PHYC10003 Physics 1

Lecture 12: linear momentum

Elastic collisions, energy and rockets

Last lecture

- Centre of Mass
- Centre of Mass-motion
- Linear momentum
- Impulse and collisions
- Conservation of momentum

Key learning objectives

Distinguish between elastic collisions, inelastic collisions, and completely inelastic collisions.

Identify a one-dimensional collision as one where the objects move along a single axis, both before and after the collision.

Apply the conservation of momentum for an isolated one-dimensional collision to relate the initial momenta of the objects to their momenta after the collision.

Identify that in an isolated system, the momentum and velocity of the centre of mass are not changed even if the objects collide.

9-6 Elastic and inelastic collisions

- Types of collisions:
- **Elastic collisions**:
  - Total kinetic energy is unchanged (conserved)
  - A useful approximation for common situations
  - In real collisions, some energy is always transferred
- **Inelastic collisions**: some energy is transferred
- **Completely inelastic collisions**:
  - The objects stick together
  - Greatest loss of kinetic energy

9-6 Inelastic and *completely* inelastic collisions

- For one dimension:
- Inelastic collision

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}.$$

Eq. (9-51)

- Completely inelastic collision, for target at rest:

$$m_1 v_{1i} = (m_1 + m_2)V$$

Eq. (9-52)

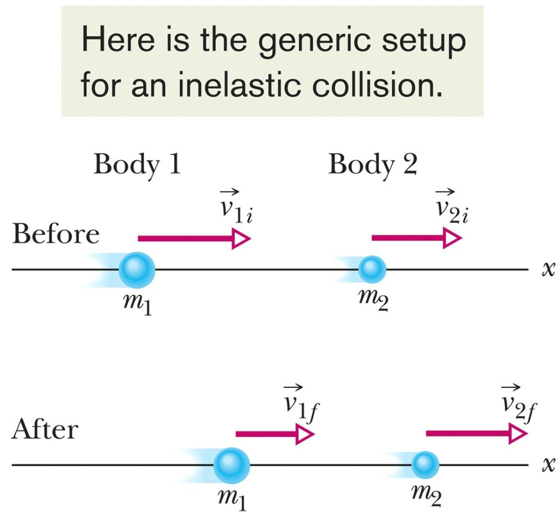


Figure 9-14

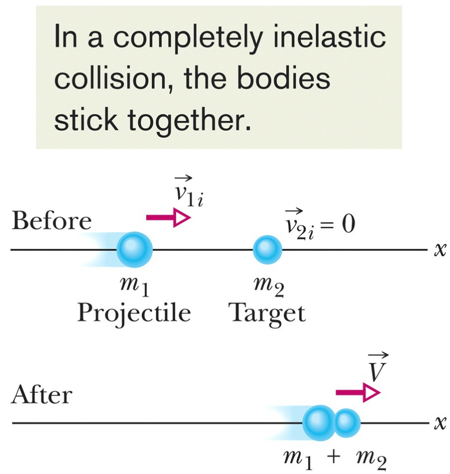
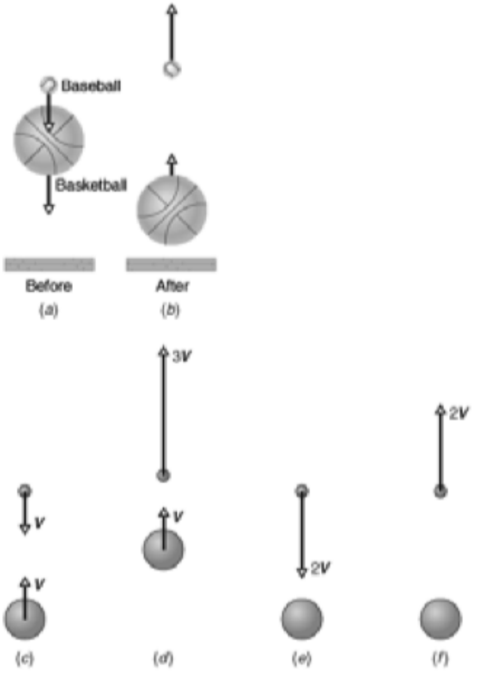


