Bond Pricing with Default Risk

A.K.A.

Credit Risk

Default ("Credit") Risk

- Most loans have credit risk: the borrower can default.
- Default risk is the risk of losing some or all of your *promised* return on investment because the borrower goes belly-up.

bankrupt or temporal cashflow payment problem

result in ① creditor take over the firm
② selling off part of the firm

Working Example

Assume that a *government* bond costing $200 promises a 5% interest rate for 1-year, i.e., $210. *Assume this is risk-free*.

- Assume you are risk-neutral.

- *I* want to borrow $200 from you. I promise to repay $210.
  - However, I may go bankrupt in 1 out of 100 cases, in which case I can repay only $50.

- I'm hiding most of the answers to force you to think a little
  - PLEASE TAKE NOTES

Q1:

- What is the expected return $E[\tilde{r}]$ on the risk-free government bond?

2|0

Q2:

Assume that a *government* bond costing $200 promises a 5% interest rate for 1 year, i.e., $210. *Assume this is risk-free*. Assume you are risk-neutral.

- *I* want to borrow $200 from you. I promise to repay $210.
  - However, I may go bankrupt in 1 out of 100 cases, in which case I can repay only $50.

- What is your *promised* rate of return on my personal bond?

5%

Q3:

- Do I promise to give you the same rate of return as the Government Bond?

Yes

Q4:

- What pay off do you <u>expect</u> from my bond?

$\frac{1}{100} \times \$50 + \frac{99}{100} \times \$210 = \$208.4$

Assume that a *<u>government</u>* bond costing $200 promises a 5% interest rate for 1 year, i.e., $210. *Assume this is risk-free*. Assume you are risk-neutral.

- *<u>I</u>* want to borrow $200 from you. I promise to repay $210.
  - However, I may go bankrupt in 1 out of 100 cases, in which case I can repay only $50.

Q5:

- If you extend this loan to me, what rate of return would you expect my bond to give you?

$$\frac{E[\tilde{CF}]}{P} - 1 \qquad \frac{208.4 - 200}{200} = 4.2\%$$

Assume that a *government* bond costing $200 promises a 5% interest rate for 1 year, i.e., $210. *Assume this is risk-free*. Assume you are risk-neutral.

- *I* want to borrow $200 from you. I promise to repay $210.
  - However, I may go bankrupt in 1 out of 100 cases, in which case I can repay only $50.

Q6:

- Assuming you are risk neutral, would you prefer to
  - make this loan to me or
  - put your money into the 5% government bond
  - or are you indifferent?

Assume that a *government* bond costing $200 promises a 5% interest rate for 1 year, i.e., $210. *Assume this is risk-free*. Assume you are risk-neutral.

- *I* want to borrow $200 from you. I promise to repay $210.
  - However, I may go bankrupt in 1 out of 100 cases, in which case I can repay only $50.

Q7:

- If you are risk neutral, how much money would you be willing to give me for my promise to pay you $210?

$$P = \sum_{t=1}^{\infty} \frac{E\left[\widetilde{CF}_t\right]}{(1 + E[\tilde{r}])^t}$$

$\frac{208.4}{1+0.05\%} = \$198.4$

Recall: $E\left[\widetilde{CF}_1\right] = 0.99 \times \$210 + 0.01 \times \$50 = \$208.40$

Q8:

Assume that a government bond costing $200 promises a 5% interest rate for 1 year, i.e., $210. Assume this is risk-free. Assume you are risk-neutral.

- *I want to borrow $200 from you. I promise to repay $210.*
  - *However, I may go bankrupt in 1 out of 100 cases, in which case I can repay only $50.*

- If a newspaper or website were quoting the yield on my bond, what would it be?

$$\frac{210}{1+YTM} = 198.47$$

$$YTM = 5.8\%$$

Q8:

- If a newspaper or website were quoting the yield on my bond, what would it be?

- This is a yield to maturity question: At the price of $198.47 and the promise of $210, it would print a promised interest rate of

Q9:

- Are these quoted rates the expected rate?

No.

