comp10002
Foundations of Algorithms

Semester Two, 2019

Strings and Algorithms

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Overview

Chapter 7 (Part II)

Pattern search

KMP pattern search

BMH pattern search

String index structures

AtN.

…56800

There is no pre-defined string type in C, and they are stored as null-terminated arrays of characters.

String operations are carried out using character pointers.

The libraries ctype.h and string.h contain useful functions, such as isalpha() and strcmp().

The function malloc() (Chapter 10) can be used to create space for new strings (and other arrays).

eg. 56780 56784 56788 56792 56796

int *p;

printf("A = %10p\n", A)

pointer the address.

56780

p = A;

since type p is int. p++, → add 4 bytes

for (p=A; p<N; p++){

}.

give character → 1/0

take two string, two pointers to array, compare char based on Ascii code.

return

null bytes =0.

char *p = "A/A ... "; // declare a pointer to char.

p

while (*p)

Chapter 7 – Program examples (Part II)

typedef char word_t[MAXCHAR+1]

type

null bytes.

- string1.c
- strcpy.c
- getword.c
- words.c
- progargs.c

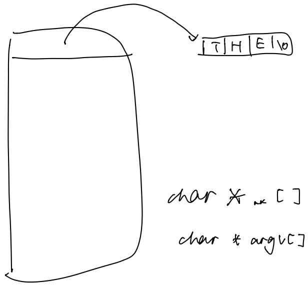
include "getword.c"

consume one word, put in one-word variable

getword (one_word, ____) != EOF

getword one by one

strcmp



argc

char * [ ]

char * argv[ ]

You may not modify either of the two strings in the first three exercises.

Exercise 1

Write a function `is_subsequence(char *s1, char *s2)` that returns `1` if the characters in `s1` appear within `s2` in the same order as they appear in `s1`. For example, `is_subsequence("bee", "abbreviate")` should be `1`, whereas `is_subsequence("bee", "acerbate")` should be `0`.

Chapter 7 – Exercises

Exercise 2

Ditto arguments, but determining whether every occurrence of a character in `s1` also appears in `s2`, and `0` otherwise. For example, `is_subset("bee", "rebel")` should be `1`, whereas `is_subset("bee", "brake")` should be `0`.

Exercise 3

Write a function `is_anagram(char *s1, char *s2)` that returns `1` if the two strings contain the same letters, possibly in a different order, and `0` otherwise, ignoring whitespace characters, and ignoring case. For example, `is_anagram("Algorithms", "Glamor Hits")` should return `1`.

Chapter 7 – Exercises

Exercise 4

Write a function `next_perm(char *s)` that rearranges the characters in a string argument and generates the lexicographically next permutation of the same letters. For example, if the string `s` is initially `"51432"`, then when the function returns `s` should be `"52134"`.

Exercise 5

If the two strings are of length n (and, if there are two, m), what is the asymptotic performance of your answers to Exercises 1–4?

Chapter 7 – Summary (Part II)

Key messages:

- Strings are stored in character arrays
- The underlying array must be declared big enough to hold the string plus a sentinel byte
- Functions to manipulate strings inevitably make use of `char*` pointers
- Arrays of `char*` are used to manipulate sets of strings, including `argv`, the initiating command line.

Pattern search

Given: A text sequence $T[0 \ldots n-1]$ and a pattern $P[0 \ldots m-1]$.

Question: Does pattern P appear as a continuous subsequence of text T? If so, where?

Sequential pattern search

$s \leftarrow 0$

while $s \leq n - m$
    for $i \leftarrow 0$ **to** $m - 1$
        if $T[s+i] \neq P[i]$
            break
    if $i = m$
        return s
    $s \leftarrow s + 1$

return *not_found*

$O(n \cdot m)$

Sequential pattern search

Running time?

a | a | a | a | a | a | a | a | a

a | a | a | b

In the worst case (what inputs?), requires $O(nm)$ time.