Figure 9-15

Double and Triple Bounce



https://www.youtube.com/watch?v=2UHS883_P60

9-6 Completely inelastic collision

- The centre of mass velocity remains unchanged:

$$\vec{v}_{com} = \frac{\vec{P}}{m_1 + m_2} = \frac{\vec{p}_{1i} + \vec{p}_{2i}}{m_1 + m_2}.$$

Eq. (9-56)

- Figure 9-16 shows freeze frames of a completely inelastic collision, showing centre of mass velocity

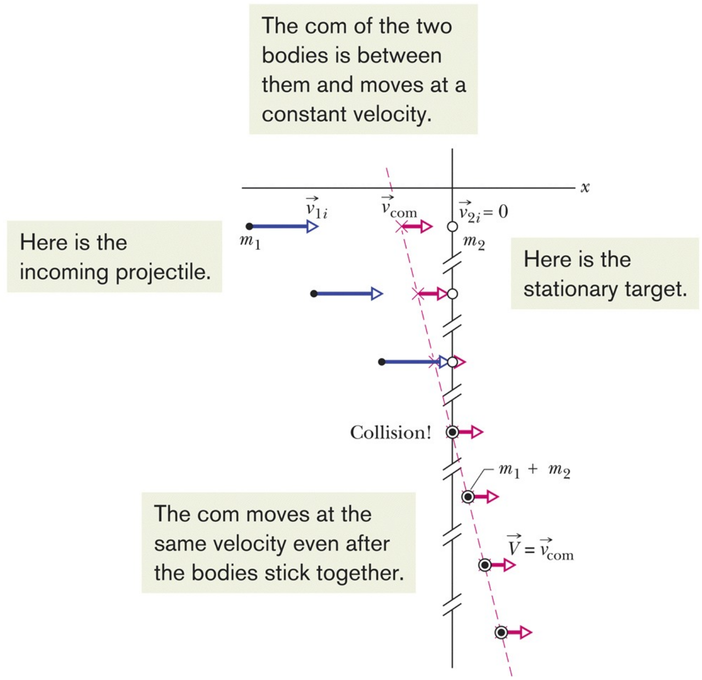


Figure 9-16

9-7 Kinetic energy in collisions

- Total kinetic energy is conserved in elastic collisions

> In an elastic collision, the kinetic energy of each colliding body may change, but the total kinetic energy of the system does not change.

- For a stationary target, conservation laws give:

$$m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f} \quad \text{(linear momentum).}$$ **Eq. (9-63)**

$$\tfrac{1}{2} m_1 v_{1i}^2 = \tfrac{1}{2} m_1 v_{1f}^2 + \tfrac{1}{2} m_2 v_{2f}^2 \quad \text{(kinetic energy).}$$ **Eq. (9-64)**

Here is the generic setup for an elastic collision with a stationary target.

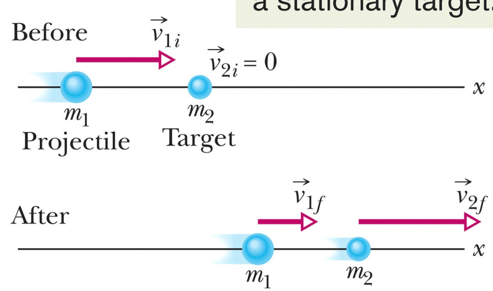


Figure 9-18

9-7 Behaviour depends on masses

- With some algebra we get:

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} \qquad \text{Eq. (9-67)}$$

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}. \qquad \text{Eq. (9-68)}$$

- Results
  - Equal masses: $v_{1f} = 0$, $v_{2f} = v_{1i}$: the first object stops
  - Massive target, $m_2 >> m_1$: the first object just bounces back, speed mostly unchanged
  - Massive projectile: $v_{1f} \approx v_{1i}$, $v_{2f} \approx 2v_{1i}$: the first object keeps going, the target flies forward at about twice its speed

9-7 Collisions and moving targets

- For a target that is also moving, we get:

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2m_2}{m_1 + m_2} v_{2i}$$ **Eq. (9-75)**

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} + \frac{m_2 - m_1}{m_1 + m_2} v_{2i}.$$ **Eq. (9-76)**

Here is the generic setup for an elastic collision with a moving target.

