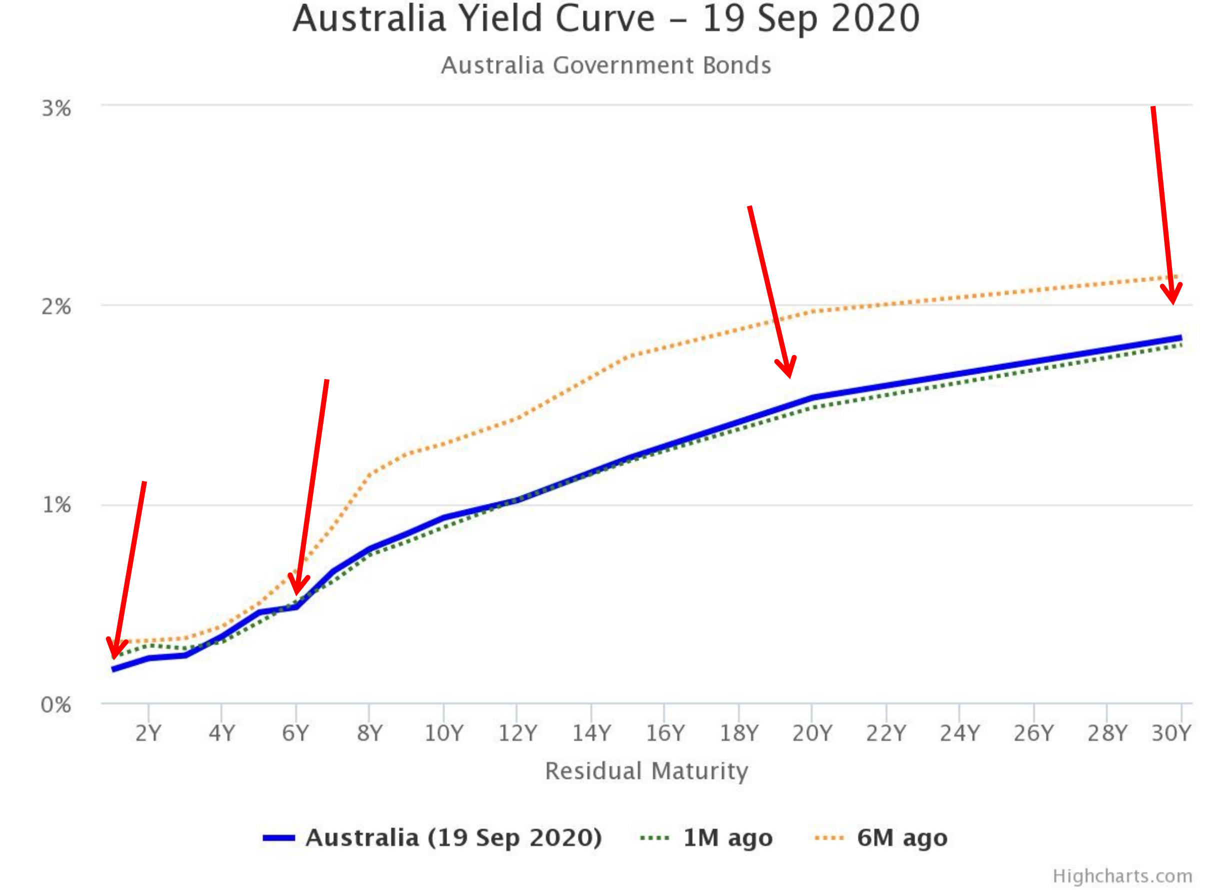
Term Structure

Dr Patrick J Kelly


Australia Yield Curve – 19 Sep 2020
Australia Government Bonds
3%
2%
1%
0%
2Y
4Y
6Y
8Y
10Y
12Y
14Y
16Y
18Y
20Y
22Y
24Y
26Y
28Y
30Y
Residual Maturity
Australia (19 Sep 2020)
1M ago
6M ago
Highcharts.com

Term Structure of Interest Rates

- A yield curve is the plot of annualized yields (Y-axis) against time-to-maturity.
  - Usually uses yields on zero coupon bonds
    - Traded → traded asset has more acurate prices
    - Implied zero rates
  - Usually Government Bonds
  - Have to be similar risk or other factors would be influencing yields
    - Could be other types of bonds, as long as the risk is similar.

- The term structure refers to the relationship between yields to maturity and maturity (term)

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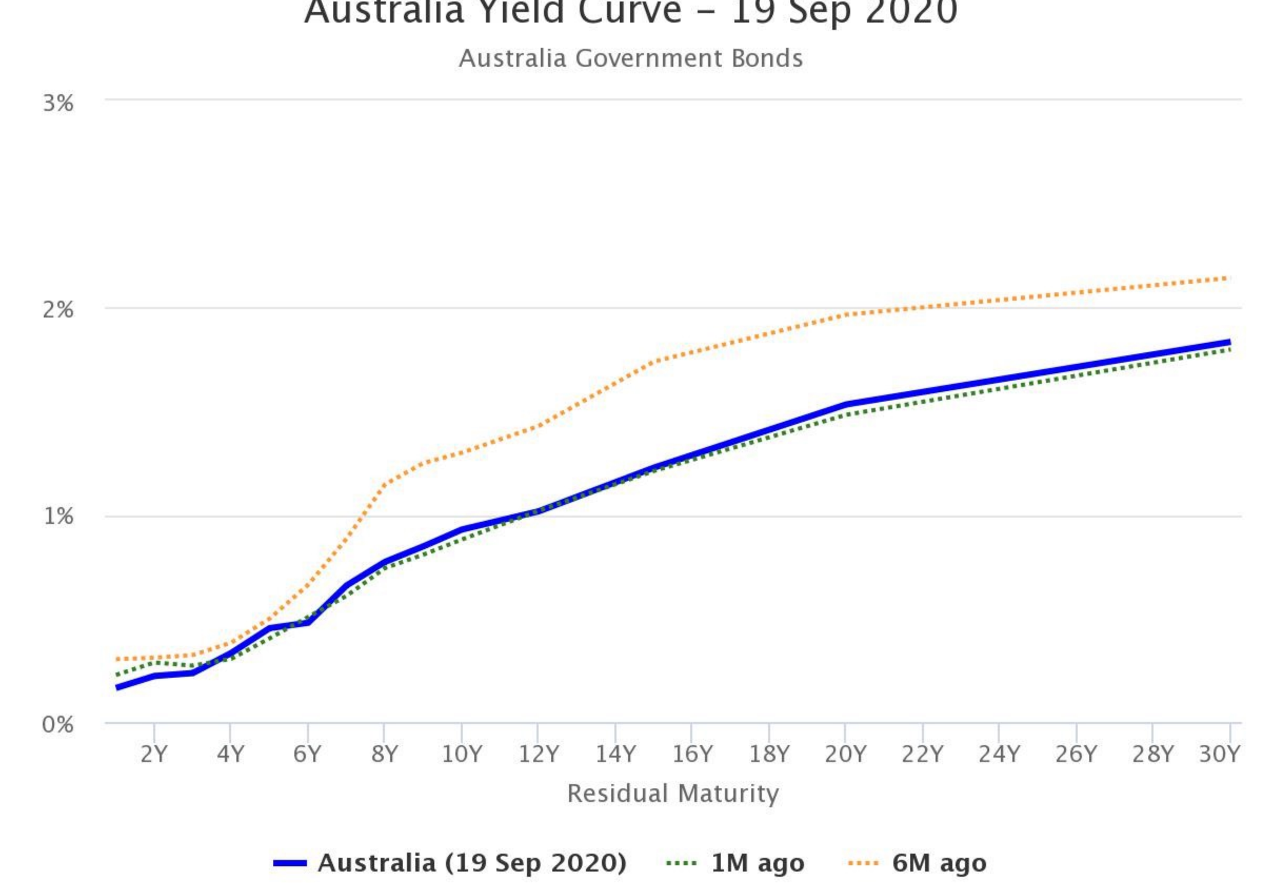