Average case (but remember, need to be careful with this!) is $O(n)$. Is linear time worst case possible? Or even sub-linear?

average do comparison of first pattern $O(n)$.

KMP pattern search

Great idea: Start with a standard first alignment, and extend a match as far as possible. If/when a mismatch occurs, shift the pattern forward as far as possible without moving past any matching prefix of the pattern.

In the cases where pattern shifts forward by less than i (the current position in P), the new i gets set accordingly.

The search location in T described by $s + i$ never moves backwards.

KMP – Example

Example: does `she shells` appear in `she sells sea shells`.

0					0					1					1				
0	1	2	3	4	5	6	7	8	9	0	1	2	3	4	5	6	7	8	9
s	h	e		s	e	l	l	s		s	e	a		s	h	e	l	l	s
s	h	e		s	h	e	l	l	s										
-1	0				1	2	3	0	0										

sh she is the same

we need to go back x steps when move forward

Variables: $s = 0$ and $i = 0$.

KMP – Example

Example: does `she shells` appear in `she sells sea shells`.

```
0        0        1        1
0 1 2 3 4 5 6 7 8 9 0 1 2 3 4 5 6 7 8 9
s h e   s e l l s   s e a   s h e l l s
s h e   s h e l l s
* * * * * X
```

Variables: $s = 0$ and $i = 5$. Mismatch

KMP – Example

Example: does `she shells` appear in `she sells sea shells`.

```
0         0         1         1
0 1 2 3 4 5 6 7 8 9 0 1 2 3 4 5 6 7 8 9
s h e   s e l l s   s e a   s h e l l s
        s h e   s h e l l s
```

Variables: $s = 4$ and $i = 1$.

KMP – Example

Example: does `she shells` appear in `she sells sea shells`.

```
0         0         1         1
0 1 2 3 4 5 6 7 8 9 0 1 2 3 4 5 6 7 8 9
s h e   s e l l s   s e a   s h e l l s
        s h e   s h e l l s
        - X
```

Variables: $s = 4$ and $i = 1$. Mismatch

KMP – Example

Example: does `she shells` appear in `she sells sea shells`.

```
0         0         1         1
0 1 2 3 4 5 6 7 8 9 0 1 2 3 4 5 6 7 8 9
s h e   s e l l s   s e a   s h e l l s
          s h e   s h e l l s
```

Variables: $s = 5$ and $i = 0$.

KMP – Example

Example: does `she shells` appear in `she sells sea shells`.

```
0         0         1         1
0 1 2 3 4 5 6 7 8 9 0 1 2 3 4 5 6 7 8 9
s h e   s e l l s   s e a   s h e l l s
          s h e   s h e l l s
          X
```

Variables: $s = 5$ and $i = 0$. Mismatch

KMP – Example

Example: does `she shells` appear in `she sells sea shells`.

```
0         0         1         1
0 1 2 3 4 5 6 7 8 9 0 1 2 3 4 5 6 7 8 9
s h e   s e l l s   s e a   s h e l l s
            s h e   s h e l l s
```

Variables: $s = 6$ and $i = 0$.

Etc.

KMP pattern search

Define $F[i]$ to be the maximum $k < i$ such that $P[0 \ldots k-1]$ matches $P[i-k \ldots i-1]$, with $F[0]$ set to be -1.

Then at each mismatch, can shift P right by i (mismatch position) minus $F[i]$ (allowance for pattern self-overlap).

If $F[i]$ is zero (common case), then pattern search resumes from mismatched location $s+i$, rather than $s+1$.

Cool!

