Multi-Factor Models

The plan:

- We've seen index models with multiple sources of covariance risk:

$$r_{i,t} - r_{f,t} = \alpha_i + \sum_{k=1}^{K} \beta_{i,k}\left(r_{k,t} - r_{f,t}\right) + e_{i,t}$$

- Multifactor asset pricing is similar:

$$E[\tilde{r}_i] - r_f = \sum_{k=1}^{K} \beta_{i,k} E\left[\tilde{f}_k\right]$$

- $\tilde{f}_k$ represent factors, possibly risk premia, but not necessarily.

inflation ...

could be GDP growth, labor income growth

The plan:

$$E[\tilde{r}_i] - r_f = \sum_{k=1}^{K} \beta_{i,k} E[\tilde{f}_k]$$

- To come up with factors ($\tilde{f}_k$) we can use:
  - Theory: ICAPM
    - Intertemporal CAPM
  - Statistics: APT
    - Arbitrage Pricing Theory
  - Data/Empirics:
    - The Fama-French 3-Factor model and many, many others.

- At their core in each of these models, factors represent risks that investors cannot diversify away.

Merton’s (1973) ICAPM

Merton's (1973) ICAPM

$$E[\tilde{r}_i] - r_f = \beta_{i,W}\left(E[\tilde{r}_W] - r_f\right) + \sum_{k=1}^{K} \beta_{i,k}\left(E[\tilde{f}_k]\right)$$

- Multi-period version of CAPM in which factors are "state variables" that determine how well the investor can do his/her optimization.

- Merton's key insight is that in addition to wealth affecting consumption, investors will try to hedge the risk of downturns.

when wealth are lower and investment opportunities are fewer

Merton's (1973) ICAPM

$$E[\tilde{r}_i] - r_f = \beta_{i,W}\left(E[\tilde{r}_W] - r_f\right) + \sum_{k=1}^{K} \beta_{i,k}\left(E[\tilde{f}_k]\right)$$

- A factor that can be anything that reflects
- current wealth
  - Labor market income, Housing value, Small business
- whether there are good investments available or not
  - Intuition: Investors with long horizons are unhappy with news that future investment opportunities are worse, and returns are lower
    - Investors place a high value to assets which are negatively correlated with long term wealth. That is, they prefer stocks with high payouts during recessions

One Proposed Version of Merton's ICAPM model

$$E[\tilde{r}_i] - r_f = \beta_{i,M}\left(E[\tilde{r}_M] - r_f\right) + \beta_{i,TB}\left(E[\tilde{r}_{TB}] - r_f\right)$$

- Where $E[\tilde{r}_{TB}]$ is the expected return on long-term Treasury bonds.
  - The intuition is that interest rates negatively affect both the cost of a firm's investments and the value of those investments.

make it more difficult to raise capital

- This is not the only possible ICAPM factor (*see previous slide*)
  - Changes in industrial production
  - (Un)expected inflation
  - Labor income growth
  - Many more possibilities. To date, more than 300 different factors have been proposed.

General note about factors

factors: risks cannot be diversified away across assets

- The state variable(s) (factors) should affect the average investor.
  - Consider a risk that in the future makes A better off and B worse off
    - B sells the risk
    - A buys it
  - Net effect is zero.

- This helps explains why LOTS of variables are correlated with returns, but do not carry any priced risk
  - For example: Industry returns comove, but not once you control for priced risks.

Arbitrage Pricing Theory (APT)

Ross (1976)

APT

- CAPM and ICAPM says start with theory when looking for factors
  - Wealth and variables that proxy for wealth are factors.
  - Variables that affect the distribution of returns are factors

- APT does not suggest factors. It says start statistical
  - Find comovement in stock returns that cannot be diversified away

APT: Approaches to Possible Factors

- Statistical: analyses of covariance of returns to uncover factors
  - We will leave this to your statistics subjects
- Fundamentals: firm characteristics that reflect exposure to systematic risks
  - Accounting ratios (market to book, price to earnings, size, …)
  - Liquidity
  - Leverage
- Macroeconomic
  - Market portfolio, growth in industrial production
  - Inflation, term premium (10-year minus 1-year govn't bond yields).
  - Default premium

Not a mistake!!

Though APT suggests a purely statistical approach to uncovering undiversifiable comovement, many studies of APT start with possible macroeconomic risks or fundamental characteristics. *For example, Chen, Roll, and Ross (1986).*

APT – Basic Intuition

- Using the Law of One Price

same timing of cash flows

- Two portfolios with the same cash flows should have the same price.