http://www.worldgovernmentbonds.com/country/australia/

The plan:

- Spot, Forward and Actual rates

- Ideas about why the term structure is not flat
  - Expectations Hypothesis
  - Liquidity Preference Hypothesis
  - Market Segmentation or Preferred Habitat

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Example

- Suppose you needed to value a new government bond with
  - one payment in 1 year of $70 per $1000 of face value and
  - a second payment in 2 years of $70 and the face value of $1000.

  - Just to make the differences more pronounced, let's suppose
    - 1 year risk-free zeros pay 6%
    - 2 year risk-free zeros pay 7% $r_{02} = 7\%$ p.a.
- Almost always when a return, interest rate, discount rate or yield is quoted, it is quoted in annualized terms
- So, this 7% is not

$$P = \frac{\$70}{1.06} + \cancel{\frac{\$1070}{1.07}}$$

Example

- Suppose you needed to value a new government bond with
  - one payment in 1 year of $70 per $1000 of face value and
  - a second payment in 2 years of $70 and the face value of $1000.

  - Just to make the differences more pronounced, let's suppose
    - 1 year risk-free zeros pay 6%
    - 2 year risk-free zeros pay 7%

$$P = \sum_{t=1}^{T} \frac{CF_t}{\left(1+r_f\right)^t} = \frac{\$70}{1.06} + \frac{\$1070}{1.07^2} = \$1000.62$$

Annualized Returns

- That is, the real 2-year rate is

$$(1.07)^2 - 1 = 0.1449$$

which is the result of a 6% rate for $r_{0\rightarrow 1}$ or $r_{0,1}$ and a 8.01% for $r_{1,2}$:

$$(1.06)\left(1 + r_{1,2}\right) - 1 = 0.1449$$

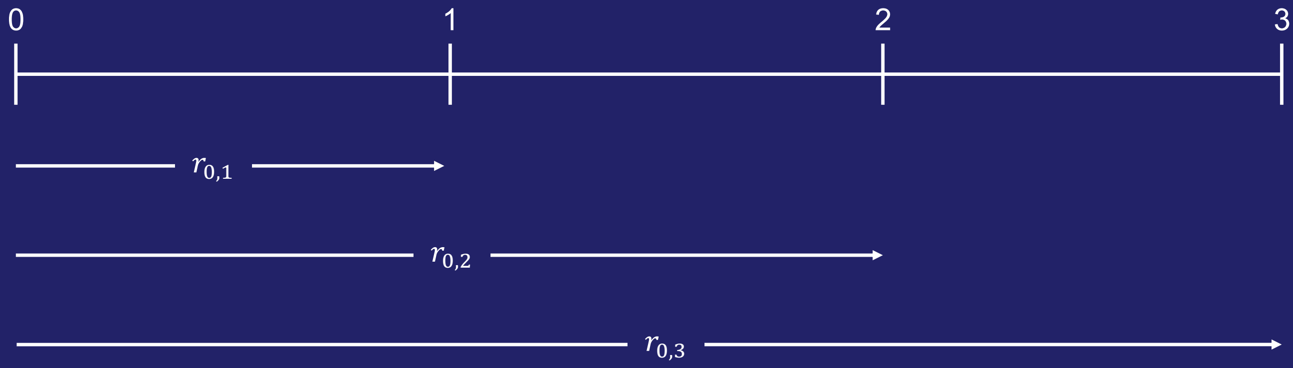
$r_{1,2}$ is a forward rate, also noted $f_{1,2}$. It is the one year interest rate that is implied by the rates from 0 to 1 and 0 to 2. $r_{1,2}$ is intentionally is not written as $\mathrm{E}[r_{1,2}]$. Forward rates are not opinions or expectations.

- 7% is the geometric average return of:
  - A 6% return
  - Followed by an 8.01% return

Spot, Forward and Actual Rates

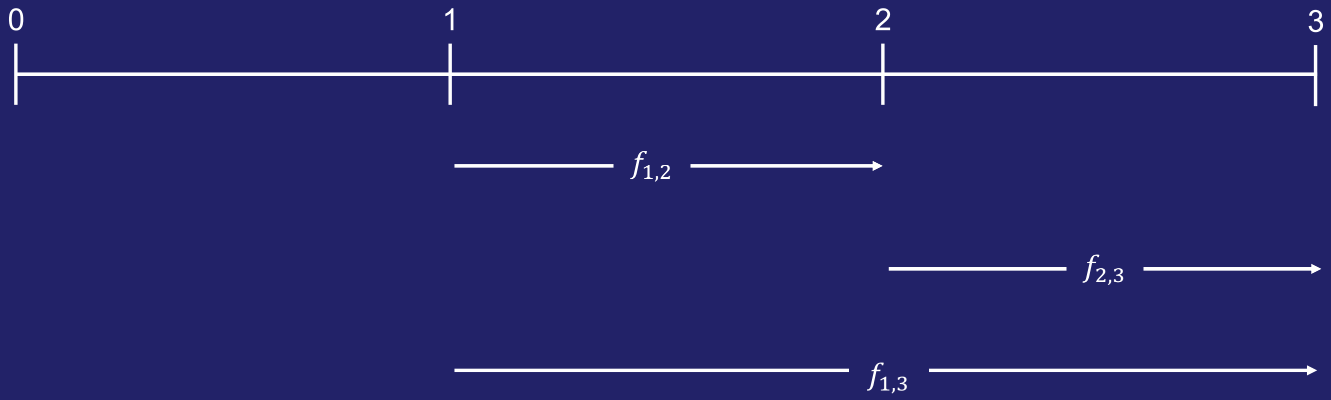
Spot Rates

- Spot rate: The interest rate for the period from today (time 0) to a future date T.



