Short Sales

Short Sales

Purpose: to profit from a decline in the price of a stock or security

Mechanics

- Borrow stock through your broker
- Sell it and deposit proceeds and margin in an account
- Closing out the position: buy the stock and return to the party from which is was borrowed

Tick-test rule

you may only short-sell on an up-tick or a zero-tick of the stock price

"you can't short sell of stock price dropping"

MICROSOFT CP (RT-ECN: MSFT)

Last Trade:	**26.12**
Trade Time:	**3:58pm ET**

→ 26.12 zero-tick
26.14 up-tick
26.11 down-tick

Bid		Ask	
Price	Size	Price	Size
26.12	24,595	**26.14**	30,363
26.11	20,770	**26.15**	19,280
26.10	20,490	**26.16**	25,430
26.09	17,645	**26.17**	10,780
26.08	9,892	**26.18**	5,980

Source: http://finance.yahoo.com/q/ecn?s=MSFT

Short Sale - Initial Conditions

Z Corp	100 Shares
50%	Initial Margin
30%	Maintenance Margin
$100	Initial Price

You borrow 100 shares of Z Corp and sell it immediately. You get $10,000 which the broker holds on to.

Sale Proceeds	$10,000
Margin & Equity	5,000
Initial Value of Stock Owed	$100 x 100 shares =$10,000

You put up $5000 in cash or other stock as collateral

liability is now a share not money

$$Margin\ Percent = \frac{Net\ equity\ in\ account}{Value\ of\ Stock\ Purchased} = \frac{Assets\ - Liabilities}{Value\ of\ Stock} = \frac{15{,}000 - 100P}{100P}$$

$$Margin\ Percent = 50\%$$

Short Sale - Maintenance Margin

Stock Price Rises to $110

The cash you put up as collateral plus the cash you got from selling the stock is your total equity.

Sale Proceeds	$10,000
Initial Margin	5,000
Value of Stock Owed	$110 x 100 shares =$11,000

$$Margin\ Percent = \frac{Assets\ - Liabilities}{Value\ of\ Stock}$$

Margin is Net Equity divided by the value of the shares

$$= \frac{\boxed{15{,}000} - \boxed{100P}}{100P} = \frac{15{,}000 - 11{,}000}{11{,}000} = \frac{4000}{11{,}000}$$

$$Margin\ Percent = 36\%$$

Short Sale - Margin Call

How much can the stock price rise before a margin call?

$$\text{Margin }\% = \frac{\text{Assets - Liabilities}}{\text{Value of Stock}} = 30\%$$

$$= \frac{15000 - 100\text{P}}{100\text{P}} = 0.30$$

$$= 15000 - 100\text{P} = 30P$$

$$\text{P} = 115.38$$

Short Sale – Your turn

V Corp	500 Shares
50%	Initial Margin
30%	Maintenance Margin
$42	Initial Price

Margin $500 \times 42 \times 0.5$
$= 10500$

Asset
$21000 + 10500$
$= 31500$

Sale Proceeds	$500 \times 42 = 21{,}000$
Your Margin (Equity)	$10500
Value of Stock Owed	$500 \times \$42 = \$21000$
Your Assets	$21000 + 10500 = \$31500$

answers are on the next slide

Short Sale – Your turn

V Corp	500 Shares
50%	Initial Margin
30%	Maintenance Margin
$42	Initial Price

Sale Proceeds	$21,000
Your Margin (Equity)	$10,500
Value of Stock Owed	$21,000
Your Assets	$21,000+10,500=31,500

Short Sale - Maintenance Margin

Stock Price Rises to $45, what is your Percent Margin?

$$\text{Margin \%} = \frac{\text{Assets - Liabilities}}{\text{Value of Stock}}$$

$$= \frac{31500 - 45 \times 500}{500 \times 45} = 40\%$$

answers are on the next slide

Short Sale - Maintenance Margin

Stock Price Rises to $45, what is your Percent Margin?

$$\text{Margin \%} = \frac{\text{Assets - Liabilities}}{\text{Value of Stock}}$$

$$= \frac{31500 - 500P}{500P} = \frac{31500 - 22500}{22500} = \frac{9000}{22500}$$

$$\text{Margin \%} = 40\%$$

Short Sale - Margin Call

How much can the stock price rise before a margin call?

$$\frac{31500 - 500P}{500P} = 30\%$$

$$\frac{31500}{500P} = 130\%$$

$$P = 48.46$$

answers are on the next slide

Short Sale - Margin Call

How much can the stock price rise before a margin call?

$$\text{Margin \%} = \frac{\text{Assets - Liabilities}}{\text{Value of Stock}} = 30\%$$

$$= \frac{31500 - 500\text{P}}{500\text{P}} = 0.30$$

$$= 31500 - 500\text{P} = 150P$$

$$\text{P} = 48.46$$

Stop Loss Orders

- Stop Buy: buy when the price rises
- Stop Sell: sell when the price drops

- Why…?
  - Use a stop buy to make sure you don't lose your shirt on a short sale
  - Use a stop sell to make sure to don't lose everything when buying on margin.

short sale theoretically can have inf losses

Investments

FNCE30001

Dr Patrick J Kelly

Laying the Foundation for Portfolio Theory

- Refresher on Calculating Returns
- Measuring Expected Returns
- Investor Preferences (Risk Aversion)
- Measuring Risk

Refresher on Calculating Returns

Refresher on Calculating Returns

- Arithmetic, geometric and dollar-weighted average returns
  - Geometric averages give a more accurate sense of returns over time
- Norms for reporting quoted returns
  - Necessary technical details
- Real vs. Nominal Returns
  - Real returns tell us what is our increase in purchasing power

Calculating (net) returns under certainty

- Ignoring transaction costs, suppose you purchase stock XZY for \$10 per share. Over the next 6 weeks, the price drops to \$9.50, but the stock also pays a dividend of \$1.00 per share.

- What is your return over this 6-week period?

$$Holding\ Period\ Return = HPR = \frac{P_1 - P_0 + D_1}{P_0}$$

$$Holding\ Period\ Return = HPR = \frac{\$9.50 - \$10 + \$1}{\$10} = 0.05\ or\ 5\%$$

Arithmetic vs. Geometric Mean (Average)

- Suppose you have a sample $r_1, \ldots, r_n$ of annual returns
- Arithmetic average return is defined as:

$$\bar{r} = \frac{1}{n}(r_1 + r_2 + \cdots + r_n) = \frac{1}{n}\sum_{i=1}^{n} r_i$$

- Geometric average return (also Time-weighted Returns) is:

$$r_G = \sqrt[n]{(1 + r_1)(1 + r_2)\cdots(1 + r_n)} - 1$$

or

$$r_G = [(1 + r_1)(1 + r_2)\cdots(1 + r_n)]^{\frac{1}{n}} - 1$$

Example

Stock	Return
A	5%
B	9%
C	-5%

- Arithmetic mean:

$$\bar{r} = \frac{1}{n}\sum_{i=1}^{n} r_i$$

equally weighted return

in real life:
invest same
amount of money

$$\bar{r} = \frac{1}{3}(.05 + .09 - .05) = .03 \; or \; 3\%$$

Example

Time	Return
2012	5%
2013	9%
2014	-5%

geometric mean return will always be less than arithmetic mean return

- Geometric mean:

$$r_G = [(1 + r_1)(1 + r_2) \cdots (1 + r_n)]^{\frac{1}{n}} - 1$$

$$r_G = [(1.05)(1.09)(.95)]^{\frac{1}{3}} - 1 = .028\ or\ 2.8\%$$

When is Which Better?

- Geometric mean is better when:
  - You want to find average returns over time

- Arithmetic mean is better when equally weighted input
  1. You want to know average returns for a group of stocks or bonds.
     - That is, you want to know the cross-sectional average stock return, and not the time-series average return.
  2. It is required by a formula (ex. variance formula)
  3. You are using past returns to make an estimate of future returns
  4. Quick and Dirty is good enough

future is similar as the past

⇒ take return from different time period, & assume independent & take average

Question - Arithmetic or Geometric?

- Average return for Telstra, Woolworths and Qantas.

  Arithmetic

- Average return for RIO Tinto from 2007 to 2018.

  Geometric

- The average return for a equally weighte portfolio of Telstra, Woolworths and Qantas from to 2007 to 2018.

① every stock in each time period : geometric

the answer ② stock average : Arithmetic

③ first arithmetic , then geometric

Dollar-weighted Return for Investment Funds

- Dollar-weighted (Average) Return is used to measure the typical return received per dollar invested in an investment company's fund.
  - Greater weight is given to returns earned when there is more money in the fund.

- If you treat inflows to and outflows from the fund as "cash flows to or from an investment," then
  - The Dollar-weighted return is just the Internal Rate of Return (IRR) of an investment fund.

return rate for NPV=0

$CF_0 + CF_1 \div (1+irr) + CF_2 \div (1+irr)^2 + \dots + CF_n \div (1+irr)^n = 0$

geometric mean return: manager of the fund earned

Example of Dollar-Weighted Returns

At the start the fund gets $1,000,000

XYZ Investment Company Flows and Performance

Period	Return	Inflow to Fund Net of Outflows	Investment Return in Dollars ($)	Net Asset Value	Net Cash Flow
2020		$1,000,000		$1,000,000	$1,000,000
2021	0.10	$300,000	$100,000	$1,400,000	$300,000
2022	-0.10	-$100,000	-$140,000	$1,160,000	-$100,000
2023	0.10	$150,000	$116,000	$1,426,000	$150,000
2024	-0.10	-$50,000	-$142,600	$1,233,400	-$1,283,400

In/Out Flow all occur at the end of the year

IRR= -1.40%

NOTE: For ease of explanation, I am flipping the signs compared to the book in order to think of this from the perspective of the fund.

After a -10% return $100K leaves the fund. Net year's returns get a lower weight.

After the +10% return, the fund gets $300K more. Next year's returns get a higher weight.

