PHYC10003 Physics 1

Lecture 11: Linear Momentum

Centre of Mass, Newton's Laws for a system of particles

Last lecture

- Potential energy
- Mechanical energy
- Reading a potential energy curve
- Work done on a system by an external force
- Conservation of energy
- Power, average and instantaneous

9-1 Centre of mass

- The motion of rotating objects can be complicated (imagine flipping a baseball bat into the air)
- But there is a special point on the object for which the motion is simple
- The center of mass of the bat traces out a parabola, just as a tossed ball does
- All other points rotate around this point

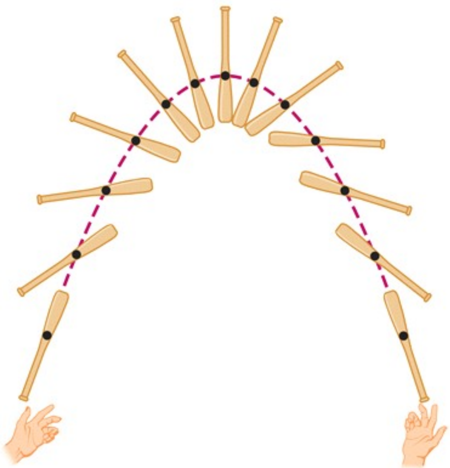


Figure 9-1

9-1 Centre of Mass

- The **centre of mass** (com) of a system of particles:

> The center of mass of a system of particles is the point that moves as though (1) all of the system's mass were concentrated there and (2) all external forces were applied there.

- For two particles separated by a distance *d*, where the origin is chosen at the position of particle 1:

$$x_{com} = \frac{m_2}{m_1 + m_2} d. \qquad \textbf{Eq. (9-1)}$$

- For two particles, for an arbitrary choice of origin:

$$x_{com} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}. \qquad \textbf{Eq. (9-2)}$$

9-1 Centre of mass

- The center of mass is in the same location regardless of the coordinate system used
- It is a property of the particles, not the coordinates

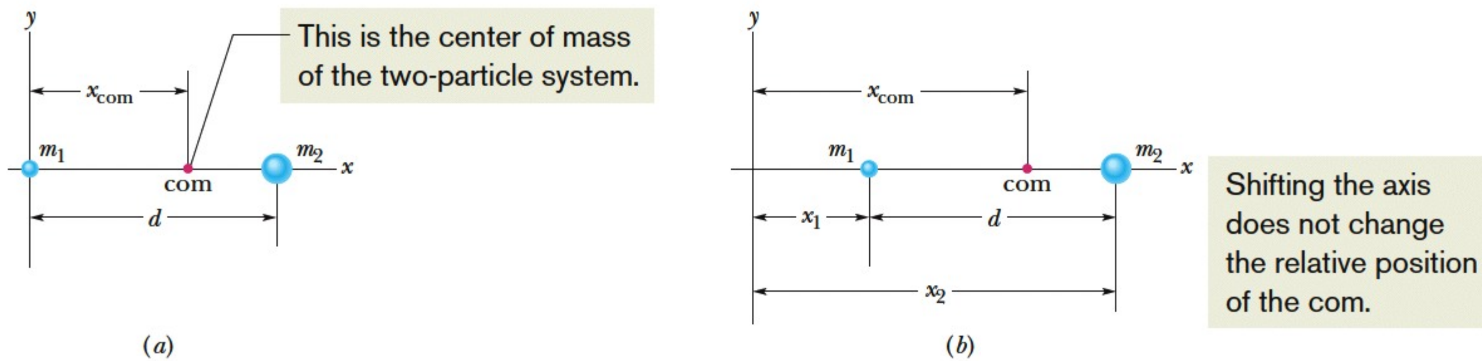


Figure 9-2

9-1 Centre of mass

- For many particles, we can generalize the equation, where $M = m_1 + m_2 + \ldots + m_n$:

$$x_{\text{com}} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + \cdots + m_nx_n}{M}$$

$$= \frac{1}{M}\sum_{i=1}^{n} m_i x_i.$$

Eq. (9-4)

