Bayesian Statistics

Sunday, 13 September 2020 7:51 pm

Model : $X_i \sim N(\theta, \sigma^2)$

$$\Rightarrow f(x_i|\theta) \propto e^{-\frac{(x_i-\theta)^2}{2\sigma^2}}$$

Prior : $\theta \sim N(\mu, \frac{\sigma^2}{n_0})$

$$\Rightarrow f(\theta) \propto e^{-\frac{(\theta-\mu)^2}{2\frac{\sigma^2}{n_0}}}$$

Posterior : $p(\theta|x_1, \dots x_n) \propto P(x_1, \dots x_n|\theta) P(\theta)$

$= \frac{P(x_1, \dots x_n|\theta) P(\theta)}{P(x_1, \dots x_n)}$ → doesn't contain θ

$\theta^2 - 2x_i\theta - x_i^2$

$$\propto \prod_{i=1}^{n} P(x_i|\theta)$$

$$\propto e^{-\frac{\sum_{i=1}^{n}(x_i-\theta)^2}{2\sigma^2}} e^{-\frac{(\theta-\mu)^2}{2\frac{\sigma^2}{n_0}}}$$

$n\bar{x}$

$$\propto e^{-\frac{1}{2\sigma^2}\left(n\theta^2 - 2\theta\sum_{i=1}^{n} x_i\right) - \frac{n_0}{2\sigma^2}(\theta^2 - 2\mu\theta)}$$

$$\propto e^{-\frac{1}{2\sigma^2}\left((n+n_0)\theta^2 - 2(n\bar{x} + n_0\mu)\theta\right)}$$

$$\propto e^{-\frac{(n+n_0)}{2\sigma^2}\left(\theta^2 - \frac{2(n\bar{x}+n_0\mu)}{n+n_0}\theta\right)}$$

$$\propto e^{-\frac{\left(\theta - \frac{n\bar{x}+n_0\mu}{n+n_0}\right)^2}{2\frac{\sigma^2}{n+n_0}}}$$

$$\sim N\left(\frac{n\bar{x}+n_0\mu}{n+n_0}, \frac{\sigma^2}{n+n_0}\right).$$