Refresher on Calculating Returns

APR and BEY

EAR and APY

History of Asset Returns

Real and Nominal Returns

Conventions for quoting returns

- Returns are always annualized with rare exception. (i.e., NOT holding period returns)
- Unless explicitly asked for a Holding Period Return always assume that returns are annualized.

- Two ways that annualization is usually done:
  - Annual Percentage Rate (also called Bond Equivalent Yield)
  - Effective Annual Rate (also called Annual Percentage Yield)

compound return

Annual Percentage Rate (Bond Equivalent Yield)

- General formula:

$$r_{APR} = r_{BEY} = r_{period} \times n_{periods\ per\ year}$$

  – If the half-year r_{period}=2%, what's the APR?

$2\% \times 2 = 4\%$

  – Suppose the interest rate for 1 quarter is 1.5%, what is the bond equivalent yield?

$$r_{APR} = 1.5\% \times 4 = 6\%$$

no compounding

- Economically, APRs and BEYs aren't useful.
  – The APR is equivalent to earning interest but not reinvesting the interest.
- To me, the only thing APR is good for is for getting r_{period}

Effective Annual Rate (EAR)

- The Effective Annual Rate (EAR) is the compound periodic rate:

$$r_{EAR} = \left(1 + r_{period}\right)^n - 1$$

n is the number of periods per year.

$$r_{EAR} = \left(1 + \frac{r_{APR}}{n}\right)^n - 1$$

Example

- Suppose the Bond Equivalent Yield (Annual Percentage Rate) on a quarterly paying corporate bond is 8%, what is the Effective Annual Rate?

n= # of coupon payments per year

$$\left(1 + \frac{8\%}{4}\right)^4 - 1$$

$$r_{EAR} = \left(1 + \frac{r_{BEY}}{n}\right)^n - 1$$

$$r_{EAR} = \left(1 + \frac{0.08}{4}\right)^4 - 1$$

$$r_{EAR} = 0.0824$$

Effective Annual Rate vs. Annual Percentage Rate

For the same amount of interest paid or received:

- Annual Percentage Rate < no compound >
  - Will be a smaller number
- Effective Annual Rate < with compound >.
  - Will be a larger number.
- Guess which is used when advertising credit card interest rates?

  APR
- Which one when advertising the rates on Savings? EAR

Example Based on ANZ Terms and Conditions

- Suppose (unrealistically) that the interest on a personal loan was 10% and the interest you could earn on a term-deposit was also 10%.

- If interest is paid monthly, how much interest do you pay/received at the end of the first 30-day month of the loan/deposit?
  - For loans, ANZ writes, "Debit interest will accrue daily on the unpaid balance of your loan account as at the end of the day. The rate applied on a particular day will be the annual percentage rate applicable to your loan for that day, divided by 365"
  - For deposits, ANZ writes, "Credit interest will accrue daily on the principal balance of a term deposit account as at the end of the day at the applicable daily interest rate, as explained below. Interest is not compounded (that is, it is not added to the principal closing balance of the account and is therefore not taken into account when calculating further interest on the account)."

https://www.anz.com.au/content/dam/anzcomau/documents/pdf/saving-transaction-products-tcs.pdf
https://www.anz.com.au/content/dam/anzcomau/documents/pdf/consumer-lending-tc.pdf

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- If interest is paid monthly, how much interest do you pay/received at the end of the first 30-day month of the loan/deposit?

accure daily

- Loan: $\$100{,}000 \times \left[\left(1 + \frac{0.10}{365}\right)^{30} - 1\right] = \$825.19$
- Deposit: $\$100{,}000 \times \frac{0.10}{365} \times 30 = \$821.92$

Pandemic Edition

Some Old, Slightly Off Topic Slides – but it underscores the difference in risk and return across different asset classes

① equity is risker than debt

② individual stock is risker than portfolio

③ long term debt is risker than short term

Mean and Standard Deviation 7/98 – 1/13 (in Local Currency)

short term

low risk (since low STD)

corporate bond is risker than government

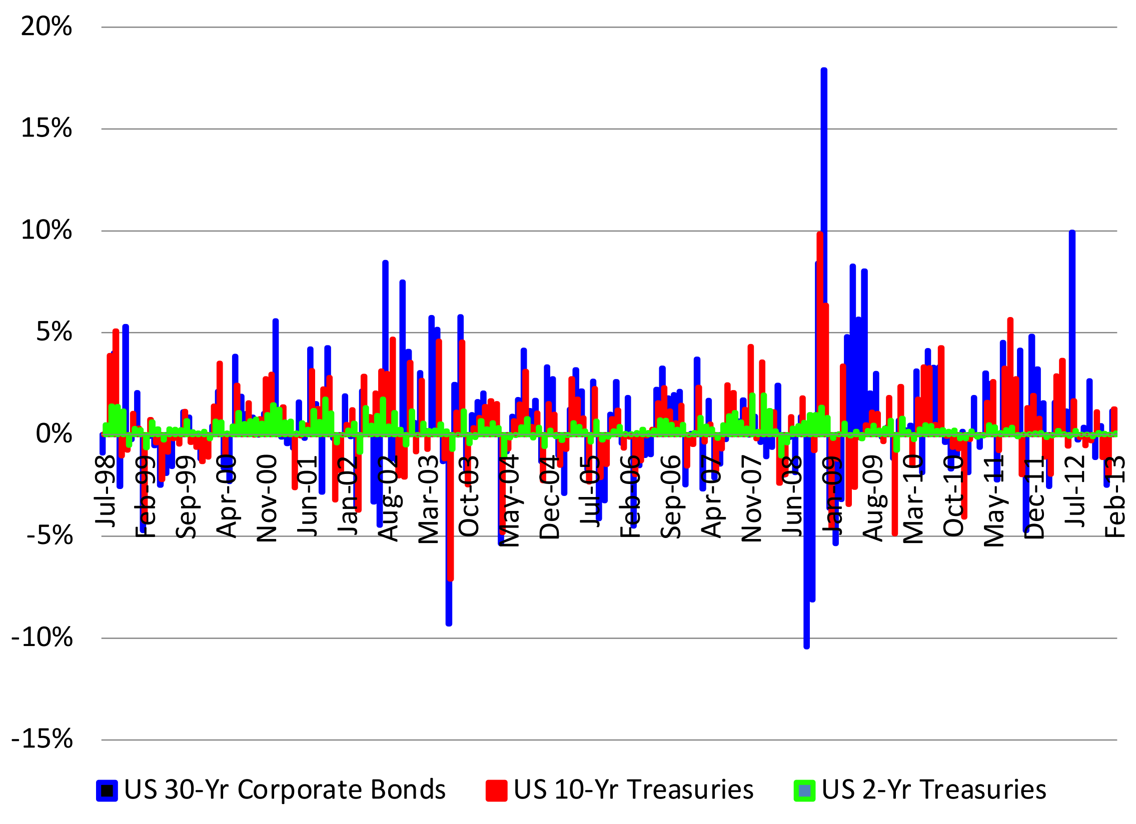
ETF

individual stock risker

Name for Charts	Mean	Geometric Mean	STD
US Money Market	0.20	0.20	0.17
German Money Market	0.21	0.21	0.12
German 2-Yr Govn't Bonds	0.26	0.26	0.41
US 2-Yr Treasuries	0.32	0.31	0.52
S&P 500	0.43	0.32	4.70
US 2-Yr Corporate Bonds	0.43	0.43	0.92
German 10-Yr Govn't Bonds	0.51	0.50	1.65
US 10-Yr Treasuries	0.51	0.49	2.25
US 30-Yr Treasuries	0.60	0.52	4.03
US 10-Yr Corporate Bonds	0.66	0.64	1.96
German 30-Yr Govn't Bonds	0.71	0.66	3.35
US 30-Yr Corporate Bonds	0.72	0.67	3.32
Russian Interbank (Money Market)	0.95	0.94	0.98
AMR	1.86	-1.49	25.55
Russia Total Market Return	3.02	2.37	11.84
Apple	3.36	2.38	13.56
Sberbank	4.93	3.09	19.82
UFA Engine	5.52	2.55	30.33

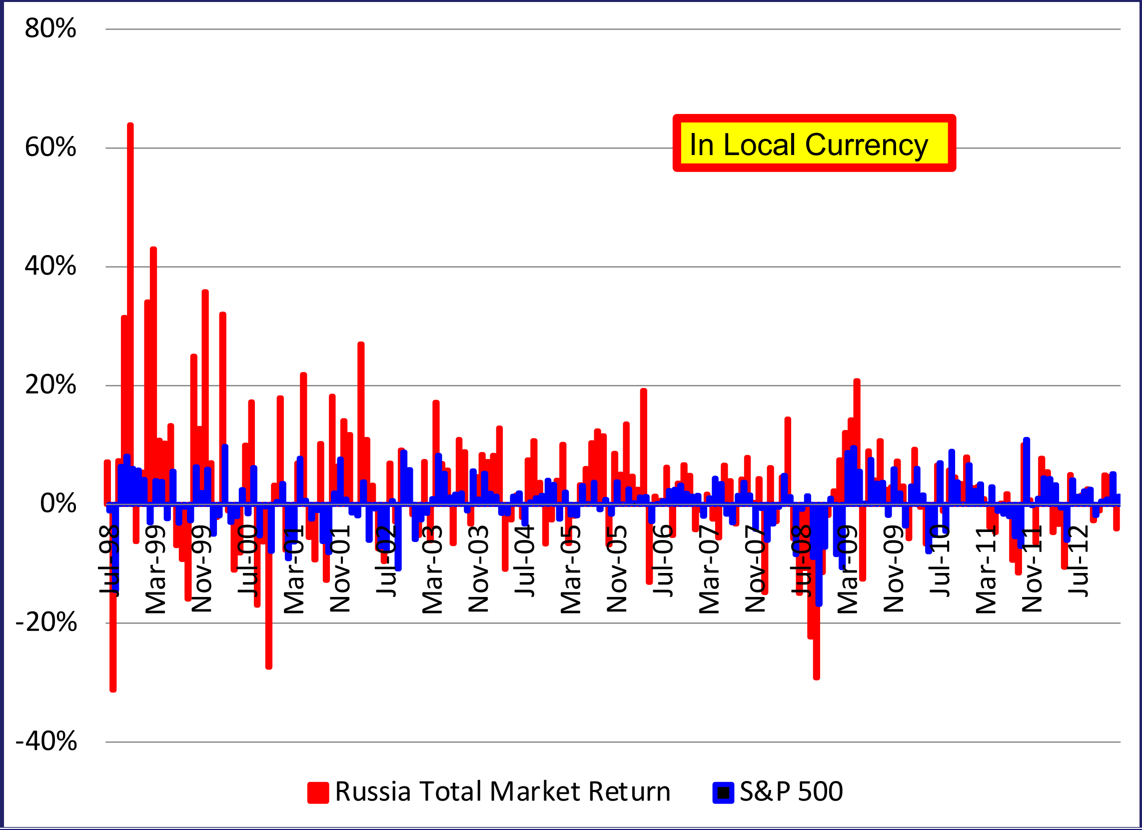
emerging market stock is riskier than developed markets

