Investments: Bond Portfolio Management

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Let's start from a concrete problem

- Suppose you work for an investment company or superannuation fund.

- Your boss expects yields to **decrease**
  - And asks you to develop a strategy to profit from this expectation.

$$\uparrow P = \sum_{i=1}^{T} \frac{E\left[\widetilde{CF}_{t+i}\right]}{(1 + E[\tilde{r}])^{i}} \downarrow$$

PAUSE: Be careful! Finance is wicked!

- Most of the texts, websites and others discussing the relation between prices and interest rates make it sound like:

$$\text{Yields} \Longrightarrow \text{Prices}$$

  – When it really is:

$$\overbrace{\text{Discount rates} \Longrightarrow \text{Prices}}^{\text{Economic}} \Longrightarrow \text{Yields}$$

$$\text{Prices} \Longrightarrow \text{Yields} \quad \text{(Definition/Mechanical)}$$

PAUSE! Something confusing

- In practice, textbooks, investors, and other smart people often conflate Yield to Maturity and $E[\tilde{r}]$
- In many places this doesn't matter because the relation between YTM and P is mathematically identical to that of $E[\tilde{r}]$ and P:

$$P = \sum_{i=1}^{T} \frac{E[\widetilde{CF}_{t+i}]}{(1+E[\tilde{r}])^i}$$

real input: $E[\tilde{CF}]$, $E[\tilde{r}]$

$$P = \sum_{i=1}^{T} \frac{Promised\ CF_{t+i}}{(1+YTM)^i}$$

real input promised CF & P.

If we assume there is no default then what is true for r_f and $E[\tilde{r}]$ is true for YTM

not default ⇒ similar for two equations

When does the difference matter?

$$P = \sum_{i=1}^{T} \frac{E[\widetilde{CF}_{t+i}]}{(1+E[\tilde{r}])^i} \qquad P = \sum_{i=1}^{T} \frac{Promised\ CF_{t+i}}{(1+YTM)^i}$$

- When we start discussing bond portfolio strategies and especially a hedging strategy called ‘immunization’, then the differences might matter.
  - We will come back to at the end of the lecture

- In the next part of lecture, I am going to follow the book and convention and discuss relations between YTM and P.
  - When I note that it is a relation is mathematical it applies just as well to P and $E[\tilde{r}]$ as it does to P and YTM.

Let's start from a concrete problem

- Suppose you work for an investment company or superannuation fund.

For this motivating question, let's temporarily assume that there is *heterogeneity of opinion*, **i.e. different investors have different opinions about future interest rates, so that prices do not yet reflect what your boss thinks.**

- Your boss expects yields to **decrease**
  - And asks you to develop a strategy to profit from this expectation.

$$\uparrow P = \sum_{i=1}^{T} \frac{E\left[\widetilde{CF}_{t+i}\right]}{\left(1 + E\left[\tilde{r}\right]\right)^{i}} \downarrow$$

Why Do Yields Change?

1. Change in the credit quality of the issuer
   - The chance a bond will make all its promised payments increases or decreases
     - For example, a firm improves its credit rating from Ba to Aa ⇒ lower prob to default ⇓ P go up ⇓ yield go down
       - Also, as you saw on last week's homework, the risk of default can change over the macroeconomic cycle

2. Change in the yield on comparable bonds.
   - "Comparable" bonds means "comparably risky" bonds
   - Why would the yield on <u>other</u> bonds affect the current bond you are looking at?
     - Arbitrage.

Working example for the next few slides

To stay focused on what happens with changes in interest rates,

- Assume:
  - Risk-free bonds
  - Flat term structure (annualized returns are the same for all periods)

- 10 year bond, with
  face value = $1000
  coupon rate = 8% paid annually

$$P = \sum_{i=1}^{T} \frac{CF_{t+i}}{\left(1 + r_f\right)^i}$$

Bond Valuation: Bonds with Coupons

- 10 year bond, with
  face value = $1000
  coupon rate = 0.08 paid annually
  r_f = 0.08   0.06   0.10

$$P = \sum_{i=1}^{10} \frac{CF_{t+i}}{\left(1 + r_f\right)^i}$$

if coupon rate = rf ⇒ P = FV

$$P = \left(\sum_{i=1}^{10} \frac{80}{(1+.08)^i}\right) + \frac{1000}{(1+.08)^{10}} = \$1000$$

Bond Valuation: Bonds with Coupons

- Example: 10 year bond, with
  par value = $1000
  coupon rate = 8% paid annually
  r_f = 0.08, 0.06, 0.10

$r_f = 0.08 \Rightarrow P = \$1000$

$r_f = 0.06 \Rightarrow P = \$1147.20$

$r_f = 0.10 \Rightarrow P = \$877.11$

Bond Prices, Discount Rates, and Yields

Prices and discount rates have an **inverse relationship**

- High discount rates $\Longrightarrow$ Low Prices $\Longrightarrow$ High Yields
- Low discount rates $\Longrightarrow$ High Prices $\Longrightarrow$ Low Yields

- When discount rates get very high the value of the bond will be very low
  - And yields will be high

- When discount rates approach zero, the value of the bond approaches the sum of the cash flows → yield will be very low.

