Index Models

The purpose

- Learn what index models are: theory-less statistical models that decompose return volatility into firm-specific and covariance related components → can't get rid of → do not have to be systematic
  - – Notice I am NOT saying systematic, but rather covariance. While most systematic risks are also covariance risks. Covariance risks do not have to be systematic.

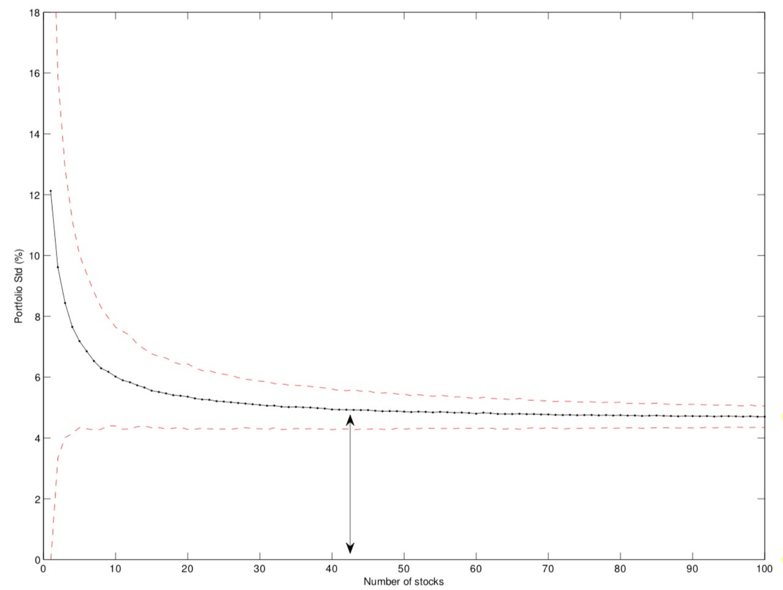
- Show you how to estimate index models with real data.

- Show you how index models can be used to simplify portfolio optimization.

- Apply index models to security selection (Stock picking)

What is an Index Model?

- Index Models are atheoretical, statistical models designed to estimate and distinguish the firm-specific and the covariance risk.



Undiversifiable/ covariance-related component of risk.

Limits to Diversification: Simulation

Avg. Std. Dev.		30%	
Avg. Correlation		0.2	
		Firm-specific	Systematic
# of Assets	Portfolio Std. Dev.	Due to Variances	Due to Covariances
2	23.24%	83.33%	16.67%
3	20.49%	71.43%	28.57%
4	18.97%	62.50%	37.50%
100	13.68%	4.81%	95.19%
1000	13.44%	0.50%	99.50%
10000	13.42%	0.05%	99.95%

Example of an index model

- An index model represents asset returns for firm *i* as a function of firm-specific ($e_{i,t}$) and *K* other asset or portfolio covariance risks.

A consistent return not explained by covariances (βs) or transitory firm-specific shocks ($e_{i,t}$).

Return due to covariances with other assets/portfolios.

$$r_{i,t} - r_{f,t} = \alpha_i + \sum_{k=1}^{K} \beta_{i,k}\left(r_{k,t} - r_{f,t}\right) + e_{i,t}$$

Firm-specific or residual return.

$$\beta_{i,k} = \frac{COV(\tilde{r}_i, \tilde{r}_K)}{VAR(\tilde{r}_K)}$$

- If the market were the only covariate (factor), then:

$$r_{i,t} - r_{f,t} = \alpha_i + \beta_i\left(r_{M,t} - r_{f,t}\right) + e_{i,t}$$

Note: The $\sim' s$ are gone because these are historic observations, not random variables from a distribution. The $t's$ indicate the observations change with time.

Pause for some notation and jargon clarification

- Your textbook is not 100% consistent in their notation.
  - When you know what you mean, it is easy to forget your reader may not.
  - I will always use $\sim' s$ for random variables and $t's$ for known observations.

- Covariance risk (me) vs. systematic risk (your book)
  - Recall that systematic risks are risks that cannot be diversified away.
  - Because this is a statistical model, we could put anything into the index model as one of the covariance risks, even something that is not a systematic risk.
    - This would be a problem for "asset pricing " (next lecture), but not a problem for the index model.
    - I want to remember that we can accidentally use non-systematic factors in index models, so I will avoid the term "systematic risk" and instead say "covariance risks".

Risk decomposition

- This form:

$$r_{i,t} - r_{f,t} = \alpha_i + \beta_{i,M}\left(r_{M,t} - r_{f,t}\right) + e_{i,t}$$

- Translates directly into risks:

$$VAR\left(\tilde{r}_i - \tilde{r}_f\right) = VAR\left(\alpha_i + \beta_{i,M}\left(\tilde{r}_M - \tilde{r}_f\right) + \tilde{e}_i\right)$$

α_i is constant, and assuming r_f is constant, then:

$$\sigma_{r_i}^2 = \beta_{i,M}^2 \sigma_{r_M}^2 + \sigma_{e_i}^2$$

covariance.

$$Total\ Risk = covariance/systematic\ risk + firm\_specific\ risk$$

Estimating Index Models with OLS

Security Characteristic Line

Estimating index models

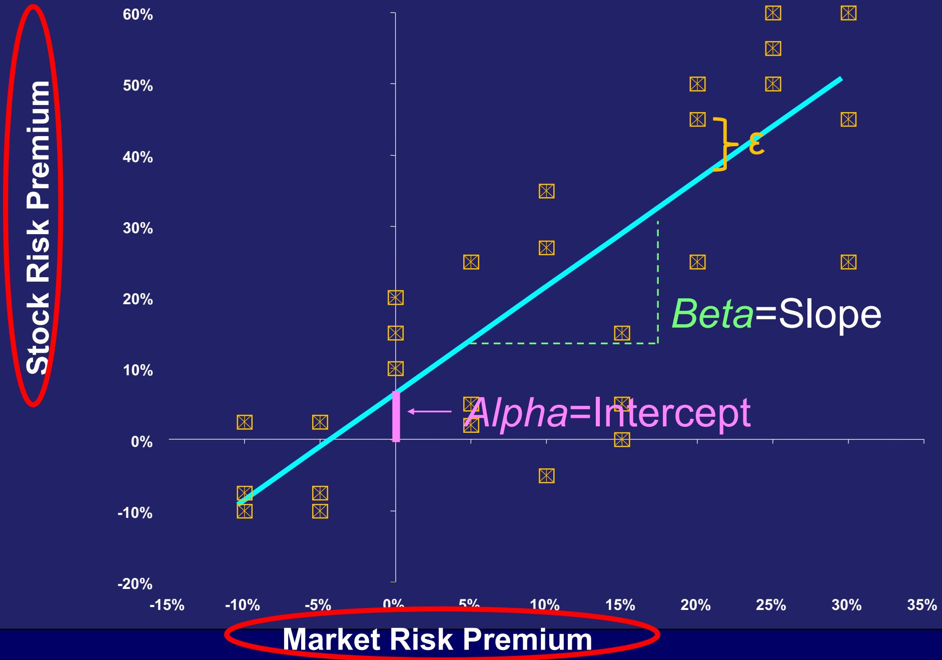
$$r_{i,t} - r_{f,t} = \alpha_i + \beta_i(r_{M,t} - r_{f,t}) + e_{i,t}$$

