PHYC10003 Physics 1

Lecture 10: Energy

Potential energy and work

Last lecture

- Energy
- Kinetic Energy
- Energy and Work
- Work done by the gravitational force
- Work done by a spring force
- Work done by a variable force
- Power

8-1 Potential energy

- **Potential energy** U is energy that can be associated with the configuration of a system of objects that exert forces on one another
- A system of objects may be:
  - Earth and a bungee jumper
  - **Gravitational potential energy** accounts for kinetic energy increase during the fall
  - **Elastic potential energy** accounts for deceleration by the bungee cord
- Physics determines how potential energy is calculated, to account for stored energy

8-1 Potential energy and work

- For an object being raised or lowered: $\Delta U = -W.$ **Eq. (8-1)**
- The change in gravitational potential energy is the negative of the work done
- This also applies to an elastic block-spring system

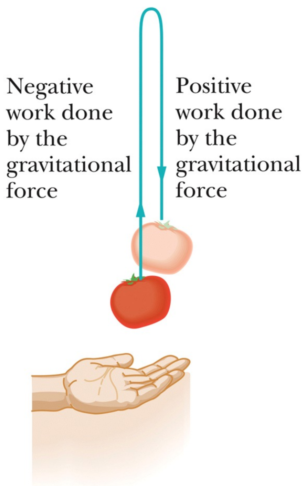


Figure 8-2

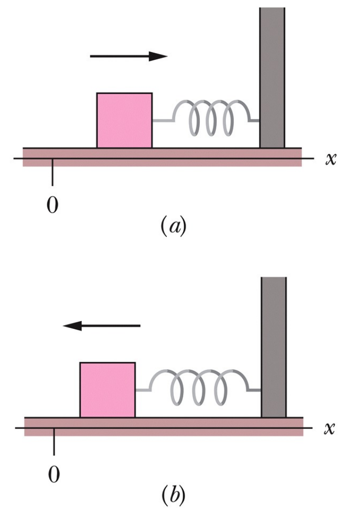


Figure 8-3

8-1 Potential energy and work

- Key points:
  1. The *system* consists of two or more objects
  2. A *force* acts between a particle (tomato/block) and the rest of the system
  3. When the configuration changes, the force does *work* W_1, changing kinetic energy to another form
  4. When the configuration change is reversed, the force reverses the energy transfer, doing work W_2
- Thus the kinetic energy of the tomato/block becomes potential energy, and then kinetic energy again

8-1 Forces- conservative & non-conservative

- **Conservative forces** are forces for which $W_1 = -W_2$ is always true
  - Examples: gravitational force, spring force
  - Otherwise we could not speak of their potential energies
- **Nonconservative forces** are those for which it is false
  - Examples: kinetic friction force, drag force
  - Kinetic energy of a moving particle is transferred to heat by friction
  - Thermal energy cannot be recovered back into kinetic energy of the object via the friction force
  - Therefore the force is not conservative, thermal energy is not a potential energy

Energy

- Energy is the ability to do work, i.e. apply a force over a distance
- Types of energy
  - Kinetic (Motion)
  - Potential (State, shape, position)
  - Thermal (kinetic energy of the molecules inside a body)
- Energy cannot be created or destroyed. It can only be converted from one form to another (e.g. potential to kinetic to thermal)

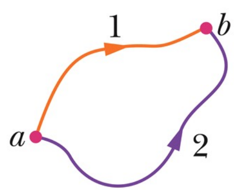
8-1 Conservative forces - closed loops

- When only conservative forces act on a particle, we find many problems can be simplified:

> The net work done by a conservative force on a particle moving around any closed path is zero.

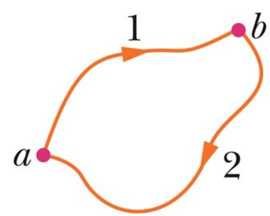
- A result of this is that:

> The work done by a conservative force on a particle moving between two points does not depend on the path taken by the particle.



(a)

The force is conservative. Any choice of path between the points gives the same amount of work.



(b)

And a round trip gives a total work of zero.

