

# Gibbs\_example\_coal

## load data

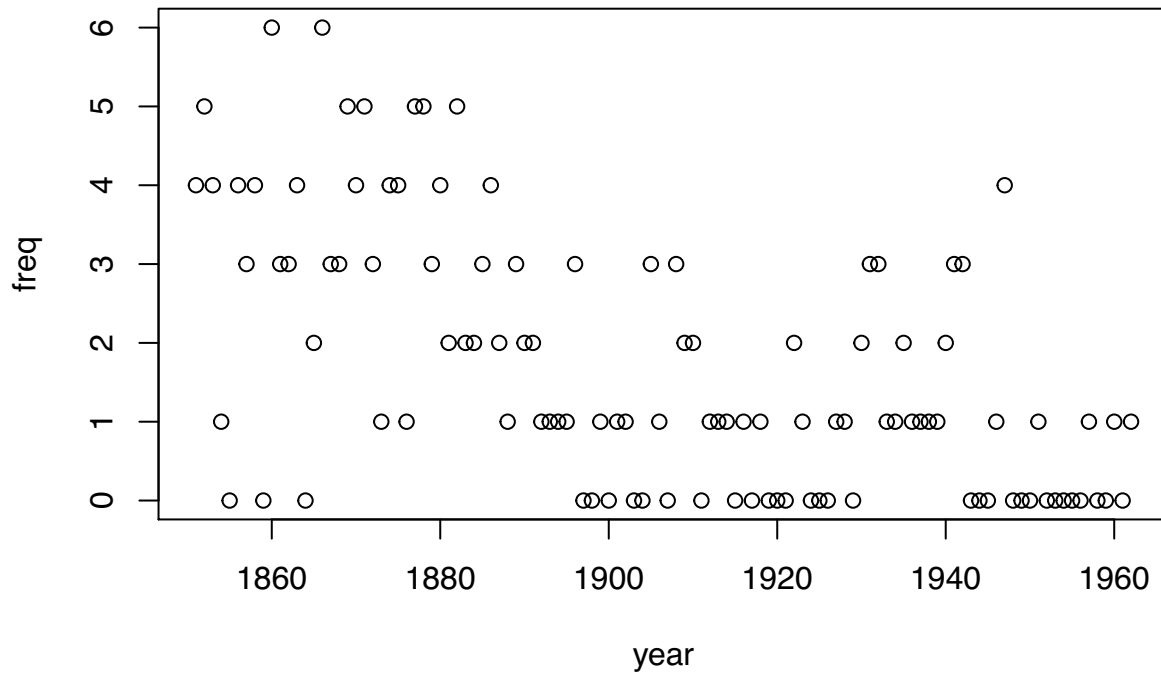
```
library(boot)
data(coal)
str(coal)

## 'data.frame': 191 obs. of 1 variable:
## $ date: num 1851 1852 1852 1852 1852 ...

when <- floor(coal)
year <- 1851:1962
freq <- sapply(year, function(x, y) sum(y==x), y=when)
str(freq)

## int [1:112] 4 5 4 1 0 4 3 4 0 6 ...

n <- length(freq)
plot(year, freq)
```



## Implement Gibbs Sampler

```
GibbsS <- function(la1, la2, M, nreps, freq){  
  # iteration  
  # (yi  
  # yi-1) data  
  # priors; uniform for M
```

```

a1 <- .5
b1 <- .001
a2 <- .5
b2 <- .001

Gsamples <- matrix(nrow=nreps, ncol=3)
Gsamples[1,] <- c(la1, la2, M)

# main loop
cfreq <- cumsum(freq)
for (i in 2:nreps) {
  la1 <- rgamma(1, a1 + cfreq[M], b1 + M)
  la2 <- rgamma(1, a2 + cfreq[n] - cfreq[M], b2 + n - M)
  M <- sample(1:n, 1, prob=(la1/la2)^cfreq*exp((la2-la1)*(1:n)))
  Gsamples[i,] <- c(la1, la2, M)
}

return(Gsamples=Gsamples)
}

```

Handwritten notes:

- $\lambda_1 \sim \Gamma(0.5, 0.001)$
- $\lambda_2 \sim \Gamma(0.5, 0.001)$
- $freq = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_{1:n} \end{bmatrix} \rightarrow cfreq = \begin{bmatrix} y_1 \\ y_1+y_2 \\ y_1+y_2+y_3 \\ \vdots \\ y_1+\dots+y_{1:n} \end{bmatrix}$
- $\sum_{i=1}^M y_i$