Monthly Returns to US Treasuries Bonds



Monthly Returns to the Russian Market and S&P 500 (US)

correlation among market



Real Return

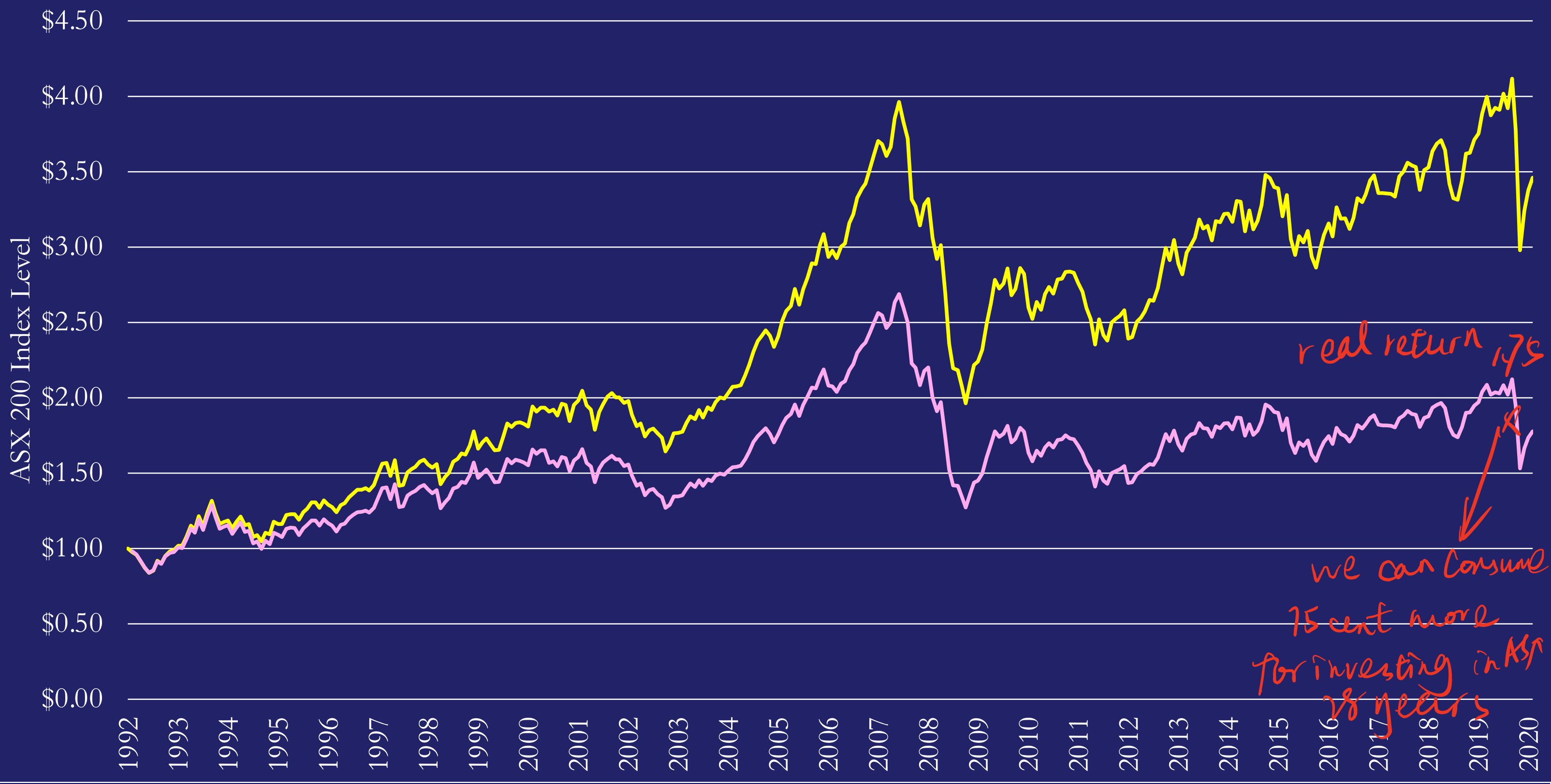
$1 invested in the ASX 200

If we are saving and investing to consume more later…

- We need to be able to measure how much more we can consume.
- We need to inflation adjust our returns so we know how much more we can consume.
- Real return:

$$r_{real} = \frac{1 + r_{nominal}}{1 + i} - 1$$

$1 invested in the ASX 200



Introducing Risk and Risk Aversion

The plan

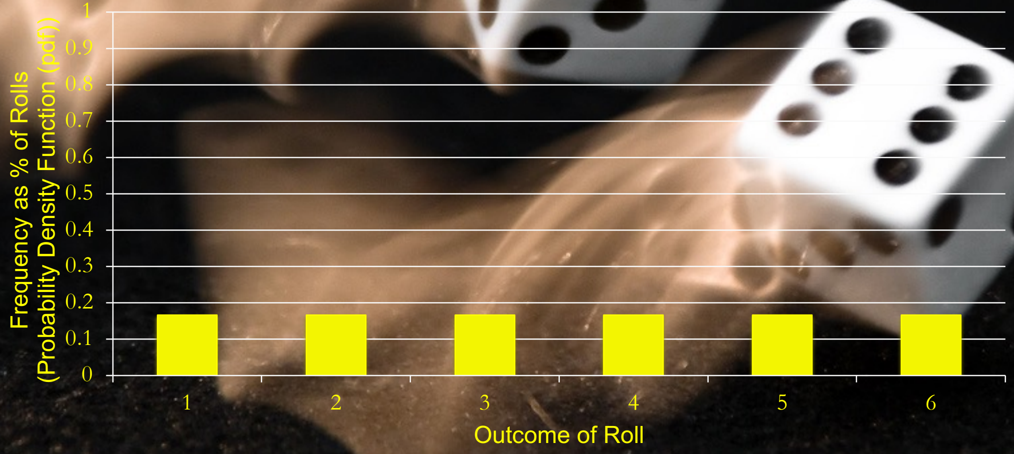
- Review probability theory to learn how to calculate expectations for random variables with known distributions:
  - Expected returns, expected dividends, expected wealth
  - Any measure that comes from a random distribution

- With randomness comes risk
- Establish that we (you, me, investors in general) are risk averse.

- Briefly discuss 3 types of investor preferences/Utility
  - Risk averse preferences
  - Risk loving preferences
  - Risk neutral preferences

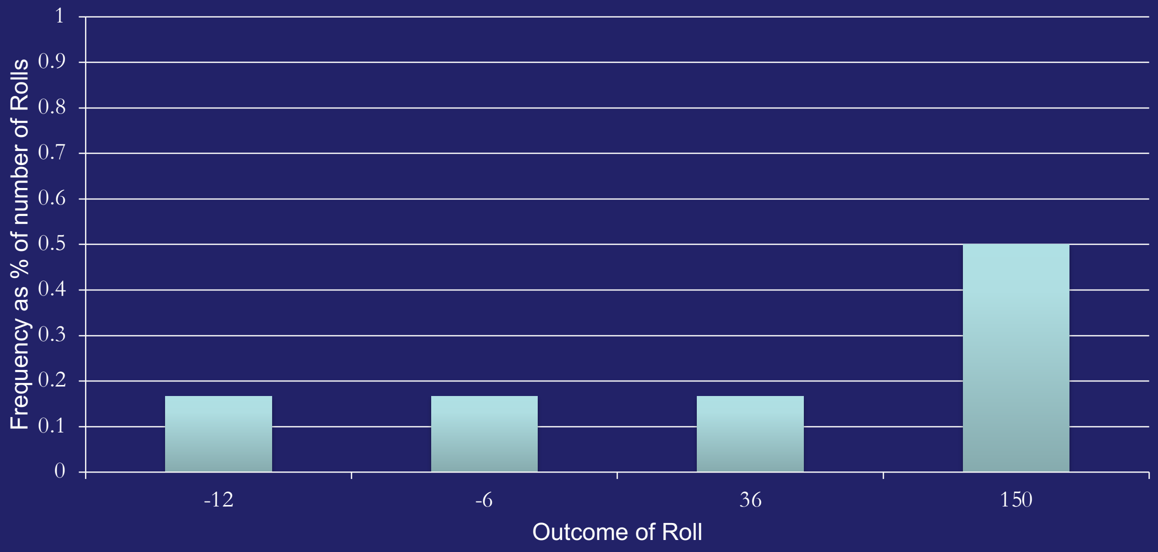
Uncertainty by example:

- Suppose you have a die.
- If you roll a die an infinite number of times, what does the distribution of outcomes look like?



Suppose you assign a dollar value to the roll…

- 1= - $6 2= $36 3= -$12 4 to 6 = $150



The Expected Value of a Random Variable

- $E_t[\quad]$ is the abbreviation for expected value.
  – Ex. $E_t[\tilde{r}_{t+1}]$.