- Index models translate directly OLS (Ordinary Least Squares) regressions you learned about in your statistics subjects.
  - When the only covariate is the market portfolio (often an index, such as the ASX200), then we call this a Market Model.
- With OLS we can estimate α_i's and β_i's and calculate what is called a Security Characteristic Line.

Security Characteristic Line (SCL)

Equation of the Security Characteristic Line:

$$E\left[\tilde{r}_i - \tilde{r}_f \,|\, r_M - r_f\right] = \hat{\alpha}_i + \hat{\beta}_i\left(r_M - r_f\right)$$



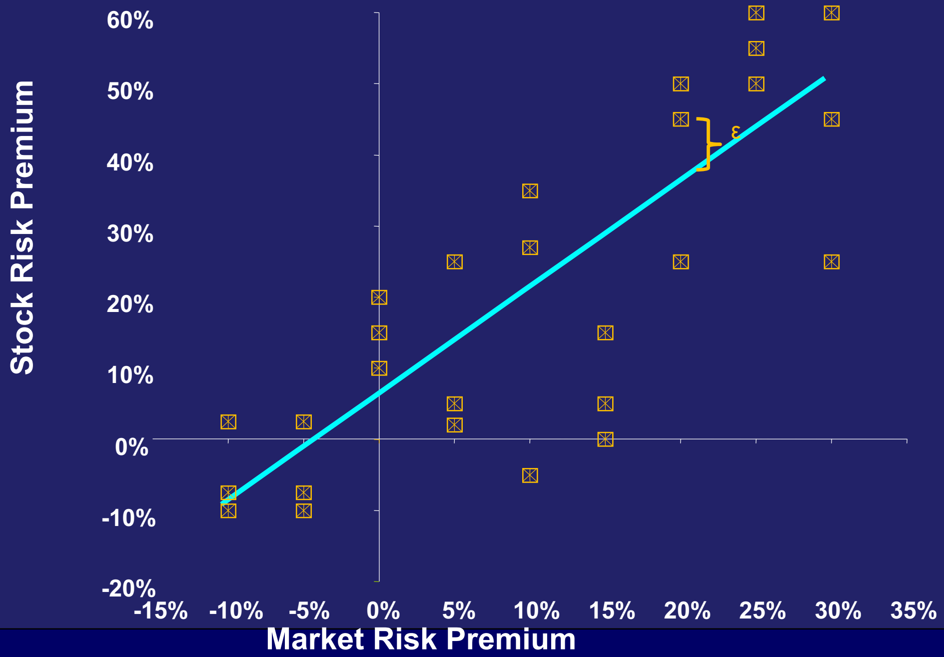
R^2 is the measure of dispersion of the data around the SCL

- It tells us how important the covariance risk is for explaining returns.

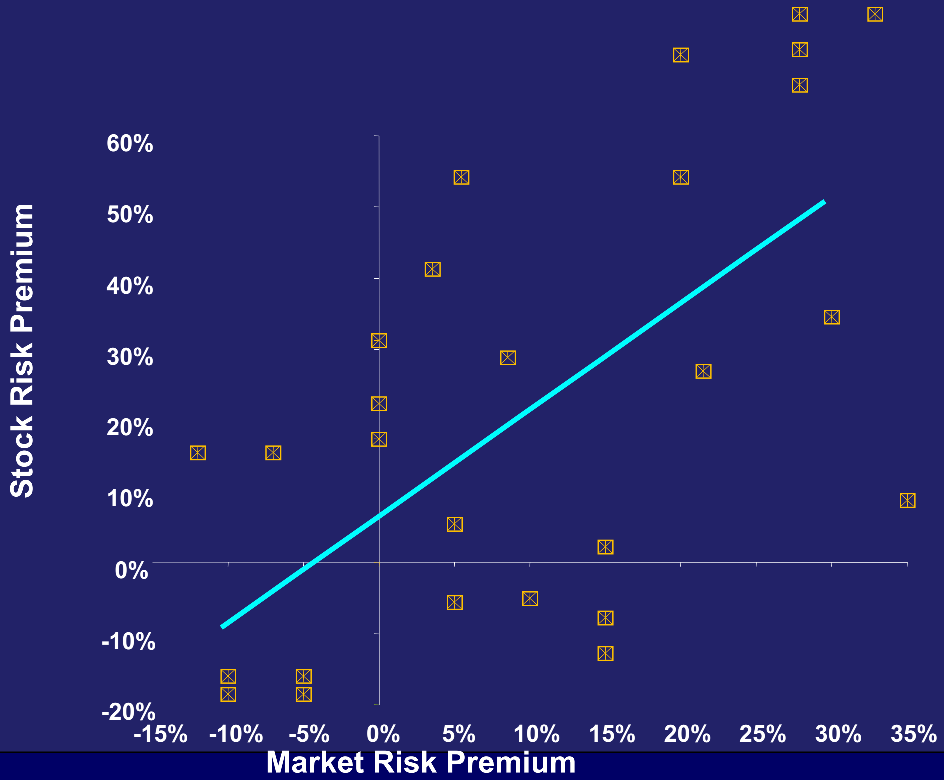
$$\underbrace{R - Square = R^2 = \rho^2}_{} = \frac{Explained\ Variance}{Total\ Variance}$$

$$\left(\underbrace{R^2 = \frac{\beta_{i,M}^2 \sigma_{r_M}^2}{\beta_{i,M}^2 \sigma_{r_M}^2 + \sigma_{e_i}^2}}_{} \right)$$