KMP pattern matching

Examples of F:

```
    0         0         1         1
    0 1 2 3 4 5 6 7 8 9 0 1 2 3 4 5

P:  s h e   s h e l l s
F: -1 0 0 0 0 1 2 3 0 0

P:  s h e   s e l l s   s h e l l s
F: -1 0 0 0 0 1 0 0 0 1 0 1 2 3 0 0

P:  a a a a a a a a
F: -1 0 1 2 3 4 5 6

P:  a b c d a b c d a b c d e f g
F: -1 0 0 0 0 1 2 3 4 5 6 7 8 0 0
```

do nothing on text
do on pattern

KMP pattern matching

With F created, doing the search is easy:

$s, i \leftarrow 0, 0$

while $s \leq n - m$
  if $T[s+i] = P[i]$
    $i \leftarrow i + 1$
    if $i = m$
      return s
  else
    $s \leftarrow s + i - F[i]$ → move back
    $i \leftarrow \max(F[i], 0)$

return *not_found*

move forward i step

for the first one
F(0) = −1
$s \leftarrow s + 0 - (-1) \Rightarrow s+1$

KMP pattern search

Building the failure function F for pattern $P[0\ldots m-1]$ makes use of very similar logic:

$s, c \leftarrow 2, 0$
$F[0], F[1] \leftarrow -1, 0$

while $s < m$
  if $P[c] = P[s-1]$
    $c, F[s], s \leftarrow c+1, c+1, s+1$
  else if $c > 0$
    $c \leftarrow F[c]$
  else
    $F[s], s \leftarrow 0, s+1$

KMP pattern search

Example: `she shells`.

```
    0       0       1       1
    0 1 2 3 4 5 6 7 8 9 0 1 2 3 4 5

P:  s h e   s h e l l s
    *   *
F: -1 0
```

Variables: $c = 0$, $s = 2$: $P[c] \neq P[s-1]$,
so $F[2] \leftarrow 0$

KMP pattern search

Example: `she shells`.

```
       0         0         1         1
       0 1 2 3 4 5 6 7 8 9 0 1 2 3 4 5

   P:  s h e   s h e l l s
       *     *
   F: -1 0 0
```

Variables: $c = 0$, $s = 3$: $P[c] \neq P[s-1]$,
so $F[3] \leftarrow 0$

KMP pattern search

Example: `she shells`.

```
      0         0         1         1
      0 1 2 3 4 5 6 7 8 9 0 1 2 3 4 5

  P:  s h e   s h e l l s
      *       *
  F: -1 0 0 0
```

Variables: $c = 0$, $s = 4$: $P[c] \neq P[s-1]$,
so $F[4] \leftarrow 0$

KMP pattern search

Example: `she shells`.

```
    0         0         1         1
    0 1 2 3 4 5 6 7 8 9 0 1 2 3 4 5

P:  s h e   s h e l l s
    *         *
F: -1 0 0 0 0
```

Variables: $c = 0$, $s = 5$: $P[c] = P[s-1]$,
so $F[5] \leftarrow 1$ and $c \leftarrow 1$

KMP pattern search

Example: `she shells`.

```
    0         0         1         1
    0 1 2 3 4 5 6 7 8 9 0 1 2 3 4 5

P:  s h e   s h e l l s
      *         *
F: -1 0 0 0 0 1
```

Variables: $c = 1$, $s = 6$: $P[c] = P[s-1]$,
so $F[6] \leftarrow 2$ and $c \leftarrow 2$

KMP pattern search

Example: `she shells`.

```
       0         0         1         1
       0 1 2 3 4 5 6 7 8 9 0 1 2 3 4 5

   P:  s h e   s h e l l s
           *         *
   F: -1 0 0 0 0 1 2
```

Variables: $c = 2$, $s = 7$: $P[c] = P[s-1]$,
so $F[7] \leftarrow 3$ and $c \leftarrow 3$

KMP pattern search

Example: `she shells`.

```
       0         0         1         1
       0 1 2 3 4 5 6 7 8 9 0 1 2 3 4 5

P:     s h e   s h e l l s
             *         *
F:    -1 0 0 0 0 1 2 3
```

