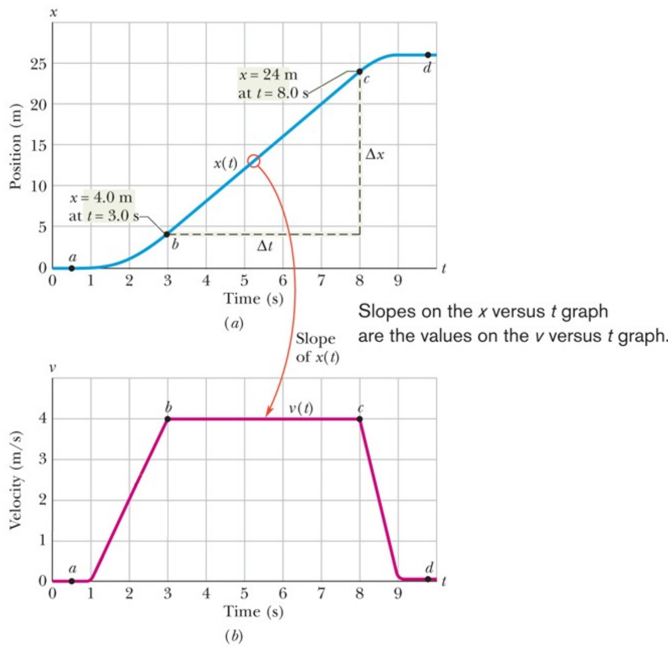


Figure 2-6

- The graph shows the position and velocity of the School of Physics lift over time.
- The slope of $x(t)$, and so also the velocity v, is zero from 0 to 1 s, and from 9s on.
- During the interval *bc*, the slope is constant and nonzero, so the cab moves with constant velocity (4 m/s).

2.3 Acceleration

- A change in a particle's velocity is **acceleration**
- **Average acceleration** over a time interval Δt is

$$a_{avg} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}$$

Eq. (2-7)

- **Instantaneous acceleration** (or just **acceleration**), *a*, for a single moment in time is:
  - Slope of velocity vs. time graph

$$a = \frac{dv}{dt}$$

Eq. (2-8)

2.3 Acceleration

- Combining Eqs. 2-8 and 2-4:

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2} \qquad \text{Eq. (2-9)}$$

- Acceleration is a vector quantity:
  - Positive sign means in the positive coordinate direction
  - Negative sign means the opposite
  - Units of (*distance*) / (*time squared*)

If the signs of the velocity and acceleration of a particle are the same, the speed of the particle increases. If the signs are opposite, the speed decreases.

2.3 Acceleration

Example If a car with velocity v = -25 m/s is braked to a stop in 5.0 s, then a = + 5.0 m/s^2. Acceleration is positive, but speed has decreased.

- Note: acceleration can be expressed in **units of g**

$$1g = 9.8 \text{ m/s}^2 \qquad (g \text{ unit})$$ **Eq. (2-10)**

Checkpoint 3

A wombat moves along an x axis. What is the sign of its acceleration if it is moving (a) in the positive direction with increasing speed, (b) in the positive direction with decreasing speed, (c) in the negative direction with increasing speed, and (d) in the negative direction with decreasing speed?

Answers: (a) + (b) - (c) - (d) +

2.3 Acceleration

Example

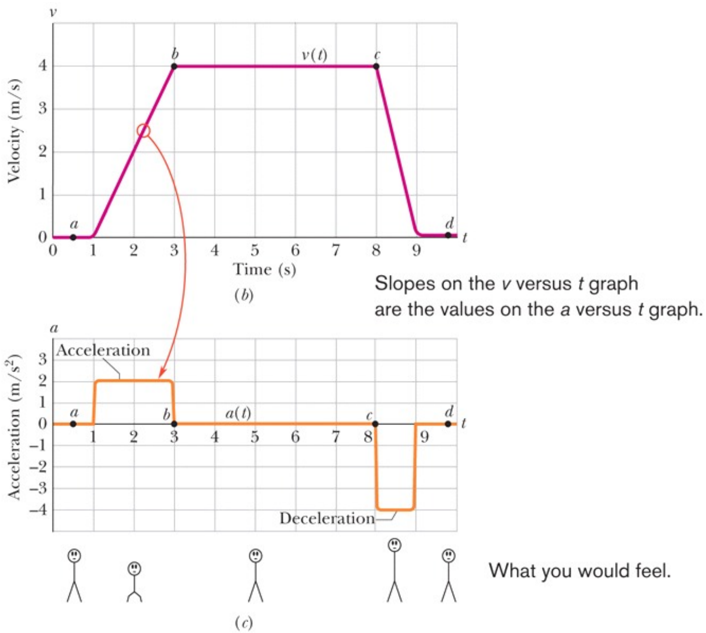


Figure 2-6

- The graph shows the velocity and acceleration of a lift over time.
- When acceleration is 0 (e.g. interval *bc*) velocity is constant.
- When acceleration is positive (*ab*) upward velocity increases.
- When acceleration is negative (*cd*) upward velocity decreases.
- Steeper slope of the velocity-time graph indicates a larger magnitude of acceleration: the lift stops in half the time it takes to get up to speed.