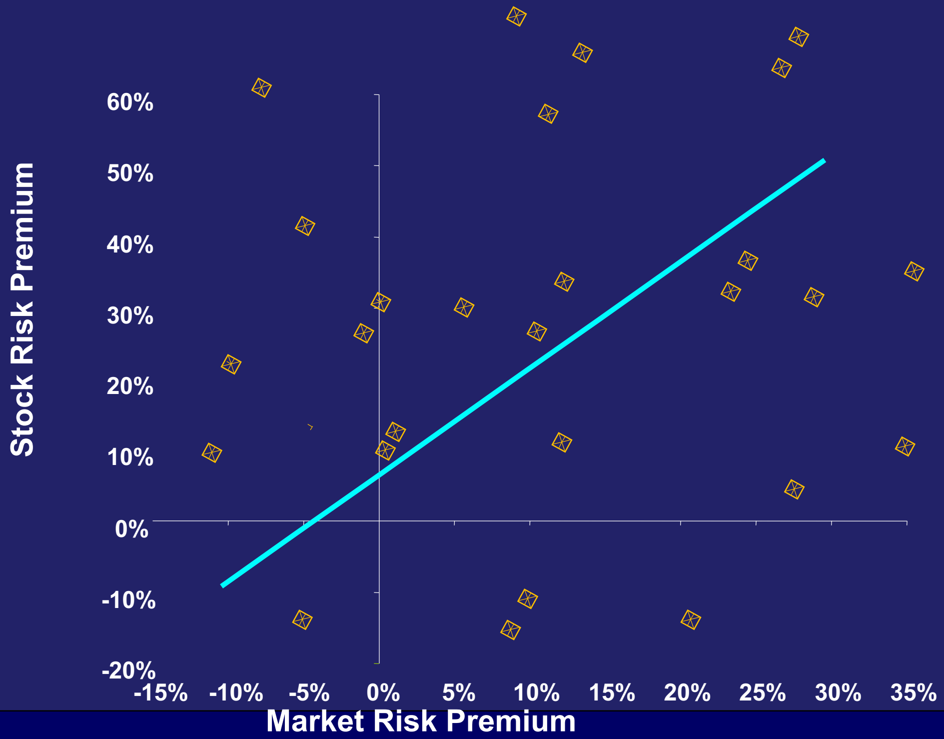
High R^2: Security Characteristic Line



Low R^2: Security Characteristic Line



Very Low R^2: Security Characteristic Line



How to Calculate Beta

$$r_{i,t} - r_{f,t} = \alpha_i + \beta_i\left(r_{M,t} - r_f\right) + \varepsilon_{i,t}$$

- Need:
  - Risk free rate → annual.
  - Stock return
  - Market return

} period should be the same

- Details, details:
  - Pay attention to the risk free rate, because that is usually stated yearly
  - What frequency are returns? Monthly
  - How long?
    - Common is to use 60 months of monthly data.
    - Or a year of daily or weekly data.

daily? Trading or non-trading day is different

How to Calculate Beta

$$r_{i,t} - r_{f,t} = \alpha_i + \beta_i\left(r_{M,t} - r_f\right) + \varepsilon_{i,t}$$

- Need:
  - Risk free rate
    - We'll use the 1 month "Bank Accepted Bills/Negotiable Certificates of Deposit"
    - Note that this is annualized. We need to make it monthly
  - Stock return
    - Woolies (WOW)
  - Market return
    - ASX 200

- Details, details:
  - Pay attention to the risk free rate, because that is usually stated yearly
  - What frequency are returns? Monthly
  - How long?
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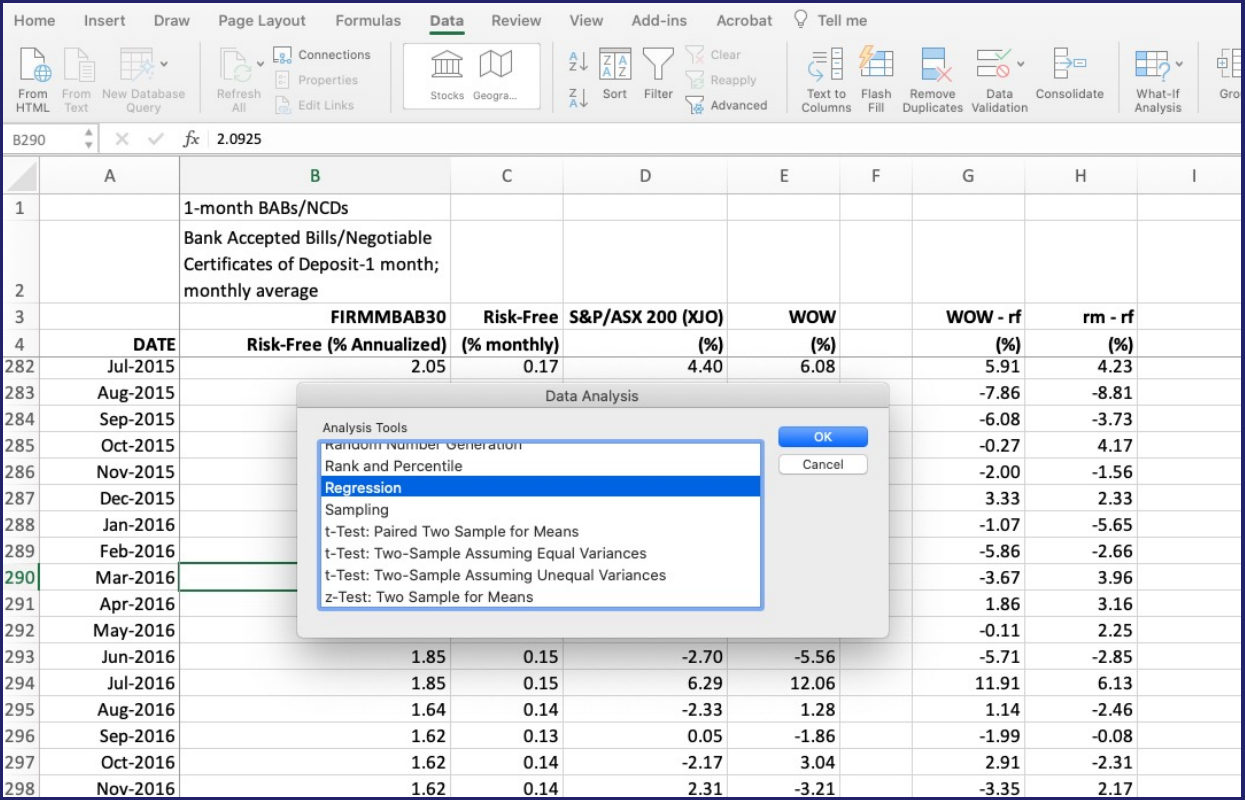
    - Or a year of daily or weekly data.

