PHYC10003 Physics 1

Lecture 13: Rotational Motion

Rigid bodies, rotational inertia & variables

Who am I?

- Harry Quiney (quiney@unimelb.edu.au)
- Theoretical Condensed Matter Physics
- Quantum mechanics of materials and imaging of molecular structure using X-rays

X-ray Free-Electron Laser (XFEL):

Linac Coherent Light Source (LCLS), Stanford University, USA

Image: LCLS

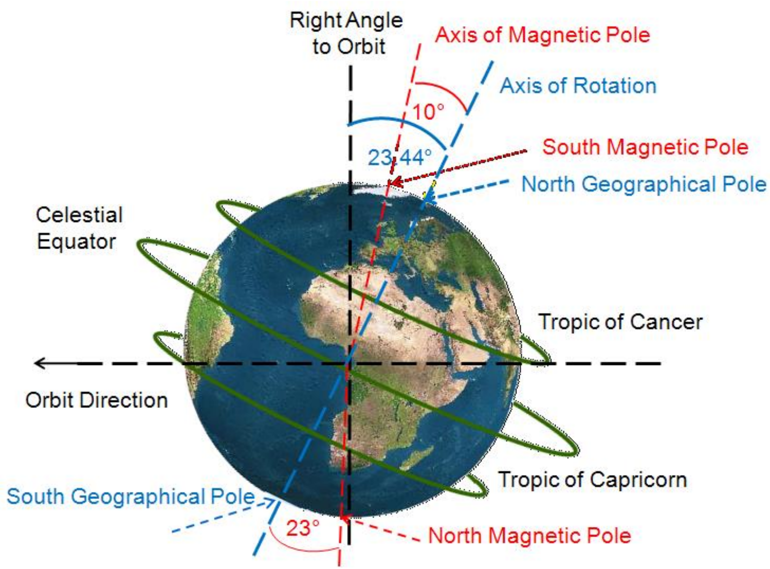
Last Lecture

- Elastic and inelastic collisions
- Conservation of energy and linear momentum
- Rockets and the rocket equation

Rotational motion

- We now look at motion of **rotation**
- We will find "*the same laws apply*": analogies.
- But we will need new quantities to express them
  - Torque
  - Rotational inertia
- A **rigid body** rotates as a unit, locked together
- We look at rotation about a **fixed axis**
- These requirements exclude from consideration:
  - The Sun, where layers of gas rotate separately
  - A rolling bowling ball, where rotation and translation occur

Rotational motion: the earth



World Geodetic System: WGS 84 sets the standard for locating position on earth (in your phone) based on rotational flattening 1:298.257223563

To a reasonable approximation, the earth can be regarded as a rigid body rotating around an axis. There are two qualifications to this:

1. The rotation of the earth causes it to bulge at the equator; it is an oblate spheroid (21 km radial difference) rather than a sphere.
2. The gravitational attraction of the sun and the moon cause a distortion of the water on the surface, giving rise to the tides.

Frictional forces cause the day to become 2ms shorter per century!

Rotational axis & angular position

- The fixed axis is called the **axis of rotation**
- Figs 10-2, 10-3 show a *reference line*
- The **angular position** of this line (and of the object) is taken relative to a fixed direction, the **zero angular position**

