PHYC10003 Physics 1

Lecture 8: Force

Newton's laws; applications & examples

Last lecture

- Friction
- Microscopic view of friction
- Drag
- Terminal velocity
- Dynamics simulation

Applying Newton's laws

- Find the resultant force by
  - Adding the vectors
  - Find components in a convenient choice of coordinate system, and summing in each direction separately.
- Apply $\vec{F} = m\vec{a}$ to find the object's acceleration

Example: Falling from a height

- The Problem: What is the greatest height from which I can fall without injuring myself?
- Step 1
  - Suppose I fall from a height H. How fast will I be going when I hit the ground?
  - Use $v^2 = u^2 + 2a(x - x_0)$.
    Here $(x - x_0) = H$, so

$$v = \sqrt{2gH}$$

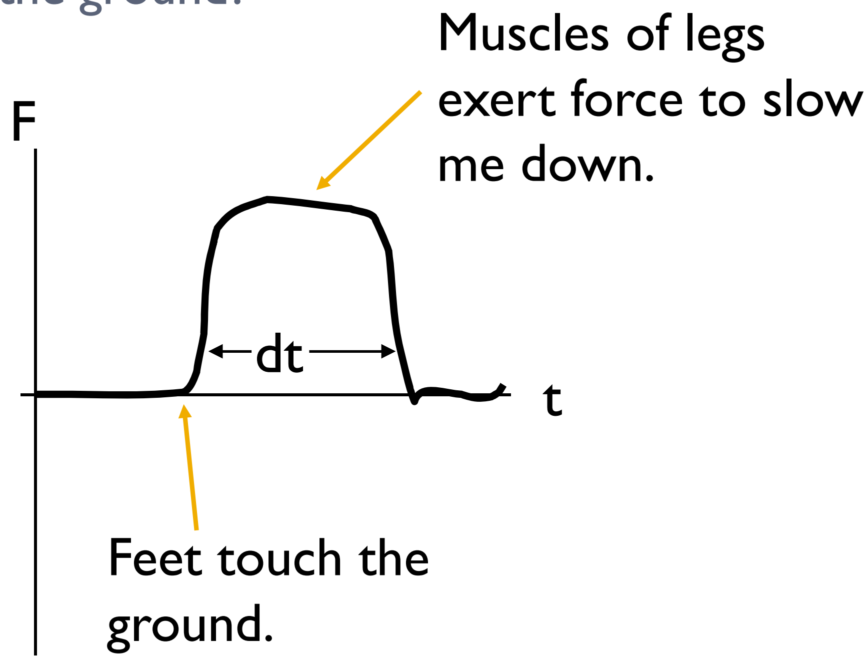
Example: Falling from a height cont…

- Step 2
  - What happens with I hit the ground?

$F = ma = m\,dv/dt$

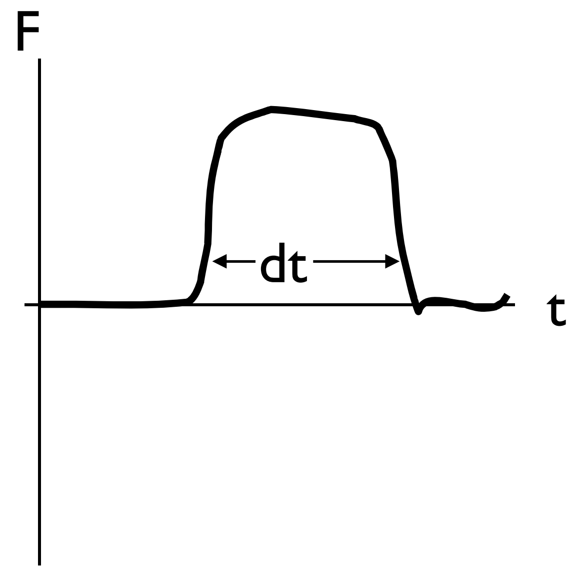
So $mdv = Fdt$

The area under the $F - t$ curve must be mdv.