- If we think about this is a dynamic setting then: Common comovements of stock returns should have the same price
  - Complete idiosyncratic price movements are not priced
  - If well diversified, only common factors affect consumption

Arbitrage is the power of APT

- No restrictive assumptions about returns or preferences
  - Just need one or several people who can arbitrage mispricing
    - And prefer more money to less money

- If 2 portfolios of securities have the same return comovement with fundamental risks, but different prices.
  - Arbitrageurs will short sell the expensive one and buy the cheap one.
    - Eventually, the price pressure will cause the cheap one to rise in price and the expensive one to drop in price.

Arbitrage Pricing Theory (APT)

- Allows multiple sources of risk:

$$E[\tilde{r}_i] - r_f = \sum_{k=1}^{K} \beta_{i,k}\left(E[\tilde{r}_k] - r_f\right)$$

- Portfolios of stocks are priced the way they are because, you can make portfolios of other stocks that are identical and arbitrage any differences in price

- **Key to APT:** well diversified portfolios that require no wealth should earn no return

arbitrage portfolio: short the expensive
invent in the cheap one
no "your" wealth

Arbitrage Pricing Theory (APT)

- If risks are important (undiversifiable) they are priced or "carry a risk premium" a priced risk is the one carries risk premium
  - If not, they are unpriced or have a "zero risk premium"

    are diversifiable

- Replication of Risk is the key to correct pricing:
  - Any 2 assets/portfolios with the same exposure to priced risk must get the same reward – if they don't then there will be Arbitrage
  - If we know the rewards on the assets associated with priced risk(s)
  - then we know the reward for all assets
  - because we can replicate their risk exposure.

APT and diversification

- APT applies directly to well-diversified portfolios. Why?
  - For arbitrage to work, there must be no firm-specific risk.
  - Without firm specific risk what is (if single factor):

$$r_{i,t} - r_{f,t} = \alpha_i + \beta_{i,k}\left(r_{M,t} - r_{f,t}\right) + e_{i,t}$$

becomes

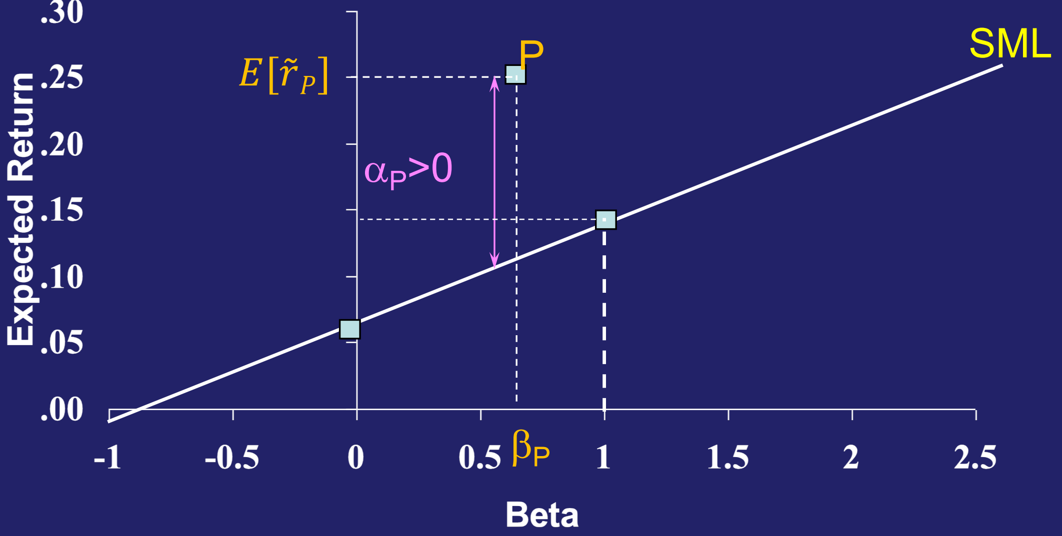
$$r_{P,t} - r_{f,t} = \alpha_P + \beta_{P,k}\left(r_{M,t} - r_{f,t}\right)$$

How to Arbitrage

How to Arbitrage with well diversified portfolios

APT market with one factor (let it be market)

$$r_{P,t} - r_{f,t} = \alpha_P + \beta_{P,k}\left(r_{M,t} - r_{f,t}\right)$$



$$\alpha_P = \left(r_{P,t} - r_{f,t}\right) - \beta_P\left(r_{M,t} - r_{f,t}\right)$$