Calculating Woolworth's Beta 1

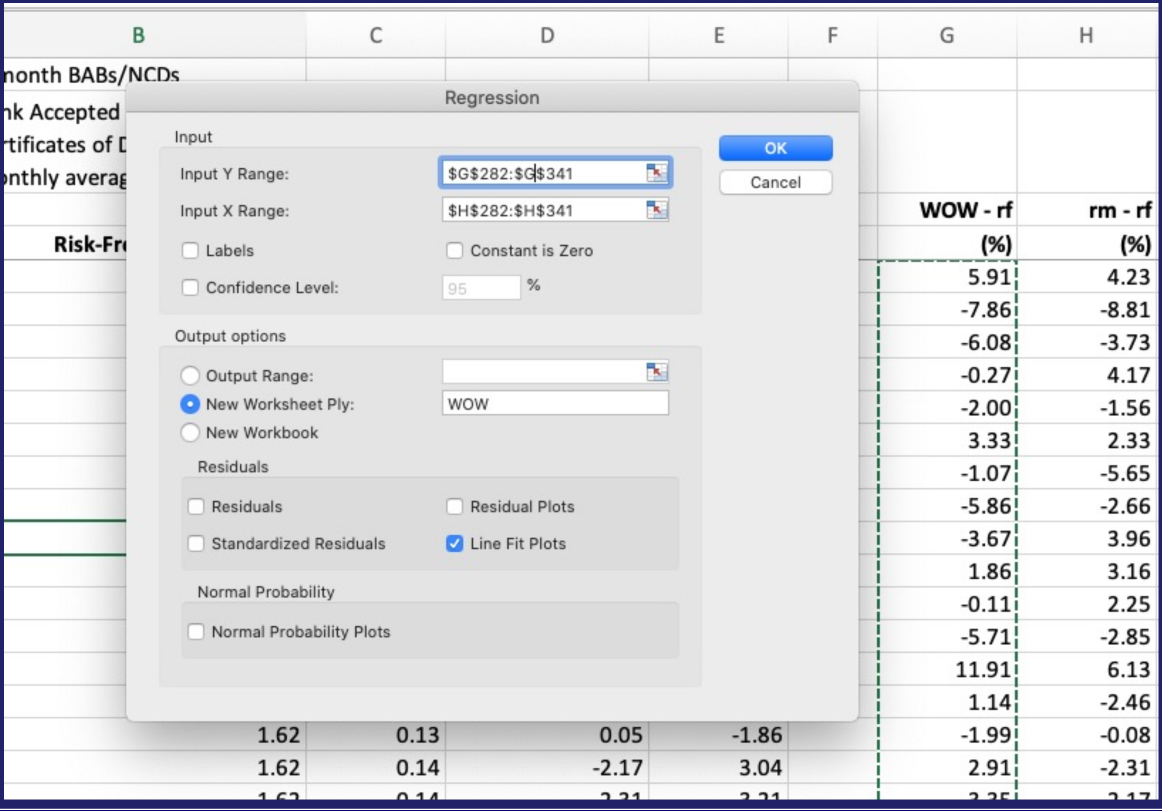
B290 f_x 2.0925

	A	B	C	D	E	F	G	H
1		1-month BABs/NCDs						
2		Bank Accepted Bills/Negotiable Certificates of Deposit-1 month; monthly average						
3		FIRMMBAB30	Risk-Free	S&P/ASX 200 (XJO)	WOW		WOW - rf	rm - rf
4	DATE	Risk-Free (% Annualized)	(% monthly)	(%)	(%)		(%)	(%)
282	Jul-2015	2.05	0.17	4.40	6.08		5.91	4.23
283	Aug-2015	2.04	0.17	-8.64	-7.69		-7.86	-8.81
284	Sep-2015	2.06	0.17	-3.56	-5.91		-6.08	-3.73
285	Oct-2015	2.04	0.17	4.34	-0.10		-0.27	4.17
286	Nov-2015	2.05	0.17	-1.39	-1.82		-2.00	-1.56
287	Dec-2015	2.07	0.17	2.50	3.51		3.33	2.33
288	Jan-2016	2.05	0.17	-5.48	-0.90		-1.07	-5.65
289	Feb-2016	2.08	0.17	-2.49	-5.68		-5.86	-2.66
290	Mar-2016	2.09	0.17	4.14	-3.49		-3.67	3.96
291	Apr-2016	2.08	0.17	3.33	2.03		1.86	3.16
292	May-2016	1.86	0.15	2.41	0.05		-0.11	2.25
293	Jun-2016	1.85	0.15	-2.70	-5.56		-5.71	-2.85
294	Jul-2016	1.85	0.15	6.29	12.06		11.91	6.13
295	Aug-2016	1.64	0.14	-2.33	1.28		1.14	-2.46
296	Sep-2016	1.62	0.13	0.05	-1.86		-1.99	-0.08
297	Oct-2016	1.62	0.14	-2.17	3.04		2.91	-2.31
298	Nov-2016	1.62	0.14	2.31	-3.21		-3.35	2.17