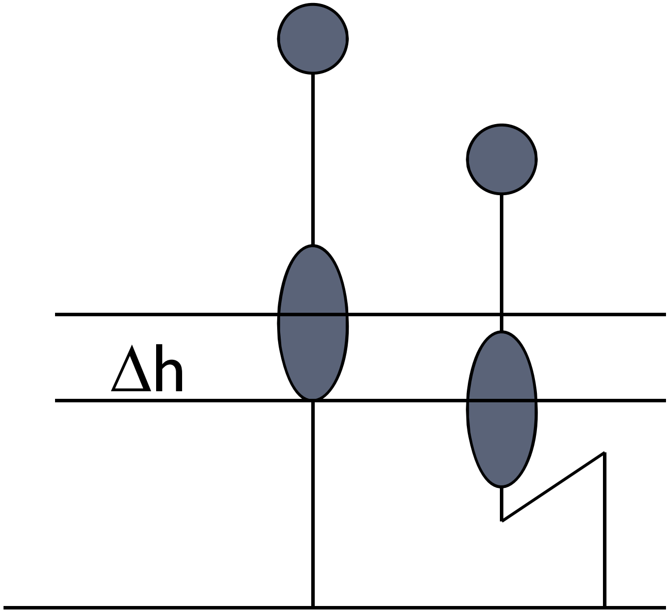
Example: Falling from a height cont…

- Step 3. Minimising harm
  - The area under the $F - t$ curve is fixed and must be equal to
    $$mdv = m\sqrt{2gH}$$
  - Minimizing injury = mimimising F.
  - Hence we need to maximize 'dt', which is the time of the interaction.
- Strategies to do this:
  - Bend knees
  - Roll



Example: Falling from a height cont…

- Assume that :
  - the decceleration, a, is constant
  - that I can bend my knees and crouch down by $\Delta h = 0.5$ m. I must decelerate from a speed of $\sqrt{2gH}$ down to zero in a distance of only Δh.
  - Applying $v^2 = u^2 + 2a(x - x_0)$, we obtain $a = \frac{gH}{\Delta h}$
  - (Eg for a 30 m drop, $d = 588$ m/s^2 or 60 times the acceleration due to gravity)



Example: Falling from a height cont…

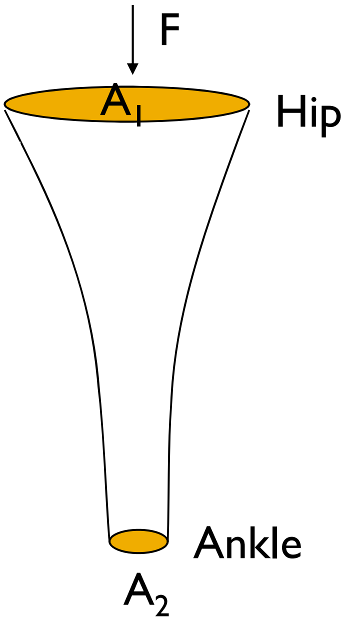
- Step 5: Calculate the force
  - Since $F = ma$, we have $F = mgH/\Delta h$
  - So, the force on the body is body weight multiplied by a factor of $H/\Delta h$
  - Recall: $H \;= $ Height of Fall
    $\Delta h =$ amount of 'crouch' (about 0.5 m).

- Step 6: What is the time of the interaction, ie the time it takes to stop?
  - Since $v = u \;+ at$, so.....
  - $dt \;= \sqrt{2gH}\Delta h/gH \;=\; 2\Delta h/\sqrt{2gH}$

Example: Falling from a height cont…

- Step 7: Consider the consequences!
  $A_2 = 4\ cm^2$ approx. so the ankle will break when the force on it $= 1.6 \cdot 10^8 \times 4 \cdot 10^{-4}) = 6.4 \cdot 10^4 N$.

A bone breaks when the pressure on it is $1.6\ \cdot\ 10^8\ N/m^2$. The pressure is $P = F/A$, so the pressure at the ankle will be the biggest because it has the smallest cross-sectional area



Example: Falling from a height cont…

- Since $F = mgH/\Delta h$ and the maximum force allowed is $6.4 \cdot 10^4 N$.
  - Using $\Delta h = 0.3\ m$ and $m = 75\ kg$, the maximum height is $H = 26\ m$!
  - Is this reasonable? Can you fall from a ten story building without injury? Is there something wrong with our analysis?
- Yes there is something wrong. The deceleration is provided by our muscles. For a $26\ m$ fall, $a = \frac{gH}{\Delta h} = 509\ m/s^2$ or about 50 times the acceleration due to gravity.
  - Can you lift 50 times your own weight? No! Hence the muscles cannot provide the necessary deceleration in the $0.5\ m$ available. For a $3\ m$ drop, $a = 59 m/s^2$ or about 6 times the acceleration due to gravity. This is possible.