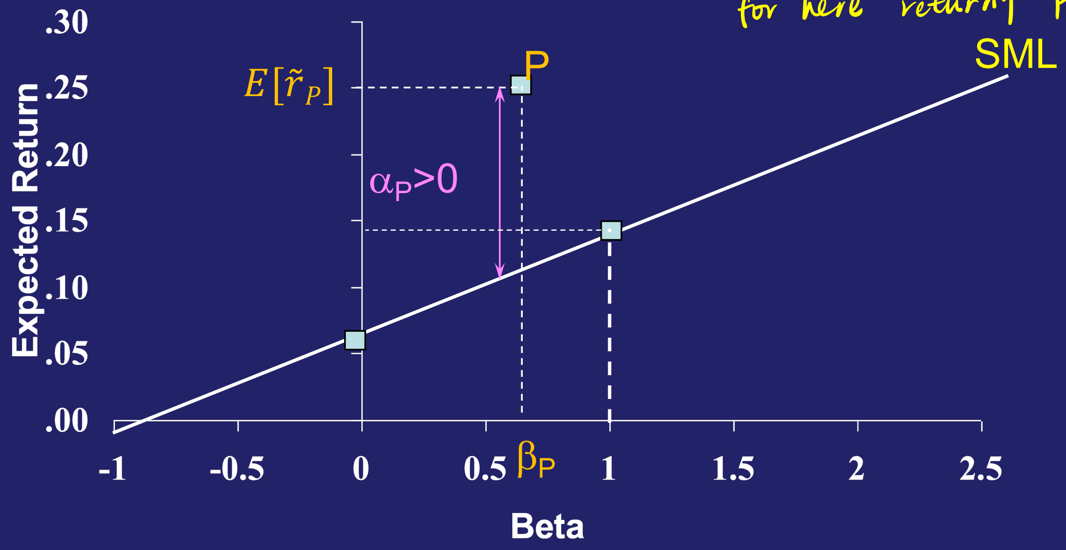
Is portfolio P underpriced or overpriced?

for bond i↑ P↓. i↓ P↑

for here return↑ P↓. → underpriced

↓

long



Intuition: Arbitrage with well diversified portfolios

- Create a long-short portfolio with a zero beta with the mispriced portfolio and other well diversified portfolio(s).
  - Strictly speaking, it doesn't matter which is correct and which is mispriced.

- What's the return on a well diversified zero-beta portfolio?
  - The risk-free rate

- Depending on whether your Zero-beta portfolio is cheap or expensive, buy it with money borrowed at the risk-free rate or short-sell it and invest at the risk-free rate.
  - An example will be clearer

key: portfolio with no risk and cost you no money

- NOTE: this example is only one way to implement this intuition.

How can we take advantage of the mispricing in the previous graph?

Arbitrage Recipe 1: no formulas

1. Choose well diversified portfolios with which you will create your arbitrage strategy.
   - Hint: if the factors themselves are portfolios, you could use those.
2. For each factor calculate weights for each of the portfolios. These weights will be the solution to a system of equations.
   - For each factor calculate weights to create a portfolio with a beta of zero toward each factor.
   - Make sure all the weights sum to zero including the risk-free
3. Pick an asset to set the weight to 1. Calculate the portfolio returns with these weights. This is your return per $1 invested.
   - If the returns are negative, flip the signs on all the weights

Arbitrage: Method 1 – Choose well diversified portfolio

Again suppose:

$\beta_P = .6,$ $\quad r_f = .06$ $\quad E[\tilde{r}_M] = 0.14$

actual $E[\tilde{r}_P] = 0.25$ $\quad$ APT $E[\tilde{r}_P] = 0.108$

Implying α= .142

- P is one of our portfolios and
- let's use the market as the other portfolio (*the market is well diversified*)

Arbitrage Method 1: Step 2 – Calculate Weights

- Calculate weights to create a portfolio with a beta of zero toward each factor (i.e. no risk)
  - Recall:

$$\beta_{portfolio} = w_P\beta_P + w_M\beta_M + w_F\beta_F = 0$$
$$0.6w_P + 1w_M + 0w_F = 0$$
$$w_M = -0.6w_P$$

  - *If you had multiple factors you would repeat this step for each factor.*

- Create a portfolio that requires zero wealth by making sure all the weights sum to zero:

$$w_P + w_M + w_F = 0$$
$$w_P - 0.6w_P + w_F = 0$$
$$w_F = -0.4w_P$$

Step 3: Calculate the portfolio returns with the weights

- $E\left[\tilde{r}_{Zero-Wealth\ Arbitrage\ Portfolio}\right] = w_P E[\tilde{r}_P] + w_M E[\tilde{r}_M] + w_F r_F$

- I'll arbitrarily pick $1 long in portfolio P.
  - Note: the book in its formula-based method picks $1 short in the risk free.