- Think of the expected value of a random variable as the average if you repeat the experiment infinitely many times.
  – Strictly speaking: the probability-weighted average

What is the expected payoff?

s: state

→ cash flow

- $E[\widetilde{CF}] = \sum_{s=1}^{\mathbb{S}} p(s) CF_s$

probability

If you haven't seen this formula before – make note.

$$E[\widetilde{CF}] = \frac{1}{6}(-12) + \frac{1}{6}(-6) + \frac{1}{6}(36) + \frac{1}{2}(150) = \$78$$

- Do you ever get \$78?
  - Nope.
  - It is an expectation, not a realization
    - Unless, by coincidence, $E[\widetilde{CF}]$ equals one of the outcomes, you will never get the $E[\widetilde{CF}]$.

What is a fair bet? *(by example)*

- Q: What would it take for the above die-bet example to become a fair bet?

paid = $E[\widetilde{CF}]$

- A: If you paid $78 to take that bet…
  - After an infinite number of games
    - Where you pay $78 and get either -12, -6, 36 or 150
  - Ultimately, you would average out at $0.
- A bet is fair if the price equals the expected payout.

Fair bet and risk neutrality

don't care risk, care Expected.

- A fair bet is a bet a risk neutral person would be willing to accept.
  - That is, if you have risk neutral preferences, you would be willing to accept a fair bet.

- However, most people are risk averse and will not accept a fair bet.
  - Let's see….

Types of Investor Preferences

Wanna bet…?

- I am offering you now the opportunity to take the following bet:
  - I throw a coin.
  - Heads – I give you $2.
  - Tails – you give me $2.

- What if we played for $10?

- Are you willing to play?
  - No? How much do I have to pay?

What if we played for $500?

Expected Wealth

- "Expected" Wealth is the probability weighted average of your wealth at the end of the gamble.

Also sometimes called "Scenario Wealth"

	Probability	Wealth in State
Good State	.50	\$500
Bad State	.50	-\$500
$E[\widetilde{W}]$		0

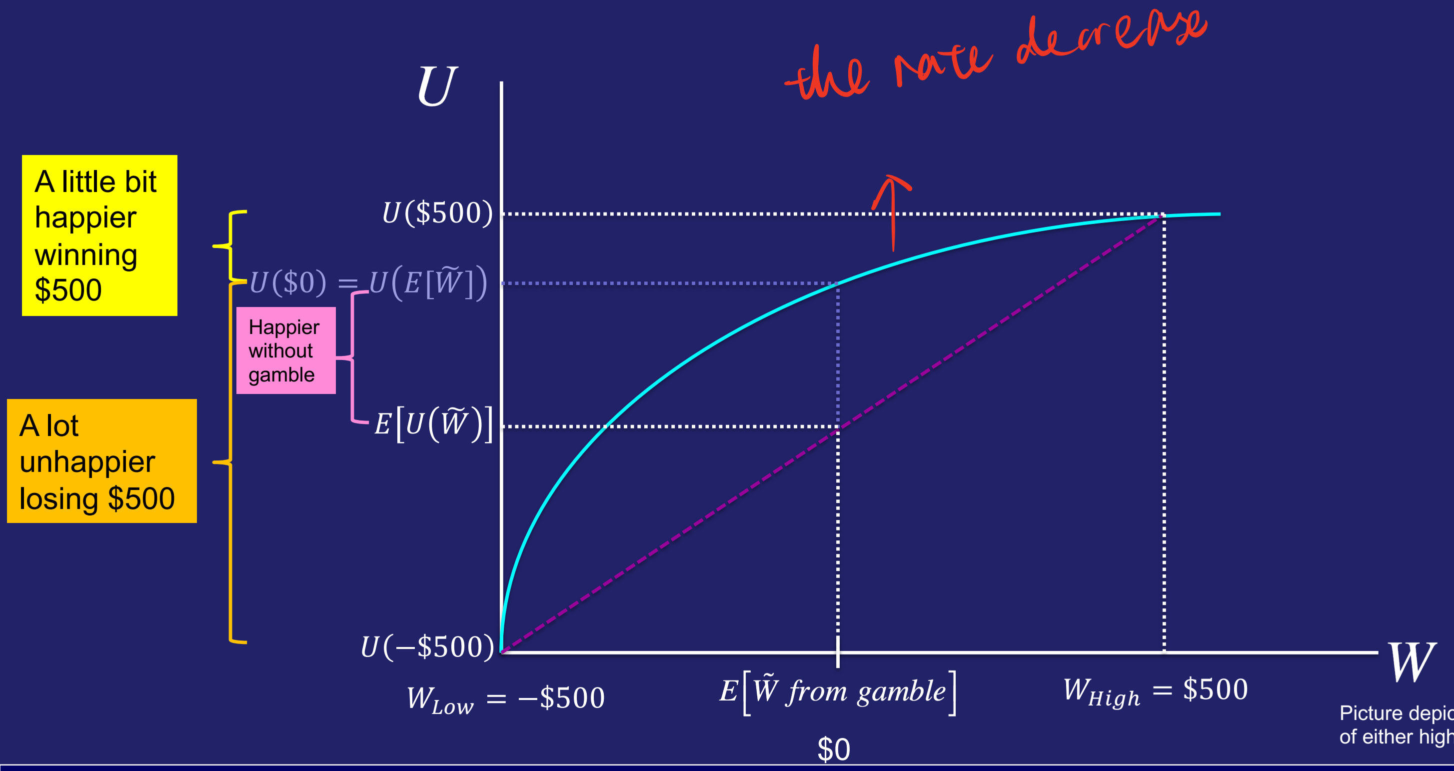
Variation Auction!

- This time at auction is a lottery ticket:
  - 50% chance you get $500
  - 50% you pay me $500.

Will a risk averse person accept a fair bet?

- No.
  - She would not be willing to accept a fair bet.
  - She needs to get compensated for the risk, in order to be willing to accept risk.