Default Premium: Definition

- Default Premium = (promised) YTM – r_f.
- What is the default premium on the loan to me?
  - Recall: Risk-free government bond promises a 5% interest rate for 1-year
  - Quoted yield on my bond is 5.8%

    Default premium = 5.8% − 5% = 0.8%
    - *Note: In this example we assumed you were risk neutral. This means the default premium is not compensation for risk!!!*
      - It is an extra amount promised in the "good state" the offsets the losses in the "bad state."
- The default premium is a bit of a misnomer. It is extra only in the good state. *Default Offset* might be a better term
  - (but no one says that)

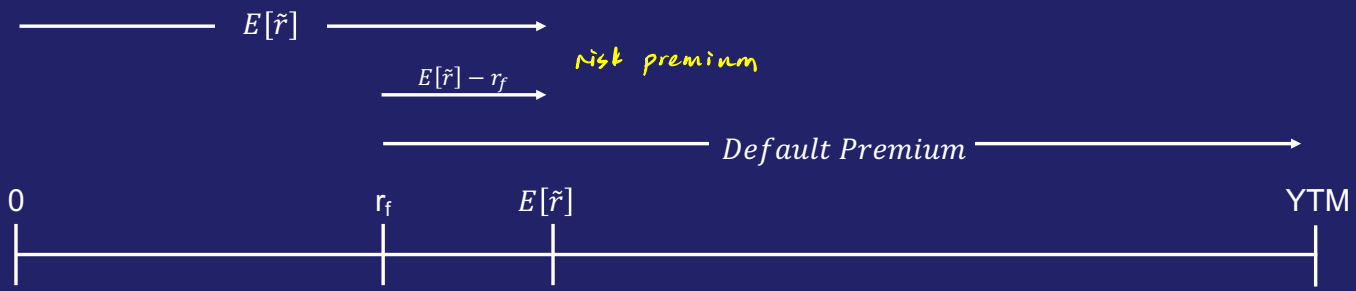
Q11:

- In the real world, would this interest rate of 5.8% really be high enough?

- Probably not.
  - In real life people are not risk neutral. They are risk averse.
    - They need a risk premium to compensate them for risk.
  - In real life there are usually liquidity premia too.

additional discount to price of bond
for the difficulty of buying and selling bond
→ high return

Default and Risk Premia: YTM = Default Premium + r_f.



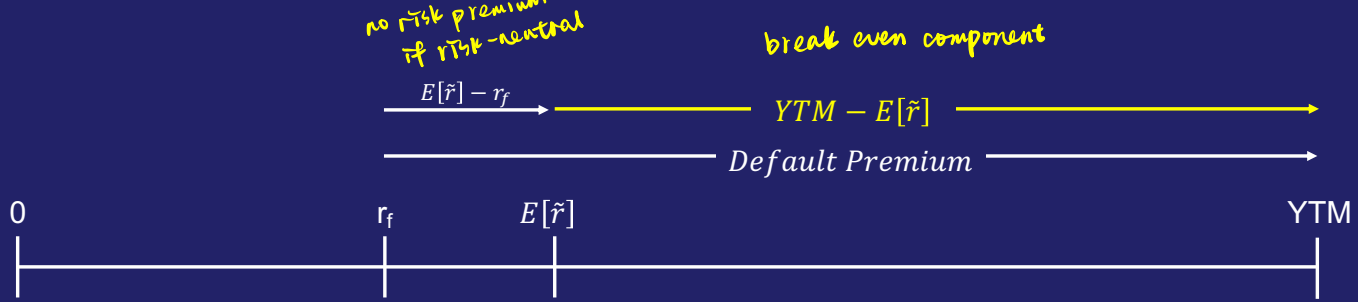
- The risk premium is extra compensation that gets you above the time premium, and it is only required to get you to participate if you are risk-averse.
  - If you repeat the investment infinitely many times, the risk premium will allow you to earn more than an investor holding risk-free government bonds will earn.

if you buy infinite times, on average you will still get risk premium but you won't get default premium

Default and Risk Premia: YTM = Default Premium + r_f.

- The Default Premium includes the risk premium, but it also contains a component ($YTM - E[\tilde{r}]$) that is compensation to make you **break even**. It would be required to get you to participate even if you were risk-neutral.
  - If you repeat the investment infinitely many times, on average ($YTM - E[\tilde{r}]$) is 0. You get a positive amount if everything goes well, and a negative amount if default occurs. That is, you come out even.

$E[YTM - E[\tilde{r}]] = 0$?