- $E\left[\tilde{r}_{Zero-Wealth\ Arbitrage\ Portfolio}\right] = 1\times25\% - 0.6\times14\% - 0.4\times6\%$

- $E\left[\tilde{r}_{Zero-Wealth\ Arbitrage\ Portfolio}\right] = 25\% - 8.4\% - 2.4\% = 14.2\%$
  - Note the sign is positive, so the weights have the correct sign. If $E\left[\tilde{r}_{Zero-Wealth\ Arbitrage\ Portfolio}\right]$<0 then flip the sign on the weights.

What's the long-short arbitrage strategy?

- With weights of
- $w_P = 1 \qquad w_M = -0.6 \qquad w_F = -0.4$

A portfolio that

- shorts the market for $0.60 and
- borrows at the risk-free $0.40 and
- invests that $1 in portfolio P
- will earn $14.2\% \times \$1 = \$0.142$.

- This method generalizes to multiple factors.
  - You will see that on your homework.

Arbitrage Method: with the book's formulas

1. Our book gives us on page 168 the weights needed for a zero-beta portfolio: just work with single factor

$$w_v = \frac{-\beta_u}{\beta_v - \beta_u} \qquad w_u = \frac{\beta_v}{\beta_v - \beta_u}$$

$$w_M = \frac{-\beta_P}{\beta_M - \beta_P} \qquad w_P = \frac{\beta_M}{\beta_M - \beta_P}$$

$$w_M = \frac{-.6}{1 - .6} = -1.5 \qquad w_P = \frac{1}{1 - .6} = 2.5$$

Arbitrage: With Book's formulas

2. Calculate the return to a strategy that shorts the risk free $1

$$r_{ZB} - r_f = \alpha_v w_v + \alpha_u w_u$$
$$r_{ZB} - r_f = \alpha_M w_M + \alpha_P w_P$$

$$r_{ZB} - r_f = 0 \times (-1.5) + .142 \times 2.5 = .355$$

- With r_f =.06 this zero-beta portfolio is a lot less expensive.
- The strategy is:
  - Short the risk free for $1 $(w_F = -1)$
  - Short M for $1.50 $(w_M = -1.5)$
  - Go long P for $2.50 $(w_P = +2.5)$
  - This zero cost/wealth portfolio generates a 35.5% return per $1 short
  - *NOTE if $r_{ZB} - r_f < 0$ flip the signs (i.e $w_M = 1.5$, $w_P = -2.5$ and $w_F = 1$)*

Core idea: CAPM, ICAPM and APT

- There are risks that you cannot get rid of.

- You must get compensation to be willing to take those risks.

- The difference between the models
  – the motivation for the risks
  – mechanism that makes stock returns reflect those risks

ICAPM vs. APT: mechanism

- CAPM and ICAPM:
  - Stocks are priced because everyone
    - believes the same thing
    - Perceives the same risks

    homogeneous expectations
  - Applies to all assets, even individual securities

- APT:
  - Stocks (really portfolios) are priced because arbitrageurs exploit mispricing for profit.
  - The act of trying to profit from mispricing corrects prices.
  - Applies exactly to well diversified portfolios and only approximately to individual assets.

Which is better?

- APT
  - has more realistic assumptions
  - BUT we don’t know what the factors are
    - Data mining?
  - Works only for well diversified portfolios

- CAPM
  - Simple to use and the model is more stable (caveat: β’s do drift)
  - Fama and French (1992, 1993) show that a model using book-to-market and size factors is better

Empirically Driven Factors

Empirically Driven Factors

$$r_{i,t} - r_{f,t} = \alpha_i + \sum_{k=1}^{K} \beta_{i,k}\left(r_{k,t} - r_{f,t}\right) + e_{i,t}$$

- The core idea behind the search for empirically driven factors:
  - Find assets (i) (usually portfolios) which have returns that are not well explained by existing models. That is $\alpha_i \neq 0$.

    part isn't explained by factors
  - Hunt for a potential covariate/factor that explains away the alpha so that with the new factor $\alpha_i = 0$.

Popular Model:
Fama and French (1993) Model

ICAPM or APT?

