Week 5: FNCE10002 Principles of Finance

THE UNIVERSITY OF MELBOURNE

Modern Portfolio Theory and Asset Pricing I

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5. Modern Portfolio Theory and Asset Pricing I

1. Examine realized returns and their variability over time
2. Describe the probability distribution approach and develop risk and return measures for securities
3. Explain the concept of risk diversification
4. Calculate and interpret the covariance of returns and correlation of returns
5. Calculate and interpret the expected return and standard deviation of a two security portfolio and examine the risk-return tradeoff given different correlations
6. Examine the effects on risk and expected return of portfolio leveraging and short selling

Required Readings: Weeks 5 – 7

- *Week 5*
  - GRAH, Ch. 6 and Ch. 7 (Sec 7.1 – 7.2)
- *Week 6*
  - Mid Semester Exam
  - *Exam Duration:* 60 minutes (no reading time)
  - *Format:* Multiple choice questions
  - *Coverage:* Weeks 1 – 4 inclusive
- *Week 7*
  - GRAH, Ch. 7 (Review Sec 7.1 – 7.2)

5.1 Risk and Return of Financial Securities

- In any period *t*, the observed (or realized) return of a security (or portfolio) can be measured as the *change* in cash flows from that security (or portfolio) divided by the initial investment in period *t* – 1
- For ordinary shares, we have…
  - $R_t = (P_t + D_t - P_{t-1})/P_{t-1}$
  - $R_t = D_t/P_{t-1} + (P_t - P_{t-1})/P_{t-1}$
  - R_t = Dividend yield + Percent price change
- For bonds, we have…
  - $R_t = (P_t + C_t - P_{t-1})/P_{t-1}$
  - $R_t = C_t/P_{t-1} + (P_t - P_{t-1})/P_{t-1}$
  - R_t = Coupon yield + Percent price change

Risk and Return of Financial Securities

- The *arithmetic* average return measures the return earned from a single, one period investment over a specific time horizon
  - $\bar{R} = (R_1 + R_2 + \ldots + R_T)/T$
- The *geometric* average return measure the return earned per period from an investment over an investor's *entire* time horizon
  - $\bar{R}_g = [(1 + R_1)(1 + R_2) \ldots (1 + R_T)]^{1/T} - 1$

Risk and Return of Financial Securities

- The observed (or realized) risk of a security (or portfolio) is measured by the variability in its realized returns around the (arithmetic) average return
- The realized variance of returns is…

$$Var(R) = \frac{1}{T-1}\sum_{t=1}^{T}\left(R_t - \bar{R}\right)^2$$

- The realized standard deviation of returns is…

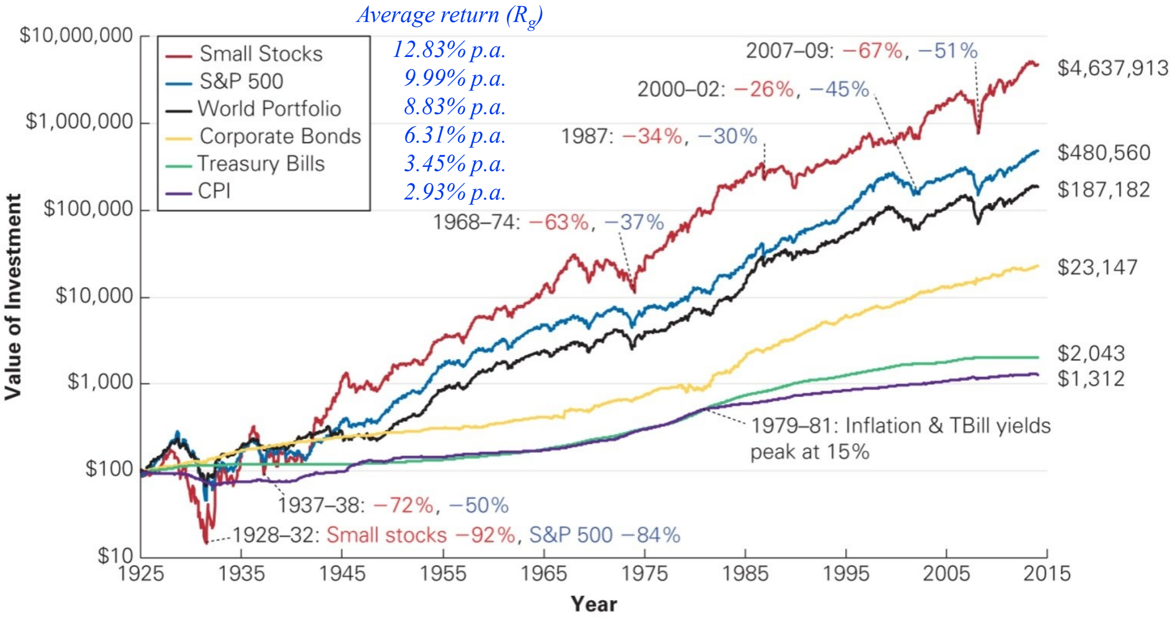
$$SD(R) = \sqrt{\frac{1}{T-1}\sum_{t=1}^{T}\left(R_t - \bar{R}\right)^2}$$

  - *Note:* The above expressions are used to calculate the historical volatilities in the graphs shown in later slides

Case Study 1: Risk and Return Over 1925-14

- How much would $100 invested in 1925 (year 0) have grown to if it were placed in the following investments (from *least* to *most* risky) until the end of 2014?
  - *Treasury bills:* An investment in one-month US Treasury bills
  - *Corporate bonds:* Long-term, AAA-rated US corporate bonds with maturities of approximately 20 years
  - *World portfolio:* International stocks from the world's major stock markets in North America, Europe and Asia
  - *Standard & Poor's 500:* The top 500 stocks by market capitalization traded on US markets
  - *Small stocks:* Stocks traded on the NYSE with market capitalizations in the bottom 20 percentile, updated quarterly

Case Study 1: Risk and Return Over 1925-14



Source: Chicago Center for Research in Security Prices (CRSP), Standard and Poor's, MSCI, and Global Financial Data. Figure 10.1, Berk and DeMarzo

Case Study 1: Risk and Return Over 1925-14

- The returns are linked geometrically to generate the graph…
  - Starting value, C_{1925} = \$100
  - Ending value, $C_{2014} = \$100(1 + R_{1926})(1 + R_{1927}) \ldots (1 + R_{2014})$
- How *else* could we find the ending value?
  - Ending value, $C_{2014} = 100(1 + \bar{R}_{g})^{n}$
  - Here, $n = 2014 - 1926 + 1 = 89$ years
- Investors should be concerned with *real* wealth growth, that is, *after removing the effects of inflation*
- The relation between nominal and real rate of returns is called the Fisher relationship

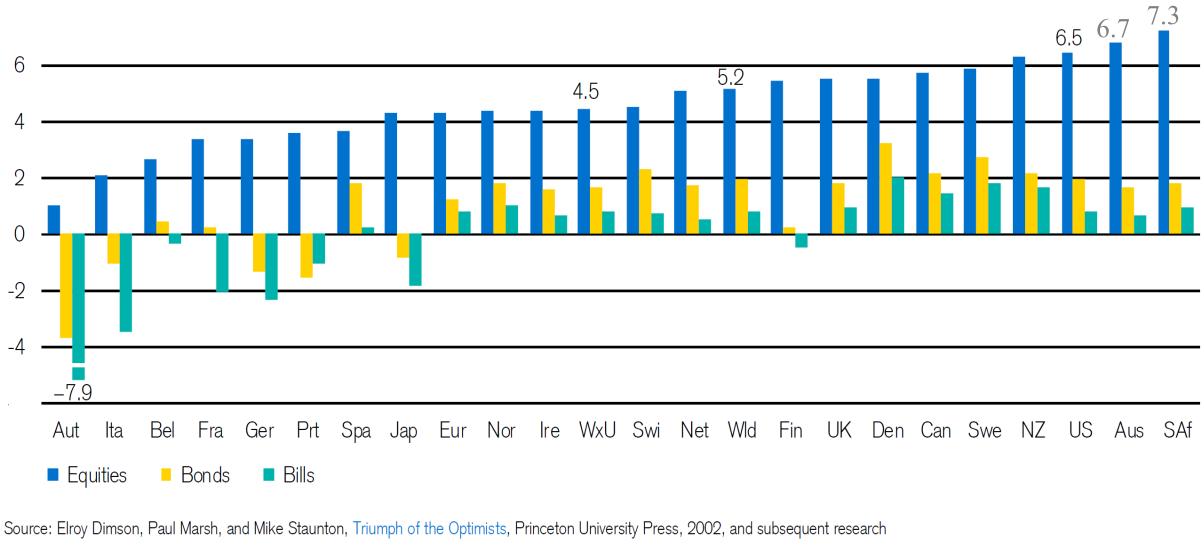
Case Study 1: Risk and Return Over 1925-14