Example: Falling from a height cont…

- Strategies for minimizing injury eg when landing from a Parachute jump:
  - Bend knees:
    - This maximizes Δh and dt
  - Roll onto one side
    - This maximizes Δh and dt and
    - Increases the area over which the force is exerted, thus reducing the pressure and the chance of injury.

- http://theconversation.com/ive-always-wondered-if-youre-in-a-falling-elevator-can-you-really-save-yourself-by-jumping-99597

Actual forces measured during a jump

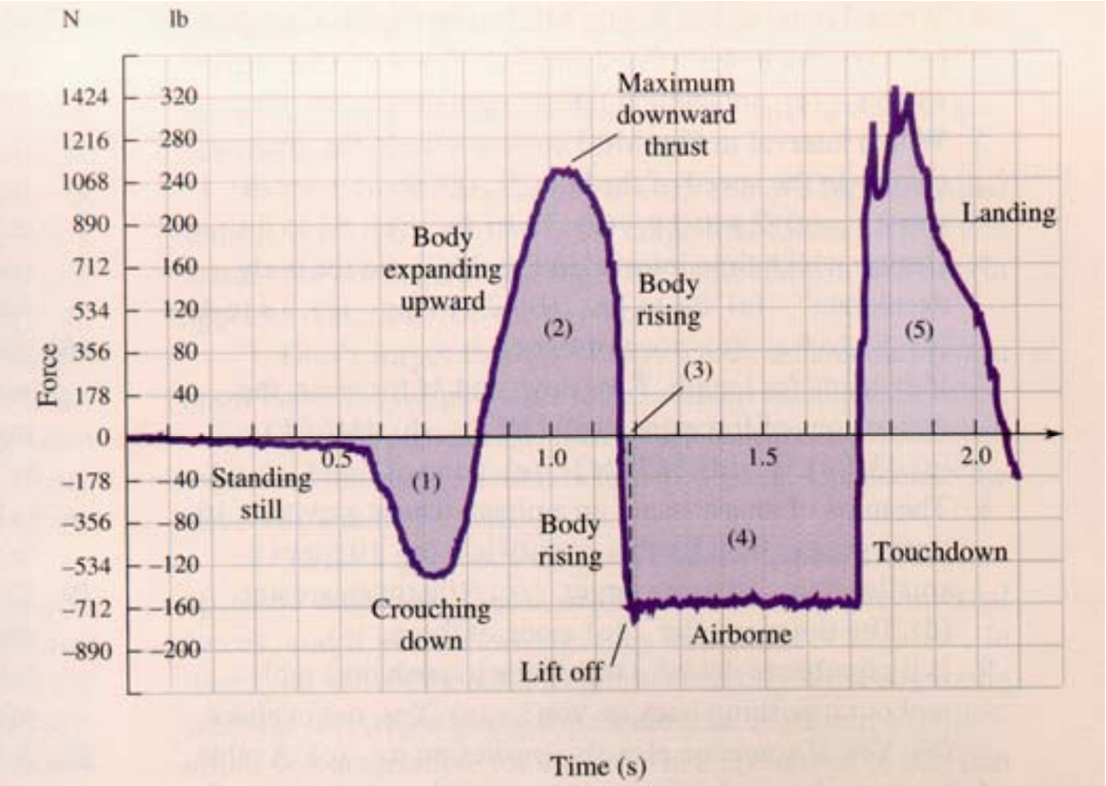


Figure Q15

Example 2: Air bags save lives

- The motion of the passengers must be stopped in a distance of 30 cm to avoid hitting the dashboard. This is our new Δh.
- Use same analysis as before:
  - Assume car is travelling at a speed v.
  - The deceleration is $a = v^2/2\Delta h$
  - $F = ma = \frac{mv^2}{2\Delta h}$
  - Assume $v = 70\ \mathrm{km/h} = 19.4$ m/s; $\Delta h = 0.3$ m; m =70 kg
    Then $F = 44\times 10^3 N$.

