

# ROBOTICS

## CHAPTER 25

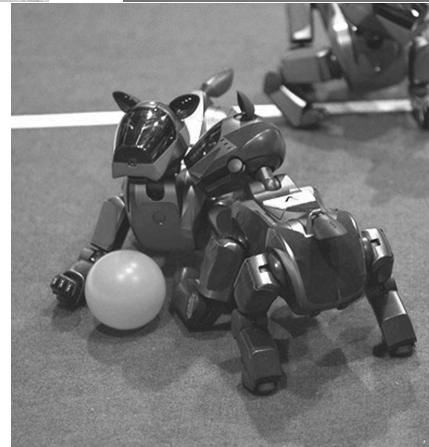
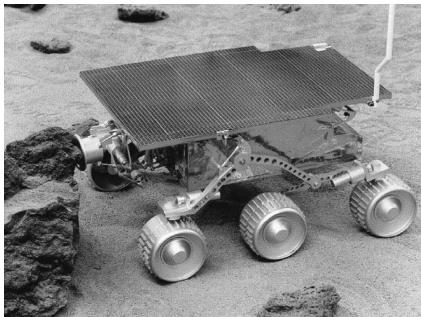
# Outline

Robots, Effectors, and Sensors

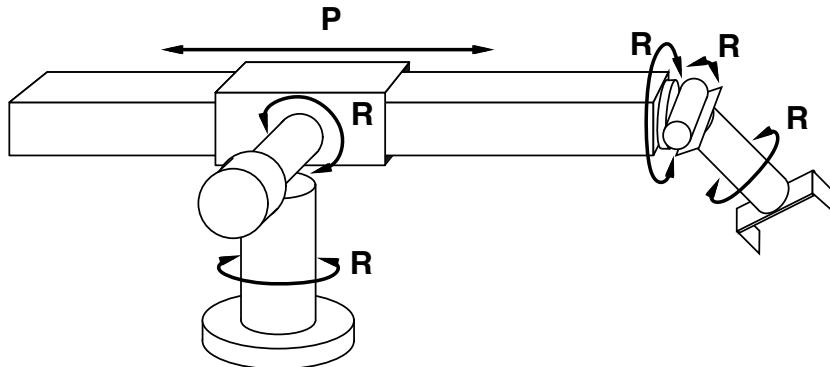
Localization and Mapping

Motion Planning

# Mobile Robots



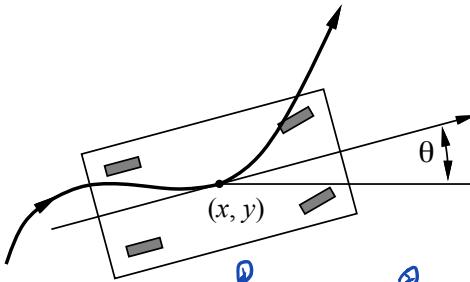
# Manipulators



Configuration of robot specified by 6 numbers  
⇒ (6 degrees of freedom (DOF))

6 is the minimum number required to position end-effector arbitrarily.  
For dynamical systems, add velocity for each DOF.

## Non-holonomic robots



A car has more DOF (3) than controls (2), so is non-holonomic;  
cannot generally transition between two infinitesimally close configurations

$x, y, \theta$

① accelerate / break      ② adjust the wheel

## Sources of uncertainty in interaction

There are two major sources of uncertainty for any interacting mobile agent (human or robotics):

- ◊ Everything they perceive (percepts)
- ◊ Everything they do (actions)

## We do not see the world as it is

Kant (1724-1804)

- ◊ Distinction between 'things-in-themselves', and 'appearances'.
- ◊ We have sensory states, but how do we know how (even if) these relate to the world?

## We do not see the world as it is

- ◊ What do you really see?
  - Small foveal region in colour.
  - Most of field of view is monochrome, at low resolution, and sensitive to motion.
  - 2D image only (binocular seldom used in practice).
- ◊ Occasionally, perceived reality breaks down:
  - hallucinations
  - optical illusions

## Uncertainty in actions

- ◊ Consider an agent action sequence as discussed in AIMA
  - Move forward 1 metre;
  - Turn right (90 degrees);
  - Move forward 2 metres;
  - Turn left;
  - Turn left;
  - Move forward 2 metres.
  - Turn left;
  - Move forward 1 metre.
- ◊ Go into an open space, close your eyes (no sensory feedback - that would be cheating!), and attempt this sequence.
- ◊ How close do you get?