Variables: $c = 3$, $s = 8$: $P[c] \neq P[s-1]$,
so $c \leftarrow F[3]$

KMP pattern search

Example: `she shells`.

```
          0         0         1         1
          0 1 2 3 4 5 6 7 8 9 0 1 2 3 4 5

      P:  s h e   s h e l l s
          *               *
      F: -1 0 0 0 0 1 2 3
```

Variables: $c = 0$, $s = 8$: $P[c] \neq P[s-1]$,
so $F[8] \leftarrow 0$

KMP pattern search

Example: `she shells`.

```
     0         0         1         1
     0 1 2 3 4 5 6 7 8 9 0 1 2 3 4 5

P:   s h e   s h e l l s
     *                 *
F:  -1 0 0 0 0 1 2 3 0
```

Variables: $c = 0$, $s = 9$: $P[c] \neq P[s-1]$,
so $F[9] \leftarrow 0$

KMP pattern search

Example: she shells.

```
       0         0         1         1
       0 1 2 3 4 5 6 7 8 9 0 1 2 3 4 5

P:     s h e   s h e l l s
       *                 *
F:    -1 0 0 0 0 1 2 3 0 0
```

Variables: $c = 0$, $s = 10$: $P[c] = P[s-1]$,
so $F[10] \leftarrow 0$ and $c \leftarrow 1$

KMP – Analysis

Named after Knuth, Morris, Pratt, who invented it in 1974.

Analysis? In main search loop, at every iteration, either:

- i goes up by one and s is unchanged; or
- s goes up by the same as i decreases; or
- s goes up by 1 and i remains zero.

In all three cases the quantity $2s + i$ increases by at least one.

But since $s < n$ and $i \leq m$, the number of loop iterations before exit with either success or failure is at most $2n + m$.

KMP – Analysis

A similar argument applies to the construction of F:

- s and c both go up by one; or
- c decreases by at least one; or
- s goes up by one and c remains zero.

In all cases, $2s - c$ increases by at least one.

But $c > 0$ and $s < m$; hence the total number of iterations is less than $2m$.

The preprocessing phase does not dominate.

Boyer-Moore-Horspool pattern search

Another clever idea: Start from the right-hand end of the pattern, and work to the left.

For each symbol v in the input alphabet, define $L[v]$ to be the shift needed to bring the rightmost location in the pattern at which v appears into the place in the text where the pattern previously ended.

If/when a mismatch occurs, the pattern can be shifted right by $L[T[s+m-1]]$ to force last character to be in alignment.

BMH – Example


```
0         0         1         1
0 1 2 3 4 5 6 7 8 9 0 1 2 3 4 5 6 7 8 9
s h e   s e l l s   s e a   s h e l l s
s h e   s h e l l s
```

Variables: $s = 0$ and $i = 9$.

BMH – Example


```
0         0         1         1
0 1 2 3 4 5 6 7 8 9 0 1 2 3 4 5 6 7 8 9
s h e   s e l l s   s e a   s h e l l s
s h e   s h e l l s
                  X
```

Variables: $s = 0$ and $i = 9$. Mismatch.

BMH – Example

```
0         0         1         1
0 1 2 3 4 5 6 7 8 9 0 1 2 3 4 5 6 7 8 9

s h e   s e l l s   s e a   s h e l l s
            s h e   s h e l l s
```

Variables: $s = 6$ and $i = 9$.

BMH – Example


```
0       0       1       1
0 1 2 3 4 5 6 7 8 9 0 1 2 3 4 5 6 7 8 9
s h e   s e l l s   s e a   s h e l l s
      s h e   s h e l l s
                        X
```

Variables: $s = 6$ and $i = 9$. Mismatch.

BMH – Example

```
0         0         1         1
0 1 2 3 4 5 6 7 8 9 0 1 2 3 4 5 6 7 8 9

s h e   s e l l s   s e a   s h e l l s
                    s h e   s h e l l s
```

Variables: $s = 10$ and $i = 9$.