Bond Valuation: Bonds with Coupons

- Example: 10 year bond, with
  par value = $1000
  coupon rate = 8% paid annually
  $r_f = 0.08, 0.06, 0.10$

$r_f = 0.08 \Rightarrow P = \$1000$

$r_f = 0.06 \Rightarrow P = \$1147.20$

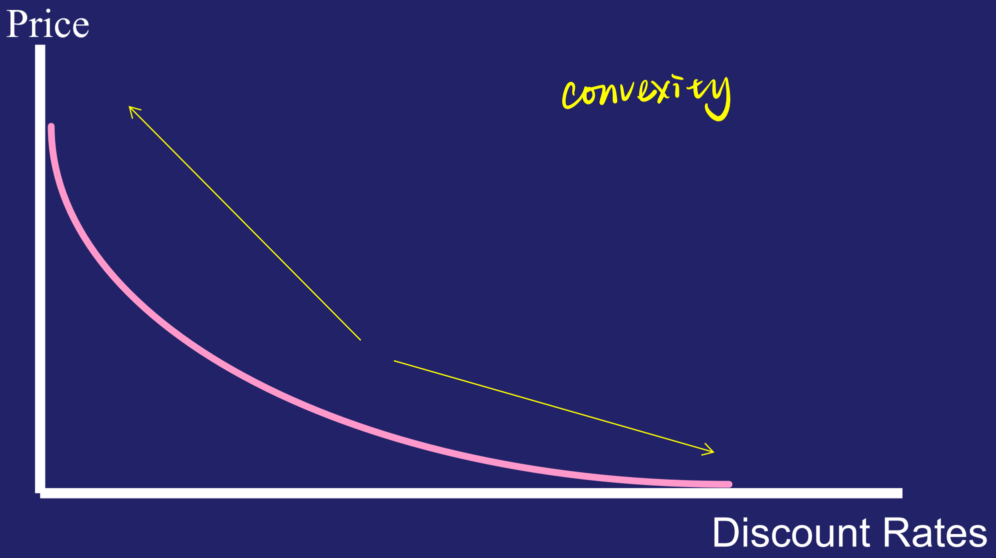
$r_f = 0.10 \Rightarrow P = \$877.11$

✩: change was different

Notice: return down 2% and price up $147.20

BUT return up 2% and price down only $122.89

Prices and Discount Rates



Notice: Decreases in discount rate increase prices more than increases in discount rates lower prices.

Prices and Yields

Mathematically, the relation between yields and price is exactly the same.

Here I am careful to keep clear that prices drive yields and ~~not yields drive prices~~.

But, people often speak as if yields drive prices.

$(\frac{1}{x})' = -\frac{1}{x^2}$

when x ↓.

slope ↑.

Notice: Increases in price lower yields less than decreases in price raise yields

Why is the change in price different?