- The Fisher relationship states…
  - $(1 + r) = (1 + r_r)(1 + i)$ *or* $(1 + r_r) = (1 + r)/(1 + i)$
  - r = Nominal rate of return (or nominal interest rate) per annum
  - r_r = Real rate of return (or real interest rate) per annum
  - i = *Expected* inflation rate per annum
- The *approximation* of the above is…
  - Nominal return (r) ≈ Real return (r_r) + Expected inflation rate (i)
  - Real return (r_r) ≈ Nominal return (r) – Expected inflation rate (i)
- Over 1925-14 the average geometric inflation rate ($\bar{I}_g$) was…
  - *CPI* in 1925 = 100 and *CPI* in 2014 = 1,312
  - $100(1 + \bar{I}_g)^{89} = 1312$
  - So, $(1 + \bar{I}_g)^{89} = 1312/100 = 13.12$
  - $\bar{I}_g = (13.12)^{1/89} - 1 = 2.93\%$ p.a.

Case Study 1: Risk and Return Over 1925-14

- *Average geometric return on an investment in US Treasury bills*
  - C_{1925} = \$100, C_{2014} = \$2,043, n = 89 years
  - $100(1 + \bar{R}_g)^{89} = 2043$
  - So, $(1 + \bar{R}_g)^{89} = 2043/100 = 20.43$
  - $\bar{R}_g = (20.43)^{1/89} - 1 = 3.45\%$ p.a.
  - *Real* geometric average return ≈ 3.45 – 2.93 = 0.52% p.a.
- *Average geometric return on an investment in US small stocks*
  - C_{1925} = \$100, C_{2015} = \$4,637,913, n = 89 years
  - $100(1 + \bar{R}_g)^{89} = 4637913$
  - $\bar{R}_g = (4637913/100)^{1/89} - 1 = 12.83\%$ p.a.
  - *Real* geometric average return ≈ 12.83 – 2.93 = 9.90% p.a.
- *How do returns vary across asset classes and across countries?*
- *Does high risk imply high realized returns?*

Real Returns Across Major Markets: 1900-17

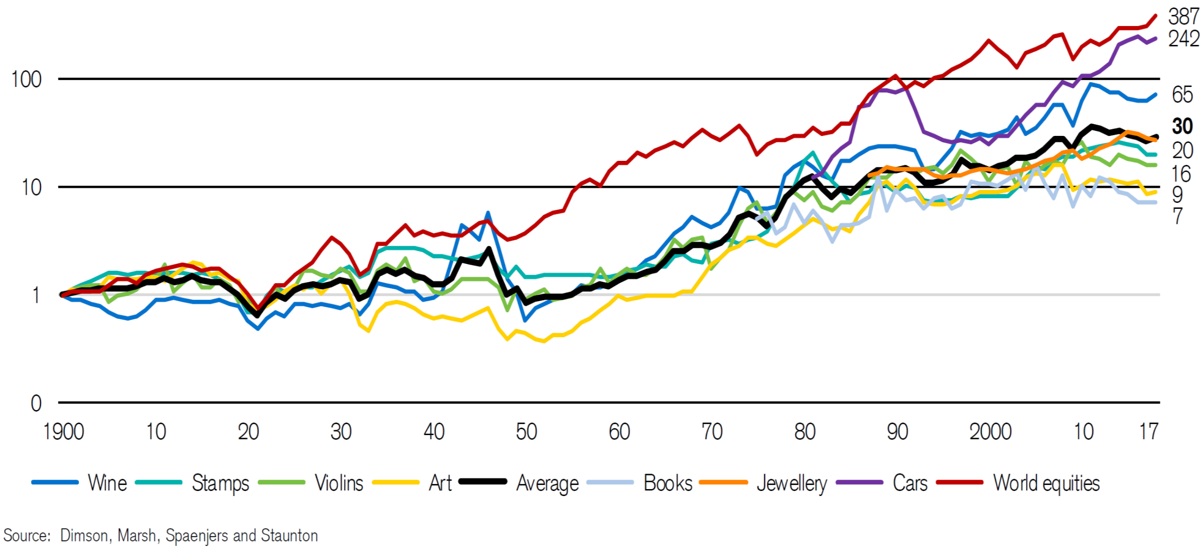


Note: Wld = World, WxU = World excluding US

Source: Dimson, E., Marsh, P. and Staunton, M., Credit Suisse Global Investment Returns Yearbook, 2018, Figure 5. Bonds and bills refer to long-term and short-term government securities, respectively.

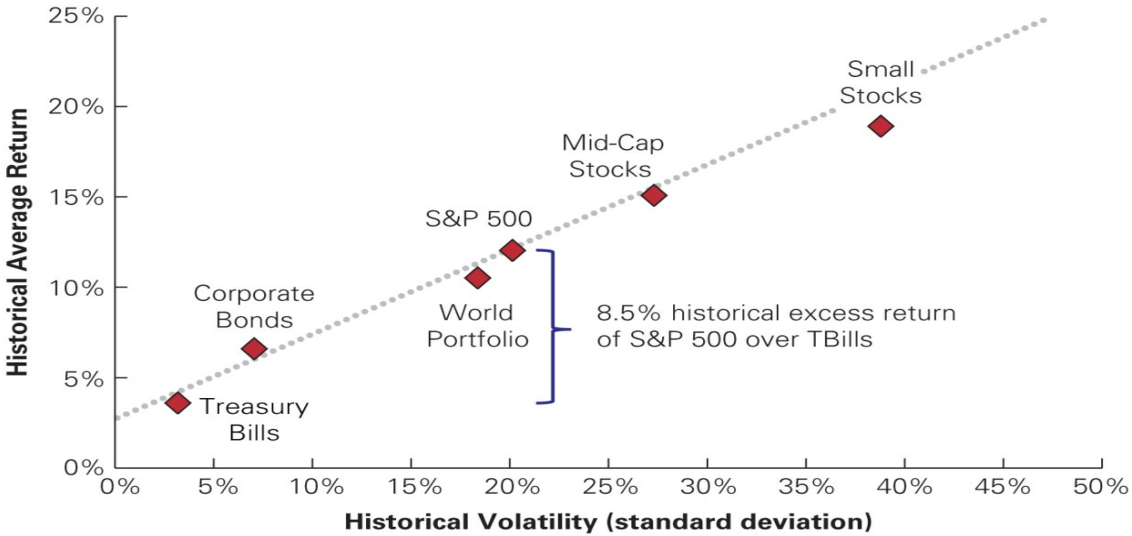
Real return ≈ Nominal return – Inflation rate

Equities vs. Alternative Investments: 1900-17



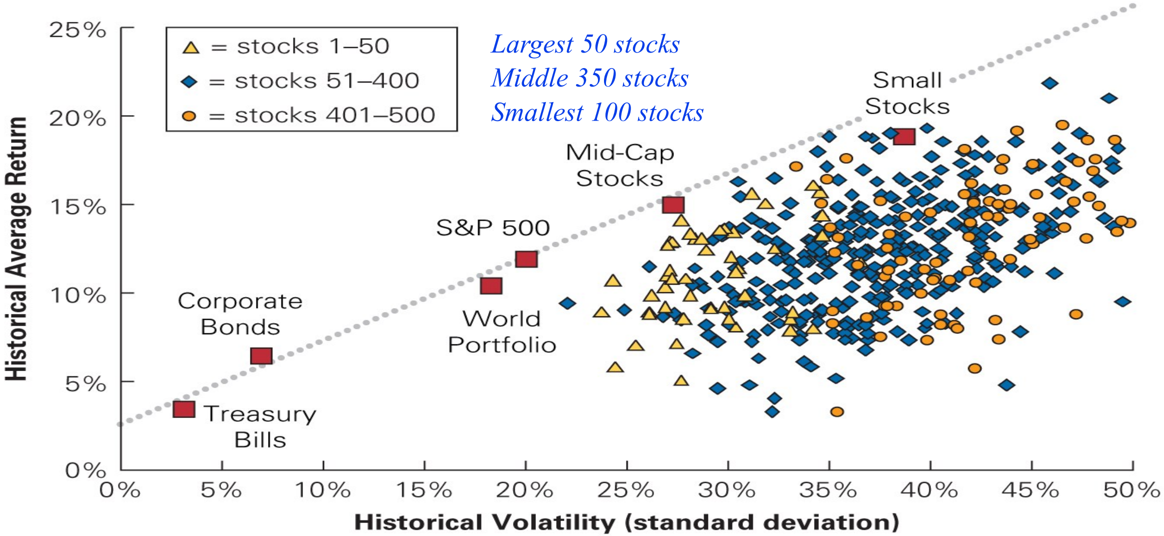
Source: Dimson, E., Marsh, P. and Staunton, M., Credit Suisse Global Investment Returns Yearbook, 2018, Figure 13. *Price indices for collectibles and world equities in real US dollars.*

Does High Risk Imply High Realized Returns?