Forward Rates

- Forward rate: The interest rate for a period from a future date *t* to another a future date *T*.



What determines forward rates?

- Forward Rates are determined from current spot rates.

- Suppose the interest rate on:
  - a 3-year zero is 5% ($r_{03} = 5\%$)
  - a 5-year zero is 7% ($r_{05} = 7\%$)

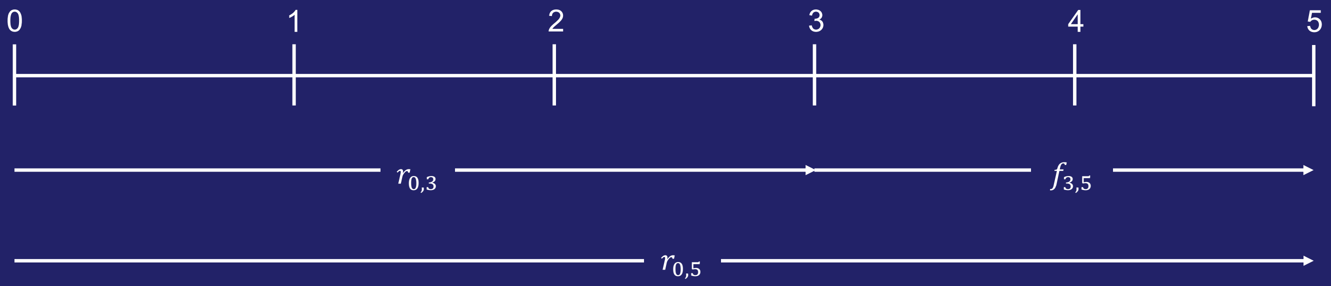
annualised !!

- What is the forward rate for a two year bond from time 3 to 5?

$(1+5\%)^3 \cdot (1+f_{35})^2 = (1+7\%)^5$

What determines forward rates?

- Example: Forward rate $f_{3,5}$



- To prevent arbitrage

$$(1 + r_{0,3})^3 (1 + f_{3,5})^2 = (1 + r_{0,5})^5$$

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Example: Forward rate $f_{3,5}$

$$\left(1 + r_{0,3}\right)^3 \left(1 + f_{3,5}\right)^2 = \left(1 + r_{0,5}\right)^5$$

- Implying:

$$(1.05)^3 \left(1 + f_{3,5}\right)^2 = (1.07)^5$$

$$f_{3,5} = \left(\frac{1.403}{1.158}\right)^{1/2} - 1$$

$$f_{3,5} = 10.1\%$$

Forward Rates Formula

- General formula for $t < T$:

$$f_{t,T} = \left[\frac{\left(1 + r_{0,T}\right)^{T}}{\left(1 + r_{0,t}\right)^{t}}\right]^{1/(T-t)} - 1$$

Try calculating forward rates on your own

- What is the forward rate from t=3 to T=6 $(f_{3,6})$, if:
  - $r_{0,3} = 4\%$
  - $r_{0,6} = 6\%$

$$f_{3,6} = \left[\frac{1.06^6}{1.04^3}\right]^{1/3} - 1 = 8.03\%$$

Try calculating forward rates on your own

- What is the forward rate from t=3 to T=6 $\left(f_{3,6}\right)$, if:
  - $r_{0,3} = 4\%$
  - $r_{0,6} = 6\%$

$$f_{t,T} = \left[\frac{\left(1 + r_{0,T}\right)^{T}}{\left(1 + r_{0,t}\right)^{t}}\right]^{1/(T-t)} - 1$$

$$f_{t,T} = \left[\frac{(1 + .06)^{6}}{(1 + .04)^{3}}\right]^{1/3} - 1 = 8.0\%$$

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Example 1: Locking in future rates

- Interest rates are:
  - a 3-year zero is 5% $(r_{03} = 5\%)$
  - a 5-year zero is 7% $(r_{05} = 7\%)$

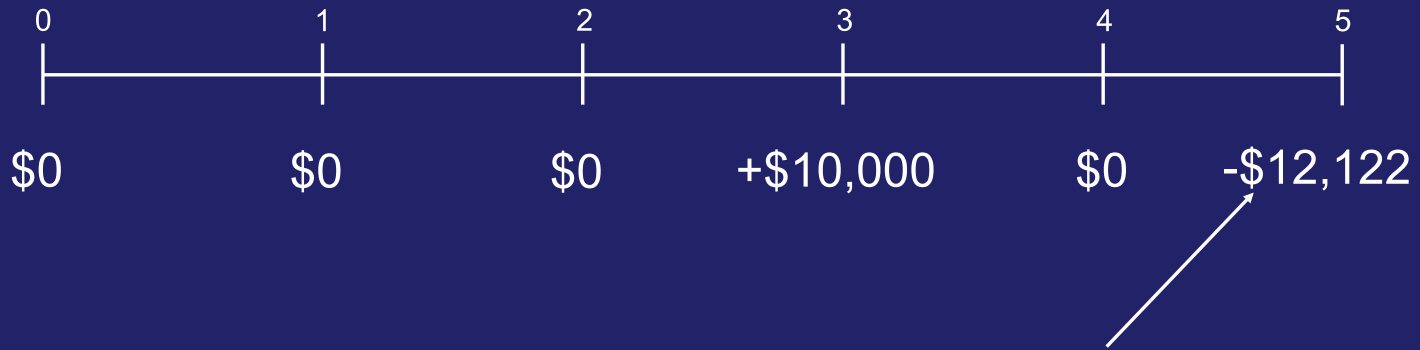
- Recall that these rates imply

$$f_{3,5} = 10.1\%$$

- Suppose you would like to borrow $10K in 3 years from time 3 to time 5, but you would like to guarantee the rate of 10.1% you could get on that loan today.

Example 1: Locking in future rates

- Suppose you would like to borrow \$10K in 3 years from time 3 to time 5, but you would like to guarantee the rate of 10.1% you could get on that loan today.



$$(1.101)^2 \times 10{,}000 = 12{,}122$$

Example 1: Locking in future rates

- Borrow \$10K in 3 years from time 3 to time 5, but you would like to guarantee the rate of 10.1% you could get on that loan today.
  - a 3-year zero is 5% ($r_{03} = 5\%$)
  - a 5-year zero is 7% ($r_{05} = 7\%$)
  - Borrow $\frac{\$10{,}000}{(1+r_{0,3})^3}$ =\$8638 today from a 5-year loan at a rate of $r_{0,5}$.
  - Invest \$8638 today in a 3-year bond at a rate of $r_{0,3}$.