Data Driven "Risk" Factors

- Research has identified portfolios of stock that appear to get extra high returns:
  - Small firms out perform large firms
    - at least they did 1926 through 1980
  - Value stocks out perform Glamour/Growth stocks

- Fama and French developed a three factor model that includes:
  - Market
  - Size factor (SMB) → designed to capture "small firms out perform large firms"
  - Value/Growth factor (HML) → designed to capture "value stock out perform growth stocks"

Related findings- Fama French 1992 - Size and Beta

No real association between β & return

	All	Low-β	β-2	β-3	β-4	β-5	β-6	β-7	β-8	β-9	High-β
Panel A: Average Monthly Returns (in Percent)											
All	1.25	1.34	1.29	1.36	1.31	1.33	1.28	1.24	1.21	1.25	1.14
Small-ME	1.52	1.71	1.57	1.79	1.61	1.50	1.50	1.37	1.63	1.50	1.42
ME-2	1.29	1.25	1.42	1.36	1.39	1.65	1.61	1.37	1.31	1.34	1.11
ME-3	1.24	1.12	1.31	1.17	1.70	1.29	1.10	1.31	1.36	1.26	0.76
ME-4	1.25	1.27	1.13	1.54	1.06	1.34	1.06	1.41	1.17	1.35	0.98
ME-5	1.29	1.34	1.42	1.39	1.48	1.42	1.18	1.13	1.27	1.18	1.08
ME-6	1.17	1.08	1.53	1.27	1.15	1.20	1.21	1.18	1.04	1.07	1.02
ME-7	1.07	0.95	1.21	1.26	1.09	1.18	1.11	1.24	0.62	1.32	0.76
ME-8	1.10	1.09	1.05	1.37	1.20	1.27	0.98	1.18	1.02	1.01	0.94
ME-9	0.95	0.98	0.88	1.02	1.14	1.07	1.23	0.94	0.82	0.88	0.59
Large-ME	0.89	1.01	0.93	1.10	0.94	0.93	0.89	1.03	0.71	0.74	0.56

Size and Book to Market

growth stock

value stock

	Book-to-Market Portfolios										
	All	Low	2	3	4	5	6	7	8	9	High
All	1.23	0.64	0.98	1.06	1.17	1.24	1.26	1.39	1.40	1.50	1.63
Small-ME	1.47	0.70	1.14	1.20	1.43	1.56	1.51	1.70	1.71	1.82	1.92
ME-2	1.22	0.43	1.05	0.96	1.19	1.33	1.19	1.58	1.28	1.43	1.79
ME-3	1.22	0.56	0.88	1.23	0.95	1.36	1.30	1.30	1.40	1.54	1.60
ME-4	1.19	0.39	0.72	1.06	1.36	1.13	1.21	1.34	1.59	1.51	1.47
ME-5	1.24	0.88	0.65	1.08	1.47	1.13	1.43	1.44	1.26	1.52	1.49
ME-6	1.15	0.70	0.98	1.14	1.23	0.94	1.27	1.19	1.19	1.24	1.50
ME-7	1.07	0.95	1.00	0.99	0.83	0.99	1.13	0.99	1.16	1.10	1.47
ME-8	1.08	0.66	1.13	0.91	0.95	0.99	1.01	1.15	1.05	1.29	1.55
ME-9	0.95	0.44	0.89	0.92	1.00	1.05	0.93	0.82	1.11	1.04	1.22
Large-ME	0.89	0.93	0.88	0.84	0.71	0.79	0.83	0.81	0.96	0.97	1.18

Fama-French Three Factor Model

$$E[\tilde{r}_i] - r_f = \beta_{M,i}\left(E[\tilde{r}_M] - r_f\right) + \beta_{SMB,i}E\left[\widetilde{SMB}\right] + \beta_{HML,i}E\left[\widetilde{HML}\right]$$

- Market Risk Premium

- SMB is a portfolio long Small stocks and short large or Big stocks
  - SMB for Small Minus Big

- HML is a portfolio long Value stocks and short Growth stocks
  - (value stocks are called "High book to market" stocks, by finance professor types and Growth stocks are called "Low book to market" stocks.
  - HML is High Minus Low.