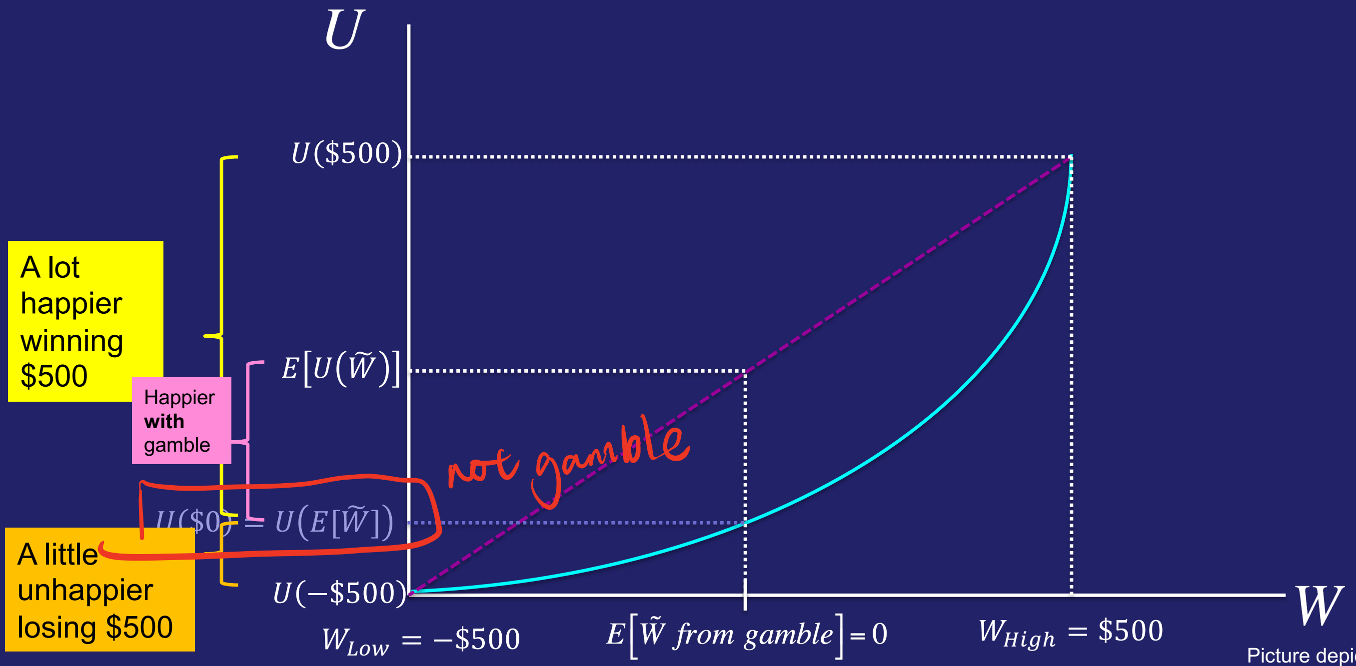
Risk Averse Utility



Picture depicts a 50% chance of either high or low wealth

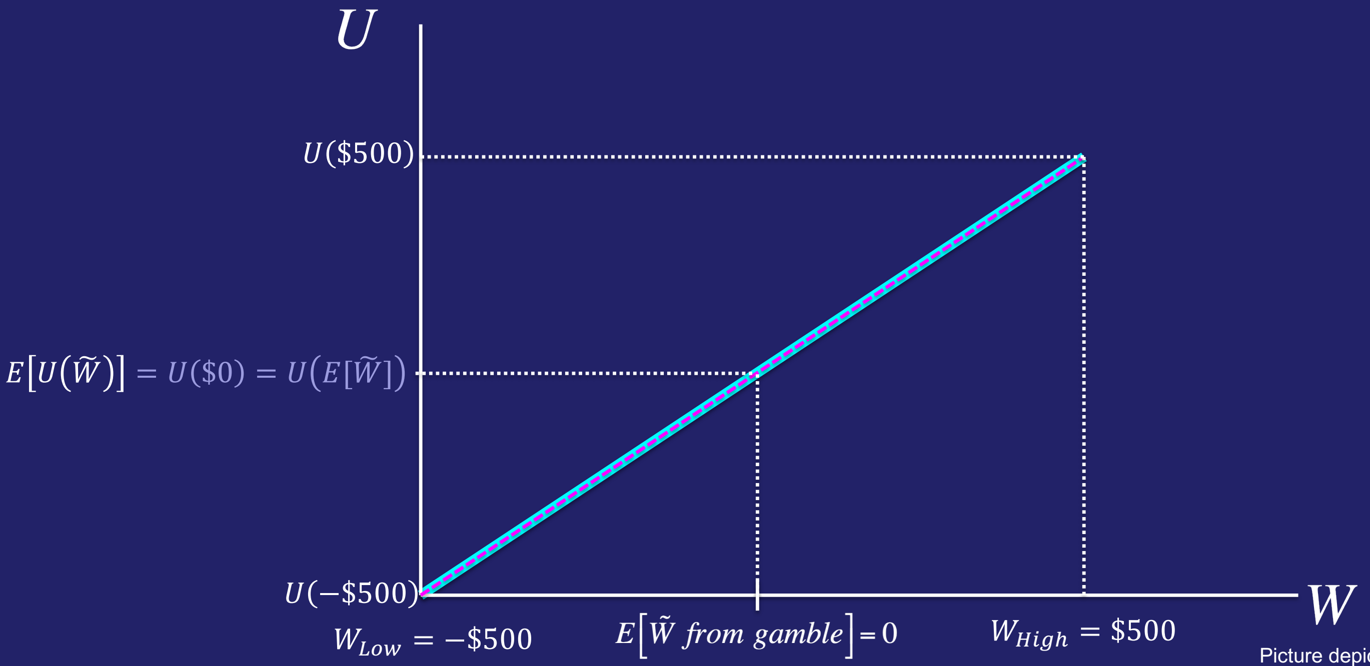
Not in your text – review from one of your Microeconomics subjects

Risk Loving Utility



Picture depicts a 50% chance of either high or low wealth

Risk Neutral Utility



Picture depicts a 50% chance of either high or low wealth

Is risk neutrality ever a good assumption?

- As we (*presumably*) just saw:
  - when bets are small relative to wealth we are less risk averse.
    - Also called Relative Risk Aversion.
  - To ensure we can lump together more and less wealthy investors, we will be focusing on returns, instead of profits in dollar terms.

similar to % of wealth

- The more risk averse we are, the more compensation for risk we demand (or the bigger discount we demand).

- Alternatively, if there is more risk, we will also demand more compensation or a bigger discount.

Risk averse preferences in finance

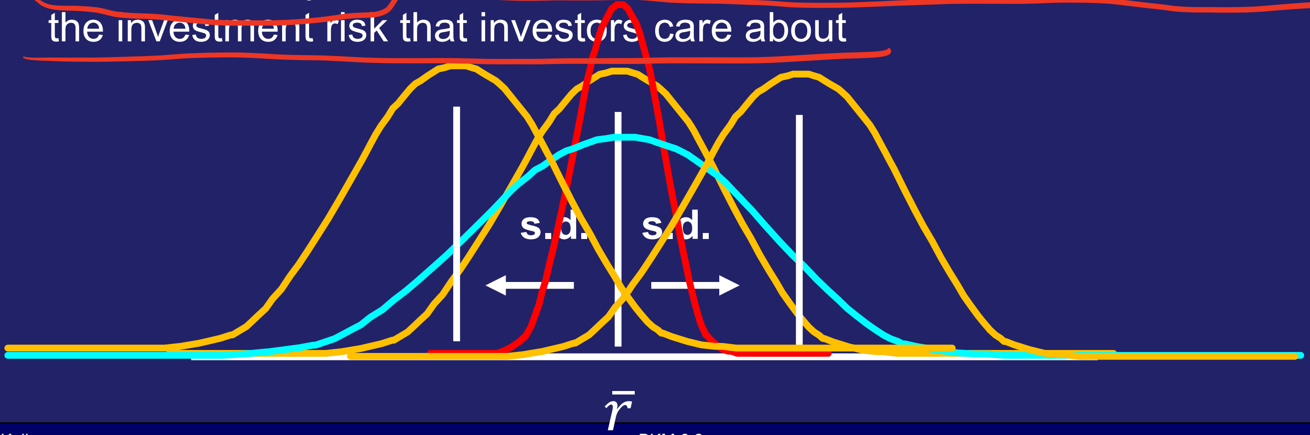
The plan

- In finance, traditionally we have focused on one form of risk-averse preferences, which follows the Mean-Variance Criterion.
  - in which investors only care about the Expected return (a.k.a. "mean") and risk as measured by variance (or equivalently standard deviation).
- The assumptions are not perfectly realistic, BUT if returns are normally distributed, then investors will behave as if they have the Mean-Variance Criterion.
- Here will will
  - explain the Mean-Variance Criterion,
  - check whether returns are roughly normal
  - Present a mean-variance utility function and look at Risk Averse, Risk Loving and Risk Neutral mean-variance maximizing preference

The mean-variance criterion

Mean-Variance Criterion

- Investors prefer more to less and dislike risk
- From this, we can build a theory of investment choice based on the expected (mean) return of an investment (higher = better) and its risk as measured by the variance of returns (lower = better)
  - This is the mean-variance criterion.
  - Critical assumption: the variance of returns is a good characterization of the investment risk that investors care about



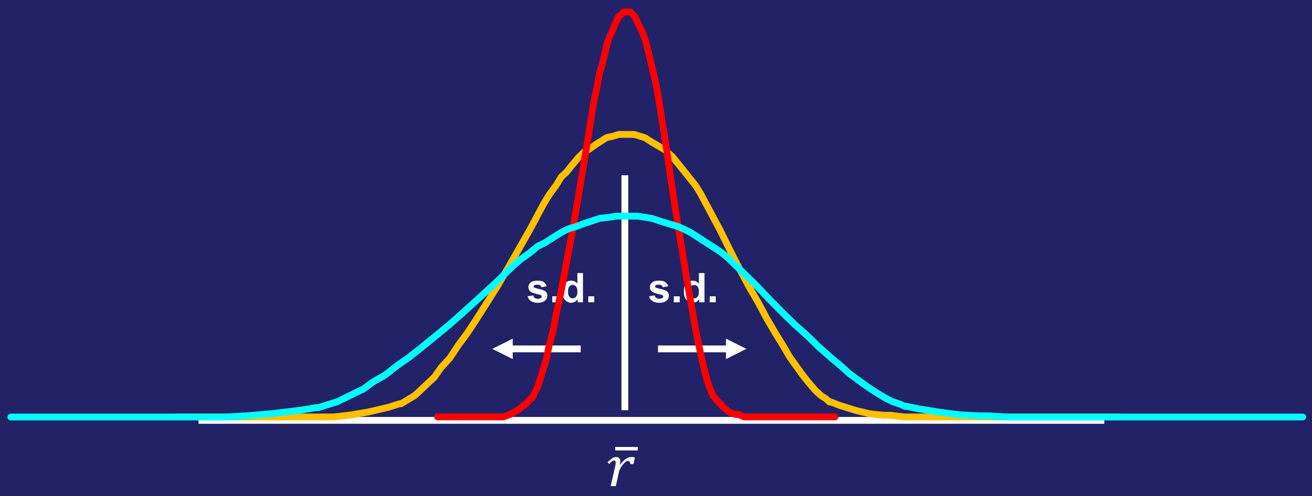
Critical Assumption

- Variance (Standard Deviation) is only characteristic of risk important to us:

$$\sigma^2 = \frac{1}{T-1}\sum_{t=1}^{T}(r_t - \bar{r})^2$$

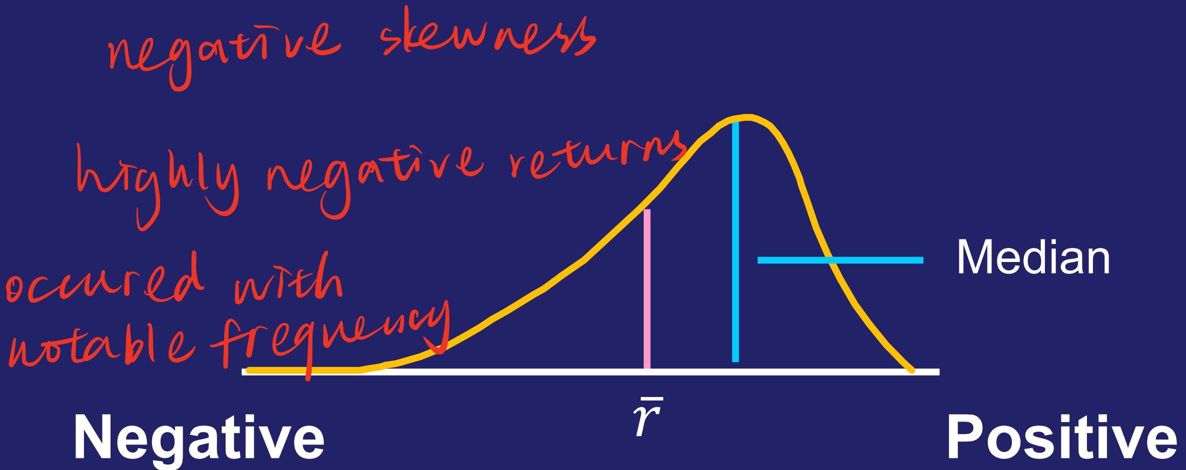
- For example Skewness doesn't matter…

Normal Distribution



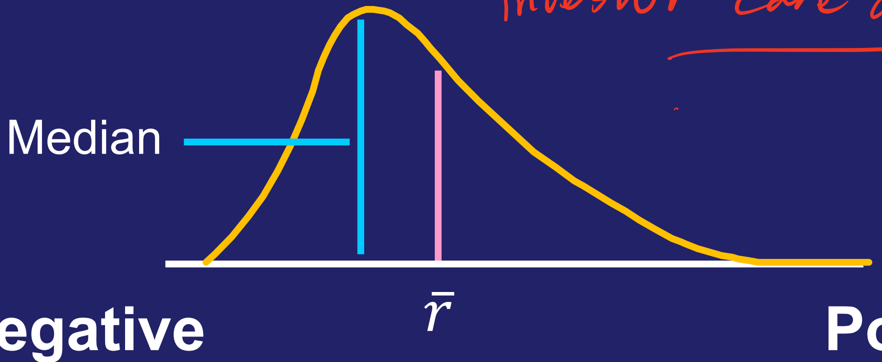
Symmetric distribution

Skewed Distribution: Large Negative Returns Possible



Skewed Distribution: Large Positive Returns Possible

investor care about skewness

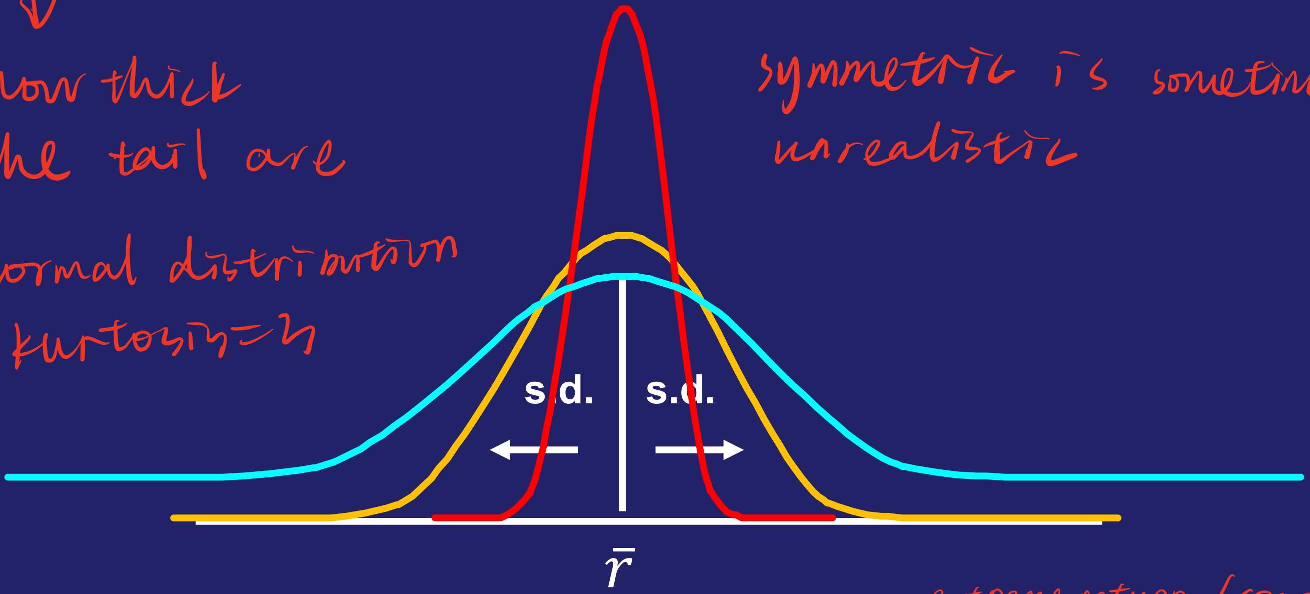


Kurtosis

how thick the tail are

normal distribution kurtosis=3

symmetric is sometimes unrealistic



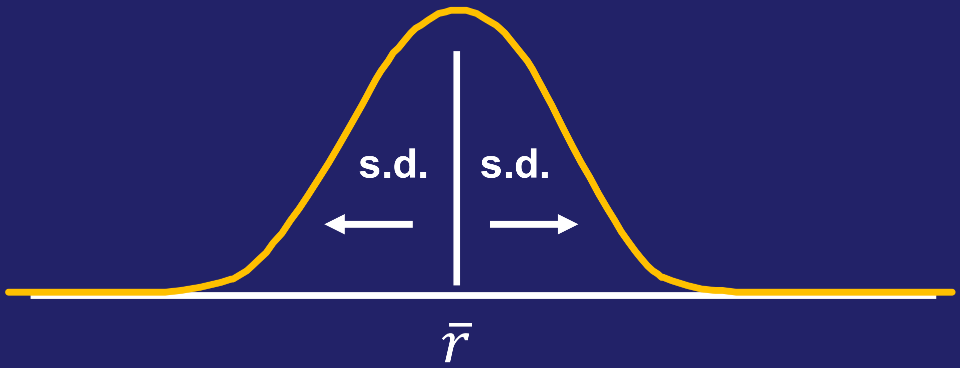
High Kurtosis → extreme return / loses occurs with a fair amount frequency

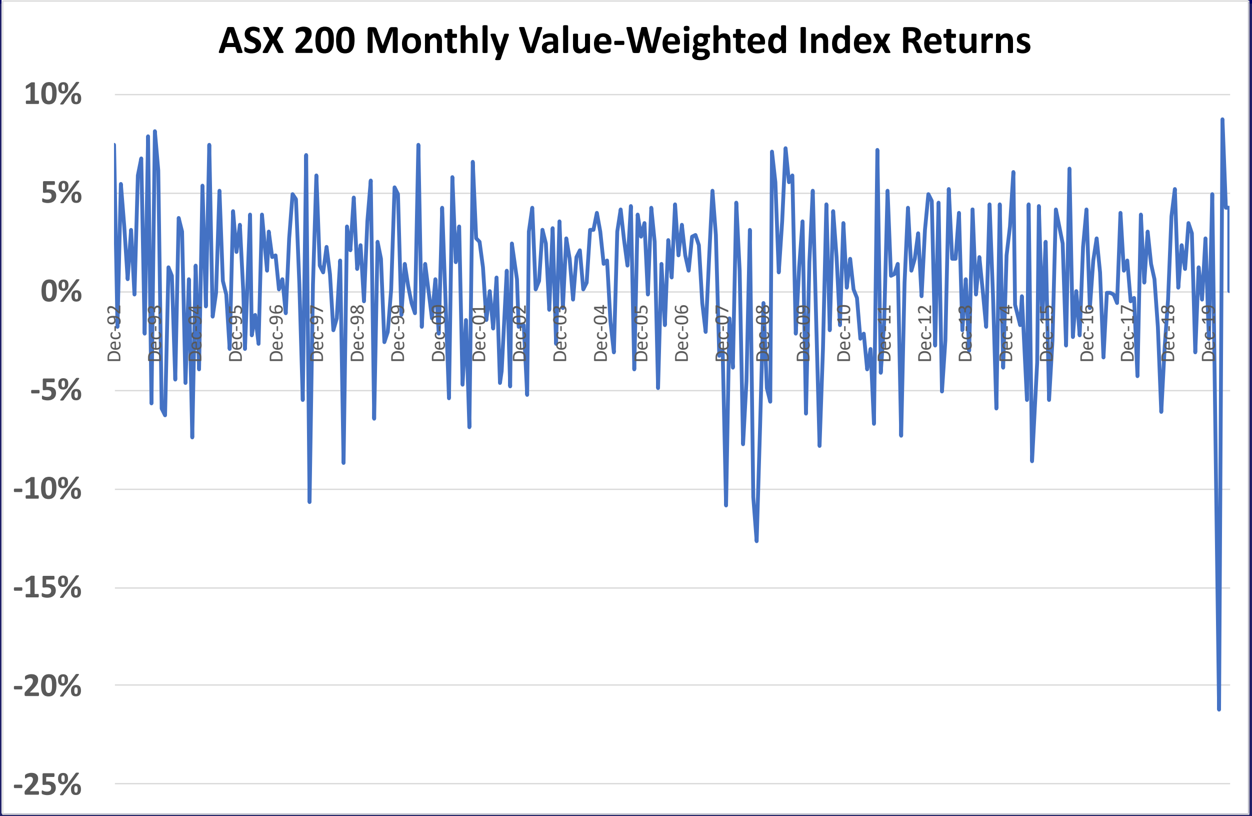
Low Kurtosis → extreme return is rare

Why do we assume the mean-variance criterion?

- If returns are normally distributed, then skewness and kurtosis won't matter, and the only thing important to investors will be the expected return (mean) and risk (variance), even if they would care about skewness and kurtosis.

- We will assume normal returns.





https://au.finance.yahoo.com/quote/%5EAXJO/history/

How does normal compare to reality?

- 0.5% monthly mean return 50 basis points.
- 0.4% monthly geometric average return
- Standard Deviation: 4.0%
- Skewness: -1.0
- Kurtosis: 2.7

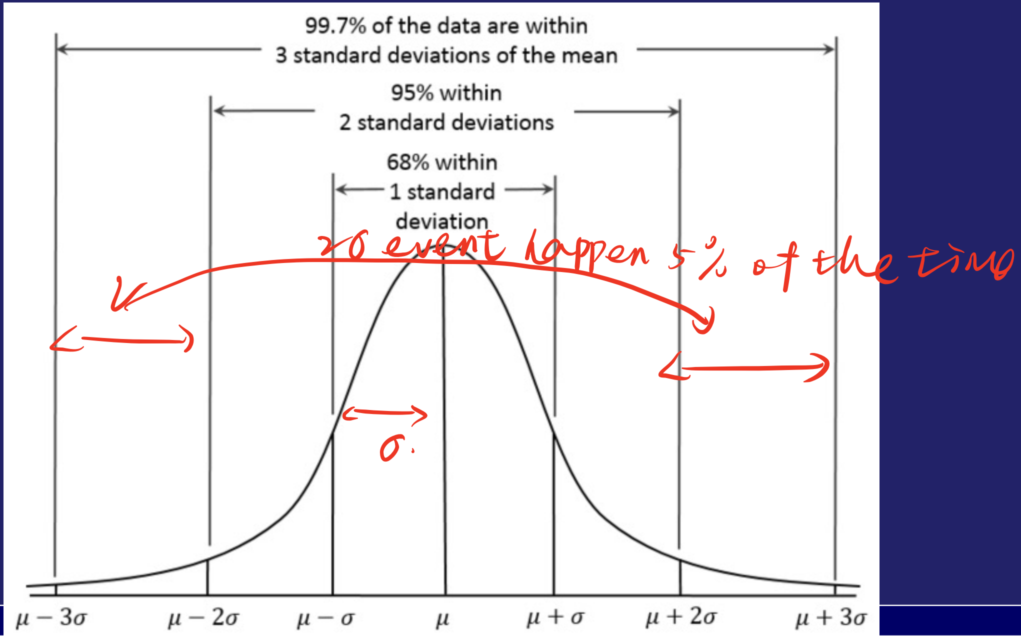
- For Normal Distributions
  - Skewness is 0
  - Kurtosis is 3
    - Note: Excess Kurtosis is 0 (your textbook calls “excess kurtosis” just “kurtosis”)

"kurtosis − 3"

In finance we usually assume returns are normally distributed

- Central Limit Theorem: when independent random variables are added the normalized sum tends toward a normal distribution
  - Read ”normalized sum” as “average”

With Mean 0.5% and StdDev=4% what's a 3 sigma event?