Air bags cont…

- When applied to the whole body, (Area = 0.1 m^2), $P = 4.4 \cdot 10^5\ \mathrm{N/m}^2$. This is just about the threshold of injury.
- What happens when you increase the velocity?
  - F is proportional to v^2, so at 100 km/h passengers may be injured.
- *Exercise*: Calculate the duration of the collision between the passenger and the inflated bag. Important because the bag can inflate unexpectedly; we want the bag to stay inflated for the minimum period of time possible.

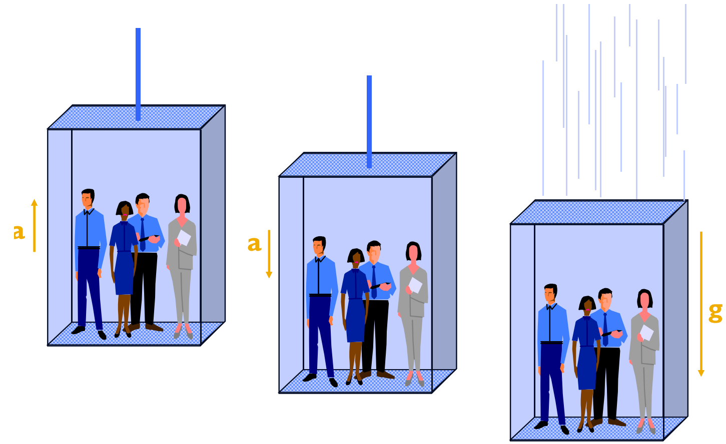
Example 3: Perception of weight

- *"Why do you feel heavy?"*
  "What determines how heavy you feel?"
  "Why would you feel lighter on the moon?"
  "What do we mean when we say someone feels weightless?"

- *"Why do you feel heavy?"*
  - awareness of inertia from force required to change your motion ($F = ma$)
  - awareness of weight from the support of your surroundings (when $a = 0, F_G = F_N$)

Perceptions of weight

- Interactions?
- Forces?
- Apply $\sum \overrightarrow{(F} = m\vec{a})$

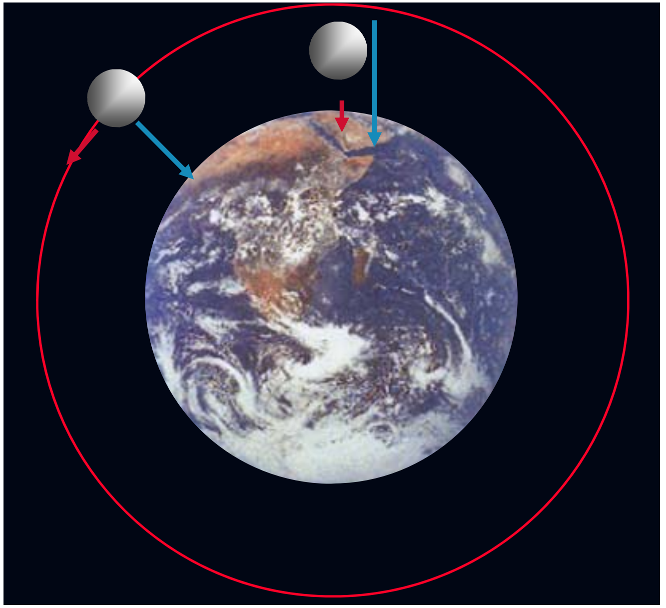


with earth

with floor

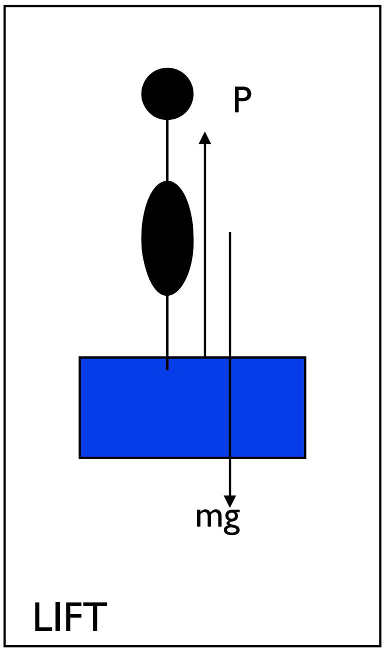
Perceptions of weight

- When $F_N = 0, a = g$, and you feel weightless.
  - You're not!
    The earth still exerts a gravitational force on you, but your sense of weight really depends on F_N.