## Sources of uncertainty in motion actions

- slippage;
  - inaccurate joint encoding;
  - rough surfaces;
  - obstacles;
  - effector breakdown (injuries);
  - ....
- ◊ All these errors accumulate without bound.
- ◊ Thus, starting out with perfect knowledge, and moving using actions with very small error, after an infinitely long action sequence a system will have infinite error in its position estimate.

## Dealing with uncertainty in motion actions

- ◊ Use sensors to verify actions (not simple).

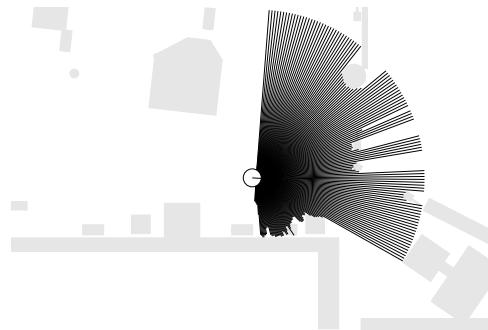
## Sources of uncertainty robotic perception

- ◊ Let's take a very simple sensor - distance sensor (laser range finder).
- ◊ Returns a number corresponding to nearest obstacle in a straight-line path, up to some finite distance.
- ◊ Has some known error (non-precise location).
- ◊ Has some known false positive/false negative rates (whether an obstacle is there or not).
- ◊ Has small spatial extent - what if partially obstructed?
- ◊ Finite time means cannot check every location, what does a reading say about the readings at neighbouring points.

# Sensors

Range finders: sonar (land, underwater), laser range finder, radar (aircraft), tactile sensors, GPS

*physical pressure sensors*



Imaging sensors: cameras (visual, infrared)

Proprioceptive sensors: shaft decoders (joints, wheels), inertial sensors, force sensors, torque sensors

*phone*

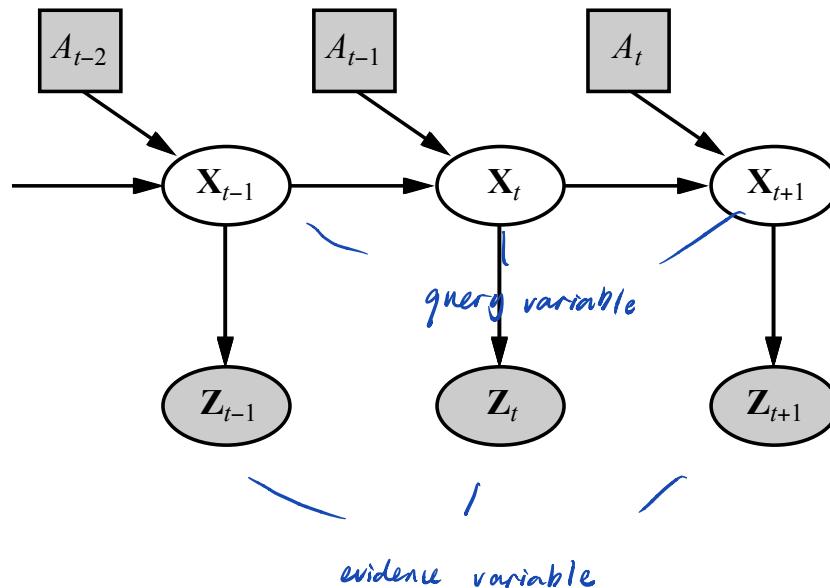
## Implications of sensor uncertainty

- ◊ Must make assumptions about the way the world behaves in order to interpret the readings at all. For example:
  - Some finite resolution sampling is sufficient to detect obstacles (consider an obstacle that consists of hundreds of long pins, sparsely distributed, pointing towards the sensor).
  - Must know something about the structure of the robot to decide what an obstacle is.
  - Given some sensor reading, only have a finite probability that it is correct
    - must have some way of dealing with this.

