Tutorial 2: Determinants and Dot Product

Determinants

Q1. Use row reduction to find the determinants of the following matrices:

(i) $\begin{bmatrix} 1 & -2 & 7 & 3 \\ 0 & 1 & -2 & 4 \\ -2 & 3 & -3 & 1 \\ -3 & 6 & -21 & 0 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{bmatrix}$ (iii) $\begin{bmatrix} 0 & 0 & 0 & a_{14} \\ 0 & 0 & a_{23} & 0 \\ 0 & a_{32} & 0 & 0 \\ a_{41} & 0 & 0 & 0 \end{bmatrix}$

(Hint: Factorise your answer in (ii) as you go.)

Q2. Let

$$H = \begin{bmatrix} \frac{1}{2} & \frac{3}{4} \\ \frac{2}{3} & -\frac{1}{3} \end{bmatrix} \qquad J = \begin{bmatrix} 2 & 1 & 3 \\ -2 & -1 & 7 \\ 1 & 0 & -2 \end{bmatrix} \qquad K = \begin{bmatrix} -1 & 2 & 0 \\ 3 & 3 & 0 \\ 2 & -1 & 1 \end{bmatrix}$$

(a) Use cofactor expansion to find:
(i) $\det(H)$ (ii) $\det(J)$. (iii) $\det(K)$.

(b) Without doing further determinant calculations, find (if defined):
(i) $\det(J^2K)$ (ii) $\det(KH)$ (iii) $\det(3J)$ (iv) $\det(K^T(J^{-1})^2)$

Q3. Suppose that

$$I = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 10 \end{bmatrix} \begin{bmatrix} 1 & -2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 1 \end{bmatrix} A.$$

(a) Write down a sequence of elementary row operations reducing A to the identity matrix I.

(b) What is $\det(A)$?

Q4. **(Exercises, Sheet 1, Q27)** A matrix P is called idempotent if $P^2 = P$. If P is idempotent and $P \neq I$ show that $\det P = 0$.

Vectors and Dot Product

Q5. Let $\mathbf{a} = (1, 2, 5)$ and $\mathbf{b} = (-3, 1, 0)$.

(a) Find the cosine of θ, the angle between $\mathbf{a}$ and $\mathbf{b}$.

(b) Find the distance between the points $(1, 2, 5)$ and $(-3, 1, 0)$.

Q6. **(Anton–Rorres p130, Q35)** Use the fact that 21375, 38798, 34162, 40223, 79154 are all divisible by 19 to show that

$$\begin{vmatrix} 2 & 1 & 3 & 7 & 5 \\ 3 & 8 & 7 & 9 & 8 \\ 3 & 4 & 1 & 6 & 2 \\ 4 & 0 & 2 & 2 & 3 \\ 7 & 9 & 1 & 5 & 4 \end{vmatrix}$$

is divisible by 19 without evaluating the determinant.