Figure 8-4

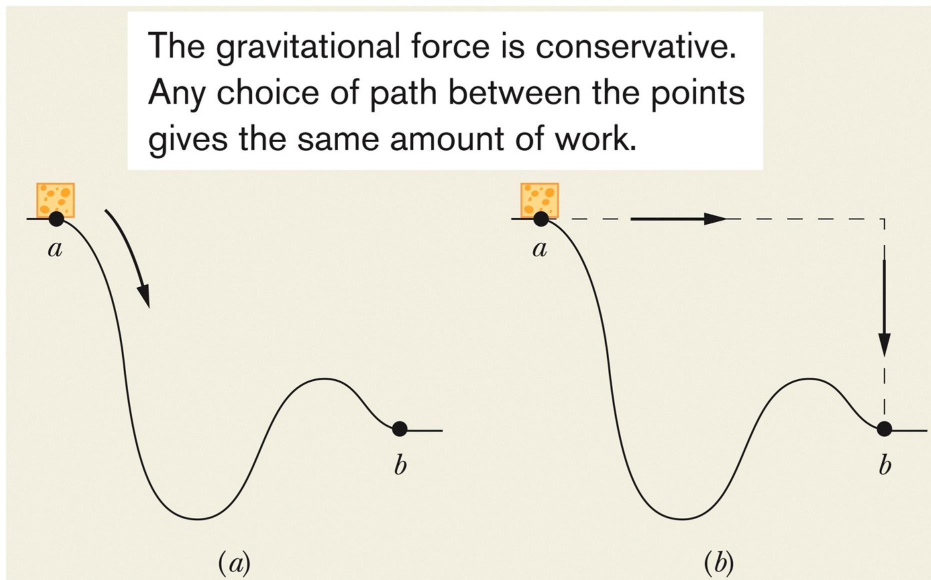
8-1 Conservative forces-simplifying features

- Mathematically:

$$W_{ab,1} = W_{ab,2}, \quad \textbf{Eq. (8-2)}$$

- This result allows you to substitute a simpler path for a more complex one if only conservative forces are involved

Figure 8-5



8-1 Work and potential energy in general

- For the general case, we calculate work as:

$$W = \int_{x_i}^{x_f} F(x)\,dx. \qquad \textbf{Eq. (8-5)}$$

- So we calculate potential energy as:

$$\Delta U = -\int_{x_i}^{x_f} F(x)\,dx. \qquad \textbf{Eq. (8-6)}$$

- Using this to calculate gravitational PE, relative to a **reference configuration** with **reference point** $y_i = 0$:

$$\boxed{U(y) = mgy} \qquad \textbf{Eq. (8-9)}$$

The gravitational potential energy associated with a particle–Earth system depends only on the vertical position y (or height) of the particle relative to the reference position $y = 0$, not on the horizontal position.

8-2 Mechanical energy

- The mechanical energy of a system is the sum of its potential energy *U* and kinetic energy *K*:

$$E_{\text{mec}} = K + U$$

Eq. (8-12)

- Work done by conservative forces increases *K* and decreases *U* by that amount, so:

Eq. (8-15)

$$\Delta K = -\Delta U.$$

- Using subscripts to refer to different instants of time:

$$K_2 + U_2 = K_1 + U_1$$

Eq. (8-17)

In an isolated system where only conservative forces cause energy changes, the kinetic energy and potential energy can change, but their sum, the mechanical energy E_{mec} of the system, cannot change.