+8638
-8638
=\$0

Example 1: Locking in future rates

- a 3-year zero is 5% ($r_{03} = 5\%$)
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- Borrow $\frac{\$10{,}000}{(1+r_{0,3})^3}$ =\$8638 today from a 5-year loan at a rate of $r_{0,5}$.
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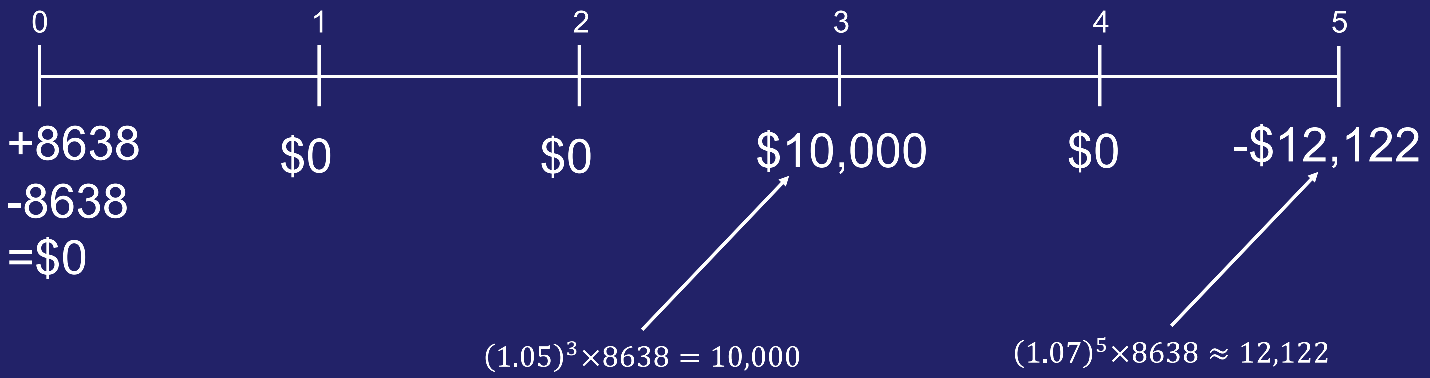
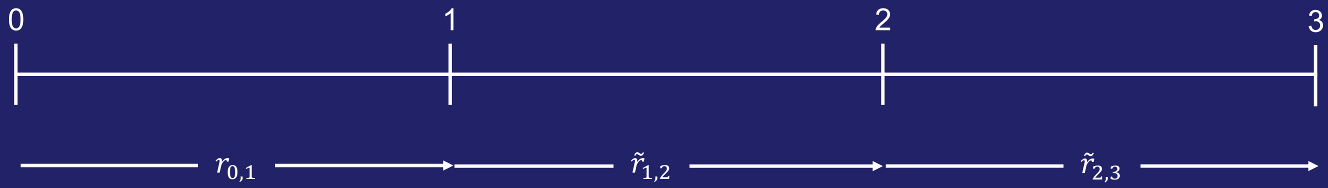


Actual Rates



"~" actual rate is not known

r.v with some distribution

Term Structure

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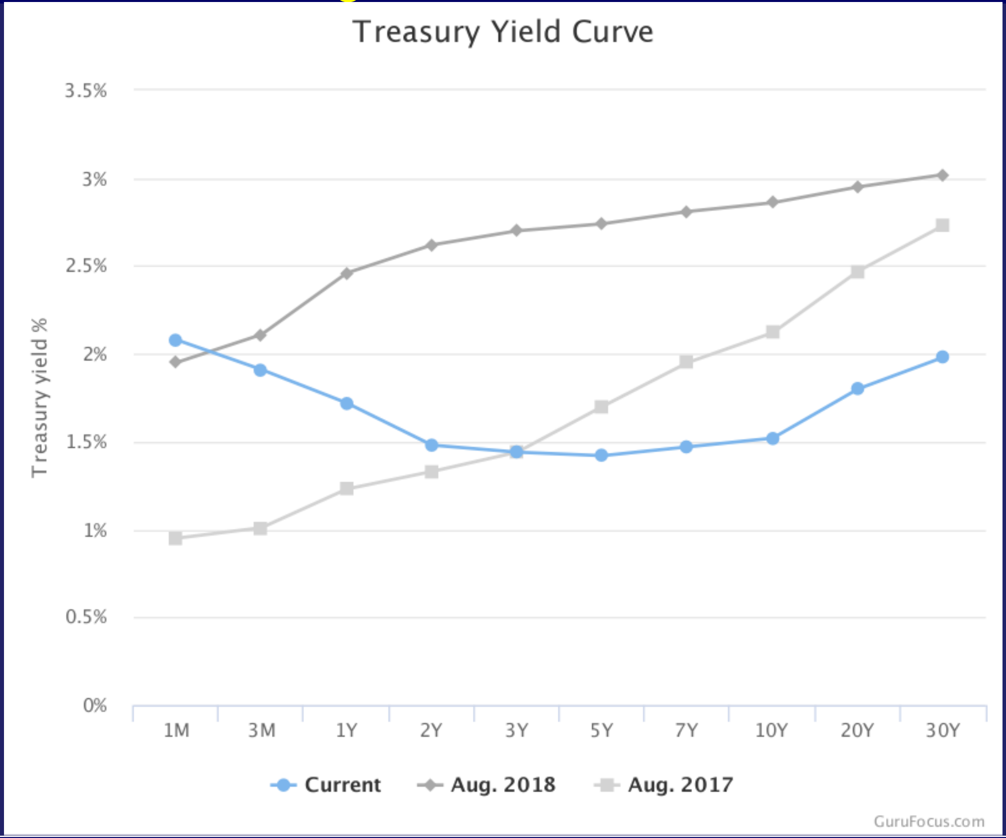
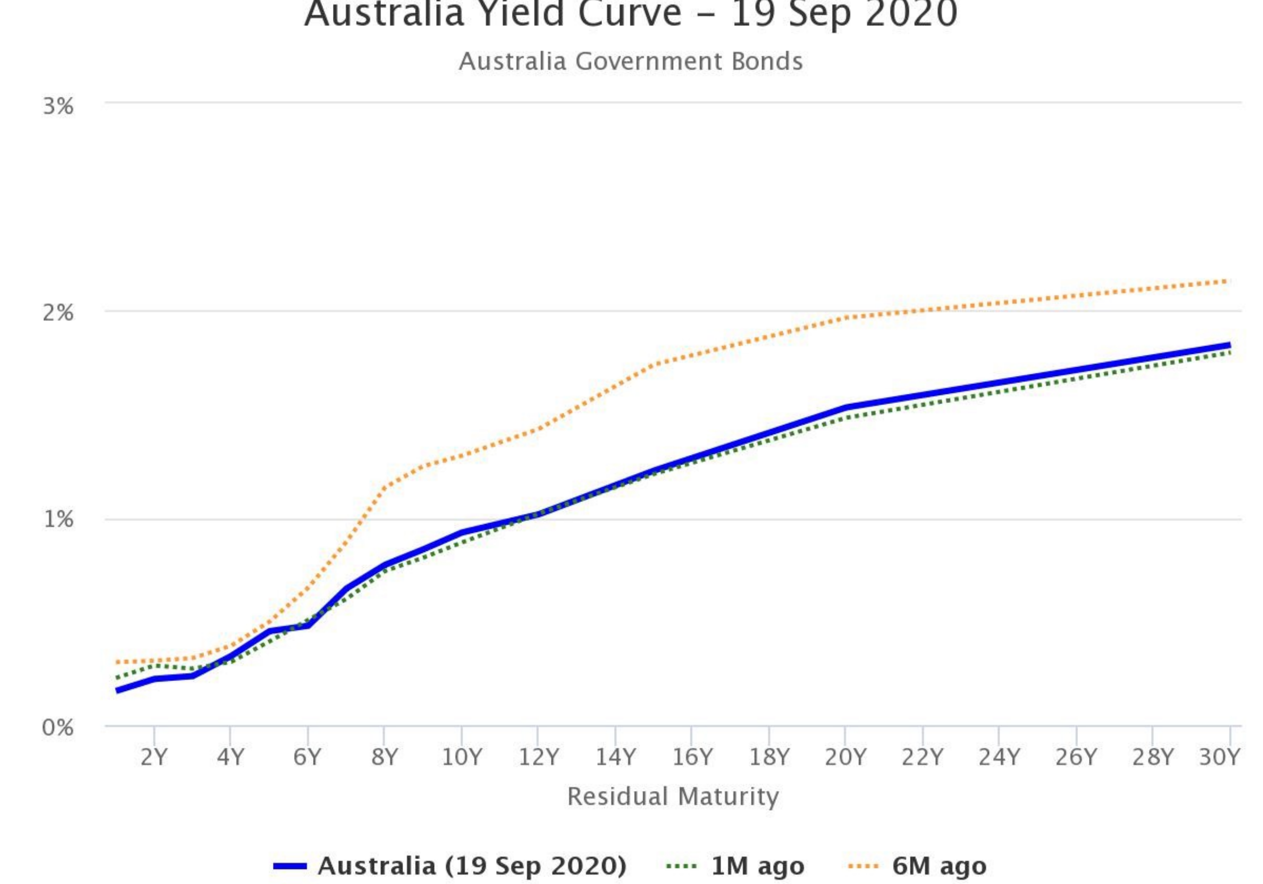
U.S. Yield Curve: 15 August 2019

