MAST30027: Modern Applied Statistics

Week 8 Lab

1. Let $X_1, \cdots, X_n$ be a random sample from a $N(\theta, \sigma^2)$ population, and suppose that the prior distribution on θ is $N(\mu, \tau^2)$. Here we assume that σ^2, μ and τ^2 are all known.

   (a) Find $p(\bar{x}, \theta)$, the joint pdf of $\bar{X}$ and θ.

   (b) Show that $p(\theta|\bar{x})$ is normal with mean and variance given by $E(\theta|\bar{x}) = \frac{n\tau^2}{n\tau^2+\sigma^2}\bar{x} + \frac{\sigma^2}{n\tau^2+\sigma^2}\mu$ and $\text{Var}(\theta|\bar{x}) = \frac{\sigma^2\tau^2}{n\tau^2+\sigma^2}$.

   (c) Show that the marginal pdf of $\bar{X}$, i.e., $p(\bar{x})$, is the pdf of $N(\mu, \frac{\sigma^2}{n} + \tau^2)$.

2. (a) Here is some code for simulating a discrete random variable Y. What is the probability mass function (pmf) of Y?

```
Y.sim <- function() {
  U <- runif(1)
  Y <- 1
  while (U > 1 - 1/(1+Y)) {
    Y <- Y + 1
  }
  return(Y)
}
```

   (b) Here is some code for simulating a discrete random variable Z. Show that Z has the same pmf as Y.

```
Z.sim <- function() {
  Z <- ceiling(1/runif(1)) - 1
  return(Z)
}
```

3. Consider the continuous random variable X with pdf given by:

$$f_X(x) = \frac{\exp(-x)}{(1+\exp(-x))^2} \qquad -\infty < x < \infty.$$

   X is said to have a standard logistic distribution. Find the cdf for this random variable. Simulate a sample of size 10 from the distribution using the inversion method.