PHYC10003 Physics 1

Lecture 9: Energy

Kinetic energy, work and power

Last lecture

- Examples - Forces
  - Falling from a height
  - Air bags save lives
  - Perception of weight
- Uniform circular motion
- Centripetal force

7-1 Energy

- Energy is required for any sort of motion
- Energy:
  - Is a scalar quantity assigned to an object or a system of objects
  - Can be changed from one form to another
  - Is *conserved* in a closed system, that is the total amount of energy of all types is always the same
- In this chapter we discuss one type of energy (kinetic energy)
- We also discuss one method of transferring energy (work)

7-1 Kinetic energy

- **Kinetic energy**:
  - The faster an object moves, the greater its kinetic energy
  - Kinetic energy is zero for a stationary object
- For an object with *v* well below the speed of light:

$$K = \tfrac{1}{2}mv^2$$ **Eq. (7-1)**

- The unit of kinetic energy is a **joule** (J)

$$1\text{ joule} = 1\text{ J} = 1\text{ kg}\cdot\text{m}^2/\text{s}^2.$$ **Eq. (7-2)**

7-1 Energy-example

Example Energy released by 2 colliding trains with given weight and acceleration from rest:

- Find the final velocity of each locomotive:

$$v^2 = v_0^2 + 2a(x - x_0).$$

$$v^2 = 0 + 2(0.26\ \text{m/s}^2)(3.2 \times 10^3\ \text{m}),$$
$$v = 40.8\ \text{m/s} = 147\ \text{km/h}.$$

- Convert weight to mass:

$$m = \frac{1.2 \times 10^6\ \text{N}}{9.8\ \text{m/s}^2} = 1.22 \times 10^5\ \text{kg}.$$

- Find the kinetic energy:

$$K = 2(\tfrac{1}{2}mv^2) = (1.22 \times 10^5\ \text{kg})(40.8\ \text{m/s})^2$$
$$= 2.0 \times 10^8\ \text{J}. \qquad \text{(Answer)}$$

7-2 Energy and Work

- Account for changes in kinetic energy by saying energy has been transferred *to* or *from* the object
- In a transfer of energy via a force, **work** is:
  - *Done on the object by the force*

Work W is energy transferred to or from an object by means of a force acting on the object. Energy transferred to the object is positive work, and energy transferred from the object is negative work.

- This is not the common meaning of the word "work"
  - To do work on an object, energy must be transferred
  - Throwing a baseball does work
  - Pushing an immovable wall does not do work

7-2 Energy and force

- Start from force equation and 1-dimensional velocity:

$$F_x = ma_x,$$

Eq. (7-3)

$$v^2 = v_0^2 + 2a_x d.$$

Eq. (7-4)

- Rearrange into kinetic energies:

$$\tfrac{1}{2}mv^2 - \tfrac{1}{2}mv_0^2 = F_x d.$$

Eq. (7-5)

- The left side is now the change in energy
- Therefore work is:

$$W = F_x d.$$

Eq. (7-6)

7-2 Work done on an object by force

To calculate the work a force does on an object as the object moves through some displacement, we use only the force component along the object's displacement. The force component perpendicular to the displacement does zero work.

- For an angle φ between force and displacement:

$$W = Fd\cos\phi$$ **Eq. (7-7)**

- As vectors we can write:

$$W = \vec{F} \cdot \vec{d}$$ **Eq. (7-8)**

- Notes on these equations:
  - Force is constant
  - Object is particle-like (rigid)
  - Work can be positive or negative