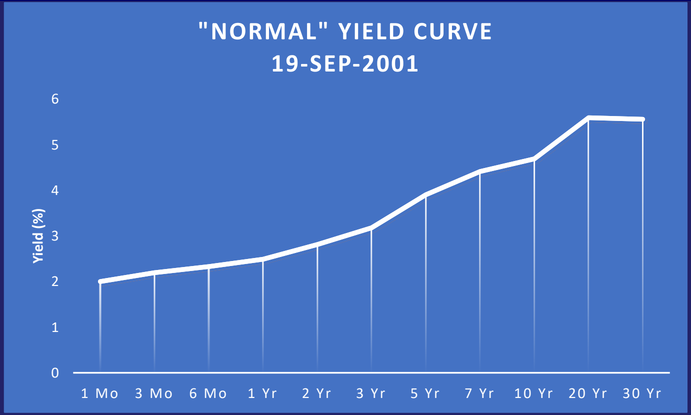


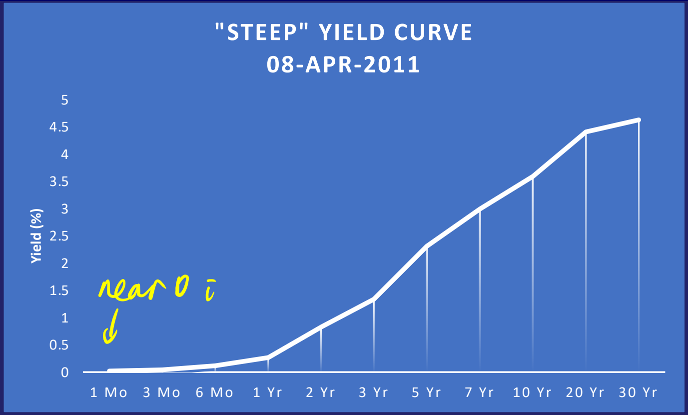
https://www.gurufocus.com/yield_curve.php

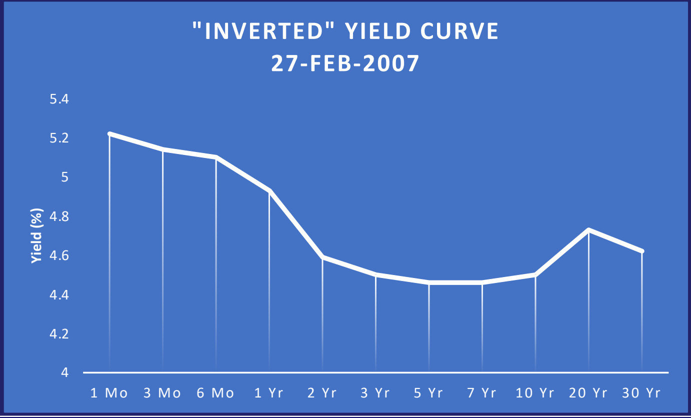


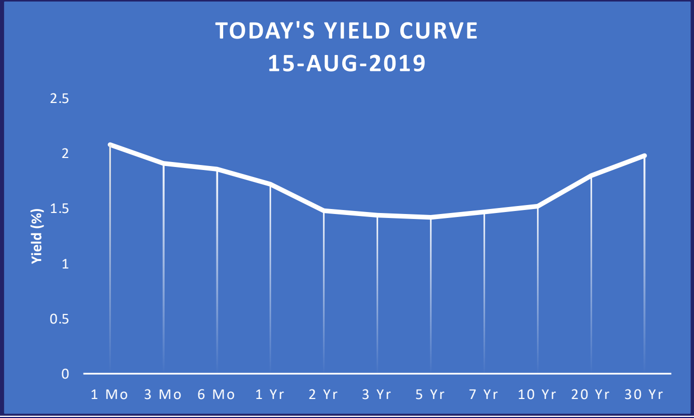
http://www.worldgovernmentbonds.com/country/australia/

Types of Yield Curves (examples are from the U.S.)