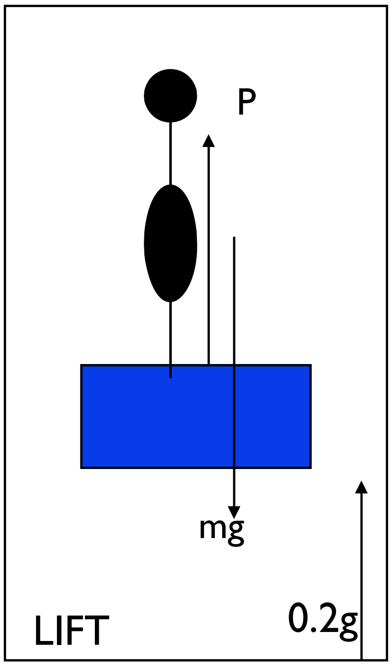


Effective weight

- Case 1:
  - $a = 0,$
  - $P = mg$

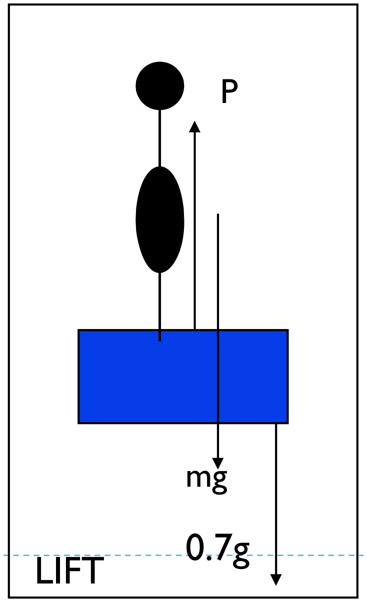


- Case 2:
  - $a = 0.2g$ upwards
  - $P - mg = ma = m(0.2g)$
    so $P = mg\,(1 + 0.2)$
  - The effective weight is 1.2 her normal weight

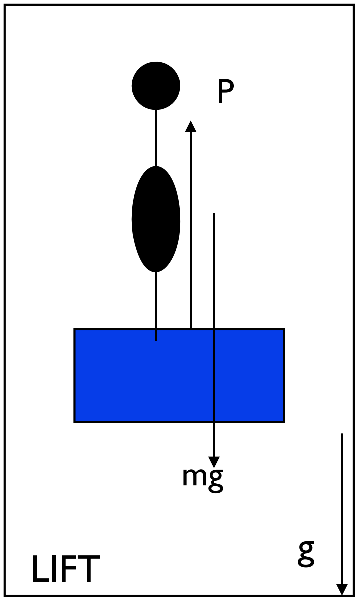


Effective weight

- Case 3:
  - $a = 0.7g$ down
  - $P = mg(1 - 0.7) = 0.3mg$

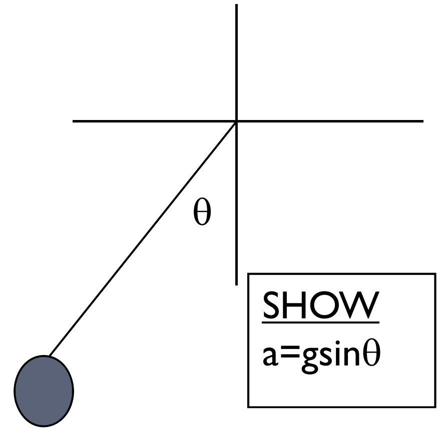


- Case 4:
  - $a = g$ down
  - $P - mg = mg$ so effective weight is zero – sense of free fall.



