School of Mathematics and Statistics
MAST20009 Vector Calculus, Semester 1 2020
Assignment 3 and Cover Sheet

Student Name	*Student Number*
Tutor's Name	*Tutorial Day/Time*

Submit your assignment via the MAST20009 website before 11am on Monday 11th May.

• This assignment is worth 5% of your final MAST20009 mark.
• Assignments must be neatly handwritten in blue or black pen on A4 paper. Diagrams can be drawn in pencil.
• Full working must be shown in your solutions.
• Marks will be deducted for incomplete working, insufficient justification of steps, incorrect mathematical notation and for messy presentation of solutions.

1. Let M be that part of the disk $x^2 + y^2 \le 4$ in the x-y plane for $x \le 0$ and $y \le 0$.

   (a) Sketch M, clearly labelling any intercepts.

   (b) Evaluate the double integral
   $$\iint_M \left(x^2 + y^2\right)^{3/2} \, dxdy.$$

2. Let D be the region in the first quadrant of the x-y plane that is bounded by the hyperbolas $x^2 - y^2 = 1$, $x^2 - y^2 = 4$ and the ellipses $\dfrac{x^2}{4} + y^2 = 1$, $\dfrac{x^2}{4} + y^2 = 4$.

   (a) Sketch D, clearly labelling any intercepts and points of intersection.

   (b) Evaluate the double integral
   $$\iint_D \frac{x^3 y}{y^2 - x^2} \, dxdy$$
   by making the substitutions $u = x^2 - y^2$ and $v = \dfrac{x^2}{4} + y^2$.

3. Let B be the solid region bounded by the hemisphere $z = -\sqrt{2 - x^2 - y^2}$ and the x-y plane for $x \ge 0$, $y \ge 0$ and $z \le 0$.

   (a) Sketch B, clearly labelling any intercepts.

   (b) Calculate the total mass of B if the mass per unit volume is
   $$\mu(x, y, z) = \frac{x^4}{1 + \left(x^2 + y^2 + z^2\right)^{7/2}} \ \text{g/cm}^3.$$

4. Let R be the solid region enclosed by the cone $z = 4 - \sqrt{x^2 + y^2}$ and the paraboloid $z = \frac{1}{9}(x^2 + y^2)$.

(a) Sketch R, clearly labelling any intercepts and points of intersection.

(b) Calculate the moment of inertia of R about the z-axis if the mass per unit volume is

$$\mu(x, y, z) = y + z.$$