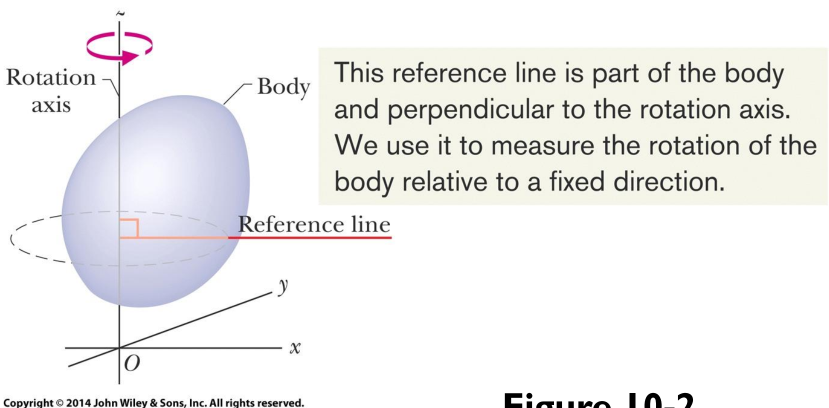


Figure 10-2

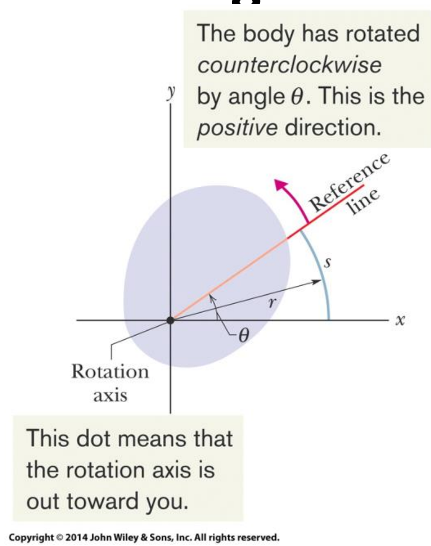


Figure 10-3

Angles and angular displacement

- Measure using **radians** (rad): dimensionless

$$\theta = \frac{s}{r} \quad \text{(radian measure).}$$

Eq. (10-1)

$$1 \text{ rev} = 360^\circ = \frac{2\pi r}{r} = 2\pi \text{ rad},$$

Eq. (10-2)

- Do not reset θ to zero after a full rotation
- We know all there is to know about the kinematics of rotation if we have $\theta(t)$ for an object
- Define angular displacement as:

$$\Delta\theta = \theta_2 - \theta_1.$$

Eq. (10-4)

Rotation – direction and sign convention

- "*Clocks are negative*":

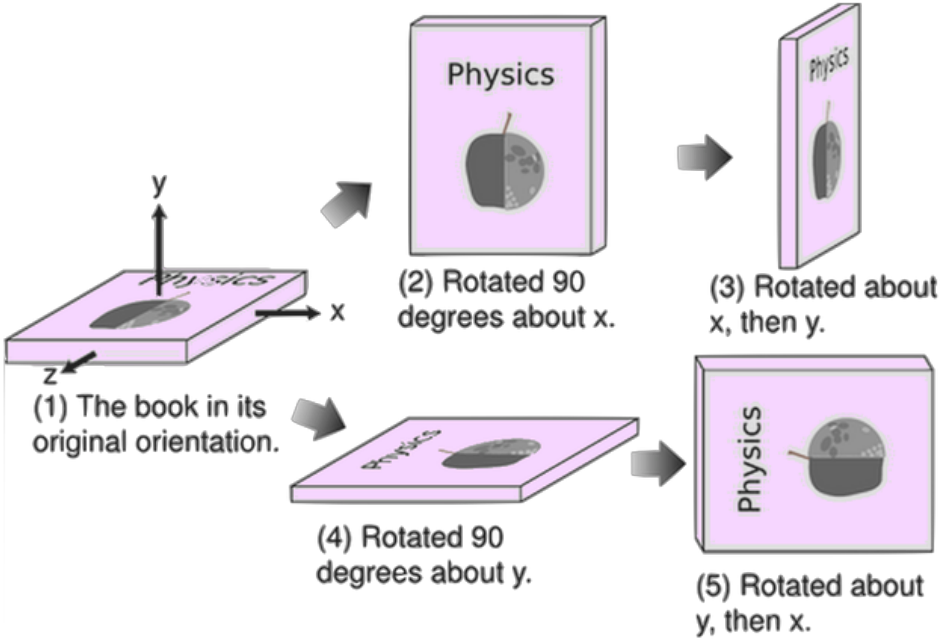
An angular displacement in the counterclockwise direction is positive, and one in the clockwise direction is negative.