## Localization—Where Am I?

An action  $A_t$  at time  $t$  results in  
state  $X_{t+1}$  and observation  $Z_{t+1}$

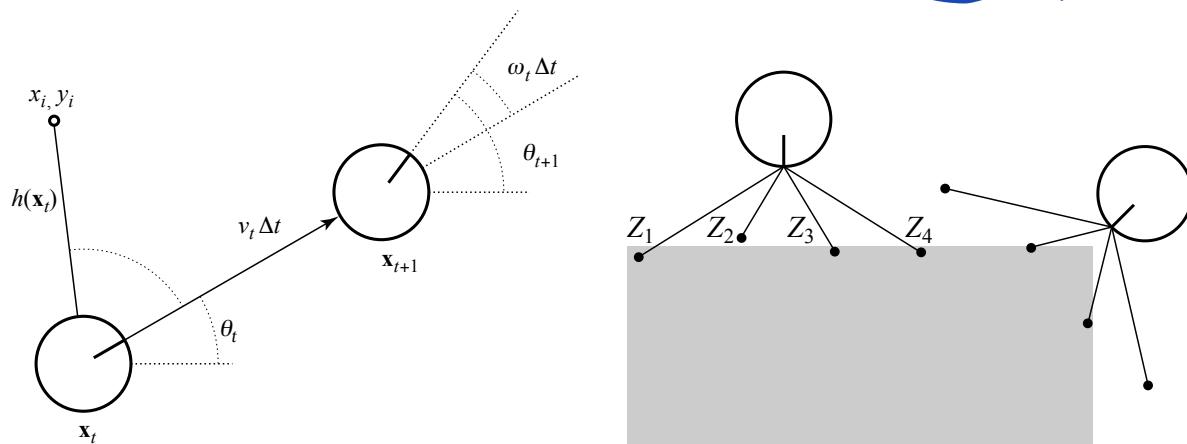
Compute current location and orientation (pose) given observations:



## Localization contd.

Sensor Model: Use observation  $h(x_t)$  of landmark  $x_i, y_i$  to estimate state  $x_t$  of robot.

Motion Model: Update state using its movements  $v_t \Delta t$  and  $\omega_t \Delta t$

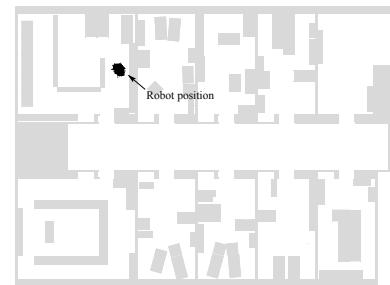
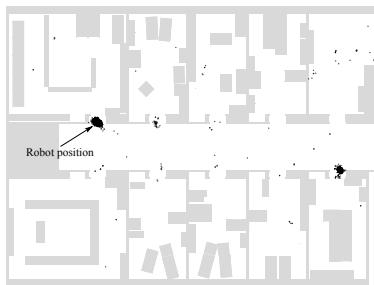
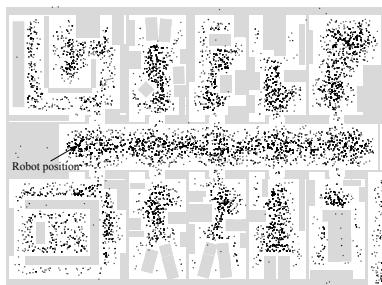


Assume Gaussian noise in motion prediction, sensor range measurements

## Localization contd.

Can use **particle filtering** to produce approximate position estimate

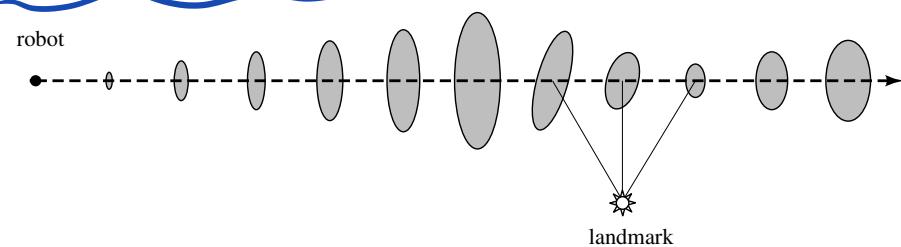
- ① Start with random samples from uniform prior distribution for robot position.
- ② Update likelihood of each sample using sensor measurements.
- ③ Resample according to updated likelihoods. *delete low likelihood*



## Localization contd.

We need to continuously update our distribution for the current state using the latest measurements.