2.4 Constant Acceleration

- In many cases acceleration is constant, or nearly so.
- For these cases, **5 special equations** can be used.
- Note that constant acceleration means a velocity with a constant slope, and a position with varying slope (unless a = 0).

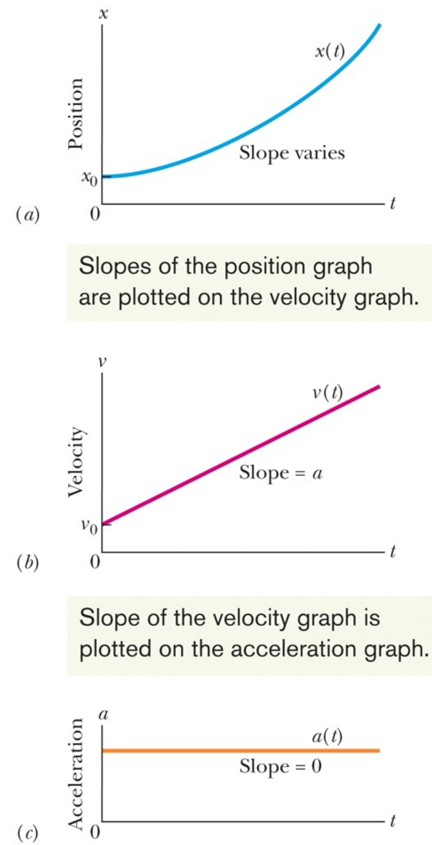


Figure 2-9

2.4 Constant Acceleration

- **First basic equation**
  - When the acceleration is constant, the average and instantaneous accelerations are equal
  - Rewrite Eq. 2-7 and rearrange

$$a = a_{avg} = \frac{v - v_0}{t - 0}$$

$$\boxed{v = v_0 + at}$$ **Eq. (2-11)**

- This equation reduces to $v = v_0$ for $t = 0$
- Its derivative yields the definition of *a*, *dv/dt*

2.4 Constant Acceleration

- **Second basic equation**
  - Rewrite Eq. 2-2 and rearrange

$$v_{\text{avg}} = \frac{x - x_0}{t - 0} \qquad x = x_0 + v_{\text{avg}} t \qquad \text{Eq. (2-12)}$$

  - Average = ((*initial*) + (*final*)) / 2: $v_{\text{avg}} = \frac{1}{2}(v_0 + v)$ **Eq. (2-13)**
  - Substitute 2-11 into 2-13

$$v_{\text{avg}} = v_0 + \frac{1}{2}at \qquad \text{Eq. (2-14)}$$

  - Substitute 2-14 into 2-12

$$\boxed{x - x_0 = v_0 t + \frac{1}{2}at^2} \qquad \text{Eq. (2-15)}$$

2.4 Constant Acceleration

- These two equations can be obtained by integrating a constant acceleration
- Enough to solve any constant acceleration problem
  - Solve as simultaneous equations
- Additional useful forms:

$$v^2 = v_0^2 + 2a(x - x_0)$$ **Eq. (2-16)**

$$x - x_0 = \tfrac{1}{2}(v_0 + v)t$$ **Eq. (2-17)**

$$x - x_0 = vt - \tfrac{1}{2}at^2$$ **Eq. (2-18)**

2.4 Constant Acceleration

- Table 2-1 shows the 5 equations and the quantities missing from them.

Table 2-1 Equations for Motion with Constant Acceleration[a]

Equation Number	Equation	Missing Quantity
2-11	$v = v_0 + at$	$x - x_0$
2-15	$x - x_0 = v_0 t + \frac{1}{2}at^2$	v
2-16	$v^2 = v_0^2 + 2a(x - x_0)$	t
2-17	$x - x_0 = \frac{1}{2}(v_0 + v)t$	a
2-18	$x - x_0 = vt - \frac{1}{2}at^2$	v_0

[a]Make sure that the acceleration is indeed constant before using the equations in this table.