Checkpoint 1

A disk can rotate about its central axis like a merry-go-round. Which of the following pairs of values for its initial and final angular positions, respectively, give a negative angular displacement: (a) -3 rad, $+5$ rad, (b) -3 rad, -7 rad, (c) 7 rad, -3 rad?

Answer: Choices (b) and (c)

Rotation: non commutativity



The order in which rotations are performed is significant: in general, rotations are ***non-commutative.***

Angular velocity

- **Average angular velocity**: angular displacement during a time interval

$$\omega_{\text{avg}} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t}, \qquad \textbf{Eq. (10-5)}$$

- **Instantaneous angular velocity**: limit as $\Delta t \rightarrow 0$

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}. \qquad \textbf{Eq. (10-6)}$$

- These equations hold for all points on a rigid body
- Magnitude of angular velocity = **angular speed**

Angular velocity

- Figure 10-4 shows the values for a calculation of average angular velocity

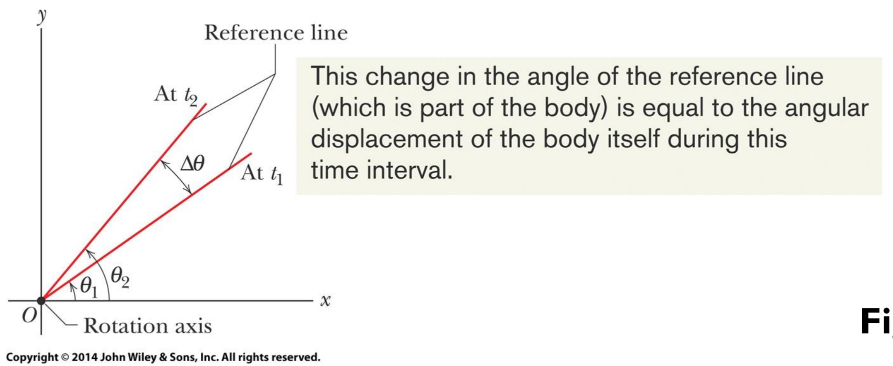


Figure 10-4

- **Average angular acceleration**: angular velocity change during a time interval

$$\alpha_{avg} = \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{\Delta\omega}{\Delta t},$$ **Eq. (10-7)**

Angular acceleration

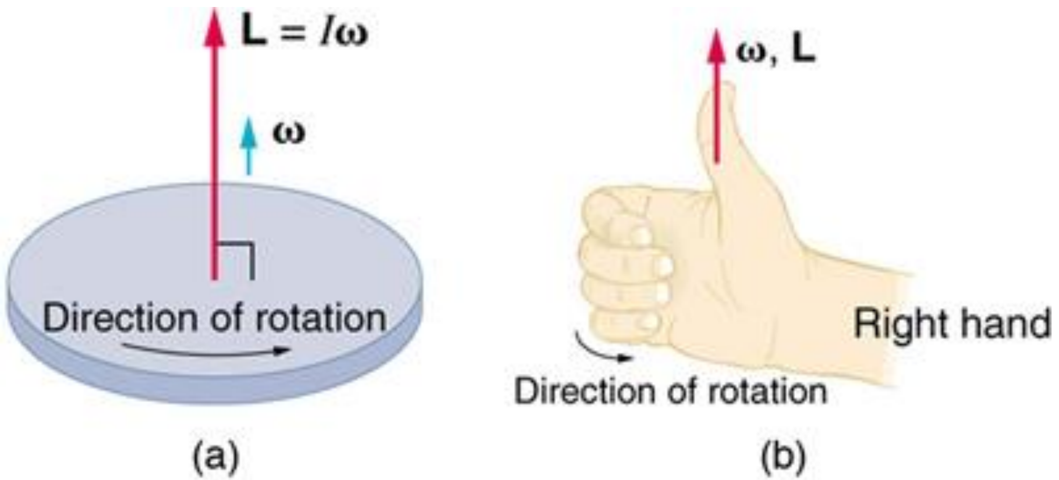
- **Instantaneous angular acceleration**: limit as $\Delta t \to 0$