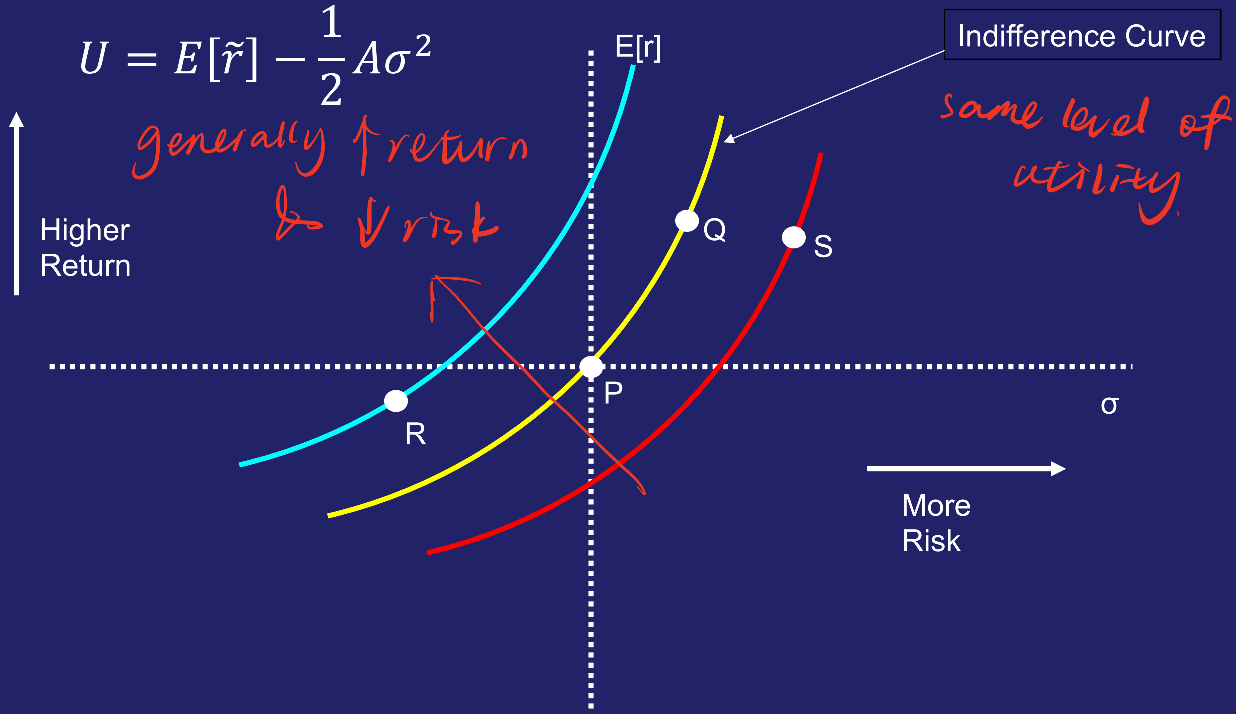


3σ event
fairly rare

The mean-variance preferences

We can show our preferences (Utility)

$$U = E[\tilde{r}] - \frac{1}{2}A\sigma^2$$



Risk Preferences

- Risk Averse
- Risk Neutral
- Risk Loving

Risk Averse

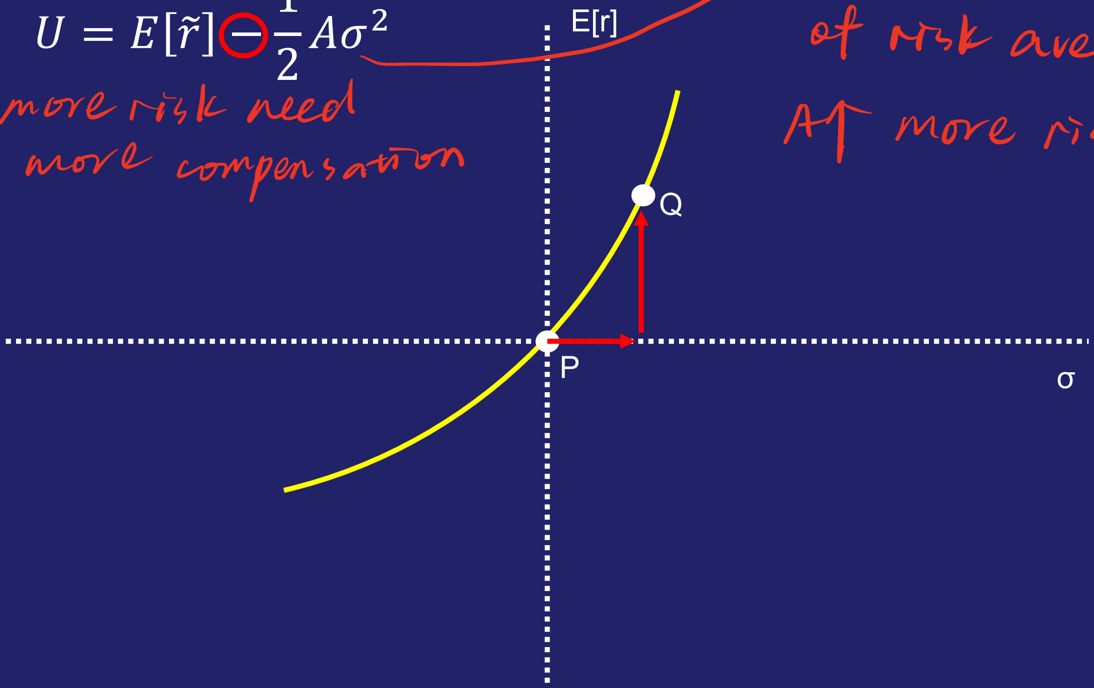
$$U = E[\tilde{r}] - \frac{1}{2} A\sigma^2$$