Figure 9-19

Checkpoint 8

What is the final linear momentum of the target in Fig. 9-18 if the initial linear momentum of the projectile is 6 kg · m/s and the final linear momentum of the projectile is (a) 2 kg · m/s and (b) −2 kg · m/s? (c) What is the final kinetic energy of the target if the initial and final kinetic energies of the projectile are, respectively, 5 J and 2 J?

Answer: (a) 4 kg m/s (b) 8 kg m/s (c) 3 J

Coefficient of restitution

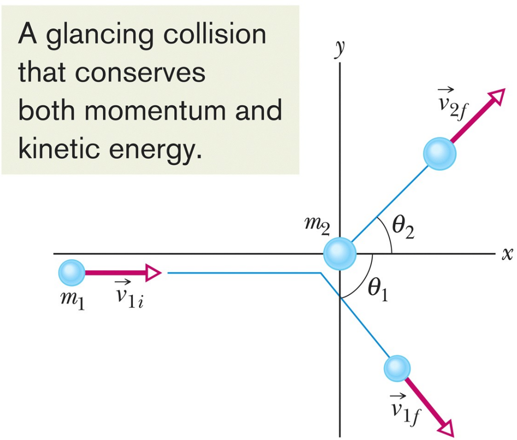
- Coefficient of restitution (e) = Relative velocity after collision / Relative velocity before collision
- Related to relative Kinetic energy

$e = 1$ perfectly elastic
$e = 0$ inelastic.
$e < 1$ (some loss inevitable)

A bouncing basketball captured with a stroboscopic flash at 25 images per second: Ignoring air resistance, the square root of the ratio of the height of one bounce to that of the preceding bounce gives the coefficient of restitution for the ball/surface impact.

9-8 Conservation of energy and momentum

- Apply the conservation of momentum along each axis
- Apply conservation of energy for elastic collisions

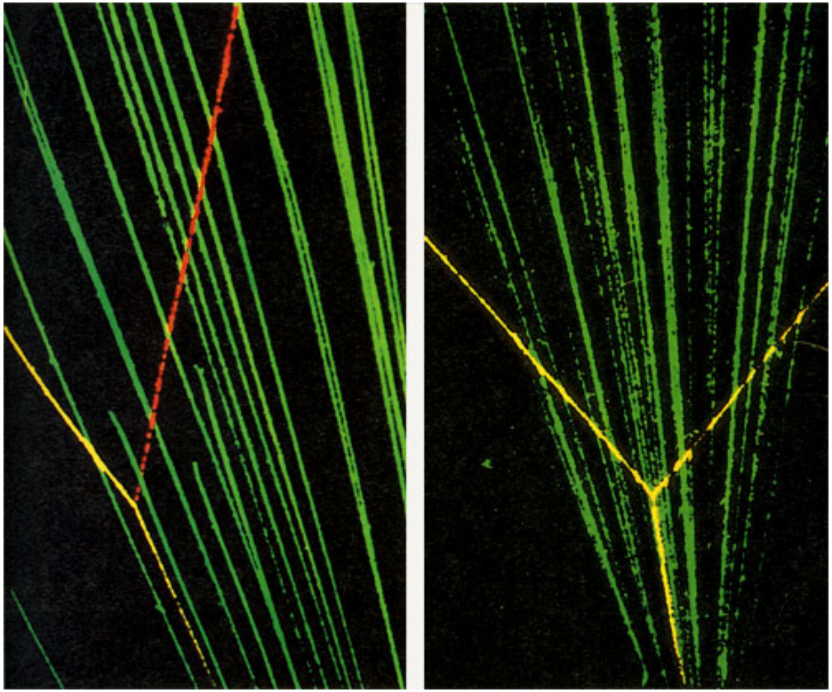


Example For Figure 9-21 for a stationary target:

Figure 9-21

- Along x: $$m_1 v_{1i} = m_1 v_{1f} \cos \theta_1 + m_2 v_{2f} \cos \theta_2,$$ **Eq. (9-79)**
- Along y: $$0 = -m_1 v_{1f} \sin \theta_1 + m_2 v_{2f} \sin \theta_2.$$ **Eq. (9-80)**
- Energy: $$\tfrac{1}{2} m_1 v_{1i}^2 = \tfrac{1}{2} m_1 v_{1f}^2 + \tfrac{1}{2} m_2 v_{2f}^2$$ **Eq. (9-81)**

Particle Collisions in Cloud/Bubble Chambers



Left: An alpha particle collides with a proton. Right: An alpha particle bounces off a nucleus in helium.