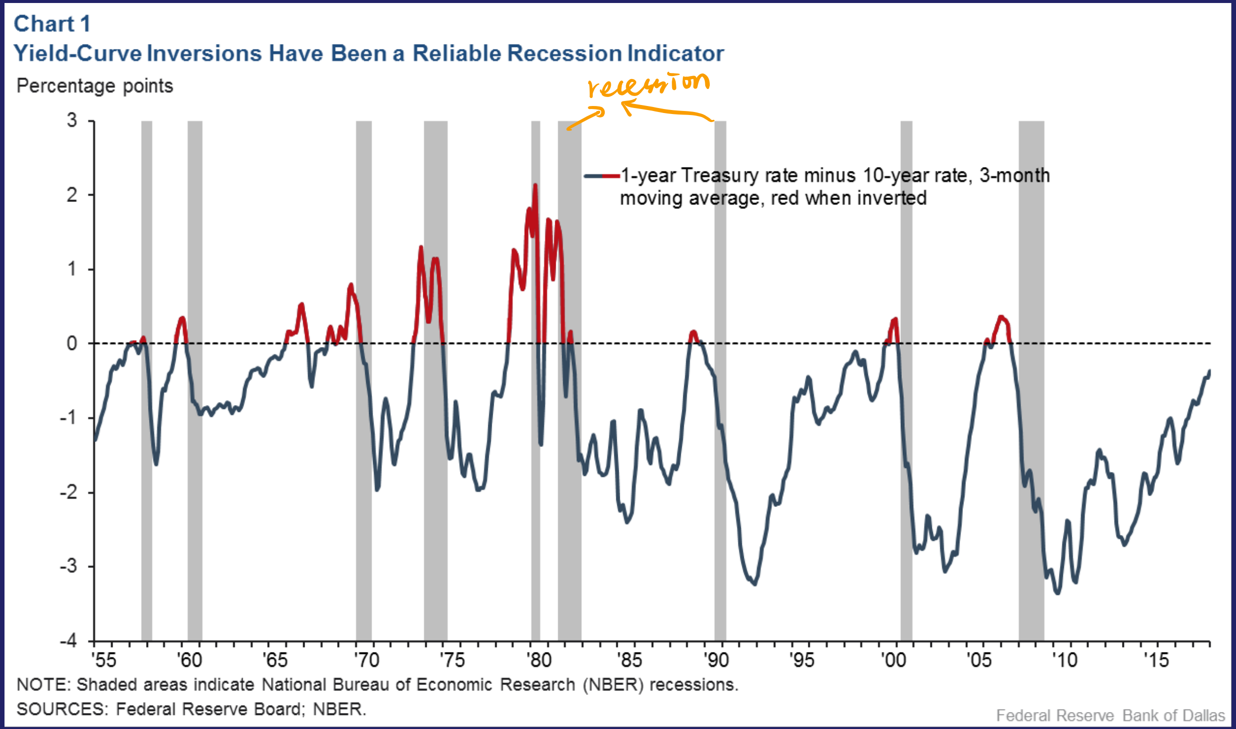






The Inverted Yield Curve

above 0:
inverted yield curve



https://www.dallasfed.org/research/economics/2019/0212

What about Australia?

- Inverted yield curves do not predict recessions

- Why?

- Here's a guess:
  - Long Term Bond rates in Australia are strongly affected by interest rates in the US.
    - So, to the extent the Australian and US economies are not perfectly corrected, the yield curve is less likely to be predictive

Australian debt may perform like a safe substitute for investor

when US i ↓, investor would put money in Australia

⇒ change in interest rate in AU may be dominated by thing abroad, not tie with AU

Explaining the shape of the yield curve

- The Expectations Hypothesis
- The Liquidity Premium Hypothesis
- Market Segmentation or Preferred Habitat Hypothesis

Expectations Hypothesis

- The yield curve derives from investor expectations about future interest rates:

$$\left(1 + r_{0,2}\right)^2 = \left(1 + r_{0,1}\right)\left(1 + E\left[\tilde{r}_{1,2}\right]\right)$$

- 2-year rates, $r_{0,2}$, are higher than 1-year rates, $r_{0,1}$, if

$$E\left[\tilde{r}_{1,2}\right] > r_{0,1}$$

if we see an inverted yield curve, then $E[\tilde{r}_{1,2}] < r_{0,1}$

investor believes interest rate will be lower in one year

Implication of the Expectations Hypothesis

- If investors are risk-neutral then

$$E\left[\tilde{r}_{t,T}\right] = f_{t,T}$$

- forward rates provide unbiased predictions about future spot rates.

one way to test Expectation hypothesis,
is to see whether forward rate is a good predictor
about future spot rate

How well do forward predict spot rates?

- Not well.
  - At least partially because investors are not risk neutral

- If investors are not risk neutral then:

risk averse

$$f_{t,T} = E\left[\tilde{r}_{t,T}\right] + risk\ premium$$

- Long-term bonds are associated with greater interest-rate risk (Duration) and demand a higher premium because investors are risk averse.

(upward sloping yield curve)

https://papers.ssrn.com/sol3/papers.cfm?abstract_id=40165

The Liquidity Premium Hypothesis

- Investors in long-term debt are exposed to more interest rate risk
  - Risk-averse investors may prefer shorter-term, more liquid bonds

$$f_{t,T} = E\left[\tilde{r}_{t,T}\right] + liquidity\ premium$$

- Borrowers may have a preference for long-term fixed rates.
  - Long-term borrows pay a premium for long-term fixed rates.

Borrowers may be willing to pay a liquidity premium in order to avoid interest risk

- The liquidity premium is larger as the maturity gets longer

A Liquidity Premium

ANZ Advance Notice Term Deposit interest rates		
	Interest at Maturity	Monthly Interest
7 days to less than 1 month	1.00% p.a.	1.00% p.a.
1 to less than 2 months	1.05% p.a.	1.05% p.a.
2 to less than 3 months	1.20% p.a.	1.20% p.a.
3 to less than 4 months	1.60% p.a.	1.60% p.a.
4 to less than 5 months	1.45% p.a.	1.45% p.a.
5 to less than 6 months	1.90% p.a.	1.89% p.a.
6 to less than 7 months	1.60% p.a.	1.59% p.a.
7 to less than 8 months	1.45% p.a.	1.44% p.a.
8 to less than 9 months	1.45% p.a.	1.44% p.a.
9 to less than 10 months	1.45% p.a.	1.44% p.a.
10 to less than 11 months	1.45% p.a.	1.44% p.a.
11 to less than 12 months	1.65% p.a.	1.64% p.a.
12 to less than 18 months		1.69% p.a.
18 to less than 24 months		1.59% p.a.
24 to less than 36 months		1.59% p.a.
36 to less than 48 months		1.64% p.a.
48 to less than 60 months		1.69% p.a.
60 months		1.69% p.a.

generally increase

https://www.anz.com.au/personal/bank-accounts/your-account/rates-fees-terms/#termdeposit