Calculating Woolworth's Beta 2

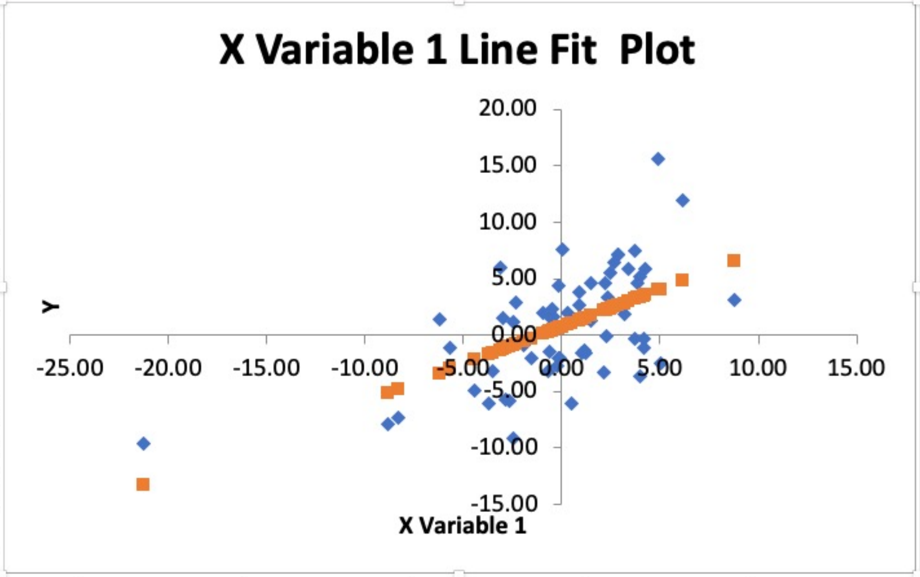


Calculating Woolworth's Beta 3



Calculating Woolworth’s Beta 4

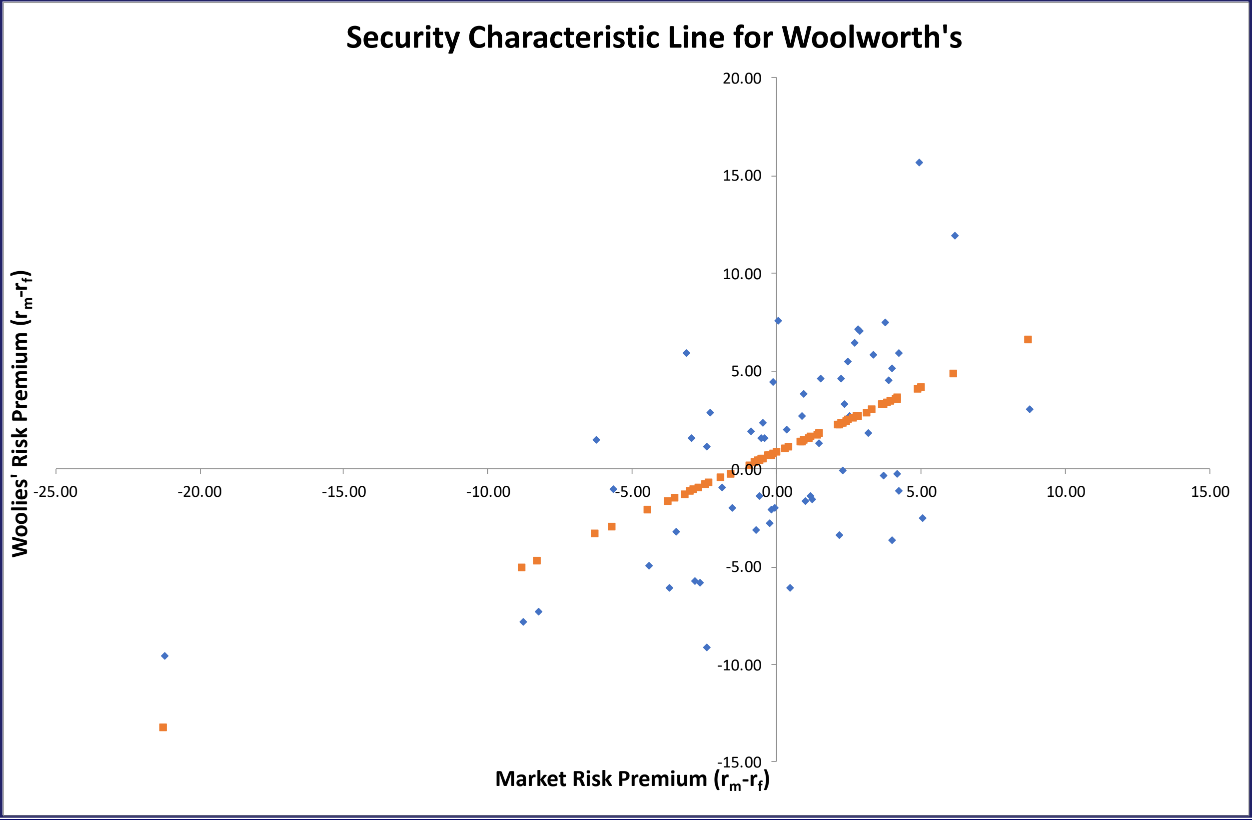
	A	B	C	D
1	SUMMARY OUTPUT			
2				
3	*Regression Statistics*			
4	Multiple R	0.584517217		
5	R Square	0.341660377		
6	Adjusted R Sq	0.330309694		
7	Standard Erro	4.091897414		
8	Observations	60		
9				
10	ANOVA			
11		*df*	*SS*	*M.*
12	Regression	1	503.9901969	503.99
13	Residual	58	971.1302178	16.743
14	Total	59	1475.120415	
15				



	A	B	C	D	E	F	G	H	I
16		*Coefficients*	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
17	Intercept	0.730114895	0.528398989	1.381749226	0.172348013	-0.3275906	1.78782039	-0.3275906	1.78782039
18	X Variable 1	0.661780179	0.120622256	5.486385341	9.4103E-07	0.420328499	0.903231859	0.420328499	0.903231859
19									

Calculating Woolworth's Beta 5

market should have beta 1



Predicting Betas

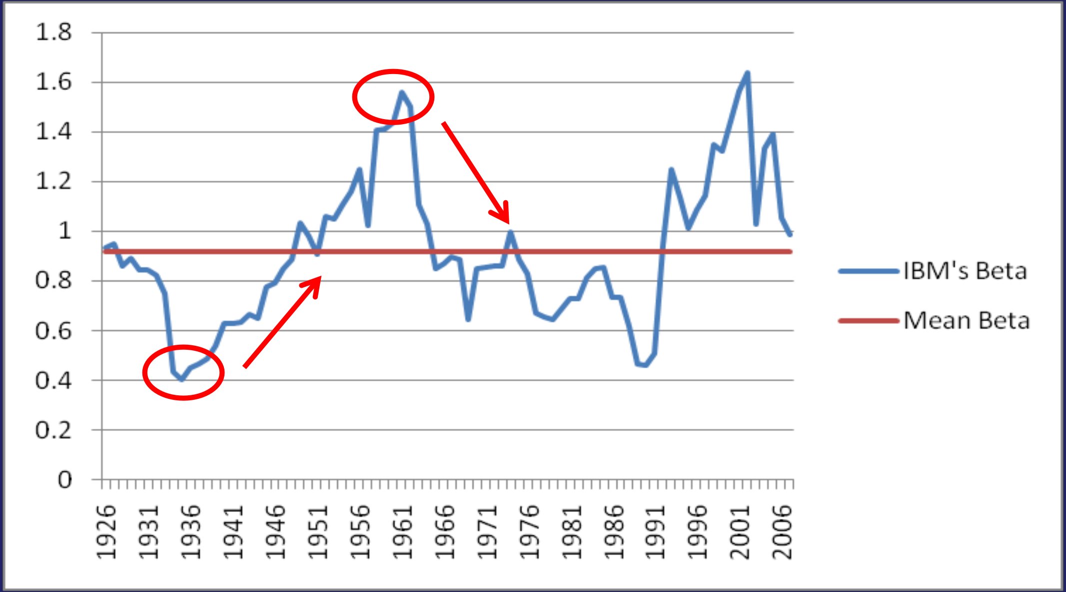
- When running a regressions, the Beta we calculate is based on historic data.