Source: CRSP and Morgan Stanley Capital International. Figure, 10.6, Berk and DeMarzo. The data are from 1926-14 showing the risk and return tradeoff for various US asset classes as well as the world portfolio. The average returns shown here are arithmetic averages.

Does High Risk Imply High Realized Returns?



Source: Chicago Center for Research in Security Prices. Figure, 10.7, Berk and DeMarzo. The data are from 1926-14 showing the risk and return tradeoff for the top 500 US stocks sorted by market capitalization. The average returns shown here are arithmetic averages.

5.2 The Probability Distribution Approach

- We assume that investors can specify the possible outcomes (C_1 through C_n) and associate *probabilities or likelihoods* (p_1 through p_n) with these outcomes. For each state, convert the cash flows into rates of returns using the initial investment of C

State	*Probability*	*Cash Flows*	*Rate of Return*
1	p_1	C_1	$R_1 = (C_1 - C)/C$
2	p_2	C_2	$R_2 = (C_2 - C)/C$
3	p_3	C_3	$R_3 = (C_3 - C)/C$
:	:	:	:
n	p_n	C_n	$R_n = (C_n - C)/C$

- *Note:* $p_1 + p_2 + \ldots + p_n = 1$, because one of these states of the world will actually occur

Measures of Risk and Expected Return

- The *expected return* is the *expected* outcome measured as the weighted average of the individual outcomes
  - $E(r) = p_1R_1 + p_2R_2 + \ldots + p_nR_n$
- The *variance* or *standard deviation* of returns is the measure of *dispersion* around the expected return
  - Greater the dispersion, higher the uncertainty and risk
- $Var(r)$ *or* $\sigma^2 = p_1[R_1 - E(r)]^2 + p_2[R_2 - E(r)]^2 + \ldots + p_n[R_n - E(r)]^2$
- $SD(r) = \sigma$
  - *Note:* Variance and standard deviation take into account returns above *and* below the expected return. Investors are typically concerned with returns *below* the expected return (that is, *downside risk*)

Measures of Risk and Expected Return

- *Example:* The current price of Stock X is \$10 and market analysts expect the following three states to occur one year from now. What is the expected return and standard deviation of return for this investment?

State of the Market	*Probability*	*Dividend*	*Price*
Sucks	0.3	\$0.50	\$8.50
Good	0.4	\$1.00	\$10.00
Awesome	0.3	\$1.50	\$10.50

Measures of Risk and Expected Return

- We first convert the cash flows into rates of return for each state
- The return distribution for Stock X is as follows

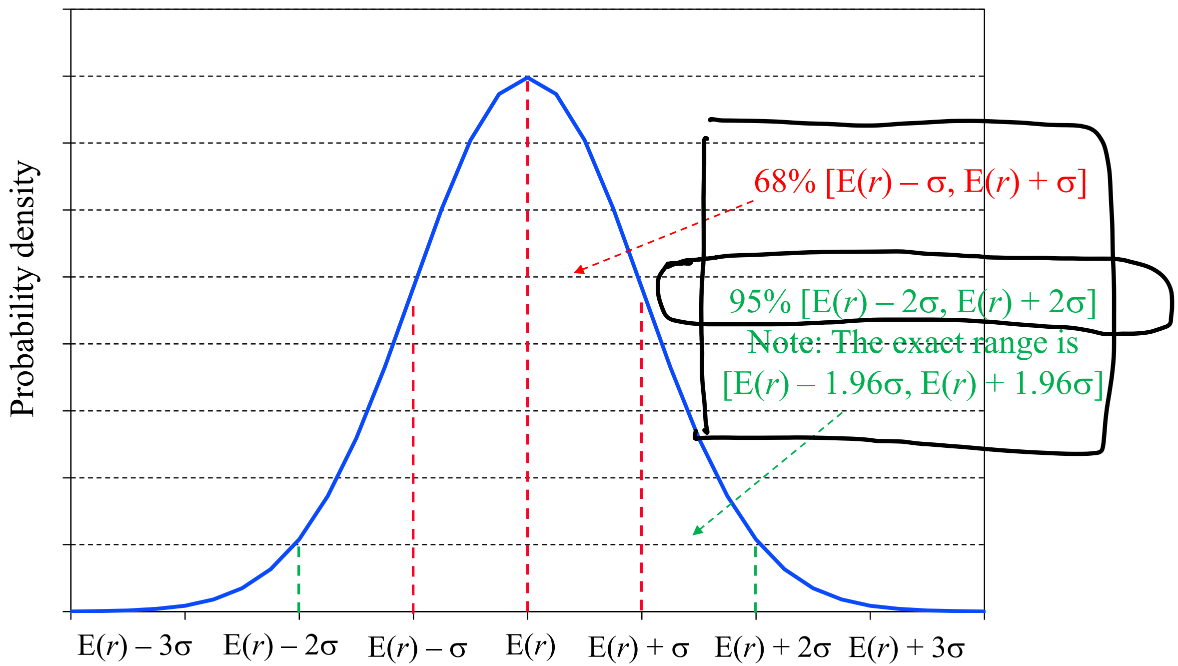
State	*Probability*	*Rate of Return, R*
Sucks	0.3	(0.50 + 8.50 – 10.00)/10.00 = –10.0%
Good	0.4	(1.00 + 10.00 – 10.00)/10.00 = 10.0%
Awesome	0.3	(1.50 + 10.50 – 10.00)/10.00 = 20.0%

- $E(r) = 0.3(-0.1) + 0.4(0.1) + 0.3(0.2) = 7.0\%$ ⇒ don't occur, the expected
- $\text{Var}(r) = 0.3(-0.1 - 0.07)^2 + 0.4(0.1 - 0.07)^2 + 0.3(0.2 - 0.07)^2$
- $\text{Var}(r) = 0.0141$
- $\text{SD}(r) = (0.0141)^{1/2} = 0.1187$ or 11.9%

Interpreting Return and Risk Measures

- A more general interpretation of the expected return and standard deviation of return requires assuming returns are *continuously and normally* distributed (see next slide)
- Properties of a normal distribution…
  - It is a bell-shaped distribution and requires *only* the expected (or mean) return and standard deviation of return to fully describe it
  - It is symmetric around its mean
  - It implies an *unlimited* downside loss potential – Is this a realistic assumption?