Effective weight

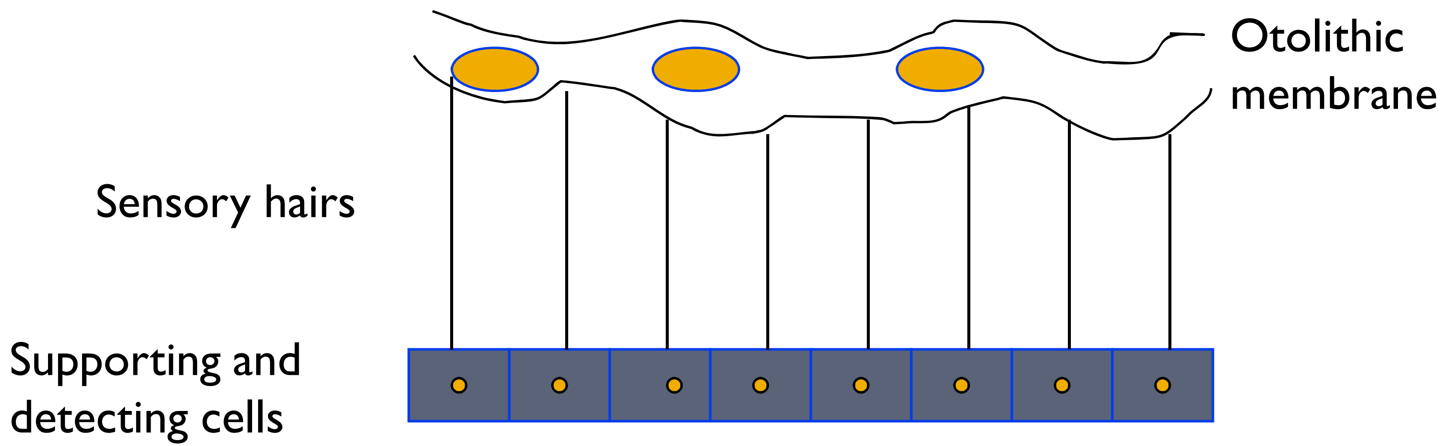
- *Orientation with respect to the gravitational field*: Inside the otic labyrinth there is an organ for sensing linear acceleration and weight. It consists of sensitive hairs connected to an elastically suspended membrane. The membrance changes position slightly if the head is tilted. The hairs then activate the support cells and thus the position of the head with respect to the direction of gravitation is conveyed to the brain.



SHOW
$a = g\sin\theta$

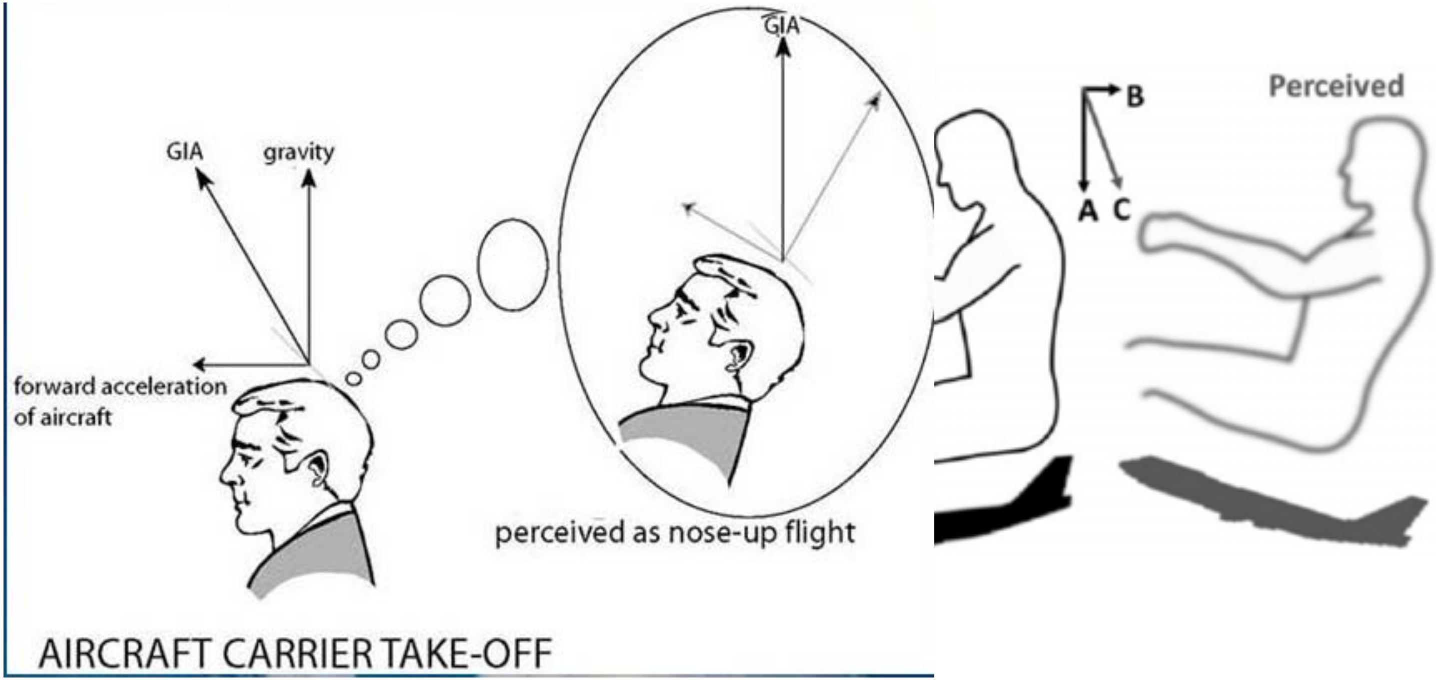
Same as watching the angle of a plum bob to the horizontal.