Table 2-1

Checkpoint 4

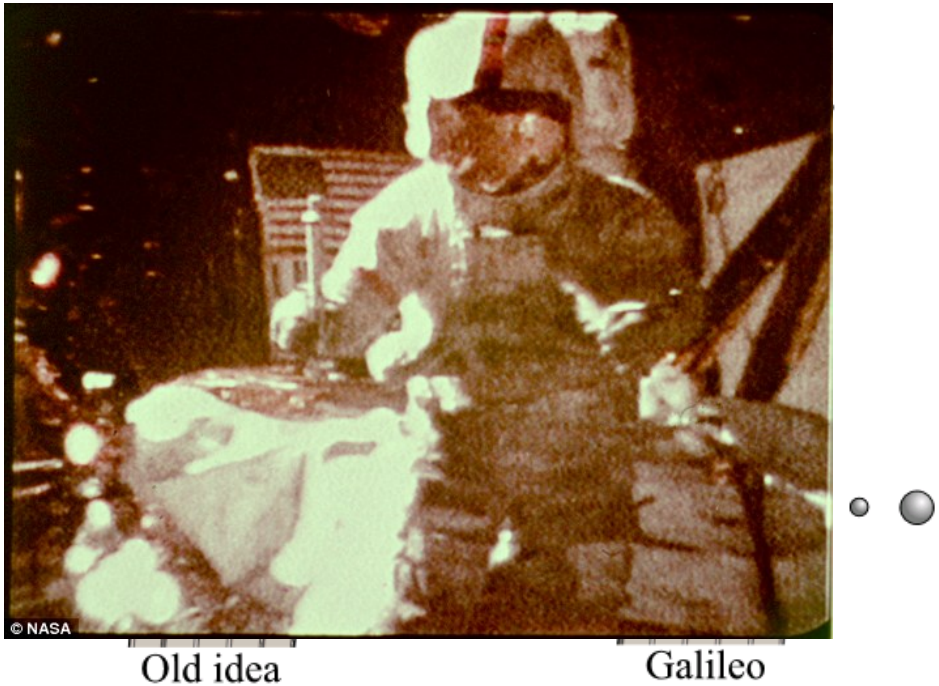
The following equations give the position $x(t)$ of a particle in four situations: (1) $x = 3t - 4$; (2) $x = -5t^3 + 4t^2 + 6$; (3) $x = 2/t^2 - 4/t$; (4) $x = 5t^2 - 3$. To which of these situations do the equations of Table 2-1 apply?

Answer: Situations 1 (a = 0) and 4.

2.5 Free-Fall Acceleration

- **Free-fall acceleration** is the rate at which an object accelerates downward in the absence of air resistance
  - Varies with latitude and elevation
  - Written as *g*, standard value of 9.8 m/s^2
  - Independent of the properties of the object (mass, density, shape, see Figure 2-12)
- The equations of motion in Table 2-1 apply to objects in free-fall near Earth's surface
  - In vertical flight (along the *y* axis)
  - Where air resistance can be neglected

2.5 Free fall experiment – Galileo and Apollo



Objects fall with the same acceleration, independent of mass, either on Earth or on the moon (**First:** Galileo, cannon balls, Pisa 1589-92; **Second:** hammer and feather, Cmdr David Scott, Apollo 15, August 2, 1972).

Images: Wikicommons

2.5 Free-fall Acceleration

- The free-fall acceleration is downward (-*y* direction)
  - Value -g in the constant acceleration equations

The free-fall acceleration near Earth's surface is $a = -g = -9.8\ \text{m/s}^2$, and the *magnitude* of the acceleration is $g = 9.8\ \text{m/s}^2$. Do not substitute $-9.8\ \text{m/s}^2$ for g.

Checkpoint 5

(a) If you toss a ball straight up, what is the sign of the ball's displacement for the ascent, from the release point to the highest point? (b) What is it for the descent, from the highest point back to the release point? (c) What is the ball's acceleration at its highest point?

Answers:

(a) The sign is positive (the ball moves upward); (b) The sign is negative (the ball moves downward); (c) The ball's acceleration is always -9.8 m/s^2 at all points along its trajectory

Summary

Position

- Relative to origin
- Positive and negative directions

Displacement

- Change in position (vector)

$$\Delta x = x_2 - x_1$$ **Eq. (2-1)**

Average Velocity

- Displacement / time (vector)

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$$ **Eq. (2-2)**

Average Speed

- Distance traveled / time

$$s_{\text{avg}} = \frac{\text{total distance}}{\Delta t}$$ **Eq. (2-3)**

Summary

Instantaneous Velocity

- At a moment in time
- Speed is its magnitude

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

Eq. (2-4)

Instantaneous Acceleration

- First derivative of velocity
- Second derivative of position

$$a = \frac{dv}{dt}$$

Eq. (2-8)

Average Acceleration

- Ratio of change in velocity to change in time

$$a_{avg} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}$$

Eq. (2-7)

Constant Acceleration

- Includes free-fall, where *a* = -*g* along the vertical axis

Equation Number	Equation	Missing Quantity
2-11	$v = v_0 + at$	$x - x_0$
2-15	$x - x_0 = v_0 t + \frac{1}{2}at^2$	v
2-16	$v^2 = v_0^2 + 2a(x - x_0)$	t
2-17	$x - x_0 = \frac{1}{2}(v_0 + v)t$	a
2-18	$x - x_0 = vt - \frac{1}{2}at^2$	v_0

Tab. (2-1)

Preparation for the next lecture

1. Read 3.1-3.3 of the text
2. You will find short answers to the odd-numbered problems in each chapter at the back of the book and further resources on LMS. You should try a few of the simple odd numbered problems from each section (the simple questions have one or two dots next to the question number).