Interpreting Return and Risk Measures



Interpreting Return and Risk Measures

- In the previous example for stock X, $E(r) = 7\%$ and $\sigma = 11.9\%$
- Assume you purchased 100 shares of Stock X, investing \$1,000
- The expected value of your investment next period is…
  - $1000[1 + E(r)] = 1000(1.07) = \$1{,}070$
- There is a 95% probability that the *realized* return will lie in the range $[E(r) - 2\sigma, E(r) + 2\sigma]$ or (–16.8%, 30.8%)
- So, there is a 95% probability that a \$1,000 investment in this security will be worth between $1000(1 - 0.168) = \$832$ and $1000(1 + 0.308) = \$1{,}308$ next period

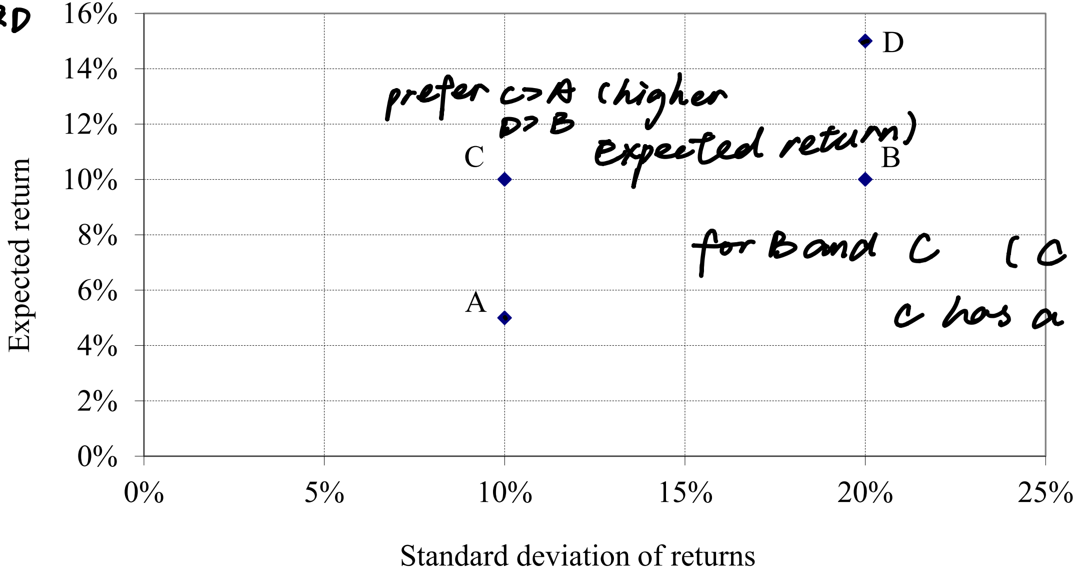
Specifying Investor Preferences

- *Investors are assumed to be risk averse* → don't like risk
  - Higher the variance or standard deviation of returns the worse off the investor
- *Illustration:* You're given the following information on four risky securities. Which securities would a risk averse investor prefer and why?

Security	*Expected Return*	*Standard Deviation of Returns*
A	5%	10%
B	10%	20%
C	10%	10%
D	15%	20%

Specifying Investor Preferences

for A & B, A&D



prefer C>A (higher
D>B
Expected return)

for B and C (C over B)

C has a lower risk level

5.3 Portfolios and Risk Diversification

there isn't low risk high return item

- A *risk averse* investor's objective is to…
  - Minimize the risk of portfolio of investments, given a desired level of expected return

or

  - Maximize the expected return of portfolio of investments, given a desired level of risk
- The simplest (and naïve) way to minimize risk is to diversify across different securities by forming a portfolio of securities
  - Portfolio risk falls as the number of securities in the portfolio increases, but…
  - Portfolio risk *cannot* be entirely eliminated using this method
  - The risk that cannot be eliminated is called *systematic risk*
  - More on this next week…

Portfolio Risk and Return: Two Securities

- A portfolio's *expected return* is the weighted average of the expected returns of its component securities
  - Note that the weights are percentages of the investor's *original* wealth invested in each security
  - It is assumed that all the available funds are invested in the two securities
- $E(r_p) = w_1 E(r_1) + w_2 E(r_2)$
  - w_j = Amount invested in security j / Total amount invested
  - *Note:* $w_1 + w_2 = 1$ and $w_1 = 1 - w_2$ (or $w_2 = 1 - w_1$)

Portfolio Risk and Return: Two Securities

- A portfolio's *variance* is the weighted average of the variance of its component securities and the *covariance* between the securities' returns…
- $\mathrm{Var}(r_p)$ *or* $\sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\sigma_{12}$
  - σ_{12} *or* $\mathrm{Cov}(r_1, r_2)$ = Covariance between securities 1 and 2
  - Note that only the *third* term can be negative
- The standard deviation of the portfolio is…
- $\mathrm{SD}(r_p)$ *or* $\sigma_p = [w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\sigma_{12}]^{1/2}$
- We first need to define the covariance of returns

covariance(x, y)

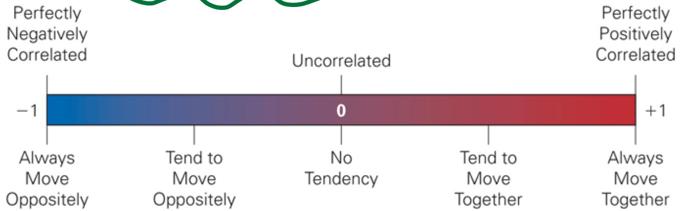
$= E\left[(X - E[X])(Y - E[Y])\right]$

5.4 Covariance Between Security Returns

- The *covariance of returns* [σ_{12} or Cov(r_1, r_2)] measures the level of *comovement* between security returns
  - $\sigma_{12} = p_1[r_{11} - E(r_1)][r_{21} - E(r_2)] + \ldots + p_n[r_{1n} - E(r_1)][r_{2n} - E(r_2)]$
  - r_{jk} = Return on security j = 1, 2 in state k = 1, 2, …, n
- $\sigma_{12} > 0$: Above (below) average returns on security 1 tend to coincide with above (below) average returns on security 2
- $\sigma_{12} < 0$: Above (below) average returns on security 1 tend to coincide with below (above) average returns on security 2
- $\sigma_{12} = 0$: Security 1's return tends to move independently of security 2's return
- Note that the magnitude of covariance changes depending on *how* returns are measured – Percentages versus decimals

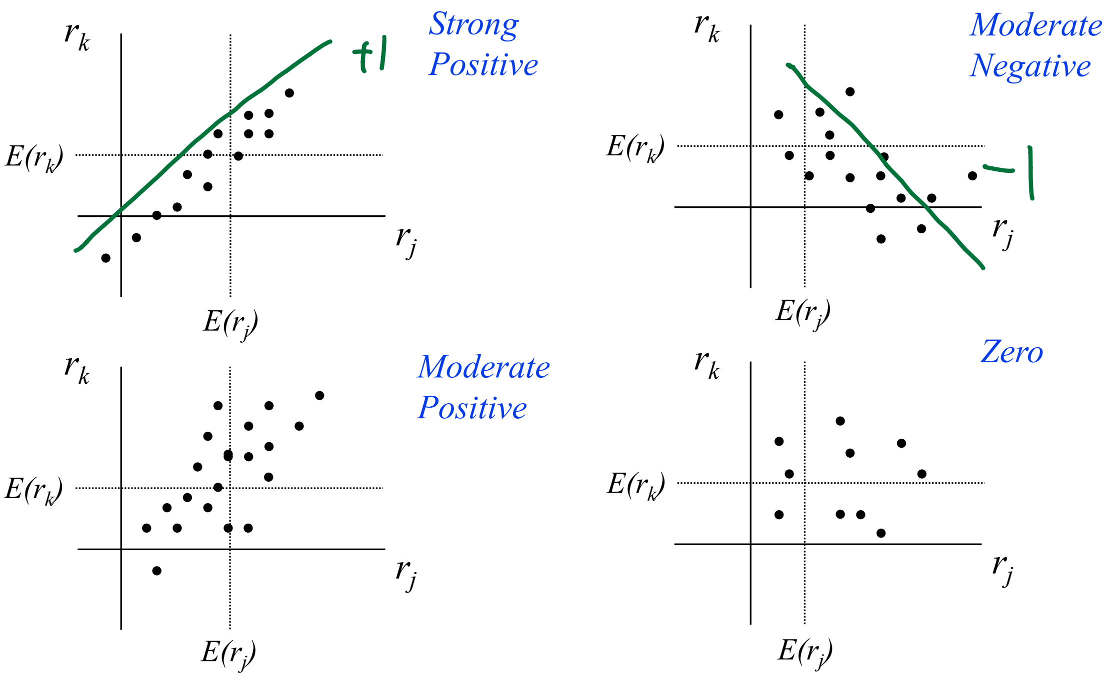
Correlation Between Security Returns

- The *correlation of returns,* ρ_{12} or Corr(r_1, r_2), is a "standardized" measure of comovement between two securities
  - $\boxed{\rho_{12} = \sigma_{12}/\sigma_1\sigma_2}$
  - *Note 1:* The sign of the return correlation is the *same* as the sign of the return covariance
  - *Note 2:* $-1 \leq \rho_{12} \leq +1$



- The covariance of returns can be rewritten as…
  - $\sigma_{12} = \sigma_1\sigma_2\,\rho_{12}$

Examples of Security Correlations



Correlation Between Security Returns

- There are *two* definitions for a portfolio's variance (and standard deviation) of returns
- Using the *covariance* of returns…
  - $\mathrm{Var}(r_p)$ *or* $\sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\sigma_{12}$
- Using the *correlation* of returns…
  - $\mathrm{Var}(r_p)$ *or* $\sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\sigma_1\sigma_2\rho_{12}$
- Which expression should one use?