Sensing weight



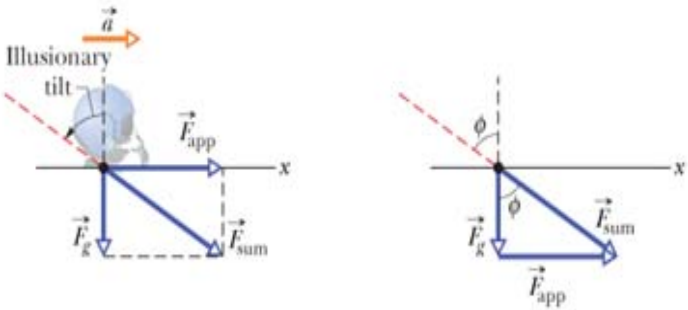
- Small organ inside the ear
- Normally mg pulls on membrane and hairs push on supporting and detecting cells.
- In free fall, hairs don't push on detecting cells. The brain interprets this as 'weightlessness'

Take off illusion



Take off illusion

- Starting from rest, the pilot undergoes constant horizontal acceleration to reach a speed of 85 m/s (306 km/h) in 90 m. What is the angle of the illusory tilt?



$$\tan\phi = F_{app}/F_g$$

$$\phi = \tan^{-1}(F_{app}/F_g)$$

Use Eq 2-16

$$v^2 = v_0^2 + 2a(x - x_0)$$

$$(85\ \mathrm{m/s^2}) = 2a_x(90\ \mathrm{m})$$

$$a_x = 40.1\ \mathrm{m/s^2}$$

$$\phi = \tan^{-1}(m(40.1\ \mathrm{m/s^2})/m(9.8\mathrm{m/s^2})) = 76^{0}$$

6-3 Centripetal force

- Recall that circular motion requires a centripetal acceleration

$$a = \frac{v^2}{R}$$ **Eq. (6-17)**

Examples You are a passenger:

- For a car, rounding a curve, the car accelerates toward the center of the curve due to a **centripetal force** provided by the inward friction on the tires. Your inertia makes you want to go straight ahead so you may feel friction from your seat and may also be pushed against the side of the car. These inward forces keep you in uniform circular motion in the car.
- For a space shuttle, the shuttle is kept in orbit by the gravitational pull of Earth acting as a centripetal force. This force also acts on every atom in your body, and keeps you in orbit around the Earth. You float with no sensation of force, but are subject to a centripetal acceleration.