Default Premium: An example with risk aversion

- What is the price and returns of the following bond? Suppose there is a very risky bond with a 50% chance of default in the next year. In exactly one year the payoffs are:
  - Good state (no default): \$100
  - Bad state (default): 0\$

- $E\left[\widetilde{CF}_t\right] = 0.5 \times \$100 + 0.5 \times \$0 = \$50$

$E[\tilde{r}_{RB}] - r_f = \beta_{RB}\left(E[\tilde{r}_M] - r_f\right)$

- $\beta_{RB} = 0.2 \; and \; E[\tilde{r}_M] = 10\% \; and \; r_f = 5\% \quad \Longrightarrow E[\tilde{r}_{RB}] = 6\%$
- Risk premium = $E[\tilde{r}_{RB}] - r_f = 6\% - 5\% = 1\%$

Default Premium: An example with risk aversion

- $P = \sum_{t=1}^{1} \frac{E[\widetilde{CF}_t]}{(1+E[\tilde{r}])^1} = \frac{50}{1.06} = \$47.1698112$

- Return in upstate: $r_{Up} = \frac{100}{47.1698112} - 1 = 1.12 \; or \; 112\%$

- Return in downstate: $r_{Down} = \frac{0}{47.1698112} - 1 = -1 \; or \; -100\%$

- Default Premium?

$$Default\ Premium = r_{Up} - r_f = 112\% - 5\% = 107\%$$

Default Premium: An example with risk aversion

- $r_f = 5\%, E[\tilde{r}_{RB}] = 6\%, \quad r_{Up} = 112\%, r_{Down} = -100\%$
- The Default Premium includes the risk premium, but it also contains a component ($YTM - E[\tilde{r}]$) that is compensation to make you **break even**. It would be required to get you to participate even if you were risk−neutral.
- What is the break-even component of up-state returns?
  - $r_{Up} - E[\tilde{r}_{RB}] = 112\% - 6\% = 106\%$
- What about the downstate?
  - $r_{Down} - E[\tilde{r}_{RB}] = -100\% - 6\% = -106\%$

The upstate and downstate breakeven components of the default premium exactly cancel out.
$0.5\times106\% + 0.5\times(-106\%)=0\%$

- You lose the same in the downstate that you gain in the up.

Default Premium: An example with risk aversion

- $r_f = 5\%, E[\tilde{r}_{RB}] = 6\%, \quad r_{Up} = 112\%, r_{Down} = -100\%$

  $.5(r_{Up} - E[\tilde{r}_{RB}]) + .5(r_{Down} - E[\tilde{r}_{RB}]) = 0.5\times 106\% + 0.5\times(-106\%) = 0\%$

- Compare: realised default risk premium
- $.5(r_{Up} - r_f) - .5(r_{Down} - r_f) = 0.5\times 107\% + 0.5\times(-105\%) = 1\%$

- The Risk Premium You lose a bit less in the downstate and gain a bit more in the up. Averaging out to a risk premium

- What part of the 107% Default Premium is the Risk Premium?
  - Answer: +1%

106% is compensation for downside loss

What lesson are we supposed to learn from this example?

- Most of the default premium is more of an offset to losses that even a risk-neutral investor would demand. not compensation for risk
- The risk premium, on the other hand, boosts both upside return AND downside return to entice risk averse investors to invest.
- One more way to say the same thing:
  - In expectation, the realizations of default premium = risk premium
  - We saw: $.5(r_{Up} - r_f) - .5(r_{Down} - r_f) = 0.5\times 107\% + 0.5\times(-105\%)$=1%

Injecting a dose of reality

Credit Ratings

- There are 2 major rating agencies that assess the probability that a corporation will default on their bonds
  - Standard and Poor's
  - Moody's
  - There are also 3 lesser ones:
    - *Fitch*
    - Australia Ratings Pty Ltd
    - *Equifax*
- These agencies rate bonds in two classes:
  - Investment grade → cooperation or fund has less probability of default
  - Speculative grade (also called junk) high default rate