- *What does the typical risk-return profile of blue chip stocks look like?*
- *Why should investors diversify across different asset classes?*

Case Study 2: Risk, Meet Return

	Mean	*Std Dev*	*Upper Range*	*Lower Range*
AOI	6.1%	12.2%	30.5%	-18.3%
ANZ	16.1%	22.0%	60.1%	-27.8%
BHP	9.6%	23.9%	57.3%	-38.1%
CBA	19.0%	19.4%	57.8%	-19.8%
NAB	13.7%	21.4%	56.5%	-29.1%
RIO	17.2%	25.8%	68.8%	-34.4%
TLS	9.2%	18.6%	46.3%	-27.9%

Note: The sample period is Jan 2009 – Dec 2018. AOI refers to the All Ordinaries Index which is often used as a proxy for the "market" portfolio. The mean and standard deviation of returns are based on monthly returns which have been annualized. The upper/lower ranges are calculated as the mean plus/minus two times the standard deviation of returns

Case Study 2: Risk, Meet Return

	Mean	*Std Dev*	*AOI*	*ANZ*	*BHP*	*CBA*	*NAB*	*RIO*
AOI	6.1%	12.2%	1.00					
ANZ	16.1%	22.0%	0.75	1.00				
BHP	9.6%	23.9%	0.62	0.35	1.00			
CBA	19.0%	19.4%	0.62	0.73	0.25	1.00		
NAB	13.7%	21.4%	0.76	0.85	0.36	0.69	1.00	
RIO	17.2%	25.8%	0.58	0.33	0.80	0.35	0.33	1.00
TLS	9.2%	18.6%	0.26	0.16	0.03	0.30	0.25	0.04

Note: The sample period is Jan 2008 – Dec 2018. AOI refers to the All Ordinaries Index which is often used as a proxy for the "market" portfolio. The mean and standard deviation of returns are based on monthly returns which have been annualized

Case Study 3: Are All Your Eggs in One Basket?

	Avg	Min	Max	Rank	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018
Australian Shares	4.4	-40.4	39.6	1	18	14.9	39.6	8.6	11.4	33	53.6	32.1	14.3	13.2	13.4	5.7
International Shares	5.1	-24.9	48	2	6.7	7.6	7.9	6	5	21.2	48	27	13.8	13	13	4.9
US Shares (S&P500)	8.1	-19.8	53.6	3	3.5	-19.8	3.5	4.7	2.2	18.8	19.9	24	13.6	11.6	12.5	4.5
Australian Property	1.1	-54	33	4	-2.6	-24.9	3.1	3.3	-0.7	14.1	19.7	15	11.8	7.9	5.7	2.9
International Property	2.9	-29.2	32.1	5	-5.3	-29.2	1.7	1.5	-1.5	13.5	7.1	9.8	3.8	4.9	3.7	1.9
Australian Bonds	5.8	1.7	14.9	6	-8.4	-40.4	-0.3	-0.4	-5.3	7.7	2.9	5	2.6	2.9	1.7	1.5
Cash	3.7	1.7	7.6	7	-25.3	-54	-2.3	-2	-11.4	4	2	2.7	2.3	2.1	0.2	-3.5

Source: http://insights.vanguard.com.au/static/asset-class/app.html. Vanguard Investment Strategy Group analysis using index data from S&P, UBS, MSCI and Barclays

5.5 Risk-Return Tradeoff: Two Securities

- Consider the second definition for a portfolio's variance (and standard deviation) of returns again…
  - $\mathrm{Var}(r_p)$ *or* $\sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\sigma_1\sigma_2\rho_{12}$
- The risk-return tradeoff in a two security portfolio depends on the level of comovement (or correlation) between their returns
- We consider the following three cases
  - *Case 1:* Return correlation of +1 (perfect positive correlation)
  - *Case 2:* Return correlation of –1 (perfect negative correlation)
  - *Case 3:* Return correlation between +1 and –1 (general case)

Risk-Return Tradeoff: Two Securities

- *Illustration:* You're given the following information on two securities.

Security	*Expected return*	*Standard deviation*
1	14.0%	20.0%
2	10.0%	12.0%

- Calculate the expected return and standard deviation of returns of portfolios with weights in security 1 of 0%, 25%, 50%, 75%, 100% for the following cases
  - The securities' returns are perfectly positively correlated
  - The securities' returns are perfectly negatively correlated
  - The securities' returns have a zero correlation

Case 1: Perfect Positive Correlation

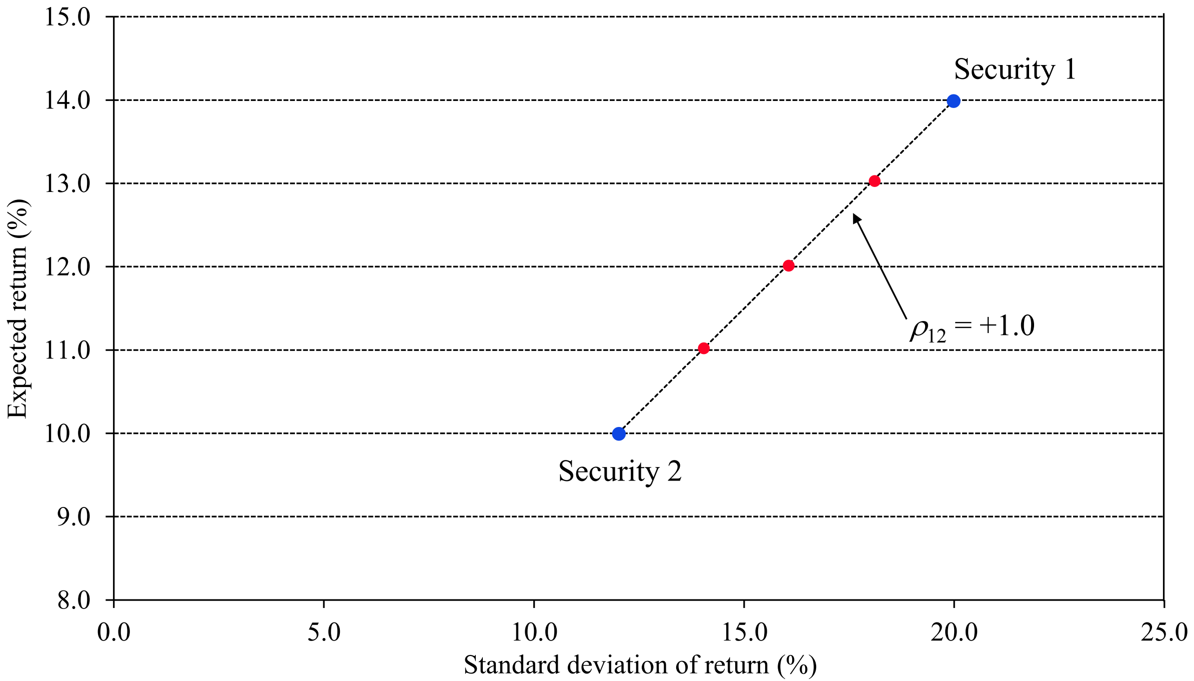
- *Case 1: Return correlation, $\rho_{12} = +1$*
- There are no gains from diversification in this case as the portfolio's risk (standard deviation) is a weighted-average of the risks (standard deviations) of the two securities
- *What is the economic intuition here?*
- The expected return and standard deviation of returns are…
  - $E(r_p) = w_1(0.14) + (1 - w_1)(0.10)$
  - $\sigma_p = \{w_1^2(0.20)^2 + (1 - w_1)^2(0.12)^2 + 2w_1(1 - w_1)(0.20)(0.12)(+1)\}^{1/2}$
  - $\sigma_p = \{[w_1(0.20) + (1 - w_1)(0.12)]^2\}^{1/2}$

Case 1: Perfect Positive Correlation

- *Case 1: Return correlation, $\rho_{12} = +1$*
- $E(r_p) = w_1(0.14) + (1 - w_1)(0.10)$
- $\sigma_p = \{[w_1(0.20) + (1 - w_1)(0.12)]^2\}^{1/2}$

Weight in 1(w_1)	*Expected return*	*Standard deviation*
0%	10.0%	12.0%
25%	11.0%	14.0%
50%	12.0%	16.0%
75%	13.0%	18.0%
100%	14.0%	20.0%