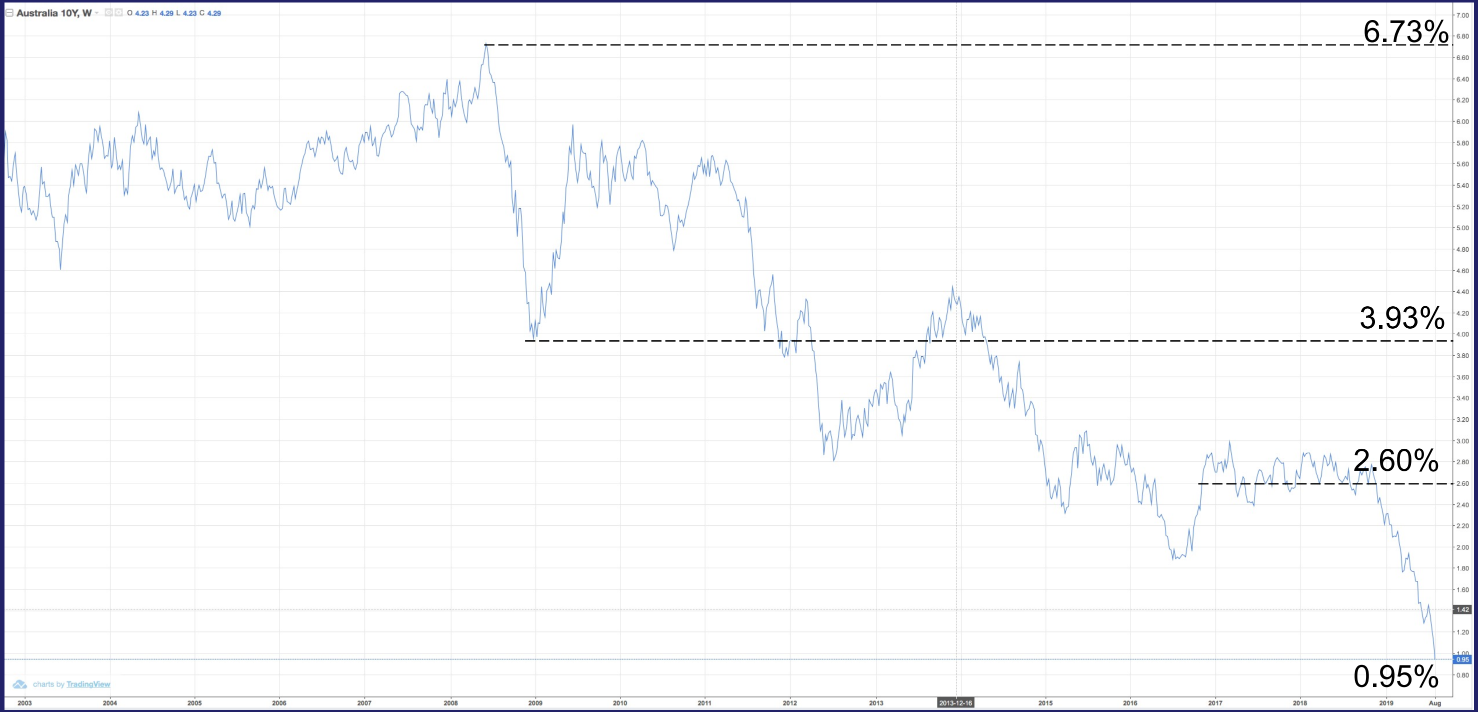
- When the discount rate goes up, the 8% coupon is (\$20) less than the 10% discount rate so the price must drop to make up for the interest you are not getting in coupons.
- When the discount rate goes down the 8% coupon is (\$20) more than the 6% discount rate so the price must increase so that the bond holder doesn't receive too much interest.
- **But why does the price increase more when the discount rate goes down than when the discount rate goes up?**
  - Answer: when the discount rate goes up to 10%, the \$20 less you get is discounted more heavily, than the \$20 more you get when the discount rate drops to 6%. So, MORE heavily discounted means the present value is less, i.e. the price drop is less. The same is true for the face value.

discount not that heavily
↓
bigger impact on valuation

Duration

A measure of interest-rate risk

Do we need a measure of interest-rate risk?



https://markets.tradingeconomics.com/tvchartexternal/pop?s=GACGB10:IND&interval=W&locale=com&originUrl=https://tradingeconomics.com/australia/government-bond-yield&AUTH=y1ODXy7p7geU2%2FpBvysmTrp%2BvyKJzhoevjBHc6yaRYI%3D

Interest Rate Risk

- Bond values change when interest/discount rates change
  - even if payments are certain,
  - bonds are risky investments, *if you plan to sell before maturity*