for scaling

A: coefficient of risk aversion

A↑ more risk averse

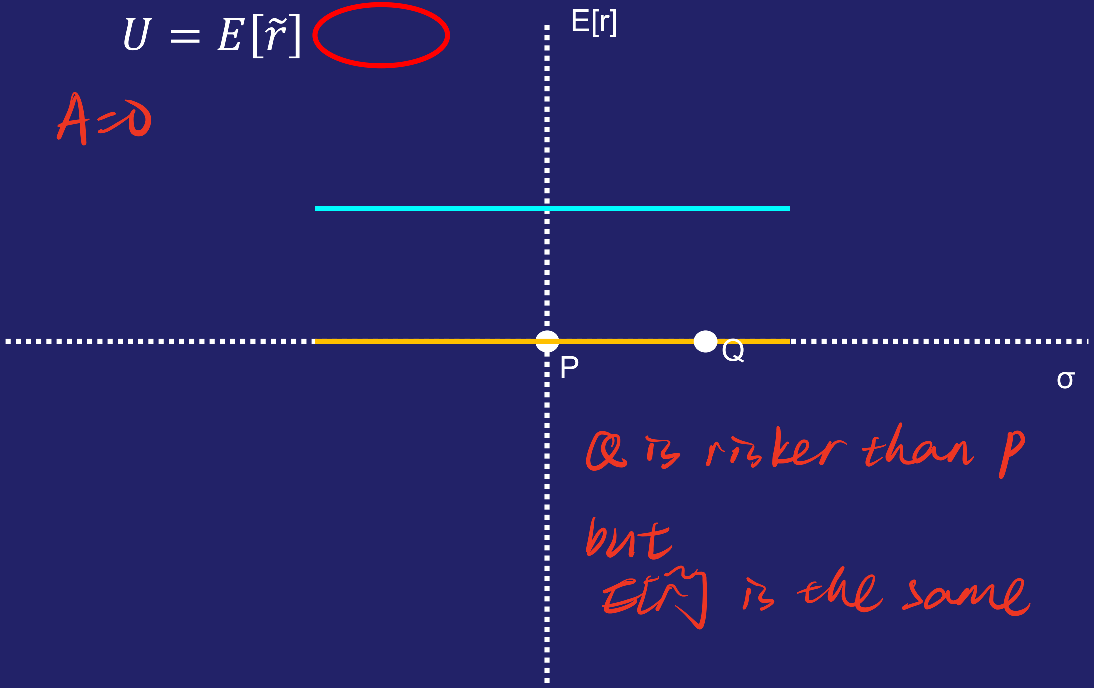
more risk need more compensation



Risk Neutral

$$U = E[\tilde{r}]$$

$A=0$



Q is riskier than P

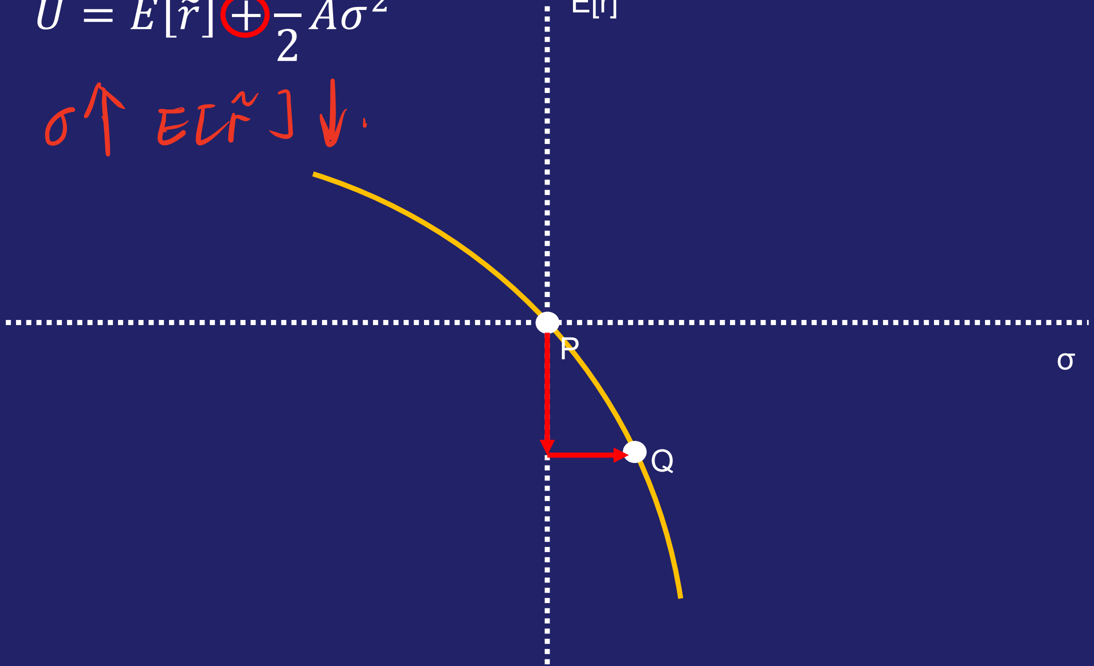
but

$E[\tilde{r}]$ is the same

Risk Loving

$$U = E[\tilde{r}] + \frac{1}{2}A\sigma^2$$

$\sigma \uparrow \; E[\tilde{r}] \downarrow$.



Calculating Expected Return and Scenario Risk

The plan: How to calculate Expected Return and Scenario Variance (and Standard Deviation)

+ a prelude to Markowitz Portfolio theory

Expected Returns

- "Expected Returns"
  - Also sometimes called "Scenario" Returns

expectation of future scenario.

- Expected returns for some risky asset A are:

$$E[\tilde{r}_A] = \sum_{s=1}^{\mathbb{S}} \underline{p(s)} r_A(s) \xrightarrow{\text{if } \mathbb{S}=2} p(1)r_A(1) + p(2)r_A(2)$$

probability

scenario / $\boxed{\text{state}}$

Example: Calculating Expected Returns

$$E[\tilde{r}_A] = \sum_{s=1}^{\mathbb{S}} p(s) r_A(s) \xrightarrow{\text{if } \mathbb{S}=2} p(1) r_A(1) + p(2) r_A(2)$$

	Pr	Stock A	Stock B
Boom	.20	.30	.08
Normal	.65	.08	.07
Bust	.15	-.05	.05
E[r]			

$$E[\tilde{r}_A] = p(Boom) r_A(Boom) + p(Normal) r_A(Normal) + p(Bust) r_A(Bust)$$

$$E[\tilde{r}_A] = .20 \times .30 + .65 \times .08 + .15 \times (-.05)$$

$$E[\tilde{r}_A] = .06 + .052 - .0075$$

$$E[\tilde{r}_A] = .1045$$

Example: Calculating Expected Returns

	Pr	Stock A	Stock B
Boom	.20	.30	.08
Normal	.65	.08	.07
Bust	.15	-.05	.05
E[r]		.1045	.069

$E[\tilde{r}_B] = 0.069$

Risk Defined - Variance

- Historic Variance

$$\sigma^2 = \underbrace{\frac{1}{T-1}}_{\text{degree of freedom adjustment}} \sum_{t=1}^{T} (r_t - \bar{r})^2$$

- Scenario Variance

$$\sigma^2 = Var(\tilde{r}) = \boxed{\sum_{s=1}^{S} p(s)(r(s) - E[\tilde{r}])^2}$$

- Standard deviation of returns is defined as:

$$\sigma = \sqrt{\sigma^2}$$

Standard Deviation

Variance

Scenario Variance

$$\sigma^2 = Var(\tilde{r}) = \sum_{s=1}^{S} p(s)(r(s) - E[\tilde{r}])^2$$

	Pr	Stock A	Stock B
Boom	.20	.30	.08
Normal	.65	.08	.07
Bust	.15	-.05	.05
E[r]		.1045	.069

Example: Calculating Scenario Variance

	Pr	Stock A	Stock B
Boom	.20	.30	.08
Normal	.65	.08	.07
Bust	.15	-.05	.05
E[r]		.1045	.069

σ_B^2

$= 0.2(0.08-0.069)^2$

$+0.65(0.07-0.069)^2$

$+0.15(0.05-0.069)^2$

$= 0.00008$

$\sigma_B = 0.00889$

$$\sigma_A^2 = \mathrm{Var}(r_A) = \sum_{s=1}^{S} p(s) \times \left(r_A(s) - E[r_A]\right)^2$$

$$\sigma_A^2 = .2\underline{(.3-.1045)}^2 + .\underline{65(.08-.1045)}^2 + \underline{.15(-.05-.1045)}^2$$

$$\sigma_A^2 = .2(.1955)^2 + .65(-.0245)^2 + .15(-.1545)^2$$

$$\sigma_A^2 = .2(.03822) + .65(.00060) + .15(.02387)$$

Example: Calculating Scenario Variance

$$\sigma_A^2 = .2(.3 - .1045)^2 + .65(.08 - .1045)^2 + .15(-.05 - .1045)^2$$

$$\sigma_A^2 = .2(.1955)^2 + .65(-.0245)^2 + .15(-.1545)^2$$

$$\sigma_A^2 = .2(.03822) + .65(.00060) + .15(.02387)$$

$$\sigma_A^2 = .007644 + .00039 + .003581$$

$$\sigma_A^2 = .01161$$

$$\sigma_A = \sqrt{\sigma_A^2}$$

$$\sigma_A = 0.10777$$

Example: Calculate the Variance for Stock B

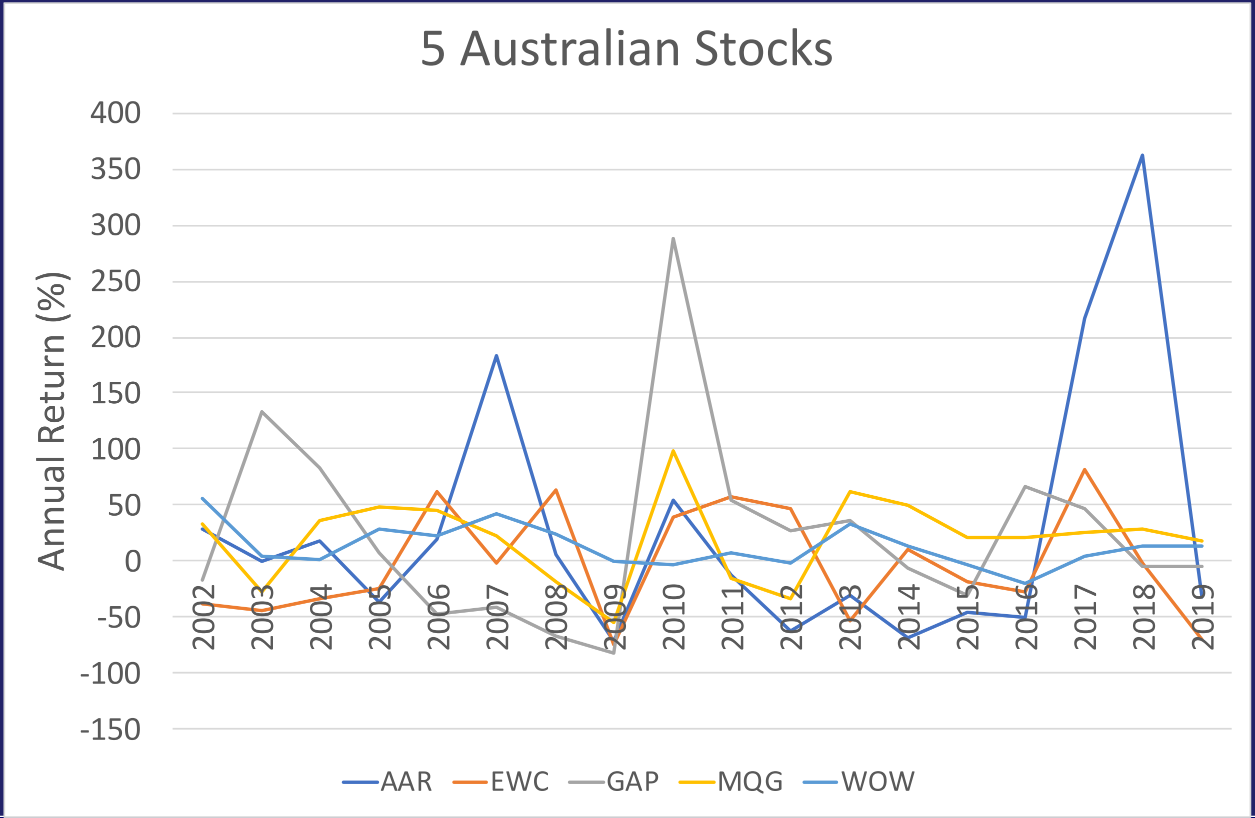
- The variance is
  .00008

- The standard deviation is
  .00889

Mean-Variance Criterion

- How do we increase the mean and reduce the variance?
- How can we improve returns?
- How can we control risk?
- Consider….

Several Stocks



Portfolio of Several Stocks

eg.
equally
weighted
portfolio