Example with the Fama-French 3 Factor Model

	O	P	Q	R
1	Google Premium (r -rf)	Rm-rf	SMB	HML
3	0.264895897	0.0195	0.0282	0.004
4	0.469887654	0.0167	0.0049	-0.0095
5	-0.046925934	0.0467	0.0411	0.0196
6	0.057802132	0.0336	0.0017	-0.0035
7	0.013079185	-0.0282	-0.0167	0.0252
8	-0.040604192	0.0211	-0.0076	0.0285
9	-0.04188935	-0.019	-0.0137	0.0171
10	0.216669043	-0.0273	-0.0395	-0.0049
11	0.257918182	0.0355	0.0301	-0.0116
12	0.058579287	0.0092	0.0258	0.0284
13	-0.02412361	0.0409	0.0277	-0.0047
14	-0.009116208	-0.0089	-0.0088	0.0144
15	0.103603497	0.0077	-0.0065	0.0123
16	0.173246407	-0.0235	-0.0104	-0.0077
17	0.084958258	0.0373	0.0098	-0.0175
18	0.021373362	0.0003	-0.0047	0.0051
19	0.039406041	0.0366	0.0533	0.0118
20	-0.165282309	-0.005	-0.0031	-0.0076
21	0.071806039	0.0154	0.0351	0.0004
22	0.068041026	0.0094	-0.0122	0.0307

SUMMARY OUTPUT

Regression Statistics	
Multiple F	0.530005
R Square	0.280905
Adjusted I	0.230736
Standard E	0.114687
Observati	47

	Coefficients	*andard Err*	*t Stat*	*P-value*
Intercept	0.037173	0.017281	2.151017	0.037137
Market	2.031364	0.632598	3.211145	0.002504
SMB	-0.61086	0.880174	-0.69402	0.491404 → not significant
HML	-1.92444	1.039474	-1.85136	0.070994

loading negative ⇒ google is growth oriented

Factor data are from: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

Partial Explanations for HML and SMB

- Petkova and Zhang (2005)
  - CAPM Betas change over time
  - HML captures changes in the CAPM market beta over time
    - Value Stocks have CAPM betas that covary positively with the market risk premium
    - Growth stocks have CAPM betas that covary negatively with the market risk premium

- Campbell and Vuolteenaho (2010)
  - Beta can be decomposed into a Discount Rate Beta and a Cash Flow Beta: a Good Beta and a Bad Beta
    - Good betas have a low price of risk and bad betas have a high price of risk
    - Value Stocks have mostly bad betas
    - Growth stocks have mostly good betas

Other explanations for HML and SMB

- Leverage
- Distress
- Macroeconomic risk
  - GDP
  - Inflation
- Behavioral Biases

The Carhart Four-Factor Model

The Carhart Model

- In a 1997 *Journal of Finance* article, Mark Carhart noted that many prior researchers had found strong evidence that managed fund managers earned persistently high returns.

$$E[\tilde{r}_i] - r_f = \alpha_i + \beta_{M,i}\left(E[\tilde{r}_M] - r_f\right) + \beta_{SMB,i}SMB + \beta_{HML,i}HML$$

  - α is large and statistically significant over short and long periods.