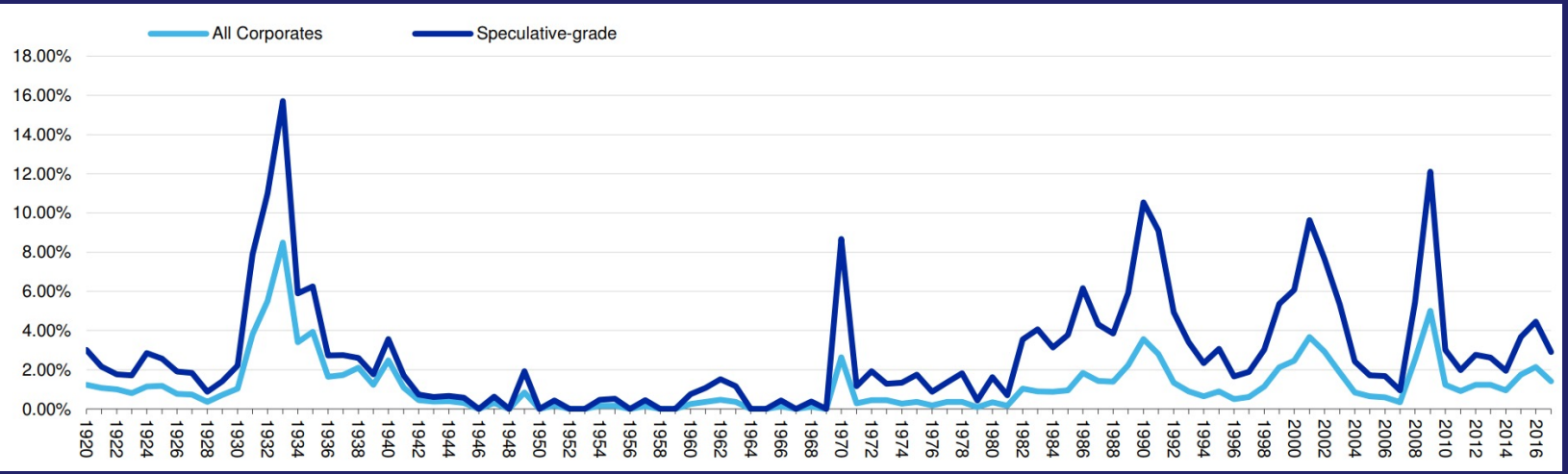
Credit Ratings

Investment Grade	Best									Worst
Moody's	Aaa	Aa1	Aa2	Aa3	A1	A2	A3	Baa1	Baa2	Baa3
Standard & Poor's	AAA	AA+	AA	AA-	A+	A	A-	BBB	BBB	BBB-
Speculative Grade										in Default
Moody's	Ba1	Ba2	Ba3	B1	B2	B3	Caa1,Caa2,Caa3,Ca, C			D
Standard & Poor's	BB+	BB	BB-	B+	B	B-		CCC		D

Remember: Default means that the bond issuer has missed a coupon payment or has violated one of the terms of the loan or bond.

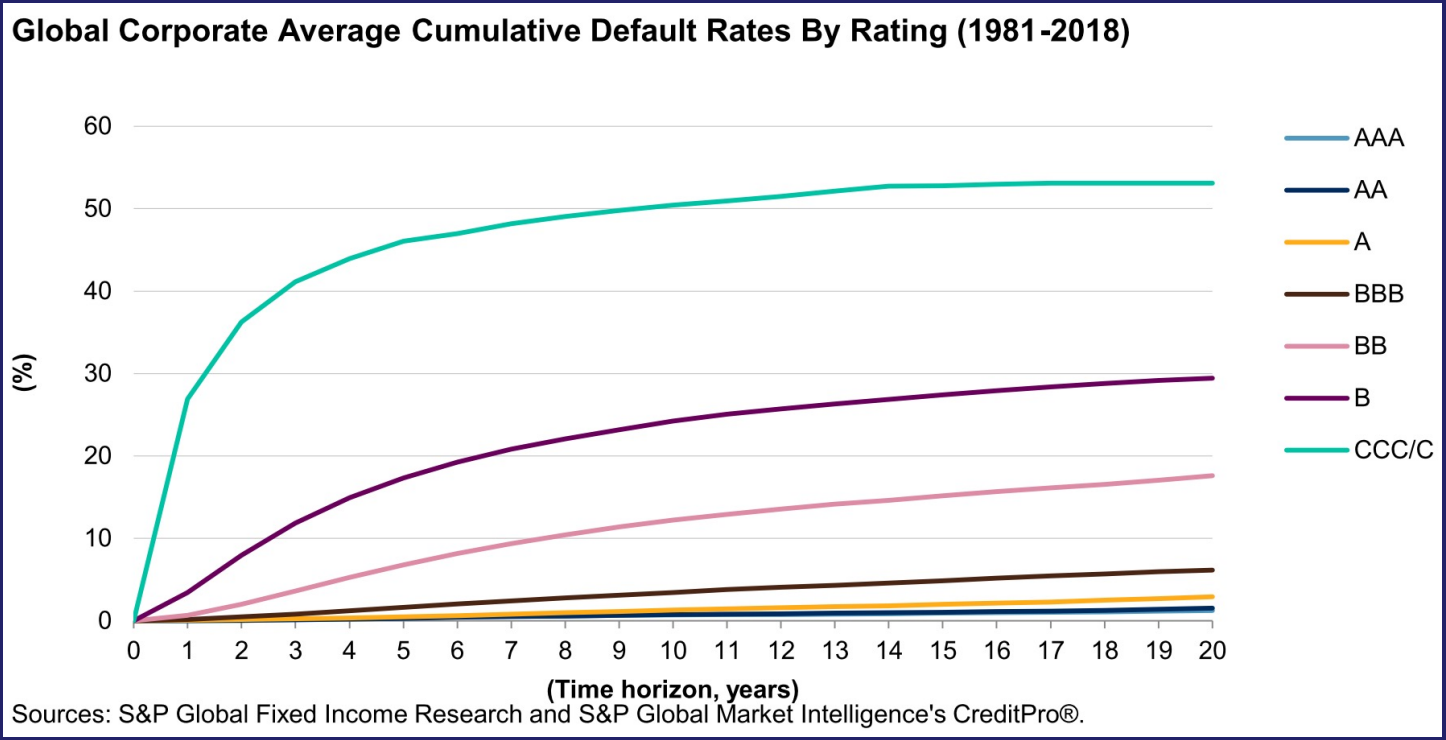
Default: When are firms more likely to miss a payment?