Case 1: Perfect Positive Correlation



Note: The red dots indicate the data points that were calculated in the previous slide

Case 2: Perfect Negative Correlation

- *Case 2: Return correlation, $\rho_{12} = -1$*
- There will be *maximum* gains from diversification in this case
- It is *always* possible to construct a *zero* risk portfolio in this case!
- *What is the economic intuition in this case?*
- The expected return and standard deviation of returns are…
  - $E(r_p) = w_1(0.14) + (1 - w_1)(0.10)$
  - $\sigma_p = \{w_1^2(0.20)^2 + (1 - w_1)^2(0.12)^2 + 2w_1(1 - w_1)(0.20)(0.12)(-1)\}^{1/2}$
  - $\sigma_p = \{[w_1(0.20) - (1 - w_1)(0.12)]^2\}^{1/2}$

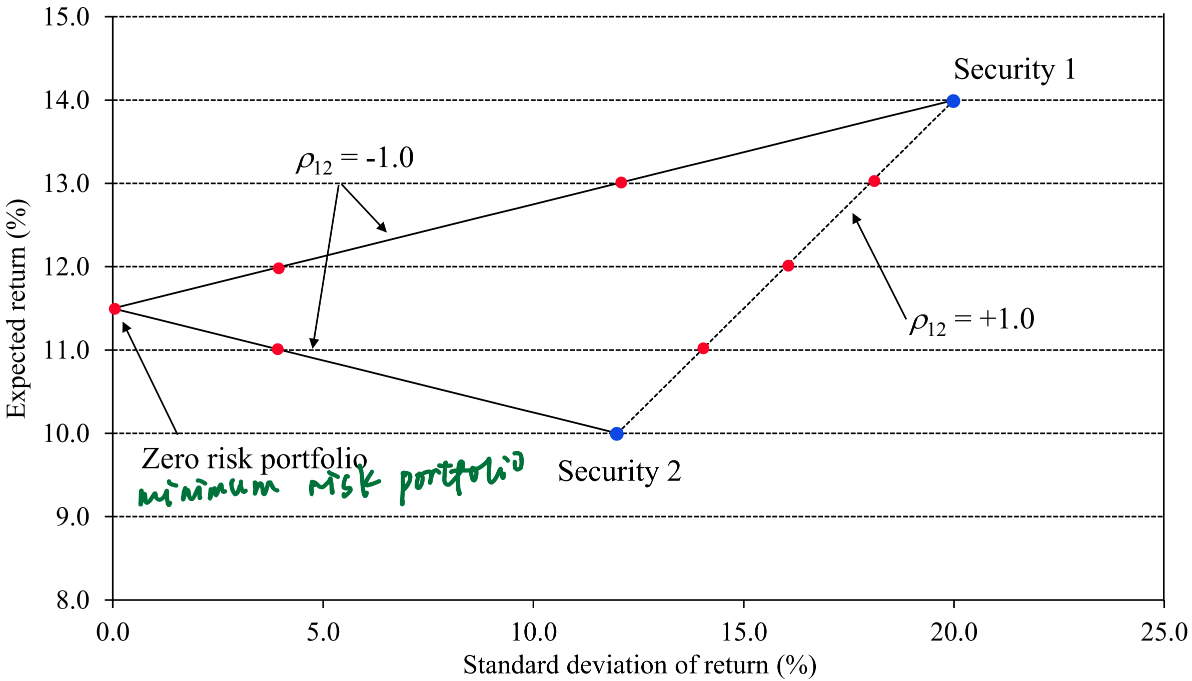
Case 2: Perfect Negative Correlation

- *Case 2: Return correlation, $\rho_{12} = -1$*
- $E(r_p) = w_1(0.14) + (1 - w_1)(0.10)$
- $\sigma_p = \{[w_1(0.20) - (1 - w_1)(0.12)]^2\}^{1/2}$

w1 = 1 − w1 ?

Weight in 1 (w_1)	*Expected return*	*Standard deviation*
0%	10.0%	12.0%
25%	11.0%	4.0%
50%	12.0%	4.0%
75%	13.0%	12.0%
100%	14.0%	20.0%

Case 2: Perfect Negative Correlation



Note: The red dots indicate the data points that were calculated in the previous slides

Case 2: Perfect Negative Correlation

- The *minimum variance (or standard deviation)* portfolio is where $\sigma_p = 0$
- When the correlation is –1 the standard deviation expression simplifies to…
  - $\sigma_p = \{w_1^2(0.20)^2 + (1 - w_1)^2(0.12)^2 + 2w_1(1 - w_1)(0.20)(0.12)(-1)\}^{1/2}$
  - $\sigma_p = \{[w_1(0.20) - (1 - w_1)(0.12)]^2\}^{1/2}$
  - $\sigma_p = w_1(0.20) - (1 - w_1)(0.12)$
- Set $\sigma_p = 0$ and solve for w_1 in the above expression…
  - $0 = w_1(0.20) - 0.12 + w_1(0.12)$
  - $0 = w_1(0.20 + 0.12) - 0.12$
  - $w_1 = 0.12/(0.20 + 0.12) = 37.5\%$ and $w_2 = 1 - w_1 = 62.5\%$
- $E(r_p) = 0.375(0.14) + 0.625(0.10) = 11.5\%$

Case 3: Return Correlation Between –1 and +1

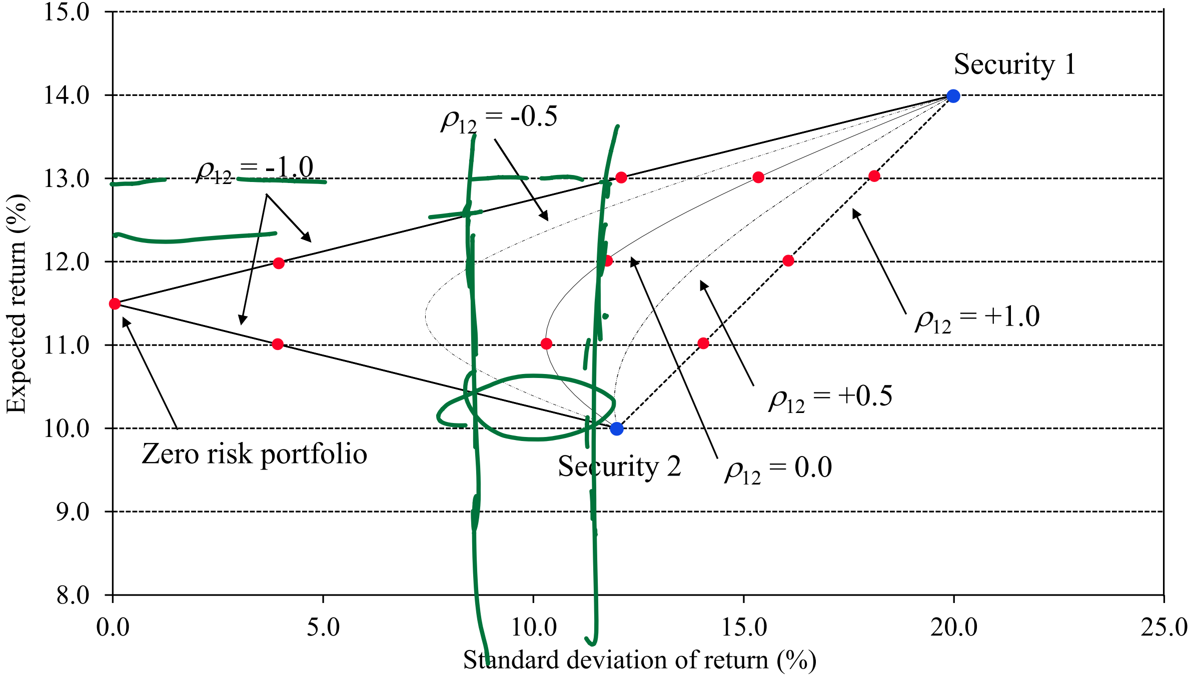
- *Case 3: Return correlation, $-1 < \rho_{12} < +1$*
- Some diversification benefits *always* exist
- $\sigma_p^{\ 2} = w_1^{\ 2}\sigma_1^{\ 2} + (1 - w_1)^2\sigma_2^{\ 2} + 2w_1(1 - w_1)\sigma_1\sigma_2\rho_{12}$
- How small or large the diversification benefits are depends on the correlation of returns
  - Lower the correlation; higher the diversification benefits!