- We usually need a future beta
  - For example, for using as a cost of capital.

- Betas tend to mean revert.

Mean Reversion as Seen with IBM's Betas

mean avg should be 1



- When Beta is below the mean (typically 1) betas tend to rise.
- When Beta is above then mean betas tend to fall

Predicting Betas: Correcting for Mean Reversion

- In order to correct for mean reversion
  - Because the average beta is 1

- We calculate the following:

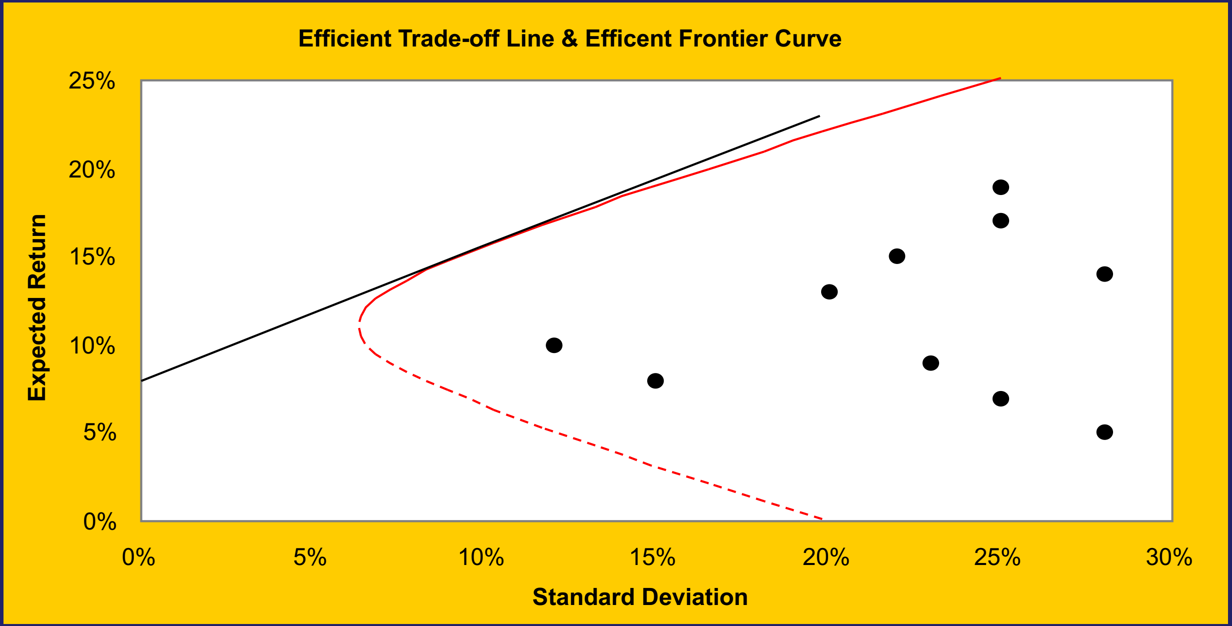
$$Adjusted\ Beta = \frac{2}{3} \times Historic\ Beta + \frac{1}{3} \times 1$$

- This is also a shrinkage estimator.
  - There are other shrinkage estimators for variances and covariances, but these are beyond the scope of this subject.

Portfolio Optimization: Using Index Models to Reduce Dimensionality

Reducing Dimensionality 1

- Consider if you want to create an efficient frontier



Reducing Dimensionality 2

- You need to calculate portfolio expected returns and variances and find the minimum:

$$E[\tilde{r}_p] = \sum_{i=1}^{N} w_i E[\tilde{r}_i]$$

$$\sigma_p^2 = \sum_{i=1}^{N} w_i^2 \sigma_i^2 + \sum_{i=1}^{N} \sum_{\substack{j=1 \\ j \neq i}}^{N} w_i w_j \sigma_{i,j}$$

Reducing Dimensionality 2

- With only 10 assets you have a variance-covariance matrix with 100 elements:

$$\begin{bmatrix} \sigma_{1,1} & \sigma_{1,2} & \sigma_{1,3} & \sigma_{1,4} & \sigma_{1,5} & \sigma_{1,6} & \sigma_{1,7} & \sigma_{1,8} & \sigma_{1,9} & \sigma_{1,10} \\ \sigma_{2,1} & \sigma_{2,2} & \sigma_{2,3} & \sigma_{2,4} & \sigma_{2,5} & \sigma_{2,6} & \sigma_{2,7} & \sigma_{2,8} & \sigma_{2,9} & \sigma_{2,10} \\ \sigma_{3,1} & \sigma_{3,2} & \sigma_{3,3} & \sigma_{3,4} & \sigma_{3,5} & \sigma_{3,6} & \sigma_{3,7} & \sigma_{3,8} & \sigma_{3,9} & \sigma_{3,10} \\ \sigma_{4,1} & \sigma_{4,2} & \sigma_{4,3} & \sigma_{4,4} & \sigma_{4,5} & \sigma_{4,6} & \sigma_{4,7} & \sigma_{4,8} & \sigma_{4,9} & \sigma_{4,10} \\ \sigma_{5,1} & \sigma_{5,2} & \sigma_{5,3} & \sigma_{5,4} & \sigma_{5,5} & \sigma_{5,6} & \sigma_{5,7} & \sigma_{5,8} & \sigma_{5,9} & \sigma_{5,10} \\ \sigma_{6,1} & \sigma_{6,2} & \sigma_{6,3} & \sigma_{6,4} & \sigma_{6,5} & \sigma_{6,6} & \sigma_{6,7} & \sigma_{6,8} & \sigma_{6,9} & \sigma_{6,10} \\ \sigma_{7,1} & \sigma_{7,2} & \sigma_{7,3} & \sigma_{7,4} & \sigma_{7,5} & \sigma_{7,6} & \sigma_{7,7} & \sigma_{7,8} & \sigma_{7,9} & \sigma_{7,10} \\ \sigma_{8,1} & \sigma_{8,2} & \sigma_{8,3} & \sigma_{8,4} & \sigma_{8,5} & \sigma_{8,6} & \sigma_{8,7} & \sigma_{8,8} & \sigma_{8,9} & \sigma_{8,10} \\ \sigma_{9,1} & \sigma_{9,2} & \sigma_{9,3} & \sigma_{9,4} & \sigma_{9,5} & \sigma_{9,6} & \sigma_{9,7} & \sigma_{9,8} & \sigma_{9,9} & \sigma_{9,10} \\ \sigma_{10,1} & \sigma_{10,2} & \sigma_{10,3} & \sigma_{10,4} & \sigma_{10,5} & \sigma_{10,6} & \sigma_{10,7} & \sigma_{10,8} & \sigma_{10,9} & \sigma_{10,10} \end{bmatrix}$$