8-2 Conservation of mechanical energy

- This is the principle of the **conservation of mechanical energy**:

$$\Delta E_{\text{mec}} = \Delta K + \Delta U = 0. \quad \textbf{Eq. (8-18)}$$

- This is very powerful tool:

> When the mechanical energy of a system is conserved, we can relate the sum of kinetic energy and potential energy at one instant to that at another instant *without considering the intermediate motion* and *without finding the work done by the forces involved.*

- One application:
  - Choose the lowest point in the system as $U = 0$
  - Then at the highest point U = max, and K = min

8-3 Force – negative gradient of potential energy

- For one dimension, force and potential energy are related (by work) as:

$$F(x) = -\frac{dU(x)}{dx}$$ **Eq. (8-22)**

- Therefore we can find the force *F*(*x*) from a plot of the potential energy *U*(*x*), by taking the derivative (slope)
- If we write the mechanical energy out:

$$U(x) + K(x) = E_{\text{mec}}.$$ **Eq. (8-23)**

- We see how *K*(*x*) varies with *U*(*x*):

$$K(x) = E_{\text{mec}} - U(x).$$ **Eq. (8-24)**

8-3 Stationary points

- To find $K(x)$ at any place, take the total mechanical energy (constant) and subtract $U(x)$
- Places where $K = 0$ are **turning points**
  - There, the particle changes direction (K cannot be negative)
- At equilibrium points, the slope of $U(x)$ is 0
- A particle in **neutral equilibrium** is stationary, with potential energy only, and net force = 0
  - If displaced to one side slightly, it would remain in its new position
  - Example: a marble on a flat tabletop

8-3 Equilibrium

- A particle in **unstable equilibrium** is stationary, with potential energy only, and net force = 0
  - If displaced slightly to one direction, it will feel a force propelling it in that direction
  - Example: a marble balanced on a bowling ball
- A particle in **stable equilibrium** is stationary, with potential energy only, and net force = 0
  - If displaced to one side slightly, it will feel a force returning it to its original position
  - Example: a marble placed at the bottom of a bowl

8-3 Potential and force - graphical relations

- Plot (a) shows the potential $U(x)$
- Plot (b) shows the force $F(x)$
- If we draw a horizontal line, (c) or (f) for example, we can see the range of possible positions
- $x < x_1$ is forbidden for the E_{mec} in (c): the particle does not have the energy to reach those points

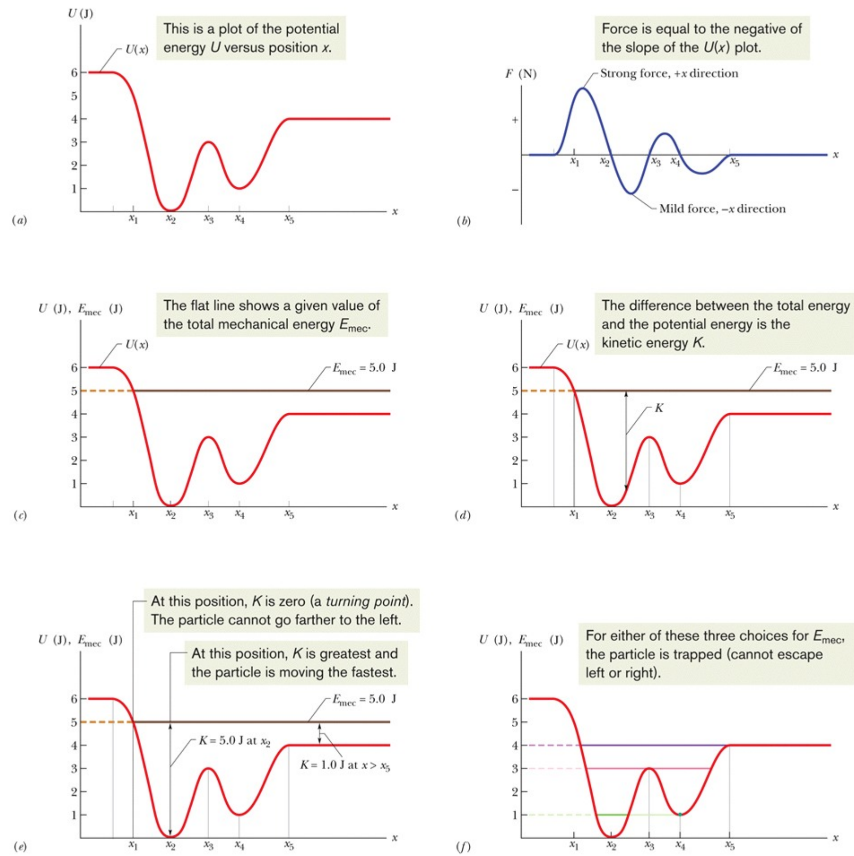


Figure 8-9

8-4 Work done on a system

- We can extend work on an object to work on a system:

> Work is energy transferred to or from a system by means of an external force acting on that system.