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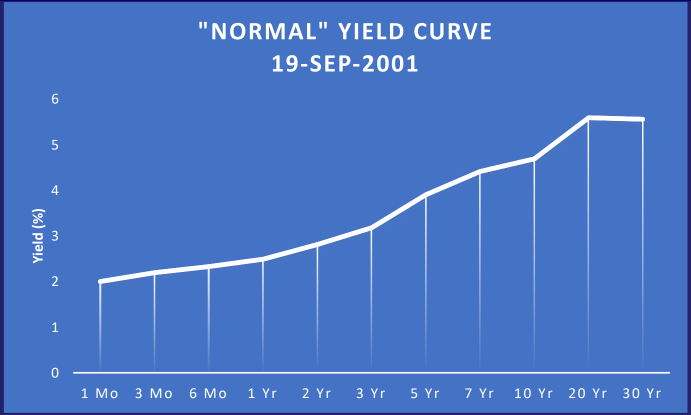
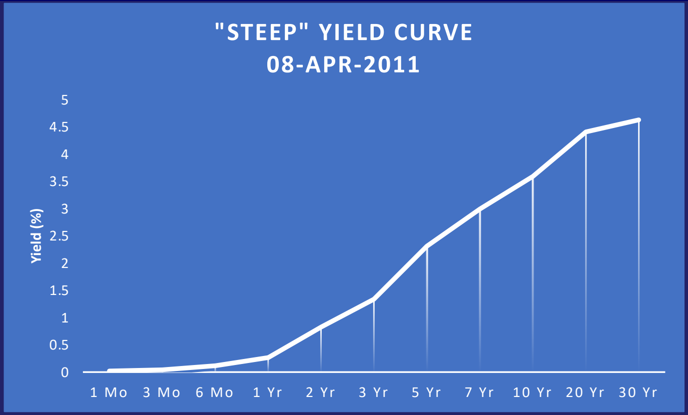
Market Segmentation or Preferred Habitat Hypothesis

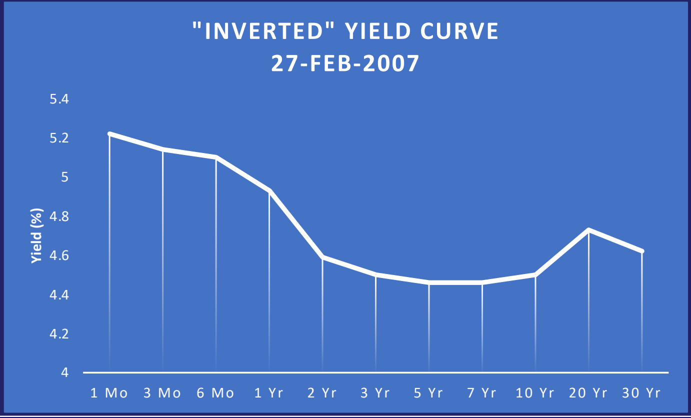
- Different types of investors may have different investing needs
  - Insurance companies are long term investors
  - Banks are short term investors

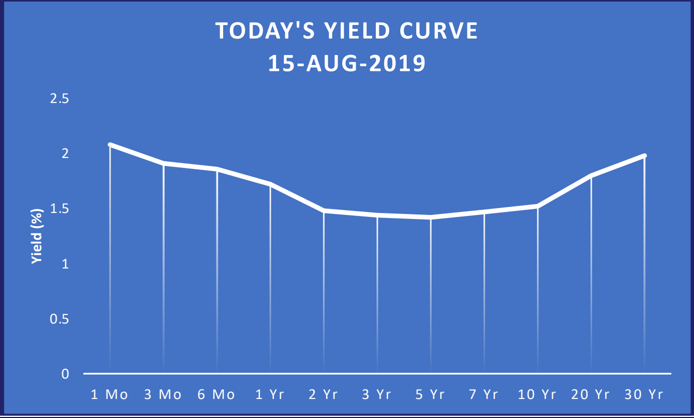
- The yields for short- and long-term debt are somewhat independent
  - The preferred habitat hypothesis says that if an investor is adequately compensated they may deviate from their preferred habitat.

Types of Yield Curves (examples are from the U.S.)









Types of Yield Curves

- **Normal Yield Curve**: Short-term rates lower. Long-term higher
  - Expectations H.: investors believe long-term rates are generally higher in the future
  - Liquidity H.: compensation for interest-rate risk of longer-term
- **Steep Yield Curve**:
  - Expectations H.: Future rates will be much higher (strong economy)
  - Liquidity H.: Short rates low due to a high demand for liquidity and high interest-rate risk.

high demand for liquidity → investor need a high interest compensation for long term debt

- **Inverted Yield Curve**: Long-term are lower than short term.
- Recession is forthcoming and investors are locking in higher long term rates or getting into long duration assets before rates drop

Why does the yield curve invert?

- Argument 1: flight to higher quality investments
  - If investors sense instability, they may increase their purchases of long-term interest baring securities
  - High demand to lock in higher interest rates puts demand pressure on the prices of long-term debt, pushing up prices and yields down.

- Argument 2:
  - Investors expect lower interest rates in the future
    - Either because a weak economy reduces the demand for capital or because
    - During recessions central banks cut interest rates

  - If interest rates are expected to go down, the greatest profit comes from investing in long duration assets.
  - Pushing prices up for long-term debt and yields down

Why do we care about the yield curve?

1. Provides information on the risk-free rates at different maturities
   – Another reason why a yield curve made up of zero coupon bonds is useful

2. Recession Forecasting → inverted yield curve

3. Bond trading strategies

Example: Different risk-free rates

- Please price a risk-free bond with an 3% annual coupon-paying bond with exactly 3 years to maturity. Government zeros are yielding 2%, 3% and 4% for 1, 2 and 3 years to maturity respectively.

$$P = \frac{3}{1.02} + \frac{3}{1.03^2} + \frac{103}{1.04^3}$$

V.

$z_{01} = 2\%$ p.a

$z_{02} = 3\%$ pa

$z_{03} = 4\%$ pa

Example: Different risk-free rates

- Please price a risk-free bond with an 3% annual coupon-paying bond with exactly 3 years to maturity. Government zeros are yielding 2%, 3% and 4% for 1, 2 and 3 years to maturity respectively.

Example: Different risk-free rates

- Please price a risk-free bond with an 3% annual coupon-paying bond with exactly 3 years to maturity. Government zeros are yielding 2%, 3% and 4% for 1, 2 and 3 years to maturity respectively.

$$P = \frac{\$3}{1.02} + \frac{\$3}{1.03^2} + \frac{\$103}{1.04^3}$$

$$P = 2.94 + 2.83 + 91.57$$

$$P = \$97.34$$

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Implied Zero Yields

- Many markets do not have zero coupon yields at all (or even most) maturities.