  – You have to estimate $55(= {}^{1}/_{2}\,(10 \times 10\ - 10) + 10)$ variances and covariances.

  – 1000 assets – it's just over half a million variances and covariances

Reducing Dimensionality 3

- Stock returns can be expressed by a factor or index model:

$$r_{i,t} - r_{f,t} = \alpha_i + \sum_{k=1}^{K} \beta_{i,k}(r_{k,t} - r_{f,t}) + e_{i,t}$$

- Then the portfolio beta for each factor *k* is:

$$\beta_{P,k} = \sum_{i=1}^{N} w_i \beta_{i,k}$$

- And portfolio variance is:

$$\sigma_P^2 = \sum_{k=1}^{K}\sum_{l=1}^{K} \beta_{k,P}\beta_{l,P} cov(r_k, r_l) + \sum_{i=1}^{N} w_i^2 var(e_i)$$

  – So with 1000 stocks and 3 factors you have 1000 variances + 9 factor variances and covariances instead of half a million.

Assuming r_f is constant and each r and e represents a time series of returns or residuals respectively.

Security Analysis with the Index Model

The Treynor-Black Model

Adding or overweighting an asset in your portfolio

- Suppose you have an investment strategy that is bench marked to K portfolios, the following index model explains your return:

$$r_{i,t} - r_{f,t} = \alpha_i + \sum_{k=1}^{K} \beta_{i,k}\left(r_{k,t} - r_{f,t}\right) + e_{i,t}$$

For example, if you have a value fund with a tilt toward mining, you might use a 2-factor model with a value ETF and a mining-sector ETF as two of your K factors/risks.

- Typically, funds will be bench marked to one portfolio, such as the ASX 200 index, so we would expect something like this:

$$r_{i,t} - r_{f,t} = \alpha_i + \beta_{i,M}\left(r_{M,t} - r_{f,t}\right) + e_{i,t}$$

underpriced $2\uparrow$

overpriced $-2.$

α_i measures how much better or worse asset or portfolio i has done compared to the benchmark portfolios

Does adding an asset to your portfolio improve reward for risk?

- Suppose you do not know the actual efficient frontier, you only know the portfolios you are benchmarked against.

- How do you know whether you should add or overweight the asset?

- Ans: Suppose you have there are *N* assets plus your benchmark, you add an asset if:
  - If alpha, α, is significantly different from zero
  - And you believe alpha will persist
    - Treynor and Black (1973) note in real life that α>0 are likely to be fleeting. Without short sale constraints, α<0 should be fleeting too.

Next year add

- Add an efficient frontier and ask how you adjust the portfolio if you can identify something that is better?

- Discuss alpha to residual risk intuition

This optimization involves forecasting

- The index model:

$$r_{i,t} - r_{f,t} = \alpha_i + \beta_i\left(r_{M,t} - r_{f,t}\right) + e_{i,t}$$

- Implies:

$$E[\tilde{r}_i] - r_f = \alpha_i + \beta_i\left(E[\tilde{r}_M] - r_f\right)$$

Treynor-Black Model: Inputs

- Requires:
  - N estimates of the securities' nonmarket risk premia, $\mathrm{E}[\tilde{\alpha}_i]$, for notational simplicity, I will call these forecasts, just "α_i"
  - N estimates of the beta in the future, $\mathrm{E}[\tilde{\beta}_i]$, but I will call these, β_i
  - N estimates of the firm-specific variances, $\sigma_{e_i}^2$
  - One estimate of the market risk premium, $(E[\tilde{r}_M] - r_f)$
  - One estimate of the market variance, σ_M^2.

- A total of $(3N + 2)$ estimates – this is the real benefit
  - With a 50 security portfolio only 152 estimates are needed as opposed to 1,325 needed with a Markowitz portfolio optimization.

Treynor-Black Optimization Procedure, 1-3

1. Compute the initial position in the "active" portfolio:

$$w_i^0 = \frac{\alpha_i}{\sigma_{e_i}^2}$$

"significance"

Information ratio
Also, appraisal ratio

close to "t-stats"

2. Rescale the weights to create the weight of each asset, i, in the active portfolio:

$$w_i = \frac{w_i^0}{\sum_{i=1}^{N} w_i^0}$$

3. Compute the alpha and beta of the active portfolio:

$$\alpha_A = \sum_{i=1}^{N} w_i \alpha_i \qquad \beta_A = \sum_{i=1}^{N} w_i \beta_i$$

Treynor-Black Optimization Procedure, 4-5

4. Compute the residual variance of the active portfolio

index model should explain all covarian

$$\sigma_{e_A}^2 = \sum_{i=1}^{N} w_i^2 \sigma_{e_i}^2$$

The index model should fully explain covariances, so residual variances have no covariance among them.