7-2 Work

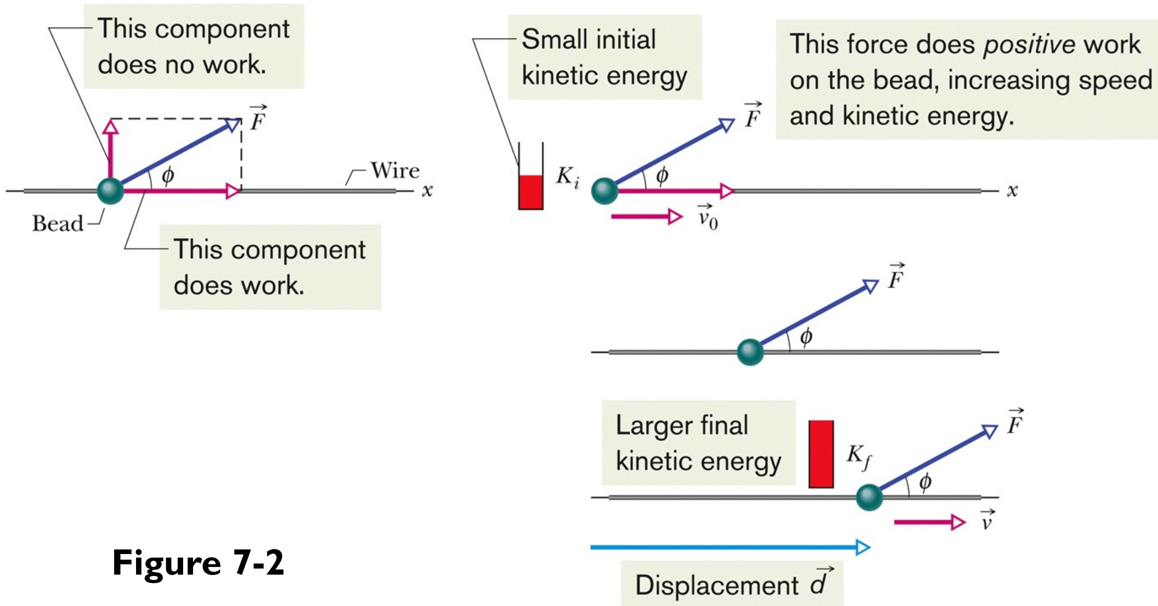

Figure 7-2

- Work has the SI unit of joules (J), the same as energy
- In the British system, the unit is foot-pound (ft lb)

7-2 Work-superposition of forces

A force does positive work when it has a vector component in the same direction as the displacement, and it does negative work when it has a vector component in the opposite direction. It does zero work when it has no such vector component.

- For two or more forces, the **net work** is the sum of the works done by all the individual forces
- Two methods to calculate net work:
  - We can find all the works and sum the individual work terms.
  - We can take the vector sum of forces (F_{net}) and calculate the net work once

7-2 Work-kinetic energy theorem

- The **work-kinetic energy theorem** states:

$$\Delta K = K_f - K_i = W, \qquad \text{Eq. (7-10)}$$

- (*change in kinetic energy*) = (*the net work done*)

- Or we can write it as:

$$K_f = K_i + W, \qquad \text{Eq. (7-11)}$$

- (*final KE*) = (*initial KE*) + (*net work*)

7-2 Conservation of energy

- The work-kinetic energy theorem holds for positive and negative work

Example If the kinetic energy of a particle is initially 5 J:

- A net transfer of 2 J to the particle (positive work)
  - Final KE = 7 J
- A net transfer of 2 J from the particle (negative work)
  - Final KE = 3 J

7-3 Work-examples

- We calculate the work as we would for any force
- Our equation is:

$$W_g = mgd\cos\phi$$ **Eq. (7-12)**

- For a rising object:

$$W_g = mgd\cos 180° = mgd(-1) = -mgd.$$ **Eq. (7-13)**

- For a falling object:

$$W_g = mgd\cos 0° = mgd(+1) = +mgd.$$ **Eq. (7-14)**

7-3 Work-path independence

- Work done in lifting or lowering an object, applying an upwards force:

$$\Delta K = K_f - K_i = W_a + W_g, \qquad \textbf{Eq. (7-15)}$$

- For a stationary object:
    - Kinetic energies are zero
    - We find:

$$W_a + W_g = 0$$

$$W_a = -W_g. \qquad \textbf{Eq. (7-16)}$$

- In other words, for an applied lifting force:

$$W_a = -mgd\cos\phi \qquad \text{(work done in lifting and lowering; } K_f = K_i\text{),}$$

Eq. (7-17)

- Applies regardless of path

7-3 Work –positive or negative?