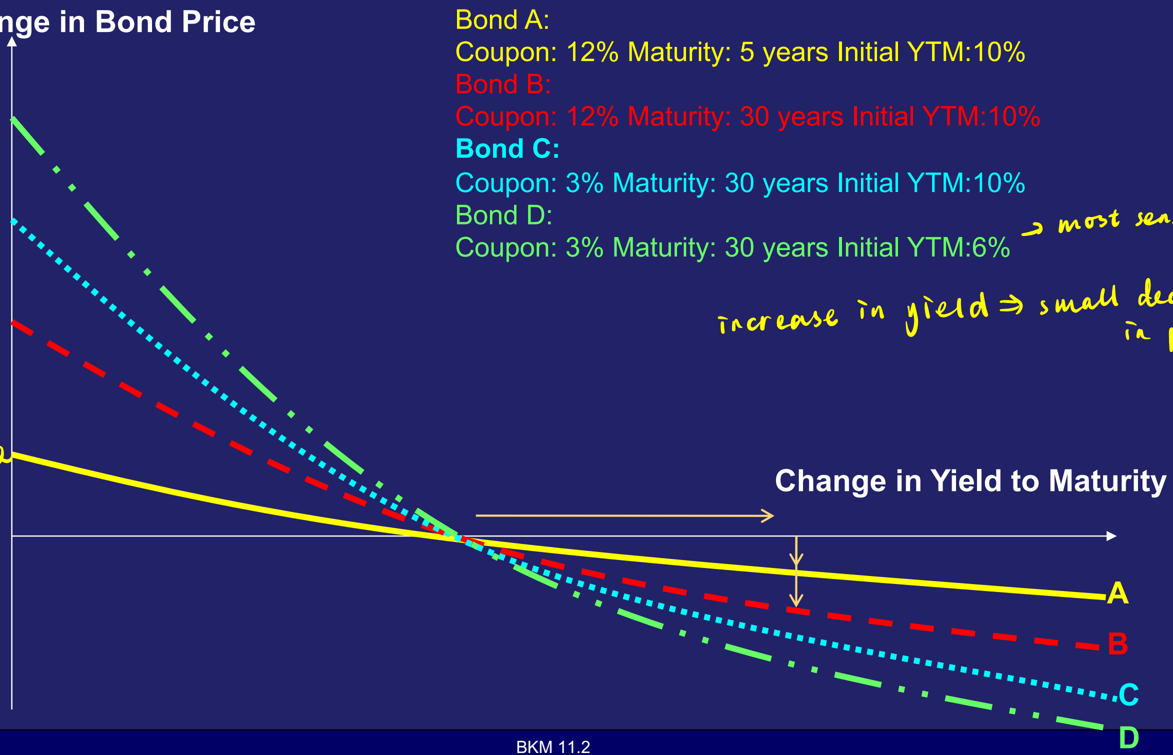
- Goal: Measure interest rate sensitivity of bonds
  - What is the change in the value of the bond for a small change in the interest rate?
  - How can the value of a bond portfolio be protected against movements in interest rates?
  - How can predictions about interest rate changes be used to increase the value of a bond portfolio?

Changes in Bond Prices

mathematical relation

duration ↑

more sensitive if
① low coupon bond
② long Maturity
③ low YTM before the change in interest rate



Bond A:
Coupon: 12% Maturity: 5 years Initial YTM:10%

Bond B:
Coupon: 12% Maturity: 30 years Initial YTM:10%

Bond C:
Coupon: 3% Maturity: 30 years Initial YTM:10%

Bond D:
Coupon: 3% Maturity: 30 years Initial YTM:6% → most sensitive

increase in yield ⇒ small decrease in price

Bond price/yield sensitivity (in English)

- Yields and prices are inversely related
- The relation between yields and prices is convex
- Prices are more sensitive to changes in discount rates
- or yields are more sensitive to changes in prices when:
  - Maturity is longer
  - Coupons are smaller
  - Discount rates/Yields are lower

Bond price/yield sensitivity examples (in math)

Example 1: Prices of 8% Coupon Bond (semi-annual payments)

	Yield to Maturity	T = 1 Year	T = 10 Years	T = 20 Years
Increase	8%	1000	1000	1000
by 1% to	9%	990,64	934,96	907,99
	Change in Price	**-0,94%**	**-6,50%**	**-9,20%**

Example 2: Prices of Zero-Coupon Bond (semi-annual compounding)

	Yield to Maturity	T = 1 Year	T = 10 Years	T = 20 Years
Increase	8%	924,56	456,39	208,29
by 1% to	9%	915,73	414,64	171,93
	Change in Price	-0,96%	-9,15%	-17,46%

An Example to try

- Your boss expects yields to **decrease**
  - And asks you to develop a strategy to profit from this expectation.
  - Which is the best to invest in and which the worst?
    - A) A 10-year maturity, 5% coupon bond
    - B) An 8-year maturity, 5% coupon bond → least sensitive
    - C) A 10-year maturity, 0% coupon bond ✓ → most sensitive
      best to invest in if we expect interest rate ↓.
    - D) An 8-year maturity, 0% coupon bond

Duration

- Bonds basically differ on two observable dimensions:
  - coupon rate
  - time to maturity

- Duration is a measure that combines these two features into one number:
  - the weighted average or effective maturity of promised cash flows

- Duration is defined as:

$$D = \sum_{t=1}^{T} w_t \times t$$

time : maturity of each cash flow

Uses of Duration

1. Summary measure of length or effective maturity for a portfolio
   - https://www.pimco.com.au/en-au/investments/australia/australian-bond-fund/inst
   - https://www.spdrs.com.au/etf/fund/spdr-sp-asx-australian-bond-fund-BOND.html