Uncertainty of the robot's state grows as it moves  
until we find a landmark.



Assumes that landmarks are *identifiable*—otherwise, posterior is multimodal

# Mapping

Localization: given map and observed landmarks, update pose distribution

Mapping: given pose and observed landmarks, update map distribution

**Simultaneous Localization and Mapping (SLAM):**

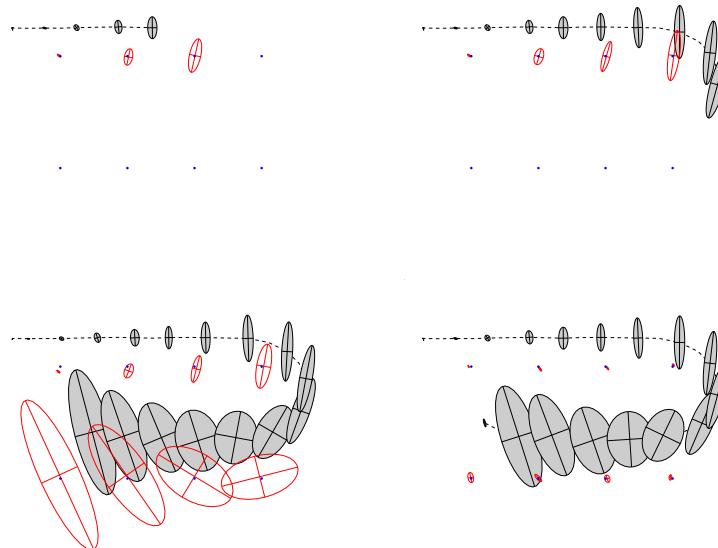
given observed landmarks, update pose and map distribution

Probabilistic formulation of SLAM:

add landmark locations  $L_1, \dots, L_k$  to the state vector,  
proceed as for localization

## Mapping contd.

Consider space with 8 identical landmarks:

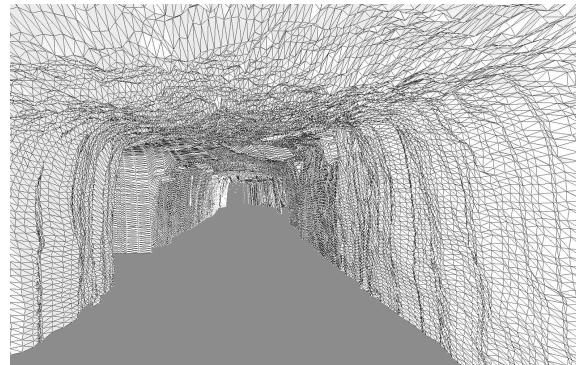


When first landmark detected again:

no uncertainty about robot or landmark positions

## 3D Mapping example

Mapping a coal mine:



## Bayesian Inference on Sensors

Need some way to determine whether an obstacle is there, given multiple measurements from a sensor.

What is Bayesian inference? (revision)

*there is an obstacle*  
↑

- A method for determining the *probability* that a hypothesis is true, given a set of measurements.
- Probability  $\approx$  belief.

## Elements of Conditional Probability: revision

The probability that  $A$  is true, *given* the condition  $B$ :

$$P(A | B) \tag{1}$$

Examples:

- $P(\text{I will get hit by a car tomorrow}) = 0.0001$
- $P(\text{I will get hit by a car tomorrow} | \text{I ride to Frankston}) = 0.01$
- $P(\text{I will get hit by a car tomorrow} | \text{I stay home all day}) = 0.0000001$

## Bayes Law: revision

Bayes Law asks:

- Given measurement  $M$ ;
- What is the probability of hypothesis  $H$ ?