5. Compute the initial weight of the active portfolio in the overall risky portfolio:

$$w_A^0 = \frac{\alpha_A / \sigma_{e_A}^2}{\left(E[\tilde{r}_M] - r_f\right) / (\sigma_M^2)}$$

Treynor-Black Optimization Procedure, 6-7

6. Adjust the initial weight allocated in the active portfolio:

$$w_A^* = \frac{w_A^0}{1 + w_A^0(1 - \beta_A)}$$

7. Calculate the weight of the passive, benchmark portfolio:

$$w_M^* = 1 - w_A^*$$

The optimized portfolio return and risk

- We can calculate the optimised risk premium on the portfolio, P, of our active and market portfolios as:

$$E[\tilde{r}_O] - r_f = w_M^*\left(E[\tilde{r}_M] - r_f\right) + w_A^*\left(\alpha_A + \beta_A\left(E[\tilde{r}_M] - r_f\right)\right)$$

$$E[\tilde{r}_O] - r_f = w_A^*\alpha_A + (w_M^* + w_A^*\beta_A)\left(E[\tilde{r}_M] - r_f\right)$$

- The variance of the optimised portfolio is then:

$$\sigma_O^2 = (w_M^* + w_A^*\beta_A)^2\sigma_M^2 + \left(w_A^*\sigma_{e_A}\right)^2$$

- The Sharpe Ratio of your new optimal portfolio is:

?

$$S_O = \sqrt{S_M^2 + \left(\frac{\alpha_A}{\sigma_{e_A}}\right)^2} = \frac{E[\tilde{r}_O] - r_f}{\sigma_O}$$

Example – Finding the New Optimal Portfolio

Asset	$E[\tilde{r}]$	Beta	σ_e
Risk-free	4%	0	0
Passive Benchmark	15%	1	15%
Stock B	30%	1.9	45%
Stock C	25%	1.2	49%
Stock D	12%	1.6	38%
Stock G	25%	0.7	22%

First find the alphas

- The index model as a forecast:

$$E[\tilde{r}_i] - r_f = \alpha_i + \beta_i\left(E[\tilde{r}_M] - r_f\right)$$

- Therefore:

$$\alpha_i = E[\tilde{r}_i] - \left\{r_f + \beta_i\left(E[\tilde{r}_M] - r_f\right)\right\}$$

$0.3 + \{0.04 + 1.9(0.15 - 0.04)\}$

$$\alpha_B = 0.30 - \{0.04 + 1.9(0.15 - 0.04)\} = 0.051$$
$$\alpha_C = 0.25 - \{0.04 + 1.2(0.15 - 0.04)\} = 0.078$$
$$\alpha_D = 0.12 - \{0.04 + 1.6(0.15 - 0.04)\} = -0.096$$ → overpriced
$$\alpha_G = 0.25 - \{0.04 + 0.7(0.15 - 0.04)\} = 0.133$$

Calculate the residual variances from standard deviations

$$\sigma_{e_B}^2 = 0.45^2 = 0.2025$$
$$\sigma_{e_C}^2 = 0.49^2 = 0.2401$$
$$\sigma_{e_D}^2 = 0.38^2 = 0.1444$$
$$\sigma_{e_G}^2 = 0.22^2 = 0.0484$$

- Step 1: Find the initial positions of each security in the portfolio
  - For example

$$w_B^0 = \frac{\alpha_B}{\sigma_{e_B}^2} = \frac{0.051}{0.2025}$$

Step 2: Rescale the weights to sum to 1

Stock	Step 1	Step 2
i	$w_i^0 = \frac{\alpha_i}{\sigma_{e_i}^2}$	$w_i = \frac{w_i^0}{\sum_{i=1}^N w_i^0}$
B	0.2519	0.0947
C	0.3249	0.1221
D	-.6648	-.2499
G	2.7479	1.0331
Sum	2.6598	1.0000

Step 3: Compute the alpha and beta of the active portfolio

$$\alpha_A = \sum_{i=1}^{N} w_i \alpha_i$$

$$= 0.0947 \times 0.051 + 0.1221 \times 0.078$$

$$+(-0.2499) \times (-0.096) + 1.0331 \times 0.133 =$$

$$\alpha_A = 0.1758$$

$$\beta_A = \sum_{i=1}^{N} w_i \beta_i$$

$$= 0.0947 \times 1.9 + 0.1221 \times 1.2$$

$$+(-0.2499) \times 1.6 + 1.0331 \times 0.7 =$$

$$\beta_A = 0.6497$$

Steps 4 & 5: active portfolio residual variance & initial weight

- Step 4: Compute the residual variance of the active portfolio

$$\sigma_{e_A}^2 = \sum_{i=1}^{N} w_i^2 \sigma_{e_i}^2$$

$$0.0947^2 \times 0.2025 + 0.1221^2 \times 0.2401$$

$$+(-0.2499)^2 \times 0.1444 + 1.0331^2 \times 0.0484 =$$

$$\sigma_{e_A}^2 = 0.0661$$

- Step 5: Compute the initial position in the active portfolio

$$w_A^0 = \frac{\alpha_A / \sigma_{e_A}^2}{\left(E[\tilde{r}_M] - r_f\right) / (\sigma_M^2)} = \frac{0.1758/0.0661}{(0.15 - 0.04)/0.0225} = 0.5441$$

Steps 6 & 7: Calculate the Optimal Portfolio Weights

- Step 6: Adjust the initial weight allocated in the active portfolio

$$w_A^* = \frac{w_A^0}{1 + w_A^0(1 - \beta_A)} = \frac{0.5441}{1 + 0.5441(1 - 0.6497)} = 0.4570$$

- Step 7: Calculate the weight of the passive, benchmark portfolio

$$w_M^* = 1 - w_A^* = 1 - 0.4570 = 0.5430$$

The optimized portfolio return and risk

- Calculate the risk premium of the optimal risky portfolio:

$$E[\tilde{r}_P] - r_f = w_A^* \alpha_A + (w_M^* + w_A^* \beta_A)\left(E[\tilde{r}_M] - r_f\right)$$

$$E[\tilde{r}_P] - r_f = 0.4570 \times 0.1758 + (0.5430 + 0.4570 \times 0.6497)(0.15 - 0.04)$$

$$E[\tilde{r}_P] - r_f = 0.1727$$

- The variance of the optimised portfolio is then:

$$\sigma_P^2 = (w_M^* + w_A^* \beta_A)^2 \sigma_M^2 + \left(w_A^* \sigma_{e_A}\right)^2$$

$$\sigma_P^2 = (0.5430 + 0.4570 \times 0.6497)^2 \times 0.15^2 + 0.4570^2 \times 0.0661$$

$$\sigma_P^2 = 0.0297$$

Compare the Sharpe Ratios

$$S_M = \frac{E[\tilde{r}_M] - r_f}{\sigma_M} = \frac{0.15 - 0.04}{0.15} = 0.7333$$

- The Sharpe Ratio of your new optimal portfolio is:

$$S_O = \sqrt{S_M^2 + \left(\frac{\alpha_A}{\sigma_{e_A}}\right)^2} = \sqrt{0.5378 + \frac{0.1758^2}{0.0661}} = 1.0021 = \frac{E[\tilde{r}_P] - r_f}{\sigma_P}$$

- Much better… if our expectation/forecasts are correct!