Week 3: FNCE10002 Principles of Finance

THE UNIVERSITY OF MELBOURNE

Valuation of Debt Securities

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3. Valuation of Debt Securities

1. Analyze loan payments and amortization schedules
2. Define and calculate effective annual interest rates
3. Examine the characteristics of debt securities
4. Outline the basic valuation principle for financial securities
5. Examine the pricing of pure discount securities
6. Examine the pricing of zero coupon and coupon paying bonds
7. Relate the coupon rate to the yield to maturity
8. Analyze the sensitivity of prices to changes in interest rates

Required Readings: Weeks 3 – 4

- *Week 3*
  - GRAH, Ch. 3 (Sec 3.7) and Ch. 4 (Sec. 4.1 – 4.4)
- *Week 4*
  - GRAH, Ch. 5 (Sec 5.1 – 5.2, 5.4b and 5.5)

3.1 Loan Payments and Amortization

- The typical bank loan involves borrowing a sum of money with the promise to make regular payments to pay off the loan over a pre-determined time horizon. The interest rate payable can be fixed or variable. In Australia, the broad categories of bank loans are…
  - Fixed rate loans
  - Variable rate loans
  - Split rate loans combination of fixed & variable
  - Interest only loans
  - Low doc loans
  - *More details:* https://www.moneysmart.gov.au/borrowing-and-credit/home-loans/choosing-a-home-loan
- Our focus here is on fixed rate loans

Case Study 1: Amortize This!

- You have borrowed \$20,000 from your bank with the loan to be repaid in equal annual installments over four years. Your bank's interest rate on this loan is 10% p.a. with interest charged (that is, compounded) annually. What annual payment would you be making on this loan? Develop a loan amortization schedule for this loan. Use this schedule to get the following information:

  (i) The principal balance outstanding at the end of year 1

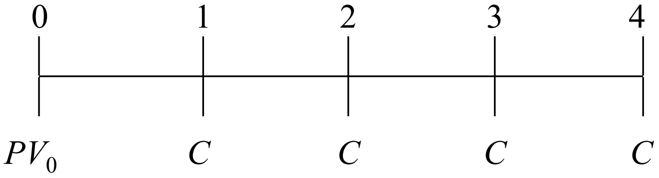
  (ii) The total interest paid in year 2

  (iii) The total principal repaid in year 3

  If you wanted to repay the loan in full at the end of year 2 what amount would you need to repay the bank?

Case Study 1: Amortize This!

- The *annual loan payment* is an ordinary annuity and its value is based on the loan amount
  - The amount includes the interest paid on the loan as well as part of the principal that is repaid to the bank
- The value of the loan outstanding today (year 0) is equal to the present value of the loan payments (why?)
  - Loan amount outstanding = PV(Loan payments)



Cash flows occur at the end of the period

Case Study 1: Amortize This!

- The loan amount today (year 0) is \$20,000. So we have:

$$PV_0 = 20000 = \left(\frac{C}{0.10}\right)\left(1 - \frac{1}{(1+0.10)^4}\right)$$

$\frac{C}{r}\left(1 - \frac{1}{(1+r)^n}\right)$

- $20000 = C \times 3.16987$
- Loan payment, $C = 20000/3.16987 = \$6{,}309.42$
- What is the economic interpretation of the amount 3.16987? ?

Case Study 1: Amortize This!

- The *loan amortization schedule* shows the interest paid, principal repaid and principal remaining over the loan's duration, as follows…
  - Interest paid = (Previous period's principal) × (Interest rate)
  - Principal repaid = Loan payment – Interest paid
  - Principal balance remaining = Previous period's principal – Principal repaid
- In year 1, we have…
  - Interest paid = 20000 × 0.10 = $2,000.00
  - Principal repaid = 6309.42 – 2000.00 = $4,309.42
  - Principal balance remaining = 20000.00 – 4309.62 = $15,690.58
- Iteratively, in year 2, we have…
  - Interest paid = 15690.58 × 0.10 = $1,569.06
  - Principal repaid = 6309.42 – 1596.06 = $4,740.36
  - Principal balance remaining = 15690.58 – 4740.36 = $10,950.22

Case Study 1: Amortize This!