- Figure 7-7 shows the orientations of forces and their associated works for upward and downward displacement
- Note that the works (in 7-16) need not be equal, they are only equal if the initial and final kinetic energies are equal
- If the works are unequal, you will need to know the difference between initial and final kinetic energy to solve for the work

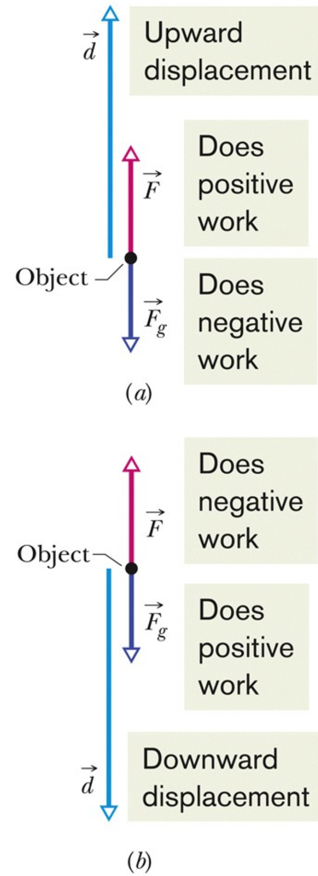

Figure 7-7

7-3 Work - example

Examples You are a passenger:

- Being pulled up a ski-slope
  - Tension does positive work, gravity does negative work

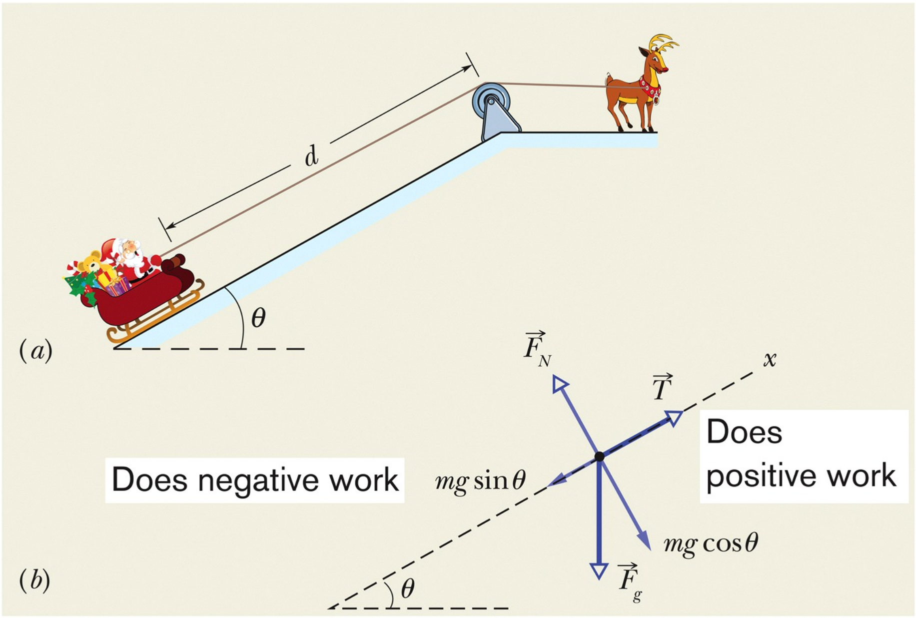

Figure 7-8

7-3 Work - example

Examples You are a passenger:

- Being lowered down in an elevator
  - Tension does negative work, gravity does positive work

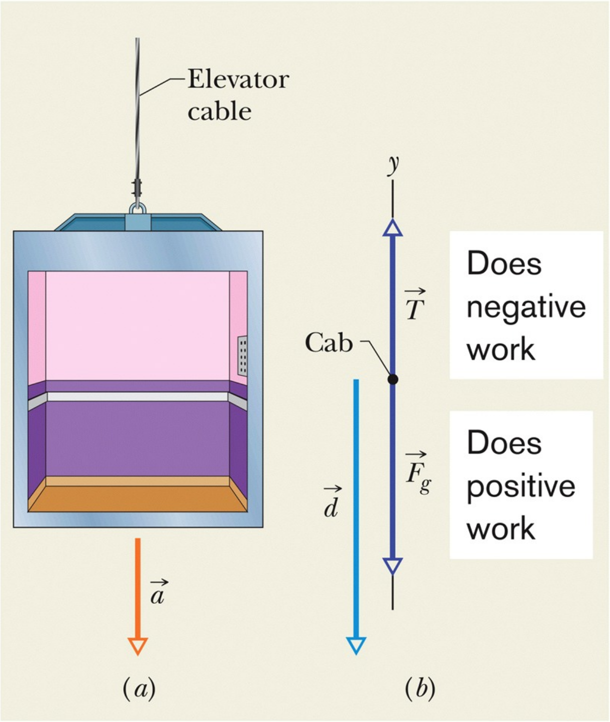

Figure 7-9

7-4 Work and Hooke's Law

- A **spring force** is the *variable force* from a spring
  - A spring force has a particular mathematical form
  - Many forces in nature have this form
- Figure (a) shows the spring in its **relaxed state**: since it is neither compressed nor extended, no force is applied
- If we stretch or extend the spring it resists, and exerts a *restoring force* that attempts to return the spring to its relaxed state