- For a system of more than 1 particle, work can change both *K* and *U*, or other forms of energy of the system
- For a frictionless system:

$$W = \Delta K + \Delta U, \quad \textbf{Eq. (8-25)}$$

$$\boxed{W = \Delta E_{\mathrm{mec}}} \quad \textbf{Eq. (8-26)}$$

Your lifting force transfers energy to kinetic energy and potential energy.

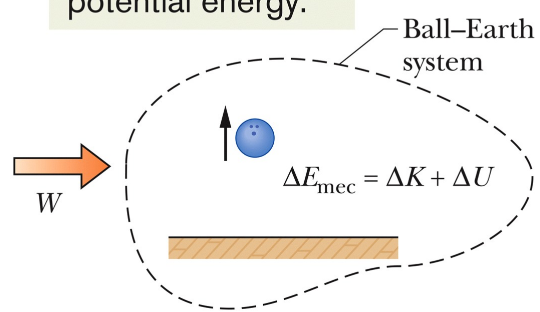


Figure 8-12

8-4 Work done on a system

- For a system with friction:

$$\Delta E_{th} = f_k d \quad \text{(increase in thermal energy by sliding).}$$ **Eq. (8-31)**

$$W = \Delta E_{mec} + \Delta E_{th}$$ **Eq. (8-33)**

- The thermal energy comes from the forming and breaking of the welds between the sliding surfaces

The applied force supplies energy. The frictional force transfers some of it to thermal energy.

So, the work done by the applied force goes into kinetic energy and also thermal energy.

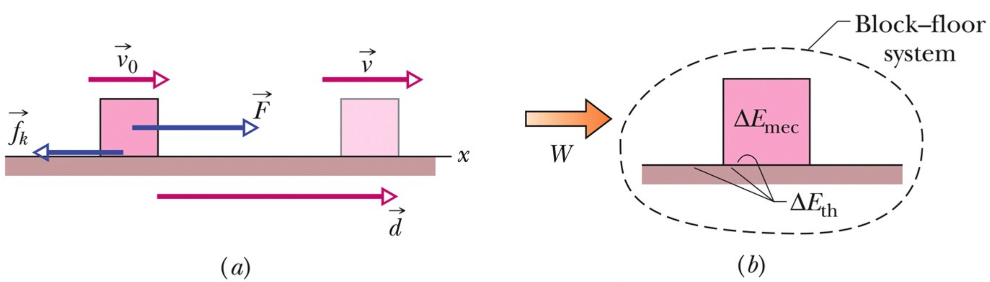


Figure 8-13

8-5 Conservation of energy in a system

- Energy transferred between systems can always be accounted for
- The **law of conservation of energy** concerns
  - The **total energy** *E* of a system
  - Which includes mechanical, thermal, and other internal energy

The total energy E of a system can change only by amounts of energy that are transferred to or from the system.

- Considering only energy transfer through work:

$$W = \Delta E = \Delta E_{\text{mec}} + \Delta E_{\text{th}} + \Delta E_{\text{int}},$$ **Eq. (8-35)**

8-5 Energy and isolated systems

- An isolated system is one for which there can be no *external* energy transfer

The total energy E of an isolated system cannot change.