Case 3: Return Correlation Between –1 and +1

- *Case 3: Return correlation, $\rho_{12} = 0$*
- $E(r_p) = w_1(0.14) + (1 - w_1)(0.10)$
- $\sigma_p = [w_1^2(0.20)^2 + (1 - w_1)^2(0.12)^2]^{1/2}$

Weight in 1(w_1)	*Expected return*	*Standard deviation*
0%	10.0%	12.0%
25%	11.0%	10.3%
50%	12.0%	11.7%
75%	13.0%	15.3%
100%	14.0%	20.0%

Summary of Risk-Return Tradeoffs



Note: The red dots indicate the data points that were calculated in the previous slides

5.6 Portfolio Risk and Return: Leveraging

- *Portfolio leveraging* refers to the strategy where an investor borrows funds at the riskfree rate of return and invests all the available funds in a risky security (or portfolio)
- *Example:* Consider the following information on a riskfree security (*F*) and risky security (*A*)

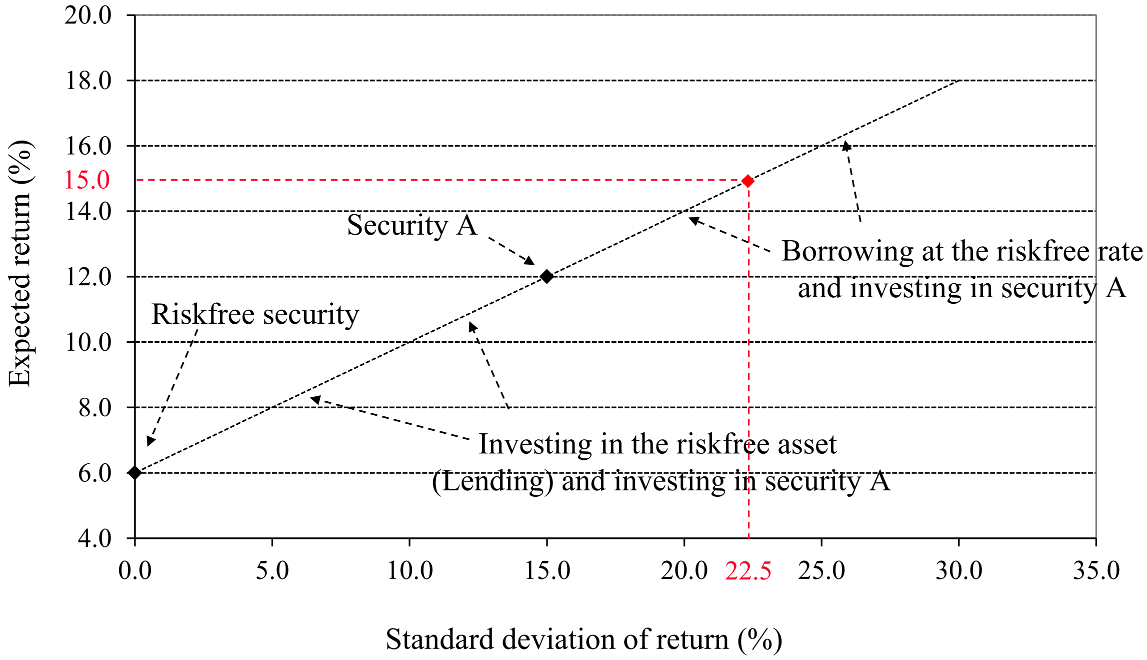
	Expected return	*Standard deviation of returns*
Riskfree security (*F*)	6.0%	0.0%
Risky security (*A*)	12.0%	15.0%

- Calculate the expected return and standard deviation of return of a portfolio if an investor with $10,000 *borrows* $5,000 at the riskfree rate and invests $15,000 in the risky security

Portfolio Risk and Return: Leveraging

- Proportion of funds invested in risky security
  - $w_A = 15000/10000 = 1.5$ or 150%
- Proportion of funds *borrowed* at the riskfree rate
  - $w_F = –5000/10000 = –0.5$
- Expected return of the portfolio…
  - $E(r_p) = –0.5(0.06) + 1.5(0.12) = 15.0\%$
- Standard deviation of return of the portfolio…
  - $\sigma_p = [w_A^2\,\sigma_A^2 + w_F^2\,\sigma_F^2 + 2w_A w_F \sigma_{AF}]^{1/2}$
  - $\sigma_p = [w_A^2\,\sigma_A^2 + w_F^2(0) + 2w_A w_F(0)]^{1/2}$
  - $\sigma_p = w_A \sigma_A$
  - $\sigma_p = 1.5(0.15) = 22.5\%$ (>> 15.0%)

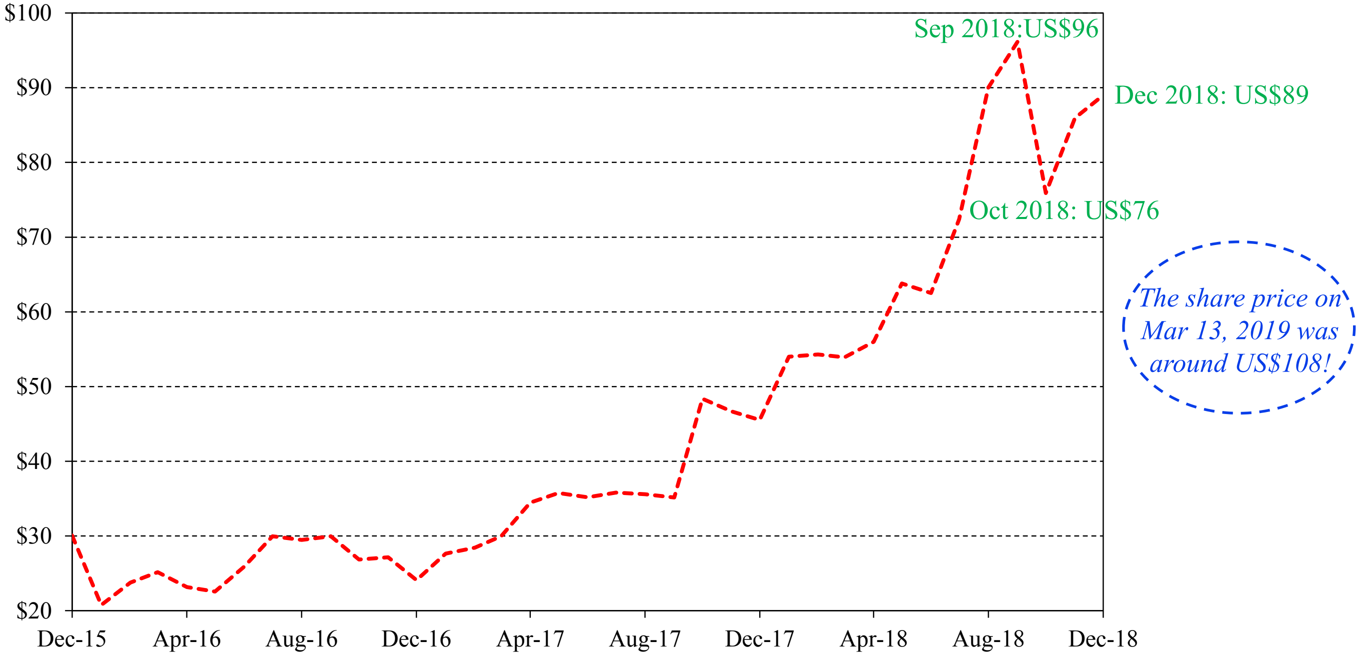
Portfolio Risk and Return: Leveraging



Portfolio Risk and Return: Short Selling

- *Short selling* refers to borrowing (typically via a broker) shares, selling them now with a *contractual* obligation to buy them back later at (an expected) lower price
  - Short selling is like *risky* borrowing to leverage a portfolio
  - This leveraging increases portfolio risk!
- Borrowing and short selling a security A and investing proceeds in security B implies $w_A < 0\%$ and $w_B > 100\%$ such that $w_A + w_B = 1$

Case Study 4: Time to Short Sell TEAM?



Source: https://finance.yahoo.com/quote/TEAM?p=TEAM. Atlassian's share price over Dec 2015 – Dec 2018.