2. Measure of price sensitivity for changes in interest rate

3. Immunization of interest rate risk (passive management)
   - Ensuring you have the cash to pay your obligations

make sure value of assets matches the value of outflows.

https://www.pimco.com.au/en-au/investments/australia/australian-bond-fund/inst

Australian Bond Fund

ETL0115AU | UPDATED 08 AUGUST 2019

INST

TYPE NAME OR APIR

OVERVIEW | YIELDS & DISTRIBUTIONS | FEES & EXPENSES | PERFORMANCE | PORTFOLIO COMPOSITION | DOCUMENTS

All data as of 31 July 2019 unless otherwise stated

Region - Duration in Years

Australia/NZ	5.54
North America	0.20
Europe - Non-EMU	0.01
Emerging Markets	0.01
Other	0.00
United Kingdom	-0.10
Europe - EMU	-0.16
Japan	-0.28

Duration %

0-1 yrs	1.38
1-3 yrs	4.43
3-5 yrs	20.02
5-7 yrs	23.90
7-8 yrs	13.19
8-10 yrs	32.25
10+ yrs	4.84
Effective Duration (yrs)	5.21

Credit Quality Exposure - Market Value %

AAA	58.96
AA	12.92
A	7.64
BBB	16.59
Sub Investment Grade	3.89

Risk Characteristics (Trailing 3 Years)

Standard Deviation[3]	2.46
Sharpe Ratio[4]	1.12
Information Ratio[5]	-0.52
Tracking Error[6]	0.41

TOP

https://www.spdrs.com.au/etf/fund/spdr-sp-asx-australian-bond-fund-BOND.html

SPDR® S&P®/ASX Australian Bond Fund

BOND

Bloomberg Code
BOND AU

Iress Code
BOND.AXW

Key Features
Relatively Low Cost±
Tradability
Transparency of Performance
Diversification^

Inception Date
26/07/2012

ISIN
AU00000BOND4

Fund Objective
The SPDR S&P/ASX Australian Bond Fund seeks to closely track, before fees and expenses, the returns of the S&P/ASX Australian Fixed Interest Index.

Index Description
The S&P/ASX Australian Fixed Interest Index Series is a broad benchmark index family designed to measure the performance of the Australian bond market, which meets certain investability criteria. The index is split across investable investment grade, Australian dollar denominated bonds issued in the local market with maturities greater than one year.

Characteristics	
Number of Holdings	143
Average Maturity in Years → higher, since maturity measures the time we receive face value payment	7.01
Current Yield	3.32%
Modified Adjusted $\boxed{\text{Duration}}$ → also put weights on time for coupon payment	6.09
Yield to Maturity	1.24%

Maturity Breakdown	Weight %
0 - 3 Years	21.68
3 - 5 Years	18.39
5 - 7 Years	15.74
7 - 10 Years	27.65
10 - 15 Years	10.58
15 - 20 Years	3.73
20 - 30 Years	2.23

Changes in Bond Prices

Two flavors of Duration

Duration is the present-value-weighted-average-maturity

$$D = \sum_{t=1}^{T} w_t \times t$$

- Exact duration
  - w_t is the present value (price) of the cash flow
- Internal duration
  - w_t uses the yield to maturity as if it were $E[\tilde{r}]$

The Internal Duration is what your book describes. This is what most people mean when they say "duration".

Also, called MacCaulay Duration

For more, see: https://www.asc.ohio-state.edu/mcculloch.2/ts/duration.htm

Exact Duration

$$D = \sum_{t=1}^{T} w_t \times t$$

where:

true present value of cash flow

$$w_t = \frac{\boxed{E\left[\widetilde{CF}_t\right]/(1 + E[\tilde{r}])^t}}{P_{bond}}$$

Can also be:

$E\left[\widetilde{CF}_t\right]/(1 + E[\tilde{r}_t])^t$

If the term structure is not flat.

and:

$$P_{bond} = \sum_{t=1}^{T} \frac{E\left[\widetilde{CF}_t\right]}{(1 + E[\tilde{r}])^t}$$

- Duration is the the present-value-weighted average time to maturity.

Internal Duration – MacCaulay Duration

$$D = \sum_{t=1}^{T} w_t \times t$$

where:

$$w_t = \frac{\left(promised\ CF_t/(1+y)^t\right)}{P_{bond}}$$

MacCaulay invented both Exact and Internal Duration, but when we say "MacCaulay Duration", this is what we mean.