BMH – Example

```
0                 0                 1                 1
0   1   2   3   4   5   6   7   8   9   0   1   2   3   4   5   6   7   8   9
s   h   e       s   e   l   l   s       s   e   a       s   h   e   l   l   s
                                        s   h   e       s   h   e   l   l   s
                                                X   *   *   *   *   *   *   *
```

Variables: $s = 10$ and $i = 2$. Mismatch.

BMH – Example


```
0         0         1         1
0 1 2 3 4 5 6 7 8 9 0 1 2 3 4 5 6 7 8 9
s h e   s e l l s   s e a   s h e l l s
                              s h e   s h e l l s
```

Variables: $s = 15$ and $i = 2$.

End of search. Only 12 character comparisons were done!

BMH – Shift array

Construct the shift array L for pattern length m and alphabet $0 \ldots \sigma - 1$:

for $v \leftarrow 0$ **to** $\sigma - 1$
    $L[v] \leftarrow m$ ← initialise to m.
for $i \leftarrow 0$ **to** $m - 2$
    $L[P[i]] = m - i - 1$

symbol

BMH – Shift array

Examples of V:

```
    0         0         1         1
    0 1 2 3 4 5 6 7 8 9 0 1 2 3 4 5

P:  s h e   s h e l l s
L:  (s,5), (h,4), (e,3), ( ,6), (l,1), (a,10)

P:  s h e   s e l l s   s h e l l s
L:  (s,5), (h,4), (e,3), ( ,6), (l,1), (a,16)

P:  a a a a a a a a
L:  (a,1), (b,8), (c,8)

P:  a b c d a b c d a b c d e f g
L:  (a,6), (b,5), (c,4), (d,3), (e,2), (f,1), (g,15)
```

BMH – Searching

Search the string:

$s, i \leftarrow 0, m-1$

while $s \leq n-m$
  if $T[s+i] \neq P[i]$
    $s, i \leftarrow s + L[T[s+m-1]], m-1$
  else if $i = 0$
    return s
  else
    $i \leftarrow i - 1$

return *not_found*

BMH – Analysis

In the worst case, back up to $O(nm)$, and not interesting.

But average case is much better, and experimentally is very fast for large alphabets (ASCII) and shortish patterns (m under 10 or so), because it can leapfrog quickly down T, looking at only a small number of characters at each leap.

Note that "average" must be from input data; these is no sense in which randomness can be introduced in to algorithm.

BMH – Variations

Plenty of extensions have been proposed:

- Scan from left to right, to get better cache/prefetch behavior;
- Use two final characters, shift by larger of two;
- Use a full $m \times \sigma$ array, so that shift amount depends on position as well as missed character;
- Use two final characters as a combination, shift by full amount m if that adjacent pair does not appear earlier in the string;
- Take into account the part of the suffix that has been matched, KMP-style.

Indexing

What happens if T is fixed and large ($m \ll n$), and there are going to be multiple independent patterns to be checked?

Is there some way of precomputing an index?

Of course there is, lots of choices...

Suffix array indexing

Consider text $T[0 \ldots n-1]$.

Suppose that $T[n] = \$$, a unique symbol smaller than any other symbol.

Define $T_i = T[i \ldots n]$ to be the ith suffix of T.

A suffix array $S[0 \ldots n-1]$ is an array of pointers $S[i]$ such that $T_{S[i]}$ lexicographically precedes $T_{S[i+1]}$.

Suffix array – Example


```
0         0         1         1
0 1 2 3 4 5 6 7 8 9 0 1 2 3 4 5 6 7
s h e # s e l l s # s h e l l s
```