Portfolio Risk and Return: Short Selling

- *Example:* You are given the following information on two stocks which have a return correlation of +0.5

Stock	*Expected Return*	*Standard Deviation of Returns*
A	10%	15%
B	20%	20%

- Calculate the expected return and standard deviation of returns for the following portfolios
  - An investor invests $5,000 each in stocks A and B (base case)
  - An investor with $10,000 borrows $5,000 worth of stock A and invests the total available funds in stock B
  - An investor with $10,000 borrows $10,000 worth of stock A and invests the total available funds in stock B

Portfolio Risk and Return: Short Selling

- *Base case:* An investor with \$10,000 invests \$5,000 each in stocks A and B implying that $w_A = w_B = 0.5$
  - $E(r_p) = 0.5(0.10) + 0.5(0.20) = 0.15$ or 15%
  - $\sigma^2_p = 0.5^2(0.15)^2 + 0.5^2(0.2)^2 + 2(0.5)(0.5)(0.15)(0.2)(0.5)$
  - $\sigma_p = (0.0231)^{1/2} = 0.152$ or 15.2%
  - There is a 95% probability that the *realized* return will lie in the range $(E(r_p) - 2\sigma_p, E(r_p) + 2\sigma_p)$ or (–15.4%, +45.4%)
- *Note:* Even though stocks A and B are positively correlated there is still some reduction in risk

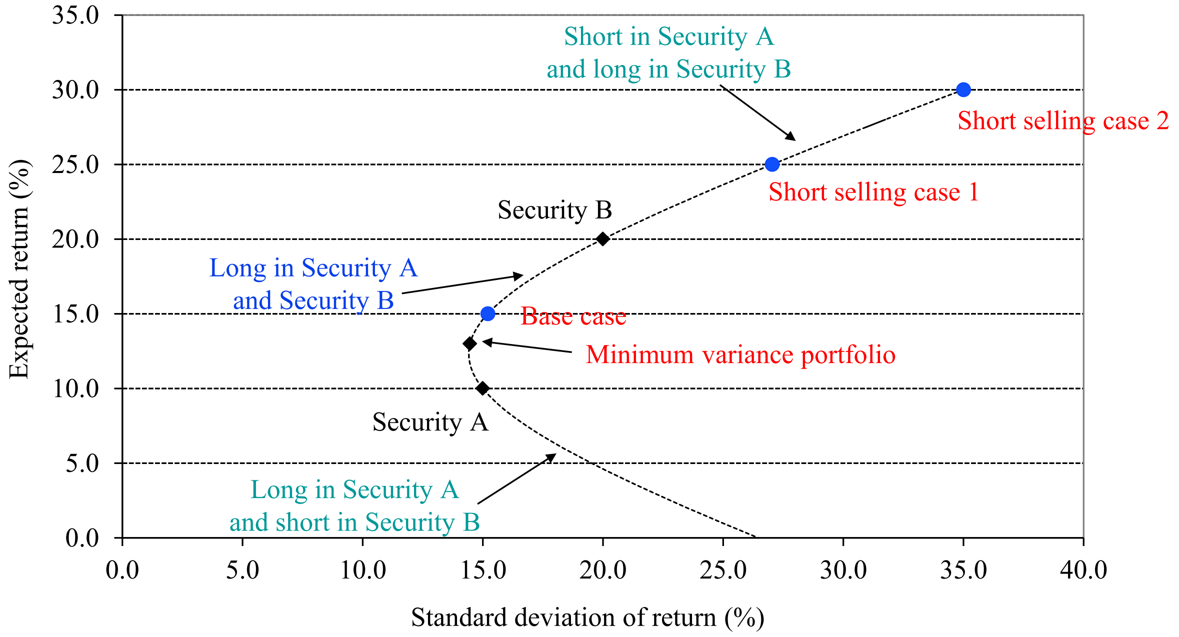
Portfolio Risk and Return: Short Selling

- *Short selling case 1:* An investor with \$10,000 borrows stock A and sells short \$5,000 worth of A, investing \$15,000 in stock B
  - $w_A = -5000/10000 = -0.5$
  - $w_B = (5000 + 10000)/10000 = 1.5$
  - $E(r_p) = -0.5(0.10) + 1.5(0.20) = 0.25$ or 25%
  - $\sigma^2_p = (-0.5)^2(0.15)^2 + 1.5^2(0.2)^2 + 2(-0.5)(1.5)(0.15)(0.2)(0.5)$
  - $\sigma_p = (0.0731)^{1/2} = 0.27$ or 27.0%
- There is a 95% probability that the *realized* return will lie in the range $(E(r_p) - 2\sigma_p, E(r_p) + 2\sigma_p)$ or (–29.0%, +79.0%)

Portfolio Risk and Return: Short Selling

- *Short selling case 2:* What happens to the portfolio's risk and expected return when the investor with \$10,000 sells short \$10,000 worth of stock A and invests \$20,000 in stock B?
  - $w_A = -10000/10000 = -1.0$
  - $w_B = (10000 + 10000)/10000 = 2.0$
  - $E(r_p) = -1.0(0.10) + 2.0(0.20) = 0.30$ or 30%
  - $\sigma^2_p = (-1.0)^2(0.15)^2 + 2.0^2(0.2)^2 + 2(-1.0)(2.0)(0.15)(0.2)(0.5)$
  - $\sigma_p = (0.1225)^{1/2} = 0.35$ or 35.0%
- There is a 95% probability that the *realized* return will lie in the range $(E(r_p) - 2\sigma_p, E(r_p) + 2\sigma_p)$ or (–40.0%, +100.0%)

Portfolio Risk and Return: Short Selling



Key Concepts

- Probability distributions represent specifications of possible outcomes in the future associated with probabilities of these outcomes occurring
- The expected return is the expected outcome measured as the weighted average of the individual returns and the standard deviation of returns is the measure of return dispersion
- A portfolio’s expected return is the weighted average of the returns of its component securities
- A portfolio’s variance is the weighted average of the variances of its component securities and the covariance between their returns
- Portfolio leveraging refers to borrowing funds at the riskfree rate and investing funds in a risky security (or portfolio)
- Short selling refers to borrowing shares, selling them at the current market price with a promise to repurchase them later at an expected lower price
- Portfolio leveraging and short selling increase the expected return of a portfolio and the risk associated with that portfolio

Formula Sheet

- Security returns
  - $R_t = (P_t + D_t - P_{t-1})/P_{t-1}$
  - $R_t = (P_t + C_t - P_{t-1})/P_{t-1}$
- Arithmetic average return
  - $\bar{R} = (R_1 + R_2 + \ldots + R_T)/T$
- Geometric average return
  - $\bar{R}_g = [(1 + R_1)(1 + R_2) \ldots (1 + R_T)]^{1/T} - 1$
- Fisher relationship
  - $(1 + r) = (1 + r_r)(1 + i)$
- Expected return on a security
  - $\mathrm{E}(r) = p_1 R_1 + p_2 R_2 + \ldots + p_n R_n$
- Variance of return and standard deviation of return on a security
  - $\mathrm{Var}(r) \textit{ or } \sigma^2 = p_1[R_1 - \mathrm{E}(r)]^2 + p_2[R_2 - \mathrm{E}(r)]^2 + \ldots + p_n[R_n - \mathrm{E}(r)]^2$
  - $\mathrm{SD}(r) = \sigma$

Formula Sheet

- Covariance of returns
  - $\sigma_{12} = p_1[r_{11} - E(r_1)][r_{21} - E(r_2)] + \ldots + p_n[r_{1n} - E(r_1)][r_{2n} - E(r_2)]$
- Correlation coefficient of returns
  - $\rho_{12} = \sigma_{12} / \sigma_1\sigma_2$
- Covariance of returns
  - $\sigma_{12} = \sigma_1\sigma_2\,\rho_{12}$
- Expected return of a two security portfolio
  - $E(r_p) = w_1E(r_1) + w_2E(r_2)$
- Return variance of a two security portfolio
  - $\sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\sigma_{12}$ *or*
  - $\sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\sigma_1\sigma_2\,\rho_{12}$

(*Note:* The formula sheets on the mid semester and final exams will contain all the formulas covered in lectures but *without* the descriptions)

Required Readings: Weeks 5 – 7

- *Week 5*
  - GRAH, Ch. 6 and Ch. 7 (Sec 7.1 – 7.2)
- *Week 6*
  - Mid Semester Exam
  - *Exam Duration:* 60 minutes (no reading time)
  - *Format:* Multiple choice questions
  - *Coverage:* Weeks 1 – 4 inclusive
- *Week 7*
  - GRAH, Ch. 7 (Review Sec 7.1 – 7.2)