- As if:
  - $E[\tilde{r}]$ = YTM.
  - Term structure is flat

Calculating Duration: an example

$$D = \sum_{t=1}^{T} w_t \times t, \qquad where \ \ w_t = \frac{promised\ CF_t/(1+y)^t}{P_{bond}}$$

- 8% bond with 3 years to maturity, 10% yield to maturity.

Time (year)	CF	Pseudo PV	weight	w x t
1	80	72.73	$\frac{72.73}{950.27}$ 0.0765	0.0765
2	80	66.12	0.0696	0.1392
3	1080	811.42	0.8539	2.5617
Sum		950.27 Real PV	1	2.7774

What is the duration of an 8% T-bond, with 1-year maturity…?

- 8% T-bond (semi-annual coupon paying) with 1 year to maturity, 10% (bond equivalent) yield to maturity

Time (half years)	CF	Pseudo PV	weight	w x t
1	40	38.10 $= \frac{40}{1.05}$	0.0388 $= \frac{38.1}{981.41}$	0.0388
2	1040	943.31 $= \frac{1040}{1.05^2}$	0.9612	1.9224
Sum		981.41	1	1.9612
In Years				0.9806

Duration – Basic Rules

1. The duration of a zero-coupon bond equals its time to maturity;

just one payment = FV payment

$$D_{zcb} = \boxed{\frac{Promised\ Face\ Value/(1+y)^T}{P_{bond}}} \times T = T$$

= 1

2. Holding maturity constant, a bond's duration is higher when the coupon rate is lower;

coupon rate ↓ ⇒ value of early coupon payment is less ⇒ lower weight of those payments ⇒ duration ↑

3. Holding the coupon rate constant, a bond's duration generally increases with its time to maturity. Duration always increases with maturity for bonds selling at par or at a premium to par;
   - For discount bonds, high $E[\tilde{r}]$ and a long time to maturity means that distant future payouts contribute little to the bond price. As such, if discount rates are high enough and the maturity long enough, near-term coupons will be more important and duration will not increase with maturity.

Duration – Basic Rules

4. Holding other factors constant, the duration of a coupon bond is higher when the bond's yield to maturity is lower;

YTM ↓ ⇒ weight of face value ↑

⇒ duration ↑

bond pays coupon but no face value (never end).

5. The duration of a level perpetuity is (1+y)/y.
   - For example, at a 10% yield, the duration of a perpetuity that pays $100 once a year for ever will equal 1.10/0.10=11 years;

$$D_{perp.} = \frac{1+y}{y}$$

A Duration Cheat

- The duration of a coupon bond equals:

$$D = \frac{1+y}{y} - \frac{(1+y) + T(c-y)}{c\left[(1+y)^T - 1\right] + y}$$

y = yield to maturity
c = coupon **rate** (not $ amount) } decimal form

Uses of Duration

- Summary measure of length or effective maturity for a portfolio
- Measure of price sensitivity for changes in interest rate
- Immunization of interest rate risk (passive management)
  - Ensuring you have the cash to pay your obligations

Interest Rate Change Sensitivity

- Q: What is the impact of a small change in discount rates on bond value?
- A: An approximation is given by the first derivative of the bond price with respect to yield to maturity:

growth yield

$$\frac{\Delta P}{P} = -D\left[\frac{\Delta(\boxed{1+y})}{1+y}\right]$$

Interest Rate Change Sensitivity

- There is a modified version for the sensitivity of a bond price to changes in yield apparently often used:

$\Delta(1+y) = \Delta y$.

$$\frac{\Delta P}{P} = -D^* \Delta y$$

where $D^* = \frac{D}{1+y}$

D* is called Modified Duration

modified duration is a measure of the sensitivity to a change in yield

- Dollar Duration:

$$\Delta P = -D^* \times \Delta y \times P$$

Interest Rate Sensitivity Example

$\Delta y = 0.01\%$

- An 8% annual coupon paying 3-year bond has a YTM of 9%; how much will its price change if YTM increases to 9.01% (one basis point)?

1. Set up the cash flows
2. Calculate PV for every cash flow
3. Calculate price of the bond
4. Calculate weights and duration
5. Calculate Modified Duration
6. Calculate price change

w.

year 1 80 $\frac{80}{1.09} =$ 0.00926

year 2 80 $\frac{80}{1.09^2} =$ 0.0085

year 3 1080. $\frac{1080}{1.09^3} =$ 0.9824

$P =$ 2.

$D = 1 \times 0.008 + 2 \times 0.00789 + 3 \times 0.9835$

$=$

$D^* = \frac{D}{1+y} = 2.7287$

Interest Rate Sensitivity - Example

- An 8% annual coupon paying 3-year bond has a YTM of 9%; what is its Duration and how much will its price change if YTM increases to 9.01% (one basis point)?