Bayes Law:

$$P(H \mid M) = \frac{P(M \mid H)}{P(M)} P(H) \quad (2)$$

## Bayes Law: revision

Bayes Law:

$$P(H | M) = \frac{P(M | H)}{P(M)} P(H) \quad (3)$$

Elements:

- $P(H | M)$ : posterior probability of  $H$ .
- $P(H)$ : prior probability of  $H$ .
- $P(M | H)$ : sensor model.  $\Rightarrow$  prob that get recording given that there is an obstacle
- $P(M)$ : normalisation factor.

Normalisation:

$$P(M) = P(M | H)P(H) + P(M | \neg H)P(\neg H) \quad (4)$$

summing over whether hypothesis is T / F

## Example

Obstacle detection:

- The odds of there being an obstacle present are 1 in 10.  
assume obstacle, actually no  
assume no obstacle, actually yes
- The detector has 5% false positive rate and 10% false negative rate.  
p(positive|no obstacle)  
p(negative|obstacle)
- Probability that an obstacle is present if the detector returns positive?
- Probability that an obstacle is present if the detector returns negative?

Priors:

$$P(\text{obstacle}) = 0.1$$

$$P(\text{no obstacle}) = 0.9$$

$$\begin{aligned} P(\text{obstacle} | \text{positive}) &= \frac{P(\text{positive} | \text{obstacle}) \times P(\text{obstacle})}{P(\text{positive})} \\ &= \frac{0.9 \times 0.1}{0.9 \times 0.1 + 0.1 \times 0.05} = 0.607 \quad (5) \end{aligned}$$

$$\begin{aligned} P(\text{obstacle} | \text{negative}) &= \frac{P(- | o) \cdot P(o)}{P(- | o) \cdot P(o) + P(- | n) \cdot P(n)} \\ &= \frac{0.1 \times 0.1}{0.1 \times 0.1 + 0.9 \times 0.95} = 0.0116 \end{aligned}$$

## Example continued

Sensor model:

$$\begin{aligned} P(\text{positive} \mid \text{obstacle}) &= 0.9 \\ P(\text{negative} \mid \text{obstacle}) &= 0.1 \\ P(\text{negative} \mid \text{not obstacle}) &= 0.95 \\ P(\text{positive} \mid \text{not obstacle}) &= 0.05 \end{aligned} \tag{6}$$

If the sensor returns positive:

$$P(\text{obstacle} \mid \text{positive}) = \frac{0.9}{0.9 \times 0.1 + 0.05 \times 0.9} \cdot 0.1 = 0.667 \tag{7}$$

If the sensor returns negative:

$$P(\text{obstacle} \mid \text{negative}) = \frac{0.1}{0.1 \times 0.1 + 0.95 \times 0.9} \cdot 0.1 = 0.0116 \tag{8}$$

why independent is important?

## Incremental form of Bayes Law

more independent measurement (even if it is noise), the better our estimate

Bayes Law can be extended to handle multiple measurements.

- Given a set of *independent* measurements  $\{M_j\}$ ;
- What is the probability of the hypothesis  $H$ ?

If measurements are *independent*, can use incremental form.

- Given the *current* probability distribution  $P(H)$ ;
- And a *new* measurement  $M$
- What is the updated probability distribution  $P(H)$ ?

Use Bayes Law in incremental form:

$$P(H) \xleftarrow{M} \frac{P(M | H)}{P(M)} P(H)$$

Sometimes called *Bayesian update rule*.

$$P(H | M) = \frac{P(M | H) \cdot P(H)}{P(M)}$$

$$\begin{aligned} P(H | M_1, M_2) &= \frac{P(M_1, M_2 | H) \cdot P(H)}{P(M_1, M_2)} \\ &= \frac{P(M_1 | H) \cdot P(H) \cdot P(M_2 | H)}{P(M_1 | H) \cdot P(M_2 | H) + P(M_1, M_2 | \neg H) \cdot P(\neg H)} \\ &\stackrel{\text{independence}}{=} \frac{P(M_1 | H) \cdot P(M_2 | H) \cdot P(H)}{P(M_1 | H) \cdot P(M_2 | H) + P(M_1 | \neg H) \cdot P(M_2 | \neg H)} \\ &\stackrel{\text{normalize by } P(M_1)}{=} \frac{P(M_1 | H) \cdot P(M_2 | H) \cdot P(H) / P(M_1)}{[P(M_1 | H) \cdot P(M_2 | H) \cdot P(H) / P(M_1)] + [P(M_1 | \neg H) \cdot P(M_2 | \neg H) \cdot P(\neg H) / P(M_1)]} \\ &= \frac{P(M_2 | H) \cdot P(H | M_1)}{P(M_2 | H) \cdot P(H | M_1) + P(M_2 | \neg H) \cdot P(\neg H | M_1)} \end{aligned}$$

