# MAST30001 Stochastic Modelling

## Assignment 1

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Please complete and sign the Plagiarism Declaration Form (available from the LMS or the department's webpage), which covers all work submitted in this subject. The declaration should be attached to the front of your first assignment.

**Don't forget** to staple your solutions and to print your name, student ID, and the subject name and code on the first page (not doing so will forfeit marks). The submission deadline is **Friday, 11 September, 2015 by 4pm** in the appropriate assignment box at the north end of Richard Berry Building.

There are 2 questions, both of which will be marked. No marks will be given for answers without clear and concise explanations. Clarity, neatness, and style count.

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1. A switch is either in the *on* or *off* position. At the discrete times $T_1$, $T_1 + T_2, \ldots$ the switch is toggled from the position it is in to the other position (so if the switch is *off* at time $T_1 - 1$, then it is *on* at time $T_1$), where $T_1, T_2, \ldots$ are i.i.d. positive integer-valued random variables. Let $X_n = 1$ if at time $n$ the switch is *on* and $X_n = 2$ if the switch is *off* at time $n$.

   (a) Assume that $T_1$ has the geometric distribution with parameter $0 < p < 1$; that is, for $k = 1, 2, \ldots$, $\mathbb{P}(T_1 = k) = (1-p)^{k-1}p$.

   i. Explain in a few sentences why $(X_n)_{n\geq 0}$ is a Markov chain.

   ii. Find the transition matrix of $(X_n)_{n\geq 0}$, analyse its state space and discuss its long run behaviour (including deriving long run probabilities where appropriate).

   (b) If now $\mathbb{P}(T_1 = 1) = 1 - \mathbb{P}(T_1 = 2) = q$, where $0 < q < 1$, explain why $(X_n)_{n\geq 0}$ is *not* a Markov chain.

2. A Markov chain $(X_n)_{n\geq 0}$ on $\{0, 1, 2, \ldots\}$ has transition probabilities for $i = 0, 1, 2, \ldots$,

$$p_{i,i+1} = 1 - p_{i,0} = \left(\frac{i+1}{i+2}\right)^{\alpha},$$

   where $\alpha > 0$. Note that this chain is irreducible.

   (a) For which values of $\alpha$ is the chain transient? Null recurrent? Positive recurrent?

   (b) Describe the long run behaviour of the chain (including deriving long run probabilities where appropriate).

   (c) If $T(i) = \min\{n \geq 1 : X_n = i\}$, find $E[T(i)|X_0 = i]$ for $i = 0, 1, \ldots$.

   (d) If $X_0 = 0$, what is the chance the chain reaches state 3 before it returns to state 0?