① survivalship bias

② momentum

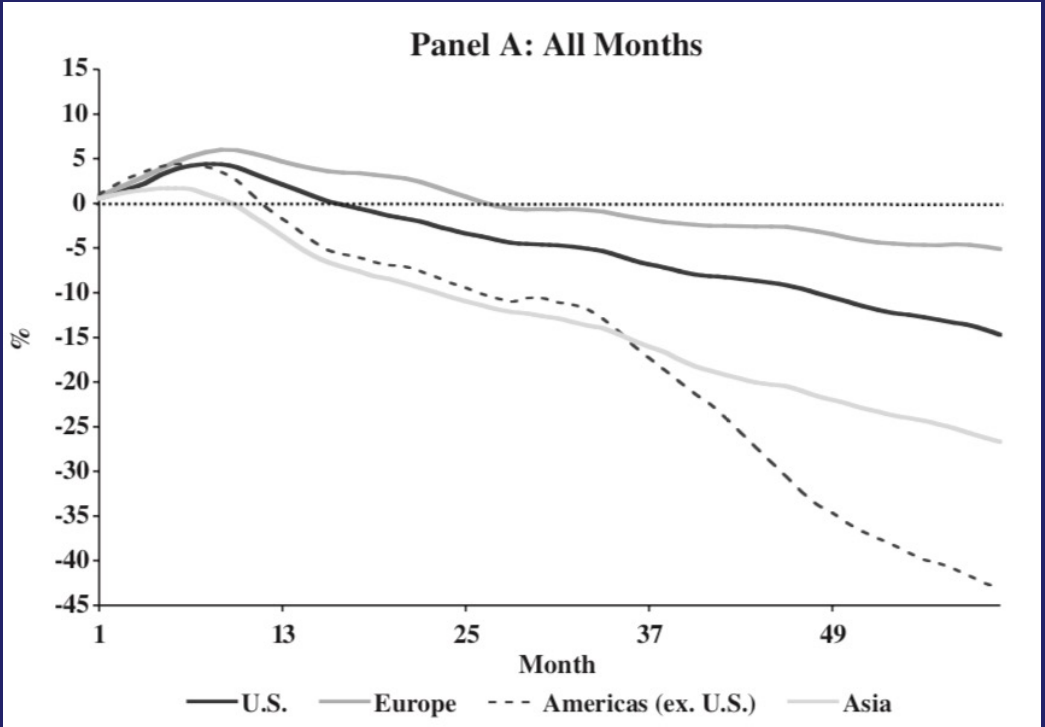
Observation: Momentum

- Stocks that do well over the past 6 to 12 months,
  – continue to do well for another 6 to 12 months

- Stocks that do poorly over the past 6 to 12 months,
  – continue to do poorly for another 6 to 12 months

- Picture on next slide…

Momentum: Cumulative performance



Griffin, Ji, Martin (2004)

Carhart's observations

- He also noted:
  - Survivorship bias: out performance due in part because poorly performing managed funds (with low α are dropped from common datasets)
  - Funds that earn higher one-year returns seem to do so because they ***coincidentally*** hold relatively larger positions in last year's winners – NOT because they were actively pursuing a momentum strategy.
    - Those that do actively pursue momentum strategies tend to underperform – mostly due to trading costs of those strategies.
- He proposed a "factor" to control for the momentum returns
  - Long past 11 month (skip-a-month) winners
    - equally weighted returns of the top 30%
  - Short past 11 month losers
    - equally weighted returns of the bottom 30%

The Carhart Model

- Once the new momentum factor is included:

$$E[\tilde{r}_i] - r_f = \alpha_i + \beta_{M,i}\left(E[\tilde{r}_M] - r_f\right) + \beta_{SMB,i}SMB + \beta_{HML,i}HML + \beta_{WML,i}WML$$

- Positive abnormal performance of managed funds (the α's) becomes largely insignificant.

Carhart - Evidence

Portfolios of Managed Funds Formed on Lagged 1-Year Return

Portfolio	Monthly Excess Return	Std Dev	CAPM Alpha	CAPM VWRF	CAPM Adj R-sq	4-Factor Model Alpha	RMRF	SMB	HML	PR1YR	Adj R-Sq
1 (high)	0.68%	5.04%	0.22%	1.03	0.834	−0.12%	0.88	0.62	−0.05	0.29	0.933
			(2.10)	(43.11)		(−1.60)	(50.54)	(23.67)	(−1.86)	(13.88)	
2	0.59%	4.72%	0.14%	1.01	0.897	−0.10%	0.89	0.46	−0.05	0.20	0.955
			(1.75)	(57.00)		(−1.78)	(66.47)	(22.95)	(−2.25)	(12.43)	
9	0.23%	4.60%	−0.21%	1.00	0.926	−0.20%	0.93	0.22	−0.10	−0.02	0.938
			(−3.24)	(67.91)		(−3.11)	(60.44)	(9.69)	(−3.80)	(−1.17)	
10 (low)	0.01%	4.90%	−0.45%	1.02	0.851	−0.40%	0.93	0.32	−0.08	−0.09	0.887
			(−4.58)	(46.09)		(−4.33)	(42.23)	(9.69)	(−2.23)	(−3.50)	
1-10 spread	0.67%	2.71%	0.67%	0.01	−0.002	0.29%	−0.05	0.30	0.03	0.38	0.231
			(4.68)	(0.39)		(2.13)	(−1.52)	(6.30)	(0.53)	(10.07)	

The Cutting Edge

Hou, Karolyi and Kho (2011)

- More recent evidence that *Cash Flow to Price* captures risk around the world.
  - And outperforms the Fama-French 3 Factor model

- Other findings include that everywhere local factors are more important – suggesting that markets are much less integrated than the world looks.

- We still don't know if or what macroeconomic risks these characteristic factors are associated with.

The Fama and French 5-Factor Model

The bleeding edge

Why is the Fama-French 3-Factor Model successful?

- Particularly, HML: the effects of SMB may have gone away, but HML seems to persist.
  - And we do not know why.