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta \omega}{\Delta t} = \frac{d\omega}{dt}.$$

Eq. (10-8)

- These equations hold for all points on a rigid body
- With right-hand rule to determine direction, angular velocity & acceleration can be written as ***vectors***
- If the body rotates around the vector, then the vector points along the axis of rotation
- Angular displacements are *not* vectors, because the order of rotation matters for rotations around different axes

Angular velocity: right hand rule



The direction of the curled fingers on the right hand determine the direction of angular velocity, ***ω*,** angular acceleration***, α*,** and angular momentum***, L*.**

Analogies – linear and angular motion

- The same equations hold as for *constant linear acceleration*, see Table 10-1
- We simply change linear quantities to angular ones!
- Eqs. 10-12 and 10-13 are the basic equations: *all* others can be derived from them

Table 10-1 Equations of Motion for Constant Linear Acceleration and for Constant Angular Acceleration

Equation Number	Linear Equation	Missing Variable		Angular Equation	Equation Number
(2-11)	$v = v_0 + at$	$x - x_0$	$\theta - \theta_0$	$\omega = \omega_0 + \alpha t$	(10-12)
(2-15)	$x - x_0 = v_0 t + \frac{1}{2}at^2$	v	ω	$\theta - \theta_0 = \omega_0 t + \frac{1}{2}\alpha t^2$	(10-13)
(2-16)	$v^2 = v_0^2 + 2a(x - x_0)$	t	t	$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$	(10-14)
(2-17)	$x - x_0 = \frac{1}{2}(v_0 + v)t$	a	α	$\theta - \theta_0 = \frac{1}{2}(\omega_0 + \omega)t$	(10-15)
(2-18)	$x - x_0 = vt - \frac{1}{2}at^2$	v_0	ω_0	$\theta - \theta_0 = \omega t - \frac{1}{2}\alpha t^2$	(10-16)

Table 10-1

Angular description of position and speed

- Linear and angular variables are related by *r*, perpendicular distance from the rotational axis
- Position (note θ *must* be in radians):

$$s = \theta r$$

Eq. (10-17)

- Speed (note ω *must* be in radian measure):

$$v = \omega r$$

Eq. (10-18)

- We can express the period in radian

$$T = \frac{2\pi}{\omega}$$

Eq. (10-20)

Radial and tangential acceleration

- Tangential acceleration (radians):

$$a_t = \alpha r$$

Eq. (10-22)

α = angular acceleration (rad s^{-2})

- We can write the radial acceleration in terms of angular velocity (radians):

$$a_r = \frac{v^2}{r} = \omega^2 r$$

Eq. (10-23)