A high-energy proton (yellow) collides with another proton in liquid hydrogen. An electron (green) spirals away.

https://www.symmetrymagazine.org/article/january-2015/how-to-build-your-own-particle-detector

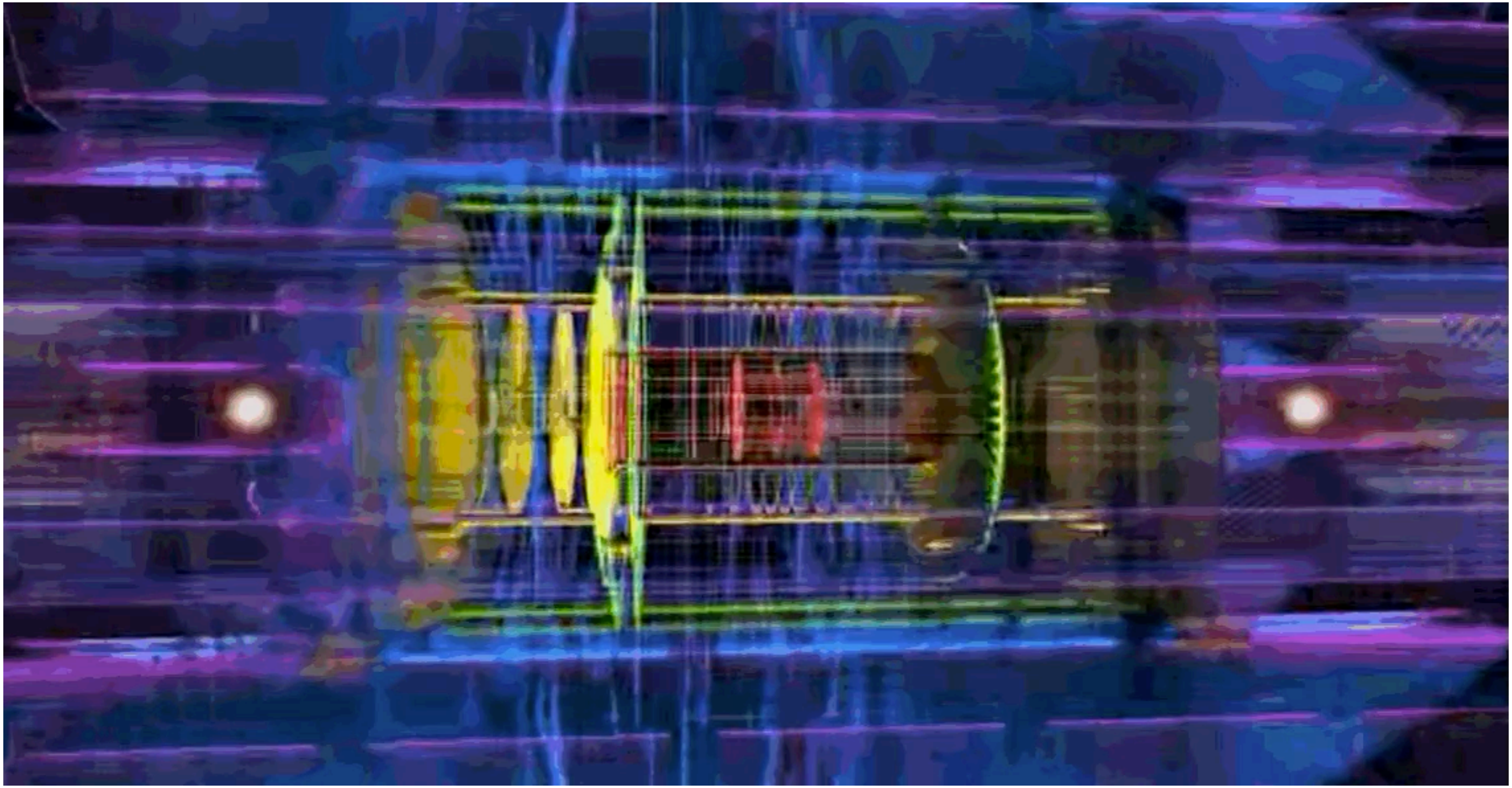
Proton-proton collision example

- A proton travelling with speed 8.2×10^5 m/s collides elastically with a stationary proton in a hydrogen target.
- One of the protons is observed to be scattered at a 60^o angle. At what angle will be the second proton be observed, and what will be the velocities of the two protons after the collision?
  - $m_A = m_B$
    $v_A = v'_A \cos\theta'_A + v'_B \cos\theta'_B$ (1)
    $0 = v'_A \sin\theta'_A + v'_B \sin\theta'_B$ (2)
    $v_A^2 = v'^2_A + v'^2_B$ (3) $\quad$ ($= 8.2 \times 10^5$ m/s and $\theta'_A = 60^o$)
- Combining the above and squaring both sides:
  - $v_A^2 - 2v'_A v_A \cos\theta'_A + v'^2_A \cos^2\theta'_A = v'^2_B \cos^2\theta'_B$
    $v'^2_A \sin^2\theta'_A = v'^2_B \sin^2\theta'_B$

Proton-proton collision example

- Use $\sin^2 \theta + \cos^2 \theta = 1$