Defaults are likely after recessions



Source: Moody's Investors Service, 15 Feb., 2018, "Cross-Sector Annual Default Study: Corporate Default and Recovery Rates, 1920 – 2017."
https://www.researchpool.com/download/?report_id=1751185&show_pdf_data=true

Default: What do the ratings mean?



Grade A and up almost Never Default, Grade CCC 46% in 5 Years

Source: https://www.spratings.com/documents/20184/774196/2018AnnualGlobalCorporateDefaultAndRatingTransitionStudy.pdf

Default rate within X years

Average Cumulative Issuer-Weighted Global Default Rates By Letter Rating, 1983-2017

Rating	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Aaa	0.00%	0.01%	0.01%	0.04%	0.07%	0.10%	0.13%	0.14%	0.14%	0.14%	0.14%	0.14%	0.14%	0.14%	0.14%	0.14%	0.14%	0.14%	0.14%	0.14%
Aa	0.02%	0.06%	0.12%	0.20%	0.31%	0.40%	0.50%	0.58%	0.66%	0.74%	0.85%	0.99%	1.13%	1.23%	1.33%	1.43%	1.54%	1.72%	1.97%	2.19%
A	0.06%	0.17%	0.36%	0.56%	0.80%	1.07%	1.35%	1.63%	1.92%	2.20%	2.48%	2.74%	3.03%	3.36%	3.75%	4.15%	4.54%	4.96%	5.30%	5.64%
Baa	0.18%	0.46%	0.78%	1.15%	1.53%	1.93%	2.30%	2.67%	3.05%	3.46%	3.93%	4.43%	4.97%	5.48%	5.99%	6.57%	7.19%	7.77%	8.31%	8.69%
Ba	0.91%	2.57%	4.57%	6.66%	8.50%	10.15%	11.63%	13.01%	14.36%	15.70%	16.87%	18.05%	19.14%	20.27%	21.49%	22.62%	23.60%	24.57%	25.67%	26.39%
B	3.44%	8.18%	13.02%	17.38%	21.40%	25.04%	28.31%	31.06%	33.49%	35.50%	37.15%	38.61%	39.99%	41.50%	42.87%	44.14%	45.44%	46.67%	47.69%	48.74%
Caa-C	10.14%	17.93%	24.55%	30.12%	34.61%	37.99%	40.99%	43.87%	46.64%	48.91%	50.70%	51.54%	51.98%	52.07%	52.29%	52.71%	52.83%	52.83%	52.83%	52.83%
IG	0.09%	0.25%	0.45%	0.67%	0.92%	1.18%	1.43%	1.68%	1.93%	2.19%	2.46%	2.74%	3.05%	3.35%	3.67%	4.01%	4.35%	4.71%	5.05%	5.33%
SG	4.19%	8.51%	12.64%	16.29%	19.44%	22.16%	24.53%	26.60%	28.47%	30.10%	31.46%	32.67%	33.79%	34.94%	36.08%	37.15%	38.12%	39.05%	40.00%	40.75%
All	1.64%	3.29%	4.80%	6.09%	7.18%	8.10%	8.88%	9.56%	10.17%	10.72%	11.21%	11.66%	12.11%	12.56%	13.01%	13.46%	13.89%	14.32%	14.73%	15.06%