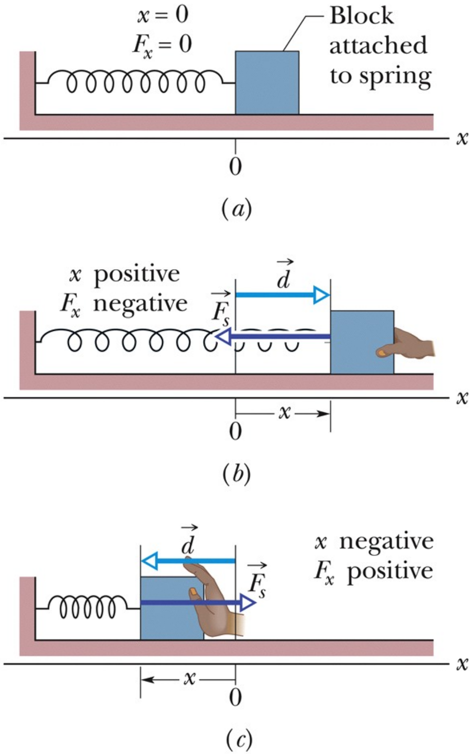

Figure 7-10

7-4 Hooke's law

- The spring force is given by **Hooke's law**:

$$\vec{F}_s = -k\vec{d} \qquad \textbf{Eq. (7-20)}$$

- The negative sign represents that the force always opposes the displacement
- The **spring constant** *k* is a is a measure of the stiffness of the spring
- This is a variable force (function of position) and it exhibits a linear relationship between *F* and *d*
- For a spring along the *x*-axis we can write:

$$F_x = -kx \qquad \textbf{Eq. (7-21)}$$

7-4 Work – compress or extend a spring

- We can find the work by integrating:

$$W_s = \int_{x_i}^{x_f} -F_x \, dx.$$ **Eq. (7-23)**

- Plug *kx* in for F_x:

$$W_s = \tfrac{1}{2}kx_i^2 - \tfrac{1}{2}kx_f^2$$ **Eq. (7-25)**

- The work:
  - Can be positive or negative
  - Depends on the *net* energy transfer

Work W_s is positive if the block ends up closer to the relaxed position ($x = 0$) than it was initially. It is negative if the block ends up farther away from $x = 0$. It is zero if the block ends up at the same distance from $x = 0$.

7-4 Work and conservation of energy

- For an initial position of x = 0:

$$W_s = -\tfrac{1}{2}kx^2 \qquad \textbf{Eq. (7-26)}$$

- For an applied force where the initial and final kinetic energies are zero:

$$W_a = -W_s. \qquad \textbf{Eq. (7-28)}$$

If a block that is attached to a spring is stationary before and after a displacement, then the work done on it by the applied force displacing it is the negative of the work done on it by the spring force.

7-5 Work - example

- We take a one-dimensional example
- We need to integrate the work equation (which normally applies only for a constant force) over the change in position
- We can show this process by an approximation with rectangles under the curve

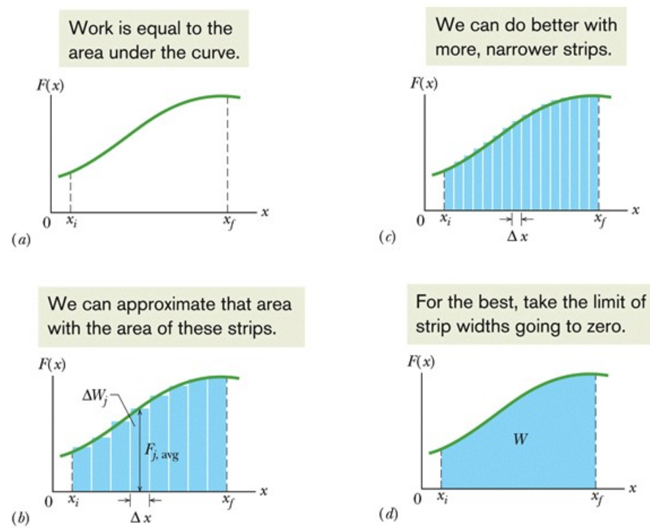

Figure 7-12

7-5 Work – three-dimensional forces

- Our sum of rectangles would be:

$$W = \lim_{\Delta x \to 0} \sum F_{j,\text{avg}}\,\Delta x. \qquad \textbf{Eq. (7-31)}$$