  - $v_A^2 - 2v'_A v_A \cos\theta'_A + v'^2_A = v'^2_B$
- Into this equation we substitute $v'^2_B = v_A^2 - v'^2_A$
  - $2v_A^2 = 2v'_A v_A \cos\theta'_A$
- Or
  - $v_A = v'_A \cos\theta'_A = (8.2 \times 10^5 m/s) \times (\cos 60^o) = 4.1 \times 10^5 m/s$
- To obtain v'^2_B we use equation (3) (conservation of kinetic energy)
  - $v'^2_B = v_A^2 - v'^2_A,\ v'_B = 7.1 \times \frac{10^5 m}{s}$
- Finally from equation (2) we obtain
  - $\sin\theta'_B = \frac{-v'_A}{v'_B} \sin\theta'_A = -0.50$, so $\theta'_B = -30^o$
  - So particle B moves at an angle below the x axis if particle A is above the axis.

Inelastic collisions: ATLAS @ Large Hadron Collider



9-9 Rockets

- Rocket and exhaust products form an isolated system
- Conserve momentum

  $P_i = P_f$
- Rewrite this as:

$$Mv = -dM\,U + (M + dM)(v + dv),$$

Eq. (9-83)

- We can simplify using relative speed, defined as: $U = v + dv - v_{\text{rel}}.$

Eq. (9-84)

The ejection of mass from the rocket's rear increases the rocket's speed.

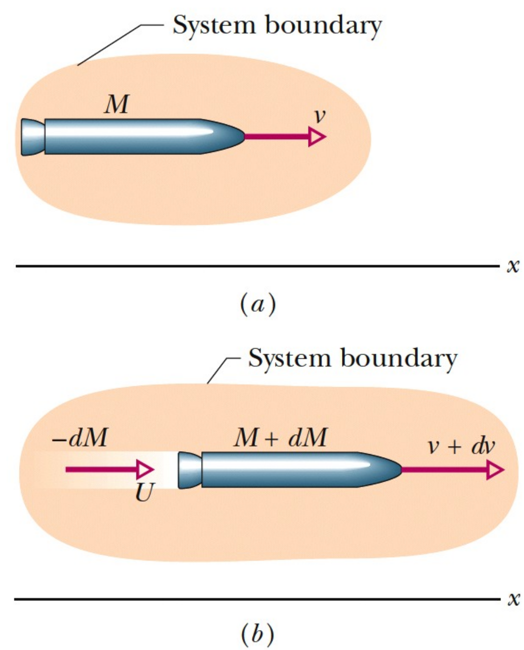


Figure 9-22

9-9 Rocket equations

- The first rocket equation:

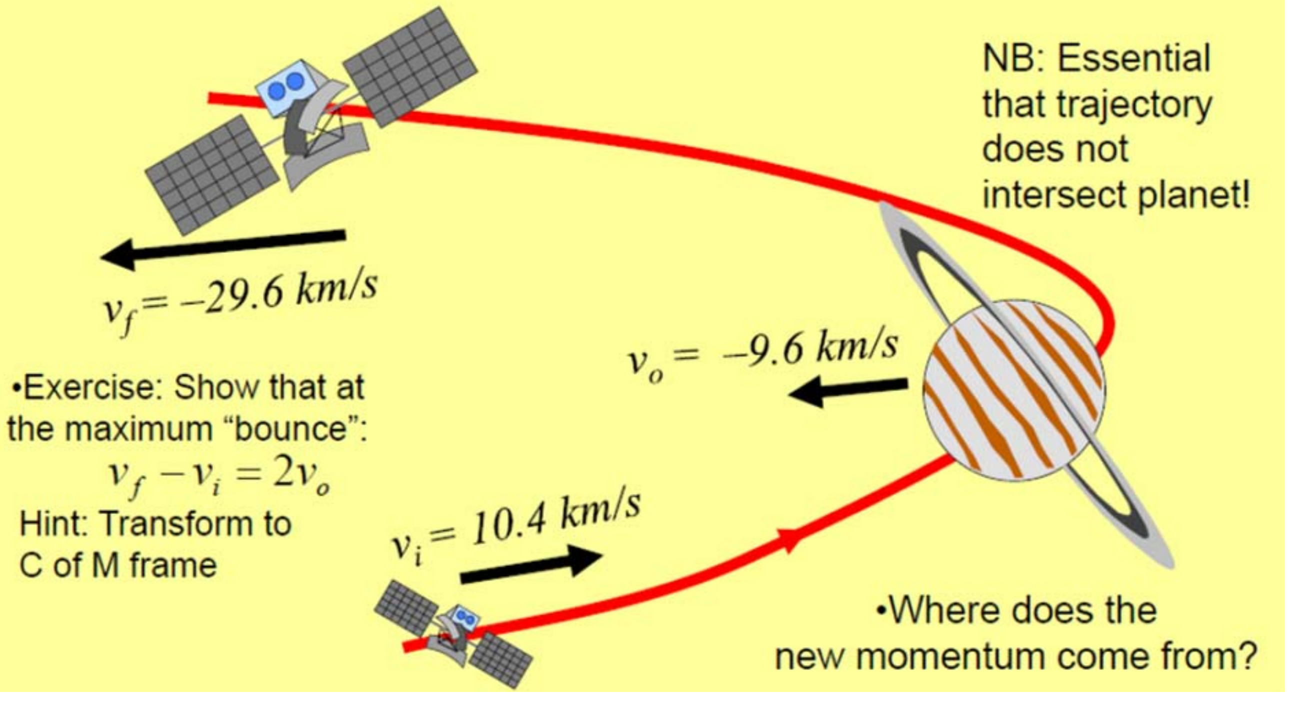
$$R v_{\text{rel}} = Ma \qquad \text{Eq. (9-87)}$$

- R is the mass rate of fuel consumption
- The left side of the equation is **thrust**, T
- Derive the velocity change for a given consumption of fuel as the second rocket equation:

$$v_f - v_i = v_{\text{rel}} \ln \frac{M_i}{M_f} \qquad \text{Eq. (9-88)}$$

Example of elastic collision: Gravitational Slingshot

- Can use gravity to bounce off a planet
- Pick up twice the orbital speed for a "head-on" collision



Summary

Motion of the Centre of Mass

- Unaffected by collisions/internal forces

Collisions in Two Dimensions

- Apply conservation of momentum along each axis individually
- Conserve K if elastic

Elastic Collisions in One Dimension

- K is also conserved

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i}$$

Eq. (9-67)

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}.$$

Eq. (9-68)

Variable-Mass Systems

$$Rv_{\text{rel}} = Ma \qquad \text{(first rocket equation).}$$

Eq. (9-87)

$$v_f - v_i = v_{\text{rel}} \ln \frac{M_i}{M_f} \qquad \text{(second rocket equation)}$$

Eq. (9-88)