Source: Moody's Investors Service, 15 Feb., 2018, "Cross-Sector Annual Default Study: Corporate Default and Recovery Rates, 1920 – 2017."
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Recovery rate for bonds that default with X years

if they did default, how much was the bond holders were able to call back

fraction of value you could sell the bond for

Average Sr. Unsecured Bond Recovery Rates By Year Prior To Default, 1983-2017*					
	Year 1	Year 2	Year 3	Year 4	Year 5
Aaa**		3.33%	3.33%	61.88%	69.58%
Aa	37.24%	39.02%	38.08%	43.95%	43.18%
A	30.36%	42.57%	44.97%	44.48%	44.17%
Baa	42.89%	44.16%	43.99%	43.79%	43.52%
Ba	44.63%	43.30%	42.13%	41.60%	41.59%
B	37.62%	36.77%	37.21%	37.71%	38.36%
Caa-C	38.10%	38.43%	38.50%	38.83%	38.86%
Investment Grade	40.04%	43.33%	43.96%	44.11%	43.85%
Speculative Grade	38.34%	38.19%	38.31%	38.66%	38.99%
All Rated	38.40%	38.47%	38.71%	39.11%	39.45%

Source: Moody's Investors Service, 15 Feb., 2018, "Cross-Sector Annual Default Study: Corporate Default and Recovery Rates, 1920 – 2017."
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How do Rating Companies Rate Bonds?

Bond Rating Agencies look at some of the following criteria:

- Coverage ratios – earnings to fixed costs
- Leverage ratios – Debt to equity
- Liquidity ratios – Current Assets/Current Liabilities (Current ratio)
- Profitability ratios – Return on Assets: EBIT/Total Assets
- Cash flow to debt

high earning / low fixed cost → more likely to pay debt.

more other debts → high prob to miss a debt payment

→ enough cash on hand

more profit → easy to repay

Financial Ratios and Bond Ratings

TABLE 9.6 Financial ratios and default risk by rating class, long-term debt

	Three-year (2002 to 2004) medians						
	AAA	AA	A	BBB	BB	B	CCC
EBIT interest coverage multiple	23.8	19.5	8.0	4.7	2.5	1.2	0.4
EBITDA interest coverage multiple	25.5	24.6	10.2	6.5	3.5		0.9
Funds from operations/total debt (%)	203.3	79.9	48.0	35.9	22.4	11.5	
Free operating cash flow/total debt (%)	127.6	44.5	25.0	17.3	8.3	2.8	(002.1)
Total debt/EBITDA multiple	0.4	0.9	1.6	2.2	3.5	5.3	7.9
Return on capital (%)	27.6	27.0	17.5	13.4	11.3	8.7	3.2
Total debt/total debt + equity (%)	12.4	28.3	37.5	42.5	53.7	75.9	113.5

Note: EBITDA is earnings before interest, taxes, depreciation and amortisation.

Source: *Corporate Rating Criteria*, Standard & Poor's, 2006. Reproduced by permission of Standard & Poor's, a division of The McGraw-Hill Companies, Inc.

Protection Against Default

Indenture: contract specifying restrictions (covenants) on the issuer to protect bondholders from "moral hazard"

- Sinking funds → issuers has to pay off part of face value with each coupon payment
- Subordination of future debt → older debt should be paid before new debt
- Dividend restrictions → prevent firm from paying out dividend before they've made coupon and face value payment
- Collateral (Secured vs. Unsecured Debt)

Morgage collateral will be the house itself → secured loan

secured debt has lower interest rate

Q12: Example with risk aversion

coupon = 3.5 per annual.

- A 3.5% annual coupon paying bond with a $100 face value, a B Rating, and 1 year to maturity has a 3% chance of defaulting in the next year.