take all positible suffix

↓

sort them base on alphabetical sequence

→ like a phonebook

insert binary search

put char precede the suffix

L[i] ·

Burrow wherer transfrog

$S[i]$

↓

suffix happens in which position

pointer, offset

i	$S[i]$	$T_{S[i]}$
0	16	$ → non byte
1	3	# s e l l s # s h e l l s $
2	9	# s h e l l s $
3	2	e # s e l l s # s h e l l s $
4	12	e l l s $
5	5	e l l s # s h e l l s $
6	1	h e # s e l l s # s h e l l s $
7	11	h e l l s $
8	13	l l s $
9	6	l l s # s h e l l s $
10	14	l s $
11	7	l s # s h e l l s $
12	15	s $
13	8	s # s h e l l s $
14	4	s e l l s # s h e l l s $
15	0	s h e # s e l l s # s h e l l s $ → $.
16	10	s h e l l s $

s

e

s

binary search

h

h

$

$

Suffix array pattern search

Looking for a pattern $P[0 \ldots m-1]$?

Use binary search in S, comparing the pattern P against the suffixes in T, examining as long a prefix of $T_{S[i]} = T[S[i] \ldots \min\{S[i]+m, n\}]$ as is necessary in each comparison, to identify a range of matches in S.

$O(m \log n) \to$ efficient.

kmp $O(m+n)$.

Takes $O(\log n)$ string comparisons via S. Each string comparison takes at most m character comparisons.

So total time is $O(m \log n)$ per search. That's very fast!

we need to compare string which is length of m

string comparisons

Remember this problem?

Exercise 6

Given: A sequence S of n symbols

Problem: Find all locations in S at which repeated subsequences of length m or more appear.

Would a suffix array be useful??

look at suffix

Complexity linear since just look at first then compare the rest of suffix

Suffix array construction

Just one small problem: generating the suffix array.

Simple approach: use an $O(n \log n)$-comparison sorting algorithm, with each comparison requiring as many as n steps.

Overall, $O(n^2 \log n)$ time average case, and $O(n^3)$ worst case. Not cheap!

Sorting strings

Ternary Quicksort: partition on one character, at depth d in the strings. Then do three recursive calls:

$tquicksort(S, n, depth)$:

$p \leftarrow T[S[i] + depth]$ for some $0 \leq i < n$
$(fe, fg) \leftarrow partition(S, n, p, depth)$
$tquicksort(S, fe, depth)$
$tquicksort(S + fe, fg - fe, depth + 1)$
$tquicksort(S + fg, n - fg, depth)$

Initial call:
$tquicksort(S, n, 0)$

Sorting strings

Analysis: tricky. But, roughly speaking, shaves a factor of up to n off execution time.

Worst case drops from $O(n^3)$ to $O(n^2)$. Average case analysis still requires randomness in the data.

Experimentally, works well on typical non-pathological texts.

Suffix array construction is an active area of algorithmic research. The best current methods take $O(n)$ time, but too much space to be fully practical. There will probably be new algorithms five years from now.

Indexing – Exercises

Exercise 7:

A KWIC index for a text (KeyWord In Context) presents a small window of words around each (case folded) word that appears in the text, in dictionary order.

Write a program that outputs a KWIC Index for the text that is provided as input. Only whole words should be indexed.

You may assume that at most 10,000 words will be input.

Indexing – Exercises

For example, for the string

```
She sells sea shells by the sea shore
```

and for a window size of two words either side, the output of the indexing program should be:

```
sea shells *by the sea
 She sells *sea shells by
    by the *sea shore
       She *sells sea shells
           *She sells sea
 sells sea *shells by the
   the sea *shore
 shells by *the sea shore
```

Indexing – Exercises

Exercise 8:

An inverted index for a text is an alphabetical listing of all of the words that appear, together with the line numbers(s) at which they appear:

Write a program that generates an inverted index for the text that is provided as input. Words should be case-folded.

You may assume that at most 10,000 words will be input.

Indexing – Exercises


```
mac:./inverted_index
She sells sea shells
by the sea shore
He sells sea shells too
sells to see her more
^D
by      : 2
he      : 3
her     : 4
more    : 4
sea     : 1, 2, 3
see     : 4
sells   : 1, 3, 4
she     : 1
shells  : 1, 3
shore   : 2
the     : 2
to      : 4
too     : 3
```