- Energy transfers may happen internal to the system
- We can write: $$\Delta E_{\text{mec}} + \Delta E_{\text{th}} + \Delta E_{\text{int}} = 0$$ **Eq. (8-36)**

- Or, for two instants of time:

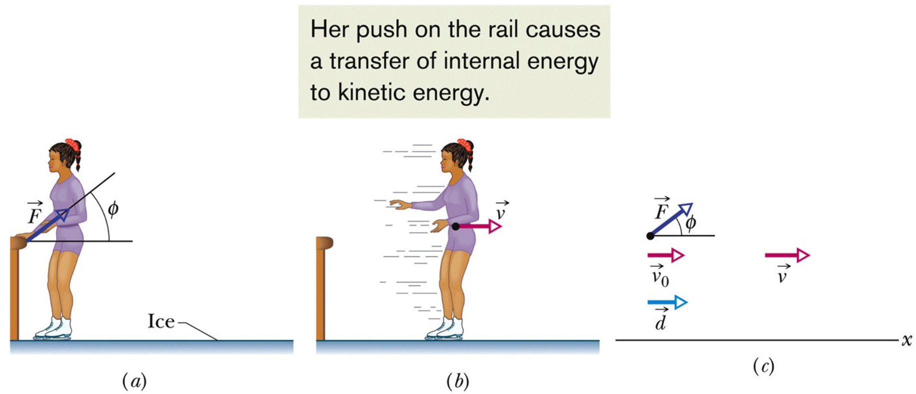
$$E_{\text{mec},2} = E_{\text{mec},1} - \Delta E_{\text{th}} - \Delta E_{\text{int}}.$$ **Eq. (8-37)**

In an isolated system, we can relate the total energy at one instant to the total energy at another instant *without considering the energies at intermediate times.*

8-5 Energy and isolated systems

- External forces can act on a system without doing work:

Figure 8-15



- The skater pushes herself away from the wall
- She turns internal chemical energy in her muscles into kinetic energy
- Her *K* change is caused by the force from the wall, but the wall does not provide her any energy

8-5 Power

- We can expand the definition of power
- In general, power is the rate at which energy is transferred by a force from one type to another
- If energy ΔE is transferred in time Δt, the **average power** is:

$$P_{\text{avg}} = \frac{\Delta E}{\Delta t}. \qquad \textbf{Eq. (8-40)}$$

- And the **instantaneous power** is:

$$P = \frac{dE}{dt}. \qquad \textbf{Eq. (8-41)}$$

Summary

Conservative Forces

- Net work on a particle over a closed path is 0

Potential Energy

- Energy associated with the configuration of a system and a conservative force

$$\Delta U = -\int_{x_i}^{x_f} F(x)\, dx. \quad \textbf{Eq. (8-6)}$$

Gravitational Potential Energy

- Energy associated with Earth + a nearby particle

$$U(y) = mgy \quad \textbf{Eq. (8-9)}$$

Elastic Potential Energy

- Energy associated with compression or extension of a spring

$$U(x) = \tfrac{1}{2}kx^2 \quad \textbf{Eq. (8-11)}$$

Summary

Mechanical Energy

$$E_{\text{mec}} = K + U$$ **Eq. (8-12)**

- For only conservative forces within an isolated system, mechanical energy is conserved

Potential Energy Curves

$$F(x) = -\frac{dU(x)}{dx}$$ **Eq. (8-22)**

- At turning points a particle reverses direction
- At equilibrium, slope of $U(x)$ is 0

Work Done on a System by an External Force

- Without/with friction:

$$W = \Delta E_{\text{mec}}$$ **Eq. (8-26)**

$$W = \Delta E_{\text{mec}} + \Delta E_{\text{th}}$$ **Eq. (8-33)**

Conservation of Energy

- The **total energy** can change only by amounts transferred in or out of the system

$$W = \Delta E = \Delta E_{\text{mec}} + \Delta E_{\text{th}} + \Delta E_{\text{int}},$$

Eq. (8-35)

Summary

Power

- The rate at which a force transfers energy
- **Average power**:

$$P_{\text{avg}} = \frac{\Delta E}{\Delta t}. \quad \textbf{Eq. (8-40)}$$

- **Instantaneous power**:

$$P = \frac{dE}{dt}. \quad \textbf{Eq. (8-41)}$$

Preparation for the next lecture

1. Read 9-1 to 9-5 of the text
2. You will find short answers to the odd-numbered problems in each chapter at the back of the book and further resources on LMS. You should try a few of the simple odd numbered problems from each section (the simple questions have one or two dots next to the question number).