- If the firm defaults on its bonds, like the recovery tables, assume that you are able to sell the defaulted bond for 37% of the remaining value.

- 1 year risk-free zeros yield 1%, and suppose the expected market return is 5% and the beta of this bond is 0.25. What is the price of the bond?

$E[r] = 0.25 \times (5\% - 1\%) + 1\% = 2\%$.

good state $100 \times (1 + 3.5\%) = \$103.5$

bad state $100 \times (1 + 3.5\%) \times 37\% = \$38.295$

$E[CF] = 103.5 \times 97\% + 38.295 \times 3\% = \$101.543855$

$P = \frac{E[CF]}{1+2\%} = 99.55$

Answer to Q12

3.5% annual coupon. 1 year to maturity and a 3% chance of default. If default, you get 37% of remaining cash flows. rf=1% E[r]=2%. What is the price of the bond?

- $P = \sum_{t=1}^{\infty} \frac{E[\widetilde{CF}_t]}{(1+E[\tilde{r}])^t}$

- $E[\widetilde{CF}_1] = .97\times103.5 + .03\times.37\times103.5 = \$101.54$

Q12 continued

$$E[\tilde{r}] = r_f + \beta\left(E[\tilde{r}_M] - r_f\right)$$

$$E[\tilde{r}] = .01 + .25(.05 - .01) = 0.02$$

$$P = \sum_{t=1}^{1} \frac{E\left[\widetilde{CF}_t\right]}{(1 + E[\tilde{r}])^1} = \frac{101.54}{1.02} = \$99.55$$

Q12: What is the return in the up state and the down state?

- Up state:
- $r_{Up} = \frac{103.5}{99.55} - 1 = 0.03968 = 3.968\%$

- Down state:
- $r_{Down} = \frac{38.295}{99.55} - 1 = -0.61532 = -61.532\%$

- What is the average return across up and down states?
- 2%

- Do we ever earn 2%?
  - No it's the average return over the up and down states.

Q12: Continued

- What's the YTM?

$$P_t = \sum_{i=1}^{T} \frac{Promised\ CF_{t+i}}{(1 + r_{YTM})^i}$$

$$99.55 = \frac{103.5}{1 + r_{YTM}}$$

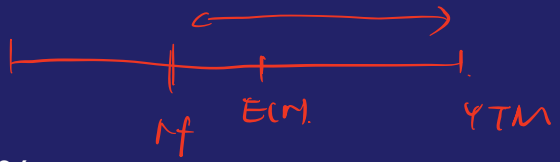
$$r_{YTM} = 3.968\%$$

Q12: What's the risk premium of this bond?

- Risk-Premium= $E[\tilde{r}] - r_f$
- Risk-Premium= $2\% - 1\%$
- What is the Risk Premium compensation for?
- Answer: The systematic component of default risk.
  - Related to the risk that this firm defaults when other things are going badly in the market.
-

Q12: What is the Default Premium?

- Default Premium = YTM – rf



- Default Premium = 3.968% – 1%=2.968%
- Why is the default premium greater than the risk-premium? Are they not both risk?
- Answer: The default premium includes completely diversifiable risk. The diversifiable component is: $Default\ Premium - E[r]$

in expectation, it is 0

YTM − E(r)

I thought we do not get compensated for diversifiable risk?

- We don't on average.
  - Let's check…

- $0.97 \times 3.968\% + 0.03 \times -61.532\% = 2\%$ = E[r]

- This excess default premium over the expected return ($YTM - E[\tilde{r}]$) is just the extra payment needed in the good state to exactly balance out for the possibility of extreme negative returns in the bad state.
  - On Average (~~$Default\ Premium$~~ YTM $- E[\tilde{r}]$) is zero
    - By the way this is the principle behind insurance companies….

Q12 Continued

- What would happen if I used the YTM to discount instead of $E[\tilde{r}]$?

$$\sum_{t=1}^{1} \frac{E[\widetilde{CF}_t]}{(1+r_{YTM})^1} = \frac{101.54}{1.03968} = \$97.66 \neq \$99.55$$

- This is because YTM is defined at the IRR of promised cash flows
  - YTM is a function of price
  - Price is not a function of YTM

$$P = \sum_{t=1}^{T} \frac{E[\widetilde{CF}_t]}{(1+E[r])^t}$$