- Solution: The bond's current price can be found as

$$P = \sum_{t=1}^{3} \frac{80}{(1.09)^t} + \frac{1000}{(1.09)^3} = \$974.69$$

- And its duration is:

$$\underset{D}{\cancel{P}} = \sum_{t=1}^{3} \frac{80/(1.09)^t}{974.69} \times t + \frac{1000/(1.09)^3}{974.69} \times 3 = 2.78$$

Example (con't)

- The expected (approximate) price change is:

$$\Delta P = -D^{*} \times \Delta y \times P$$

$$\Delta P = -\frac{2.78}{1.09} \times 0.0001 \times 974.69$$

$$\Delta P = -\$0.25$$

- In the event of a 0.01% (1 basis point) increase in interest rates, the bond loses $0.25 in value

Example (con't)

- Using duration as the measure of interest rate sensitivity we obtain the new price of the bond as

$$P_{new} = \$974.69 - 0.25 = \$974.44$$

- Using the exact formula, we obtain

larger changes

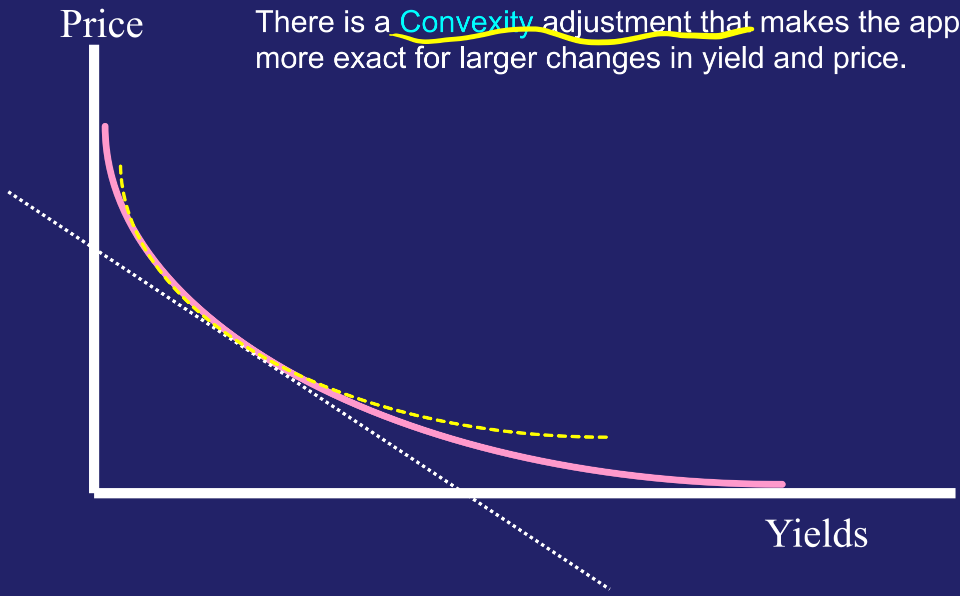
$\Rightarrow$ estimate will be off

$$P = \sum_{t=1}^{3} \frac{80}{(1.0901)^t} + \frac{1000}{(1.0901)^3} = \$974.44$$

Convexity

With Duration we approximate the slope with a tangent line.

There is a Convexity adjustment that makes the approximation more exact for larger changes in yield and price.



Convexity

$$\frac{\Delta P}{P} = -D^{*}\Delta y + \frac{1}{2}\times convexity\times(\Delta y)^2$$

where

$$convexity = \frac{1}{P(1+y)^2}\sum_{t=1}^{T}\frac{CF_t}{(1+y)^t}(t^2+t)$$

Investors like convexity because more convex bonds increase in price more when yields drop than they decrease in price when yields rise.

Uses of Duration

- Summary measure of length or effective maturity for a portfolio
- Measure of price sensitivity for changes in interest rate
- Immunization of interest rate risk (passive management)
  - Ensuring you have the cash to pay your obligations

Bond Management Strategies

Why do we need an interest rate risk measure?

- Insurance companies that sell annuities → receive fixed payment from clients, promise to pay out level cash flows over a number of year
  - Guarantee a fixed growth rate over a period
  - Investment can be withdrawn at any time
    - Might be charges or penalties
  - The insurance company promises a fixed amount

- To do this insurance companies often invest in long-term bonds
  - Investors are tempted to withdraw when yields increase
    - And Prices are low! yield ↑ , P ↓ ⇒ investors withdraw

- Ability to hedge is important – so we need to measure risk.