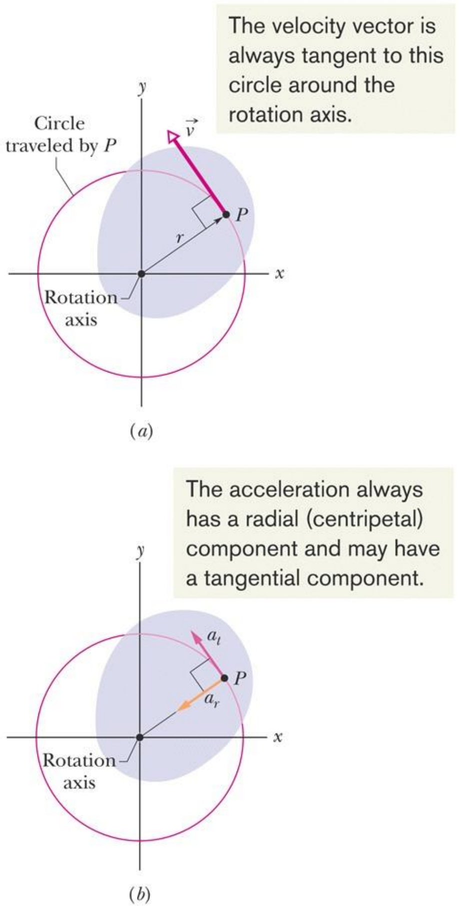


Figure 10-9

Checkpoint: merry-go-round

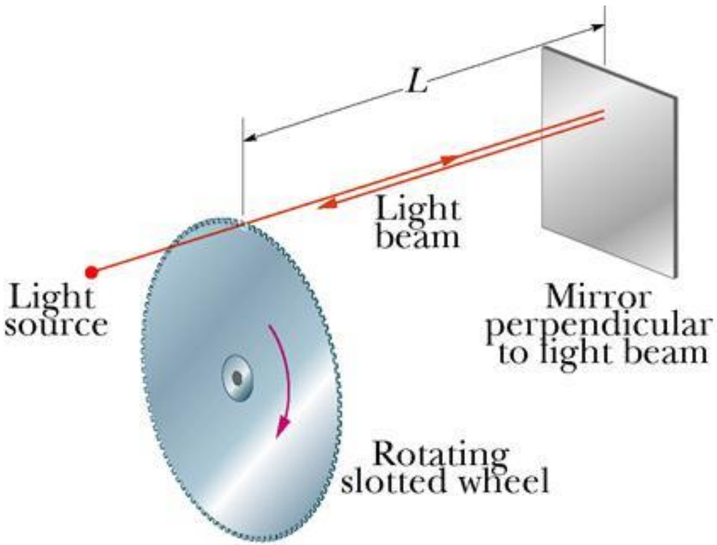
Checkpoint 3

A cockroach rides the rim of a rotating merry-go-round. If the angular speed of this system (*merry-go-round + cockroach*) is constant, does the cockroach have (a) radial acceleration and (b) tangential acceleration? If ω is decreasing, does the cockroach have (c) radial acceleration and (d) tangential acceleration?

Answer: (a) yes (b) no (c) yes (d) yes

The Magic Roundabout, BBC-TV (1963)

Example: measure the speed of light



Radius of the notched wheel=5.0cm
Number of slots around wheel =500
L = 500 m

If the speed of light is $c=3.0 \times 10^8$ ms^{-1}

(a) What is the angular speed of the wheel?
(b) What is the speed of a point on the rim of the wheel?

Solution:

(a). In the time, t (s), that it takes the light to pass a notch, reflect off the mirror and return through the next notch, the wheel passes through an angle of $\theta = 2\pi/500$ rad.

That time is $t = 2L/c = 3.34\times10^{-6}$ s and $\omega = \theta/t = 3.8\times10^{3}$ rad $s^{-1} = 1.27\times10^{6}$ rpm!

(b) If r is the radius of the wheel then the linear speed of a point on the wheel is:

$v = \omega r = 1.9\times10^{2}$ ms^{-1} = 684 km/h!

It is actually possible to buy an electric motor capable of rotating at 10^6 rpm!
(Image: Celeratron, ETZ Zurich)

Summary

Angular Position

- Measured around a **rotation axis**, relative to a **reference line**:

$$\theta = \frac{s}{r}$$ **Eq. (10-1)**

Angular Displacement

- A change in angular position

$$\Delta\theta = \theta_2 - \theta_1.$$ **Eq. (10-4)**

Angular Velocity and Speed

- Average and instantaneous values:

$$\omega_{avg} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t},$$ **Eq. (10-5)**

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}.$$ **Eq. (10-6)**

Angular Acceleration

- Average and instantaneous values:

$$\alpha_{avg} = \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{\Delta\omega}{\Delta t},$$ **Eq. (10-7)**

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}.$$ **Eq. (10-8)**