- In three dimensions, we find the center of mass along each axis separately:

$$x_{\text{com}} = \frac{1}{M}\sum_{i=1}^{n} m_i x_i, \qquad y_{\text{com}} = \frac{1}{M}\sum_{i=1}^{n} m_i y_i, \qquad z_{\text{com}} = \frac{1}{M}\sum_{i=1}^{n} m_i z_i.$$

Eq. (9-5)

9-1 Centre of mass (vector)

- More concisely, we can write in terms of vectors:

$$\vec{r}_{com} = \frac{1}{M}\sum_{i=1}^{n} m_i \vec{r}_i, \qquad \textbf{Eq. (9-8)}$$

- For solid bodies, we take the limit of an infinite sum of infinitely small particles → integration!
- Coordinate-by-coordinate, we write:

$$x_{com} = \frac{1}{M}\int x\,dm, \qquad y_{com} = \frac{1}{M}\int y\,dm, \qquad z_{com} = \frac{1}{M}\int z\,dm,$$

Eq. (9-9)

- Here *M* is the mass of the object

9-1 Centre of mass- continuous bodies

- We limit ourselves to objects of uniform density, ρ, for the sake of simplicity

$$\rho = \frac{dm}{dV} = \frac{M}{V}, \qquad \textbf{Eq. (9-10)}$$

- Substituting, we find the center of mass simplifies:

$$x_{com} = \frac{1}{V}\int x\,dV, \qquad y_{com} = \frac{1}{V}\int y\,dV, \qquad z_{com} = \frac{1}{V}\int z\,dV.$$

Eq. (9-11)

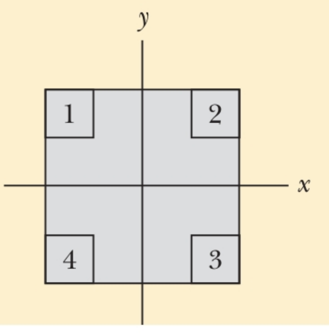
- You can bypass one or more of these integrals if the object has symmetry

9-1 Centre of mass-symmetry

- The center of mass lies at a point of symmetry (if there is one)
- It lies on the line or plane of symmetry (if there is one)
- It need not be on the object (consider a doughnut)

Checkpoint 1

The figure shows a uniform square plate from which four identical squares at the corners will be removed. (a) Where is the center of mass of the plate originally? Where is it after the removal of (b) square 1; (c) squares 1 and 2; (d) squares 1 and 3; (e) squares 1, 2, and 3; (f) all four squares? Answer in terms of quadrants, axes, or points (without calculation, of course).



Answer: (a) at the origin (b) in Q4, along y=-x (c) along the -y axis

(d) at the origin (e) in Q3, along y=x (f) at the origin

9-1 Centre of mass-example

Example Subtracting

- Task: find com of a disk with another disk taken out of it:
- Find the com of each individual disk (start from the bottom and work up)
- Find the com of the two individual coms (one for each disk), treating the cutout as having negative mass
- On the diagram, com_C is the center of mass for Plate *P* and Disk *S* combined
- com_P is the center of mass for the composite plate with Disk *S* removed

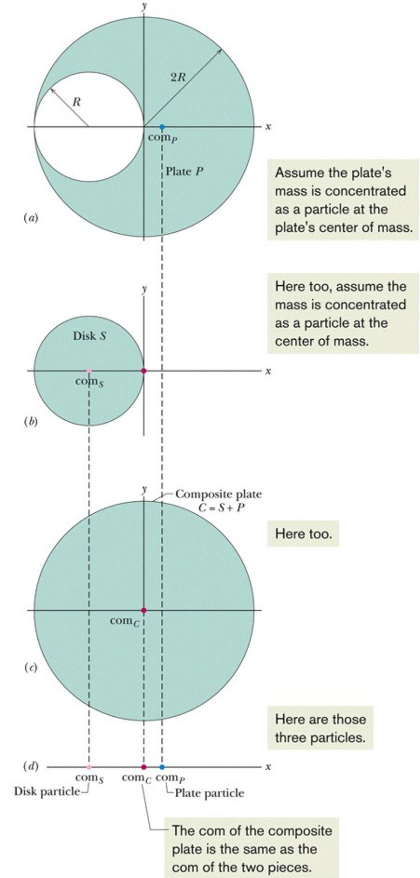


Figure 9-4

9-2 Centre of mass-motion

- Center of mass motion continues unaffected by forces internal to a system (collisions between billiard balls)
- Motion of a system's center of mass:

$$\vec{F}_{\text{net}} = M\vec{a}_{\text{com}} \qquad \text{(system of particles).}$$

