MAST30027: Modern Applied Statistics

Week 9 Lab

1. Suppose that $X \sim \text{bin}(n, \theta)$ and $\theta \sim \text{beta}(a, b)$. That is, $X|\theta$ has the pmf

$$p_{X|\theta}(x) = \binom{n}{x} \theta^x (1-\theta)^{n-x}$$

and θ has pdf

$$f_\theta(x) = \beta(a, b)^{-1} x^{a-1} (1-x)^{b-1}.$$

The marginal distribution of X is given by

$$p_X(x) = \int_0^1 p_{X\theta}(x) f_\theta(\theta) d\theta.$$

X is said to have a beta-binomial distribution. It is possible, but not easy, to work out p_X for a beta-binomial. However, it is easy to estimate it using simulation.

Generate a sample of size 1000,000 from a beta-binomial with $n = 10$, $a = 2$ and $b = 3$. Use it to estimate the pmf of X.

2. Suppose that Z follows a truncated exponential distribution, which has the pdf $p(z) = \frac{e^{-z}}{1-e^{-1}}$, $0 < z < 1$. Its theoretical mean and variance are known to be $E(Z) = 0.418$ and $\text{Var}(Z) = 0.079$.

   (a) Construct an rejection sampling algorithm to generate a sample of observations from the truncated exponential distribution.

   (b) Write an R program to implement the algorithm in (a) and use it to generate a sample of 10000 observations. Plot a histogram of the sample. Calculate the sample mean and variance, and compare them with the theoretical mean and variance.

   (c) Show that the following algorithm also simulates from the truncated exponential distribution.

   $1°$ Generate U from Unif(0,1);

   $2°$ If $U > e^{-1}$ then deliver $Z = -\ln(U)$; otherwise go to $1°$.