Tutorial 6: Bases and coordinates

Q1. Consider the following ordered basis for $\mathbb{R}^2$: $\mathcal{B} = (\mathbf{b}_1, \mathbf{b}_2)$ where $\mathbf{b}_1 = (1,2)$ and $\mathbf{b}_2 = (-1,1)$. On the xy-plane, draw axes parallel to the basis vectors $\mathbf{b}_1$ and $\mathbf{b}_2$, marking multiples of $\mathbf{b}_1$ and $\mathbf{b}_2$ on these axes. Call the axes a and b respectively. Write the vector $(-2,5)$ in terms of the ordered basis $\mathcal{B}$. What is the coordinate vector $[(-2,5)]_{\mathcal{B}}$? Mark $(-2,5)$ on your graph, clearly showing the $\mathcal{B}$-coordinates.

Q2. Consider the ordered basis for $\mathbb{R}^3$, $\mathcal{B} = (\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3)$, where $\mathbf{b}_1 = (1,0,1)$, $\mathbf{b}_2 = (2,1,3)$, and $\mathbf{b}_3 = (1,2,1)$.

(a) Find the coordinate vector $[\mathbf{a}]_{\mathcal{B}}$ for $\mathbf{a} = (3,5,4)$.

(b) Find the vector $\mathbf{v} = (x,y,z) \in \mathbb{R}^3$ with coordinate vector $[\mathbf{v}]_{\mathcal{B}} = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$.

Q3. In each part, find the coordinate vector of $\mathbf{a}$ with respect to the given ordered basis $\mathcal{B}$.

(a) $\mathbf{a} = 2 - x + x^3$, $\mathcal{B} = (1, x, x^2, x^3)$ in $\mathcal{P}_4$;

(b) $\mathbf{a} = \begin{bmatrix} 3 & -2 \\ 0 & 4 \end{bmatrix}$, $\mathcal{B} = \left(\begin{bmatrix} -1 & -1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -2 \\ 2 & 4 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix} \right)$ in $M_{2,2}$.

Q4. For each of the following subspaces (i) find a spanning set, and (ii) determine if the spanning set is a basis for the subspace. If not, use the column method to find a basis for the subspace.

What is the dimension of each subspace?

(a) $A = \{(a+2b-c+2e, b+c+3d+2e, c+2d+e, a-b+3c+5d+3e) \mid a,b,c,d,e \in \mathbb{R}\} \subset \mathbb{R}^4$

(b) $B = \{a + (b-4a)x^2 + (a-2b)x^3 \mid a,b \in \mathbb{R}\} \subset \mathcal{P}_3$

Q5. Are the following functions linear transformations?

(a) $T\colon \mathbb{R}^2 \to \mathbb{R}$ given by $T(x,y) = (x+y)^2$

(b) $T\colon \mathbb{F}_2^2 \to \mathbb{F}_2$ given by $T(x,y) = (x+y)^2$