Eq. (9-14)

$$F_{\text{net},x} = Ma_{\text{com},x} \qquad F_{\text{net},y} = Ma_{\text{com},y} \qquad F_{\text{net},z} = Ma_{\text{com},z}.$$

Eq. (9-15)

- Reminders:
  1. F_{net} is the sum of all *external* forces
  2. *M* is the total, constant, mass of the **closed** system
  3. a_{com} is the *center of mass* acceleration

9-2 Centre of mass-example

Examples Using the center of mass motion equation:

- Billiard collision: forces are only internal, $F = 0$ so $a = 0$
- Baseball bat: $a = g$, so com follows gravitational trajectory
- Exploding rocket: explosion forces are internal, so only the gravitational force acts on the system, and the com follows a gravitational trajectory

  as long as air resistance can

  be ignored for the fragments.

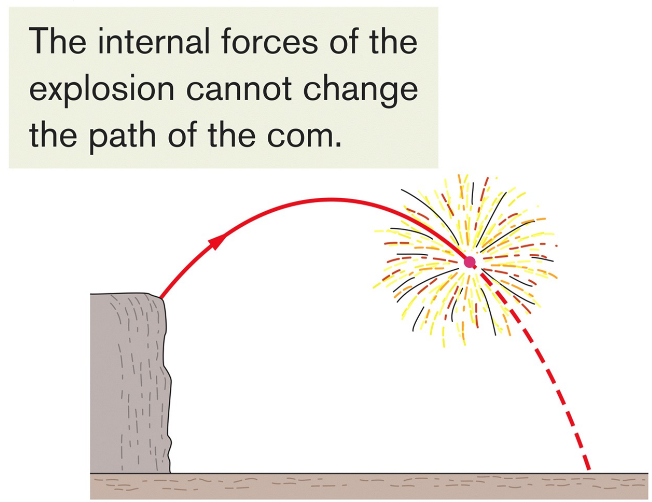
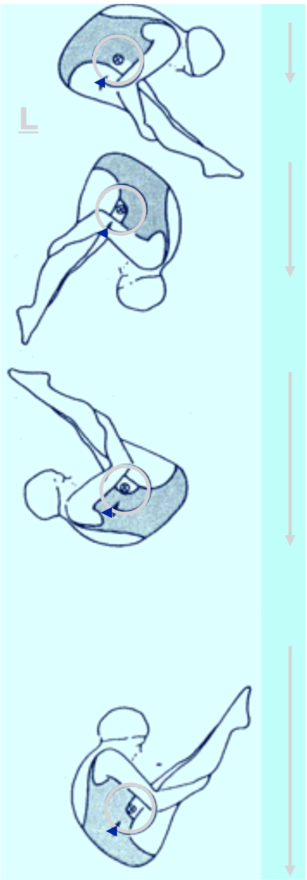
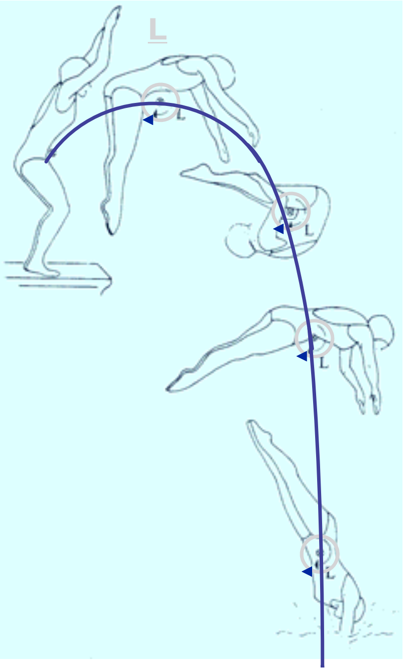


Figure 9-5

9-2 Motion of the CoM





9-3 Linear momentum

- The **linear momentum** is defined as:

$$\vec{p} = m\vec{v}$$

Eq. (9-22)

- The momentum:
  - Points in the same direction as the velocity
  - Can only be changed by a net external force

The time rate of change of the momentum of a particle is equal to the net force acting on the particle and is in the direction of that force.