## Example

Obstacle detection (again):

- The odds of there being an obstacle present are 1 in 10.
- The detector has 5% false positive rate and 10% false negative rate.  
 $P(+|N) = 0.05$        $P(-|O) = 0.1$
- What is the probability that an obstacle is present if the detector returns:
  - One positive?
  - Two positives?
  - Two positives and a negative?

$$\text{1st positive } P(O|+) = \frac{P(+|O) \cdot P(O)}{P(+|O) \cdot P(O) + P(+|N) \cdot P(N)} = \frac{0.9 \times 0.1}{0.9 \times 0.1 + 0.05 \times 0.9} = 0.67$$

$$\begin{aligned}\text{2nd positive } P(O|+,+) &= \frac{\underset{\text{updated prior}}{P(+|O) \cdot P(O|+)}}{P(+|O) \cdot P(O|+) + P(+|N) \cdot P(N|+)} \\ &= \frac{0.9 \times 0.67}{0.9 \times 0.67 + 0.05 \times 0.33} = 0.97\end{aligned}$$

## Example

Time series:

Time	$M$	$P(\text{obs})$	$P(\text{noobs})$
0	-	0.10	0.90
1	positive	0.67	0.33
2	positive	0.97	0.03
3	negative	0.79	0.21

## Motion Planning

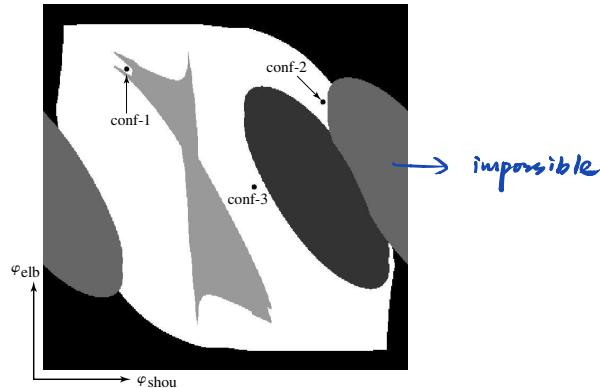
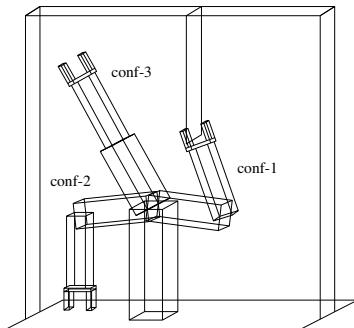
Now we know:

- where we are
- where the obstacles are
- where we want to go

Next challenge – How do we get there?

# Motion Planning

Idea: plan in **configuration space** defined by the robot's DOFs



Solution is a point trajectory in free C-space



# Configuration space planning

Basic problem:  $\infty^d$  states! Convert to finite state space.

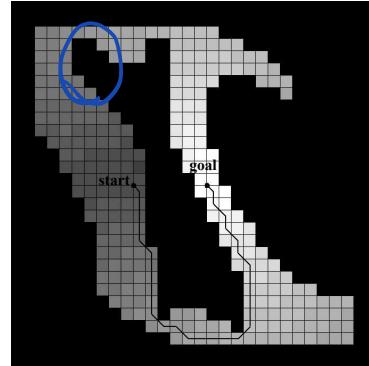
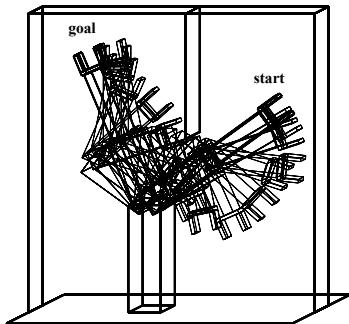
## Cell decomposition:

divide up space into simple **cells**,  
each of which can be traversed “easily” (e.g., convex)