- It is possible to construct using implied zero yields from the prices of existing coupon bonds.
  - We get these market prices from the various traded coupon bonds
  - This approach is called "bootstrapping"

Bootstrapping the Yield Curve

- Suppose we are able to observe the market prices of three coupon bonds
  - The bonds have exactly 1, 2 and 3 years before their respective maturity dates

- To keep it simple, we will also assume that these bonds pay annual coupons

- Use the following new notation:
  - $P_{0,T}$ is the price of a T-year bond at time 0
    - $P_{0,2}$ is the price of a 2-year bond at time 0
  - C_T is the (fixed) coupon paid on a T-year bond
    - C_2 is the (fixed) coupon paid on a 2-year bond

Bootstrapping the yield curve

- The price of each coupon bond should be:

$$P_{0,1} = \frac{C_1 + 100}{1 + r_{0,1}} \Rightarrow r_{0,1}$$

$$P_{0,2} = \frac{C_2}{1 + r_{0,1}} + \frac{C_2 + 100}{(1 + r_{0,2})^2} \Rightarrow r_{0,2}$$

$$P_{0,3} = \frac{C_3}{1 + r_{0,1}} + \frac{C_3}{(1 + r_{0,2})^2} + \frac{C_3 + 100}{(1 + r_{0,3})^3} \Rightarrow r_{0,3}$$

- We can solve for $r_{0,1}$, $r_{0,2}$, $r_{0,3}$ in this system of equations sequentially

Bootstrapping the yield curve: numerical example

- Suppose we observe the following bond yields (YTM):
  - 1-year bond with 12.00% coupon: 9.65% pa
  - 2-year bond with 9.75% coupon: 10.174% pa
  - 3-year bond with 6.50% coupon: 10.351% pa

$r_{01} = 9.65\%$ pa

$\frac{9.75}{1.10174} + \frac{109.75}{(1.10174)^2}$

- Use bootstrapping to identify the zero-rate curve

- First, (except for the 1-year bond) we need to know what the prices are first….

Bootstrapping the yield curve: get the prices

- The 1-year bond is effectively a zero
- $r_{0,1} = 9.650$

- Using the bond yields, we calculate the bond prices that lead to these yields as follows:

(1+YTM) (1+YTM)^2

$$P_{0,2} = \frac{9.75}{1.10174} + \frac{109.75}{1.10174^2} = \$99.265847$$

$$P_{0,3} = \frac{6.50}{1.10351} + \frac{6.50}{1.10351^2} + \frac{106.50}{1.10351^3} = \$90.482004$$

Bootstrapping the yield curve: numerical example

- We solve iteratively

$$P_{0,2} = \$99.265847 = \frac{9.75}{1.0965} + \frac{109.75}{\left(1+r_{0,2}\right)^2}$$

$$\frac{109.75}{\left(1+r_{0,2}\right)^2} = 90.373918$$

$$r_{0,2} = 1.2143990^{0.5} - 1 = 10.20\%$$

Bootstrapping the yield curve: numerical example

- We solve iteratively

$$P_{0,3} = 90.482004 = \frac{6.50}{1.0965} + \frac{6.50}{1.102^2} + \frac{106.50}{(1+r_{0,3})^3}$$

$$r_{0,3} = 10.375\%$$

Some things to keep in mind

- Zero rates from risk-free debt can be used as discount rates for other risk-free debt.
  - And (of course) zero rates can be used as the the risk-free rate for risky bonds (and stock!)

- Zero rates from corporate debt imbed default risk and cannot be used directly as discount rates
  - Although, if default rates and recovery rates are similar between the corporate debt used to calculate the zero and the cash flows from the bond being discounted, then errors probably will not be too high.

Riding the Yield Curve

A strategy that profits from risk and liquidity premia

If the interest rates don’t increase

If the expectations hypothesis is not true

Riding the Yield Curve: you believe the curve won't change

$$f_{t,T} = E\left[\tilde{r}_{t,T}\right] + liquidity\ premium$$

2 year bond will be more heavily discounted than one year bond

- The Strategy buys a long-term bond and sells it before maturity
- Riding the yield curve allows the bond holder to profit from the decline yield that happens over the life of a bond when there is a normal, upward sloping yield curve that is stable.
- Implicitly (explicitly?) this strategy assumes that there is a risk or liquidity premium that drives upward sloping curve
  - If the expectations hypothesis is driving the upward sloping yield curve, then this strategy won't work.

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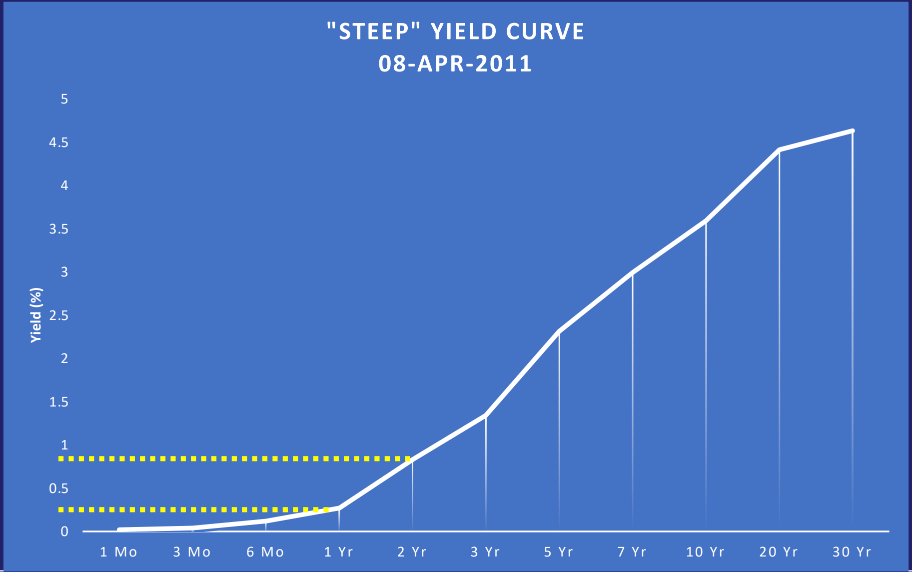
Riding the Yield Curve: you believe the curve won't change

$$f_{t,T} = E\left[\tilde{r}_{t,T}\right] + liquidity\ premium$$

- The Strategy buys a long-term bond and sells it before maturity

if at year 1, interest rate for 1 year bond becomes higher, P ↓. no additional profit



yield ↓.

P ↑

From Long-Term Predictability: The Effect is Pervasive!

- High yield/valuation → High Expected Returns
  - Not low future cash flows or lower prices

- Stocks: High D/P →High Returns
  - Not low corporate profits

- US Treasuries: Rising Yield Curve →High 1-year Returns for long-term bonds,
  - not rising interest rates

- Bonds/Credit Spreads: High Yields →High Returns,
  - not default (i.e. lower profits)

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Riding the Yield Curve

- A $100,000 180-day Treasury note yields 3% annually and sells for $98,542.12, while a 90-day bill that yields 1% sells for $99,754.03.

if the yield curve stays the same, in 90 days, this 180-day Treasury note should be priced $99754.03

- Buying the longer-term security and holding it for 90 days and then selling it for the existing 90-day price (which we assumed would not change) results in earnings of $1,211.91.

$$\frac{\$1211.91}{\$98,542.12} \times \frac{365}{90} = 4.99 > 1\%$$

- More generally:
  - The steeper the yield curve, the more the profits
  - If rates rise, you could lose money.

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