- We can write Newton's second law thus:

$$\vec{F}_{\text{net}} = \frac{d\vec{p}}{dt}.$$

Eq. (9-23)

9-3 Linear momentum

The linear momentum of a system of particles is equal to the product of the total mass M of the system and the velocity of the center of mass.

- Taking the time derivative we can write Newton's second law for a system of particles as:

$$\vec{F}_{\text{net}} = \frac{d\vec{P}}{dt} \qquad \text{(system of particles)}, \qquad \textbf{Eq. (9-27)}$$

- The net external force on a system changes linear momentum
- Without a net external force, the *total* linear momentum of a system of particles cannot change

9-4 Collisions and impulse

- In a collision, momentum of a particle can change
- We define the **impulse *J*** acting during a collision:

$$\vec{J} = \int_{t_i}^{t_f} \vec{F}(t)\, dt$$ **Eq. (9-30)**

- This means that the applied impulse is equal to the change in momentum of the object during the collision:

$$\Delta\vec{p} = \vec{J} \qquad \text{(linear momentum–impulse theorem).}$$ **Eq. (9-31)**

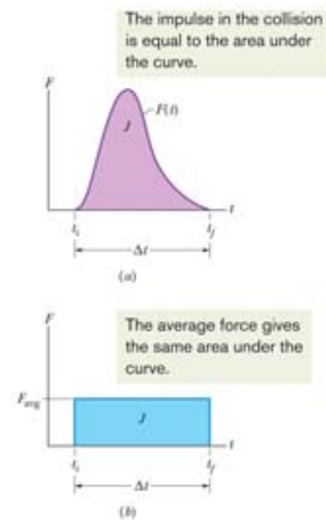
- This equation can be rewritten component-by-component, like other vector equations

9-4 Collisions and impulse

- Given F_{avg} and duration, $J = F_{avg}\,\Delta t$

Eq. (9-35)

- We are integrating: we only need to know the area under the force curve



Checkpoint 4

A paratrooper whose chute fails to open lands in snow; he is hurt slightly. Had he landed on bare ground, the stopping time would have been 10 times shorter and the collision lethal. Does the presence of the snow increase, decrease, or leave unchanged the values of (a) the paratrooper's change in momentum, (b) the impulse stopping the paratrooper, and (c) the force stopping the paratrooper?

Answer: (a) unchanged (b) unchanged (c) decreased

9-4 Impulse and projectiles

- For a steady stream of *n* projectiles, each undergoes a momentum change *Δp*

$$J = -n\,\Delta p,$$ **Eq. (9-36)**

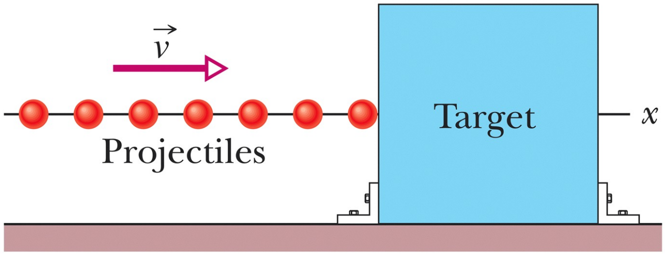


Figure 9-10

- The average force is:

$$F_{\text{avg}} = \frac{J}{\Delta t} = -\frac{n}{\Delta t}\Delta p = -\frac{n}{\Delta t} m\,\Delta v.$$ **Eq. (9-37)**

- If the particles stop:

$$\Delta v = v_f - v_i = 0 - v = -v,$$ **Eq. (9-38)**

- If the particles bounce back with equal speed:

$$\Delta v = v_f - v_i = -v - v = -2v.$$ **Eq. (9-39)**

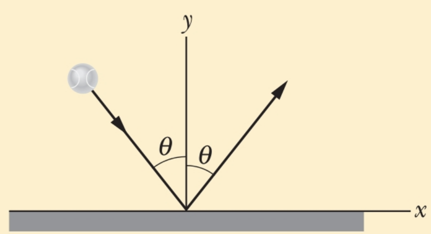
9-4 Collisions with a surface

- The product *nm* is the total mass for *n* collisions so we can write:

$$F_{\text{avg}} = -\frac{\Delta m}{\Delta t}\Delta v. \qquad \textbf{Eq. (9-40)}$$