## Simulate two Markov Chains (of length 500) using Gibbs Sampler

```

# sample size
nreps <- 500

# initial values
la1 <- 1
la2 <- 1
M <- 100

GibbsS1 = GibbsS(la1, la2, M, nreps, freq)

# initial values
la1 <- 3
la2 <- 3
M <- 50

GibbsS2 = GibbsS(la1, la2, M, nreps, freq)

```

## trace plot

```

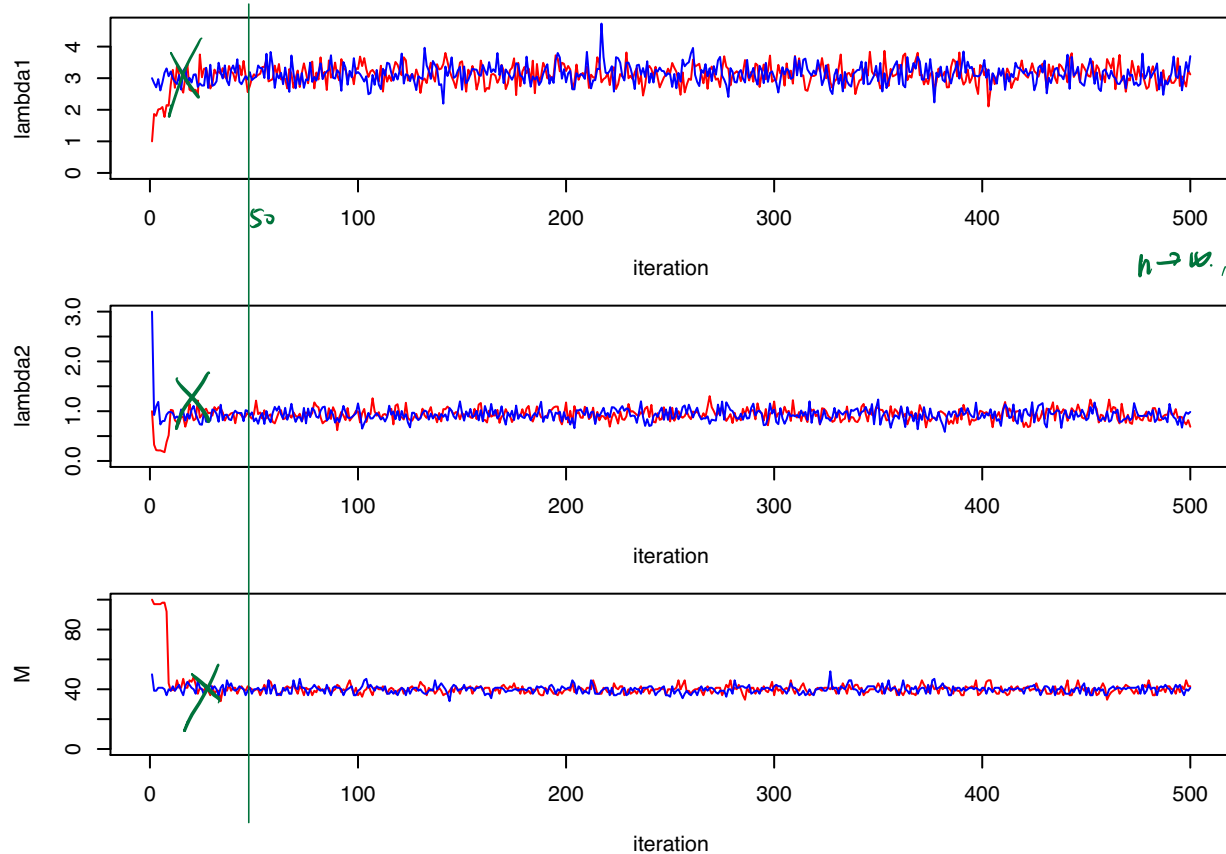
par(mfrow=c(3,1), mar=c(4,4,1,1))
plot(1:nreps, GibbsS1[,1], type="l", col="red", ylim = c(0, max(GibbsS1[,1],GibbsS2[,1])),
     xlab = "iteration", ylab = "lambda1")
points(1:nreps, GibbsS2[,1], type="l", col="blue")

plot(1:nreps, GibbsS1[,2], type="l", col="red", ylim = c(0, max(GibbsS1[,2],GibbsS2[,2])),
     xlab = "iteration", ylab = "lambda2")
points(1:nreps, GibbsS2[,2], type="l", col="blue")

plot(1:nreps, GibbsS1[,3], type="l", col="red", ylim = c(0, max(GibbsS1[,3],GibbsS2[,3])),