## Skeletonization:

identify finite number of easily connected points/lines  
that form a graph such that any two points are connected  
by a path on the graph

## Cell decomposition example

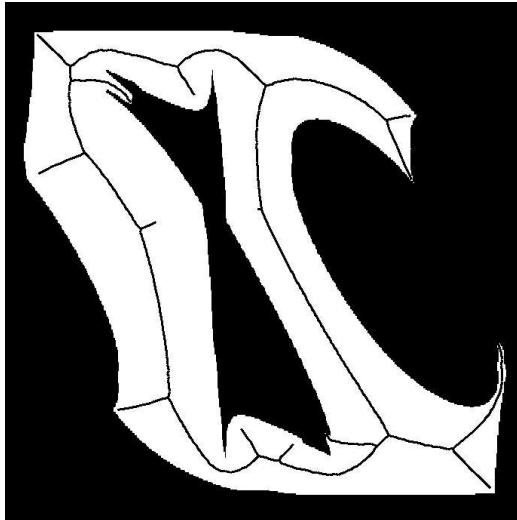


Problem: may be no path in pure freespace cells <sup>may</sup>  $\Rightarrow$  lose some details  
Solution: recursive decomposition of mixed (free+obstacle) cells

## Skeletonization: Voronoi diagram

Voronoi diagram: locus of points equidistant from obstacles

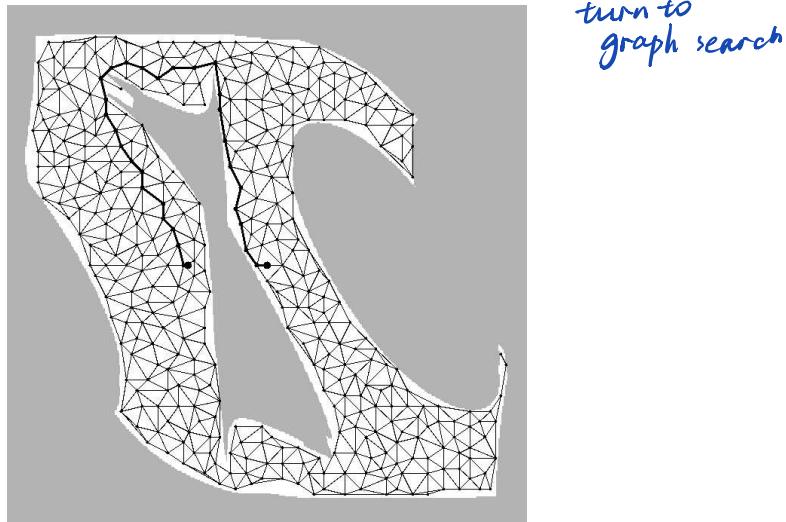
*equal distance from obstacles*



Problem: doesn't scale well to higher dimensions

## Skeletonization: Probabilistic Roadmap

A probabilistic roadmap is generated by generating random points in C-space and keeping those in freespace; create graph by joining pairs by straight lines



Problem: need to generate enough points to ensure that every start/goal pair is connected through the graph

## Summary

- ◊ Percepts and actions are both subject to uncertainty.
- ◊ We cannot interpret our percepts without having a model of what they mean, and without (partially invalid) assumptions about how they perform.
- ◊ Uncertainty in robot perception.
- ◊ Incremental form of Bayes Law.
- ◊ Motion planning.

## Implications for AI

If you can't rely on your perceptions or your actions, does that mean that Agent methods we have discussed are of no use?

- Many problems don't have uncertainty for perceptions and actions, e.g., scheduling, planning, game-playing, text-based machine translation.
- Can incorporate standard agent methods within a system that handles uncertainty, i.e., re-plan if something goes wrong.
- Can apply uncertainty handlers to whole system - e.g., Bayesian inference.

Certainly for autonomous robots and computer vision interaction with an environment creates many problems that cannot be easily handled with conventional AI techniques.