Checkpoint 5

The figure shows an overhead view of a ball bouncing from a vertical wall without any change in its speed. Consider the change $\Delta\vec{p}$ in the ball's linear momentum. (a) Is Δp_x positive, negative, or zero? (b) Is Δp_y positive, negative, or zero? (c) What is the direction of $\Delta\vec{p}$?



Answer: (a) zero (b) positive (c) along the positive *y*-axis (normal force)

9-5 Conservation of linear momentum

- For an impulse of zero we find:

$$\vec{P} = \text{constant} \quad \text{(closed, isolated system).}$$

Eq. (9-42)

- Which says that:

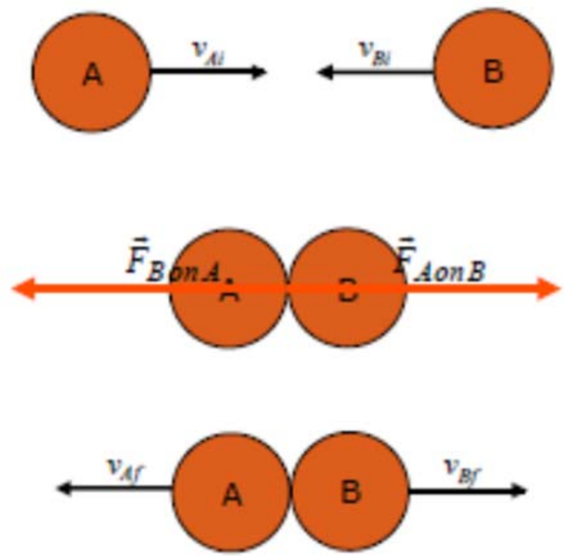
If no net external force acts on a system of particles, the total linear momentum $\vec{P}$ of the system cannot change.

- This is called the **law of conservation of linear momentum**
- Check the components of the net external force to know if you should apply this

If the component of the net *external* force on a closed system is zero along an axis, then the component of the linear momentum of the system along that axis cannot change.

9-5 Conservation of momentum

- Newton's 2nd law
  - Particle A
    $$\frac{(d\vec{p}_A)_x}{dt} = (\vec{F}_{\mathrm{B\ on\ A}})_x$$
  - Particle B
    $$\frac{(d\vec{p}_B)_x}{dt} = (\vec{F}_{\mathrm{A\ on\ B}})_x$$
- Newton's 3rd law
  $$(\vec{F}_{\mathrm{B\ on\ A}})_x = -(\vec{F}_{\mathrm{A\ on\ B}})_x$$
- Combining the two results
  $$\frac{d(\vec{p}_{Ax} + \vec{p}_{Bx})}{dt} = 0$$



Summary

Linear Momentum & Newton's 2nd Law

- Linear momentum defined as:

$$\vec{P} = M\vec{v}_{\text{com}}$$

Eq. (9-25)

- Write Newton's 2nd law:

$$\vec{F}_{\text{net}} = \frac{d\vec{P}}{dt}$$

Eq. (9-27)

Conservation of Linear Momentum

$$\vec{P} = \text{constant} \quad \text{(closed, isolated system).}$$

Eq. (9-42)

Collision and Impulse

- Defined as:

$$\vec{J} = \int_{t_i}^{t_f} \vec{F}(t)\, dt$$

Eq. (9-30)

- Impulse causes changes in linear momentum

Inelastic Collision in 1D

- Momentum conserved along that dimension

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}.$$

Eq. (9-51)

Preparation for the next lecture

1. Read 9-6 to 9-9 of the text
2. You will find short answers to the odd-numbered problems in each chapter at the back of the book and further resources on LMS. You should try a few of the simple odd numbered problems from each section (the simple questions have one or two dots next to the question number).