```

```
xlab = "iteration", ylab = "M")
points(1:nreps, GibbsS2[,3], type="l", col="blue")
```

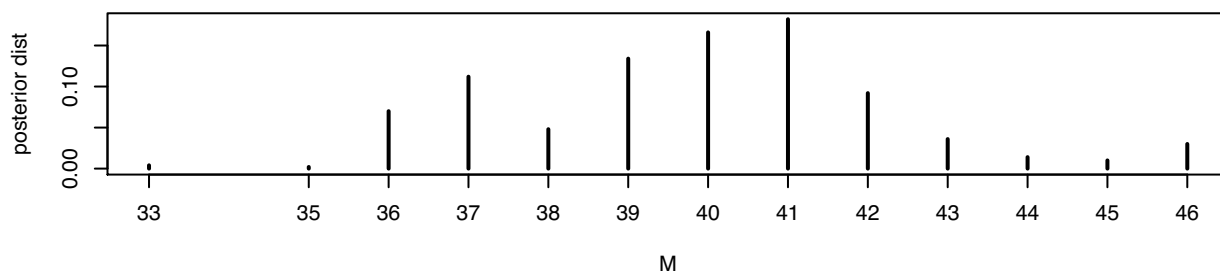
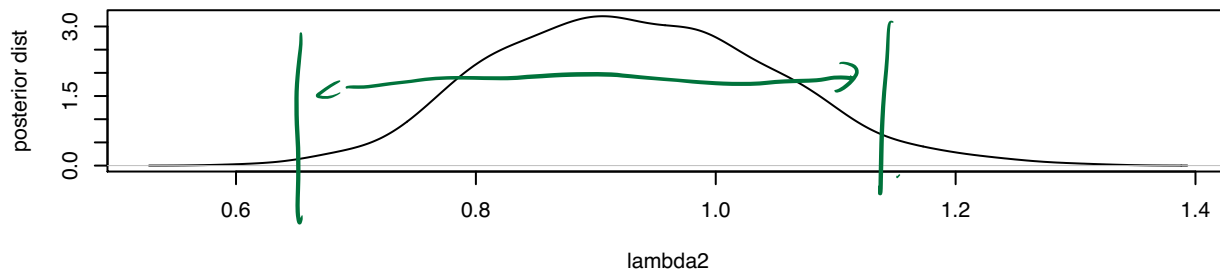
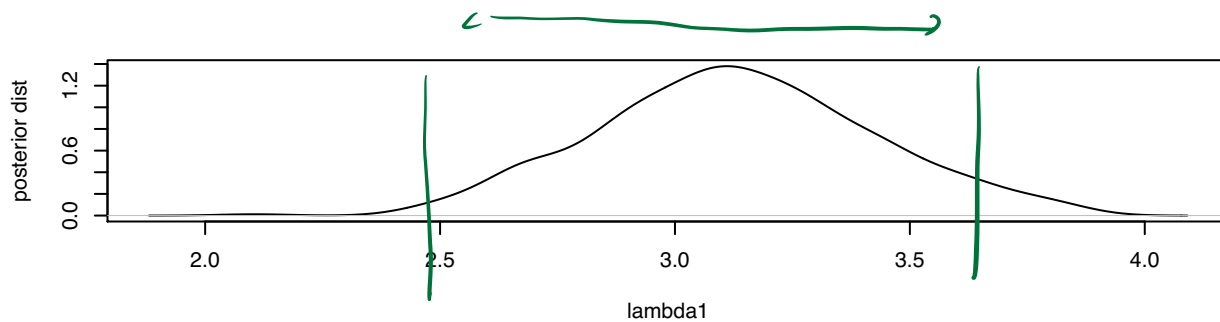


Posterior distribution after discarding the first 50 samples as a burn-in.

```
par(mfrow=c(3,1), mar=c(4,4,1,1))
plot(density(GibbsS1[-(1:50),1]), ylab="posterior dist", xlab="lambda1", main="")
plot(density(GibbsS1[-(1:50),2]), ylab="posterior dist", xlab="lambda2", main="")
plot(table(GibbsS1[-(1:50),3])/nreps, ylab="posterior dist", xlab="M", main="")
```

$\lambda_1 \neq \lambda_2$

95% credible interval for  $\lambda_1$



changes happen around 39, 40, 41.