6-3 Centripetal force

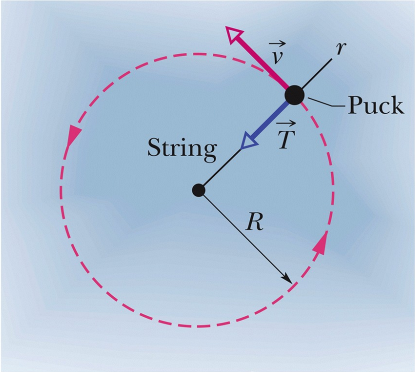
- Centripetal force is not a new *kind* of force, it is simply an application of force

$$F = m\frac{v^2}{R}$$

Eq. (6-18)

A centripetal force accelerates a body by changing the direction of the body's velocity without changing the body's speed.

- For the puck on a string, the string tension supplies the centripetal force necessary to maintain circular motion



The puck moves in uniform circular motion only because of a toward-the-center force.

Figure 6-8

6-3 Centripetal force: example

Example Bicycle going around a vertical loop:

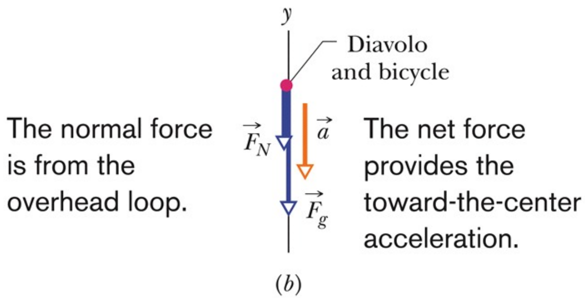


Figure 6-9

- At the top of the loop we have:

$$-F_N - mg = m\left(-\frac{v^2}{R}\right).$$

Eq. (6-19)

- Solve for v and plug in our known values, including $F_N = 0$ for the minimum answer:

$$v = \sqrt{gR} = \sqrt{(9.8\ \mathrm{m/s^2})(2.7\ \mathrm{m})}$$
$$= 5.1\ \mathrm{m/s}.$$

6-3 Centripetal force: example

Example Car in a banked circular turn:

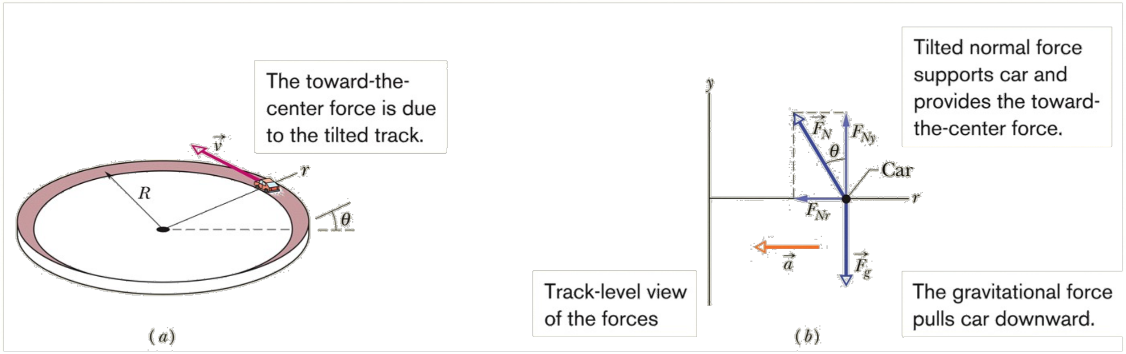


Figure 6-11

- Sum components along the radial direction:

$$-F_N \sin\theta = m\left(-\frac{v^2}{R}\right). \quad \text{Eq. (6-23)}$$

- Sum components along the vertical direction:

$$F_N \cos\theta = mg. \quad \text{Eq. (6-24)}$$

- Divide and replace (sin θ)/(cos θ) with tangent.

$$\theta = \tan^{-1}\frac{v^2}{gR}$$

Summary

- Examples - Force
  - Falling from a height
  - Air bags save lives
  - Perception of weight
- Uniform circular motion
- Centripetal force

$$F = m\frac{v^2}{R}$$

Preparation for the next lecture

1. Read 7-1 to 7-6 of the text
2. You will find short answers to the odd-numbered problems in each chapter at the back of the book and further resources on LMS. You should try a few of the simple odd numbered problems from each section (the simple questions have one or two dots next to the question number).