Bond Management Strategies

- Interest Rate Risk: Portfolios of bonds can be aversely affected by changes in interest rates
  - Interest rate increases lower the value of the portfolio
  - Interest rate decreases reduce the earning power of reinvested interest

  ⇒ although raise the overall value of portfolio

- Solution: immunization

Immunization

- Match duration of assets and liabilities

- Cash flow matching and dedication
  - Matching = single period
  - Dedication = multiple periods
  - Perfect solution but very hard to do

Pre-Example: Immunization

- Note: the duration of two assets with the same yield to maturity is just the weighted average of the duration of the 2 assets

$$D_{portfolio} = \sum_{n=1}^{N} \frac{MV_n}{\sum_{n=1}^{N} MV_n} D_n$$

MV_n is the Market Value of the amount invested in asset *n*.

- Example: Suppose you invest 25% of your money in an asset with a duration of 1 and 75% in an asset with a duration of 4

- The duration of your portfolio is:

  0.25 x 1 + 0.75 x 4 = 3.25

Immunization Example

- Suppose you are the CFO for a company that must make a payment to renew plant and equipment in 5 years of $22,040. The interest rate is 8%. Use 2 year zeros and an annual perpetuity paying the market rate to form a portfolio to immunize the liability.

- *Note: the Duration of a perpetuity paying the market rate is:*

$$D_{perpetuity} = \frac{1+y}{y}$$

Immunization Example

- The duration of the liability is: 5 → duration of zero coupon bond = timing of payments
- The duration of the perpetuity is:

$$D_{perpetuity} = \frac{1+y}{y} = \frac{1.08}{.08} = 13.5$$

- Find the weight to immunize:

$$2w + 13.5(1 - w) = 5$$

zeros: $w = 0.73913$ Perps: $(1 - w) = 0.26087$

- The present value of the liability is \$15,000.05 $= \frac{22040}{(1+8\%)^5}$

- So to immunize put \$11,086.99 in zeros and \$3913.06 in the perpetuity

Real Life: Problems with Immunization

- Duration protects against small rate changes only
  - The greater the convexity the more this statement is true
- The passage of time alters duration of our assets and liabilities
  - but not (likely to be) by exactly the same amount

may need to immunization

- Immunization is based on nominal not real rates

- Duration assumes flat yield curve with parallel shifts in rates
  - The steeper the yield curve the more important it is to use exact duration.

Active Bond Portfolio Management

Let’s start from a concrete problem

- Suppose you work for an investment company or superannuation fund.

- Your boss expects yields to **decrease**
  - And asks you to develop a strategy to profit from this expectation.

$$P = \sum_{i=1}^{T} \frac{E\left[\widetilde{CF}_{t+i}\right]}{\left(1 + E[\tilde{r}]\right)^{i}}$$

Active Bond Management

- Tries to find sources of profit rather than reduction of risk
  1. – Interest rate forecasting $i\downarrow \Rightarrow P\uparrow \Rightarrow$ high duration assets
     - If declines are forecast shift to high duration bonds
     - If increases are forecast shift to low duration bonds

       $i\uparrow \Rightarrow P\downarrow \Rightarrow$ low duration assets
  2. – Identification of mispriced bonds
     - Actual risk is higher or lower than is priced

Another way to think about duration

- Duration can also be thought of as the percentage change in a bond's price for a percentage change in a bond's required yield

$$\frac{\Delta P}{P} = -D\left[\frac{\Delta(1+y)}{1+y}\right]$$

$$\%\Delta P = -D\%\Delta(1+y)$$

$$D = -\frac{\%\Delta P}{\%\Delta(1+y)}$$

Thinking about duration this way…

- Suppose you had a bond with coupon rates that change when the required return changes.
  - These are called floating rate bonds.

- Does the price change when the yield changes?

since it is floating rate, so the price doesn't change

- No!
  - Because the coupon payments change by a large enough amount to compensate for the change in required interest rates.
  - Duration is zero.
    - The percentage change in price is zero.
  - *In real life, the percentage change in price will be really small. Because coupons change several times a year, not instantaneously.*

Exact vs. Internal Duration: When does it matter?

- If the yield curve is steep.

- Example: Suppose you have a 10-year bond with a 10% annual coupon. Suppose the yield curve starts at
  - 2% for 1-year CF
  - 4% for 2-year CF
  - …
  - 20% for 10-year CF

FV=100
C=10

$$\frac{10}{1.02} + \frac{10}{1.04^2} + \frac{10}{1.06^3} + \text{——}$$

- YTM? ~~6.13%~~ 15.44%
- Exact Duration=5.40
- Internal Duration = ~~12.55 !!~~ 6.19

later year cash flow will have lower weight in duration