- As an integral this is:

$$W = \int_{x_i}^{x_f} F(x)\,dx \qquad \textbf{Eq. (7-32)}$$

- In three dimensions, we integrate each separately:

$$W = \int_{r_i}^{r_f} dW = \int_{x_i}^{x_f} F_x\,dx + \int_{y_i}^{y_f} F_y\,dy + \int_{z_i}^{z_f} F_z\,dz. \qquad \textbf{Eq. (7-36)}$$

- The work-kinetic energy theorem still applies!

7-6 Power

- **Power** is the time rate at which a force does work
- A force does *W* work in a time Δt; the **average power** due to the force is:

$$P_{\text{avg}} = \frac{W}{\Delta t} \quad \textbf{Eq. (7-42)}$$

- The **instantaneous power** at a particular time is:

$$P = \frac{dW}{dt} \quad \textbf{Eq. (7-43)}$$

- The SI unit for power is the watt (W): 1 W = 1 J/s
- Therefore work-energy can be written as (*power*) x (*time*) e.g. kWh, the kilowatt-hour

7-6 Power – rate of change of work

- Solve for the instantaneous power using the definition of work:

$$P = \frac{dW}{dt} = \frac{F\cos\phi\,dx}{dt} = F\cos\phi\left(\frac{dx}{dt}\right),$$

$$P = Fv\cos\phi. \qquad \textbf{Eq. (7-47)}$$

- Or:

$$P = \vec{F}\cdot\vec{v} \qquad \textbf{Eq. (7-48)}$$

Checkpoint 3

A block moves with uniform circular motion because a cord tied to the block is anchored at the center of a circle. Is the power due to the force on the block from the cord positive, negative, or zero?

Answer: zero (consider $P = Fv\cos\phi$, and note that $\phi = 90°$)

Summary

Kinetic Energy

- The energy associated with motion

$$K = \frac{1}{2}mv^2$$ **Eq. (7-1)**

Work

- Energy transferred to or from an object via a force
- Can be positive or negative

Work Done by a Constant Force

$$W = Fd\cos\phi$$ **Eq. (7-7)**

$$W = \vec{F}\cdot\vec{d}$$ **Eq. (7-8)**

- The **net work** is the sum of individual works

Work and Kinetic Energy

$$\Delta K = K_f - K_i = W,$$ **Eq. (7-10)**

$$K_f = K_i + W,$$ **Eq. (7-11)**

Summary

Work Done by the Gravitational Force

$$W_g = mgd\cos\phi$$ **Eq. (7-12)**

Work Done in Lifting and Lowering an Object

$$W_a + W_g = 0$$

$$W_a = -W_g.$$ **Eq. (7-16)**

Spring Force

- Relaxed state: applies no force
- Spring constant *k* measures stiffness

$$\vec{F}_s = -k\vec{d}$$ **Eq. (7-20)**

Spring Force

- For an initial position *x* = 0:

$$W_s = -\tfrac{1}{2}kx^2$$ **Eq. (7-26)**

Summary

Work Done by a Variable Force

- Found by integrating the constant-force work equation

$$W = \int_{x_i}^{x_f} F(x)\, dx$$

Eq. (7-32)

Power

- The rate at which a force does work on an object
- Average power:

$$P_{\text{avg}} = \frac{W}{\Delta t}$$

Eq. (7-42)

- Instantaneous power:

$$P = \frac{dW}{dt}$$

Eq. (7-43)

- For a force acting on a moving object:

$$P = Fv \cos \phi.$$

Eq. (7-47)

$$P = \vec{F} \cdot \vec{v}$$

Eq. (7-48)

Preparation for the next lecture

1. Read 9-1 to 9-5 of the text
2. You will find short answers to the odd-numbered problems in each chapter at the back of the book and further resources on LMS. You should try a few of the simple odd numbered problems from each section (the simple questions have one or two dots next to the question number).