Year	*Annual payment*	*Interest paid*[1]	*Principal repaid*[2]	*Principal remaining*[3]
0	–	–	–	$20,000.00
1	$6,309.42	$2,000.00	$4,309.42	$15,690.58 (i)
2	$6,309.42	$1,569.06 (ii)	$4,740.36	$10,950.22
3	$6,309.42	$1,095.02	$5,214.40 (iii)	$5,735.82
4	$6,309.42	$573.58	$5,735.84	$0.00
Totals	$25,237.66	$5,237.66	$20,000.00	

[1] Interest paid = Previous period's principal × Interest rate

[2] Principal repaid = Loan Payment – Interest paid

[3] Principal balance remaining = Previous period's principal – Principal repaid

Case Study 1: Amortize This!

- The amount you would need to pay at the end of year 2 to repay the loan in full is the principal balance remaining at that time (PV_2). This equals the present value of the *remaining* loan payments in years 3 and 4 (why?)
- The principal balance outstanding at the end of year 2 is…

$$PV_2 = \left(\frac{6309.42}{0.10}\right)\left(1-\frac{1}{(1+0.10)^2}\right) = \$10,950.22$$

3.2 The Effective Annual Interest Rate

- Interest is often not earned or paid on an annual basis
- When interest is compounded more often than once a year we need to calculate the *effective* annual interest rate in order to make a like-for-like comparison
- The effective annual interest rate (*EAR* or r_e) is the *annualized* rate that takes account of compounding *within the year*
  - $r_e = (1 + r/m)^m - 1$
  - r is the annual percentage rate (*APR*) or the stated interest rate
  - m is the compounding frequency: 1 for annual, 2 for semi-annual, 12 for monthly, and so on
  - r/m is the *per period* interest rate taking into account the compounding frequency

The Effective Annual Interest Rate

- Some properties of the effective annual interest rate, r_e
  - $r = r_e$ *only* when the compounding frequency is one year ($m = 1$), otherwise r_e will *always* exceed r
  - r_e increases as the compounding frequency increases
- *Continuous compounding* refers to the compounding frequency becoming larger with m approaching infinity
- As m becomes larger, in the limit $(1 + r/m)^m$ approaches e^r
- The effective annual rate with *continuous* compounding…
  - $r_e = e^r - 1$
  - *Note:* The exponential constant, $e = 2.71828...$ (this is the e^x key on your calculator)

Case Study 2: Don't Bank On It

- Refer back to Case Study 1. Your bank's interest rate on the loan was 10% p.a. with interest charged annually. Your uncle's just told you that his bank only charges an interest rate of 9.8% p.a. The interest, however, is compounded on the monthly balance even though you'd be making annual payments. Overhearing this conversation, your second cousin tells you that her bank's even better because it only charges an interest rate of 9.7% p.a. On further questioning, you find out that at her bank interest is compounded on the daily balance even though you'd be making annual payments. Which bank offers the best interest rate?

Case Study 2: Don't Bank On It

- To make a decision we need to calculate the effective annual interest rate taking into account the frequency with which interest is compounded
- Your bank (r = 10.0%, m = 1)
  - $r_e = r =$ 10.0%
- Uncle's bank (r = 9.8%, m = 12)
  - Monthly rate = 0.098/12 = 0.81667%
  - $r_e = (1 + 0.098/12)^{12} - 1 =$ 10.252%
- Second cousin's bank (r = 9.7%, m = 365)
  - Daily rate = 0.097/365 = 0.02658%
  - $r_e = (1 + 0.097/365)^{365} - 1 =$ 10.185%
- *Which bank offers the best rate?*