- Central to HML is the calculation of Book To Market or Market to Book:

$$\frac{Market\ Value}{Book\ Value} = \frac{(price \times shares\ outstanding + Market\ Value\ of\ Debt)}{Book\ Value} = \frac{M}{B}$$

It is hard to accurately measure the Market Value of Debt, so the success of the model probably doesn't have much to do with the debt part.

What is Book to Market

- The value of the equity of a firm:

$$M_t = \sum_{\tau=1}^{\infty} \frac{E\left[\widetilde{D}_{t+\tau}\right]}{(1+E[\tilde{r}])^{\tau}}$$

D = Dividend for the Firm
Y = Earnings for the Firm
B = Book value of the Firm
$B_t - B_{t-1}$= Investment

- Note that using the clean surplus relation:

$$D_t = Y_t - (\overbrace{B_t - B_{t-1}}^{\text{reinvestment}})$$

All earnings not reinvested in the firm are paid out as dividends

Implications

- Substitute:

$$M_t = \sum_{\tau=1}^{\infty} \frac{E[\tilde{Y}_{t+\tau} - (B_{t+\tau} - B_{t+\tau-1})]}{(1 + E[\tilde{r}])^{\tau}}$$

$$\frac{M_t}{B_t} = \frac{\sum_{\tau=1}^{\infty} \frac{E[\tilde{Y}_{t+\tau} - (B_{t+\tau} - B_{t+\tau-1})]}{(1 + E[\tilde{r}])^{\tau}}}{B_t}$$

M/B is a noisy proxy for E[r] because it responds to both expectations about earnings and investment

- Fix Y and B:
  - Low M or low M/B implies high E[r]

Size

- Fix M and B:
  - High Y implies high E[r]

Profitability

- Fix M, Y and B_{t-1}:
  - High growth in book value (investments) implies low E[r]

Investment

The new 5 factor model

- Market
- SMB
- HML
- Profitability (RMW): Robust Profits minus Weak Profits
- Investment (CMA): Conservative investment minus Aggressive investment

$$E[\tilde{r}_i] - r_f = \beta_{M,i}\left(E[\tilde{r}_M] - r_f\right) + \beta_{SMB,i}SMB + \beta_{HML,i}HML$$
$$+\beta_{RMW,i}RMW + \beta_{CMA,i}CMA$$

Practitioner Asset Pricing Models

Proprietary Risk Models

- MSCI-Barra and MSCI’s RiskMetrics,
- JP Morgan’s Measurisk,
- BlackRock Inc.'s BlackRock Solutions unit,
- DST Systems Inc., Fimalac S.A.'s Algorithmics unit,
- Moody Corporation's KMV unit, and
- SunGard Data Systems Inc

Barra's "factors"

Classes of factors

- Market Variability
- Earnings Variability
- Low Valuation and "Unsuccess"
- Immaturity and Smallness
- Growth Orientation
- Financial Risk

BARRA's risk factors

- Market Variability
  - Historic beta
  - Historic sigma
  - Share turnover
  - Trading volume
  - Stock price
  - Historic alpha
  - Cumulative price range over one year

BARRA's risk factors

- Earnings variability
  - Variance of earnings
  - Variance of cash flow
  - Covariance of earnings and price

- Low Valuation and "Unsuccess"
  - EPS growth
  - Recent change in earnings
  - Relative strength (momentum)
  - Book to price
  - Dividend cuts
  - Return on equity

BARRA’s risk factors

- Immaturity and Smallness
  – Log of total assets
  – Log of market capitalization
  – Net plant/common equity
- Growth Orientation
  – Dividends to earnings
  – Dividend yield
  – Growth in total assets
  – Earnings to price over 5 years
- Financial Risk
  – Book value of leverage and Market value of leverage
  – Debt to assets
  – Cash flow to liabilities

A perspective on this mess of measures

- Some of these measures are capturing systematic risks
  - historic beta is the main candidate
  - Leverage
- Others largely firm-specific risks
  - variance of earnings
  - Variance of cash flows
- Others are characteristics that *might* be associated with systematic risks (also might not).
  - Book to price
  - Market capitalization
  - Most of the others

With all these possible models, why do we still teach CAPM?

- CAPM is not a bad approximation
  - As long as the firm or project is not extreme value or growth
- The theory makes sense and provides good intuition
  - Investors care about the risks that affect their wealth
- Alternatives (the Fama-French model) are not clearly better.
- Your potential boss learned it at university.
  - As we saw earlier, 90% of financial officers in corporations use it.