Matching Cash Flows With Interest Rates

- *Recall Rule 4 from Week 1:* The interest rate used to compound or discount cash flows *must* match their periodicity
- Some examples (assume r = 12%)…
- Annual cash flows; interest compounded annually, use 12%
- Monthly cash flows; interest compounded monthly, use 12/12 = 1%
- Quarterly cash flows; interest compounded quarterly, use 12/4 = 3%
- Annual cash flows; interest compounded quarterly requires calculating the *effective annual* interest rate
  - Effective annual rate, $r_e = (1 + 0.12/4)^4 - 1 = 12.551\%$
- Annual cash flows; interest compounded monthly also requires calculating the *effective annual* interest rate
  - Effective annual rate, $r_e = (1 + 0.12/12)^{12} - 1 = 12.6825\%$

Matching Cash Flows With Interest Rates

- Some examples continued (assume $r = 12\%$)…
- Semi annual cash flows; interest compounded semi annually; use $0.12/2 = 6\%$
- Semi annual cash flows; interest compounded monthly requires calculating the *effective semi annual* interest rate
  - Monthly rate $= 0.12/12 = 1\%$
  - Effective semi annual rate $= (1 + 0.01)^6 - 1 = 6.152\%$
- Quarterly cash flows; interest compounded monthly requires calculating the *effective quarterly* interest rate
  - Monthly rate $= 0.12/12 = 1\%$
  - Effective quarterly rate $= (1 + 0.01)^3 - 1 = 3.03\%$

Matching Cash Flows With Interest Rates

- Some examples continued (assume $r = 12\%$)…
- Cash flows every two years; interest compounded annually requires calculating the *effective two-year* interest rate
  - Effective two-year rate = $(1 + 0.12)^2 – 1 = 25.44\%$
- Cash flows every two years; interest compounded quarterly requires first calculating the *effective annual* interest rate and then the *effective two-year* interest rate
  - Effective annual rate = $(1 + 0.12/4)^4 – 1 = 12.551\%$
  - Effective two-year rate = $(1 + 0.12551)^2 – 1 = 26.667\%$
- Cash flows every two years; interest compounded monthly requires first calculating the *effective annual* interest rate and then the *effective two-year* interest rate
  - Effective annual rate = $(1 + 0.12/12)^{12} – 1 = 12.6825\%$
  - Effective two-year rate = $(1 + 0.126825)^2 – 1 = 26.9734\%$

3.3 The Valuation Principle

- *The price of a security today is the present value of all future expected cash flows discounted at the "appropriate" required rate of return (or discount rate)*
- Market price today, $P_0 = PV$(Future expected cash flows)
- The valuation problem is to…
  - Estimate the price; given the future cash flows and required rate of return, *or*
  - Estimate the required rate of return; given the future cash flows and market price

3.4 Characteristics of Debt Securities

- Short term debt securities (or discount debt securities)
  - They mature within the year – typically in 90 and 180 days
  - The issuer has *contractual* obligation to make the promised payment at maturity
  - *Examples:* Treasury Bills, Bank Bills, etc
- *Face (or maturity or par) value* is the dollar amount paid at maturity
  - Denoted as F_n (or FV_n)
- No other payments are made to investors (that is, debtholders)
  - The interest (or return) earned by investors is implicit and equals the difference between the price paid (P_0) and face value (F_n)

Characteristics of Debt Securities

- Long term debt securities
- Long term debt securities mature after several years
- They may or may not promise a regular interest (or coupon) payment
- The issuer has a *contractual* obligation to make *all* promised payments
- *Examples*
    - Treasury (or Commonwealth) bonds issued by the Commonwealth of Australia
    - Corporate bonds issued by companies

Characteristics of Debt Securities

- *Face (or par) value* is the dollar amount paid at maturity
  - Usually $1,000 or its multiples and denoted as F_n (or FV_n)
  - *Note:* The textbook uses M for maturity value
- *Coupon (or interest) rate* is the interest rate promised by the issuer, expressed as a percentage of the *face value*
- *Coupon (or interest) payment* denoted as C the periodic (annual or semi-annual) payment made to bondholders
  - Coupon payment, C = Coupon rate × Face value

收益率?

- *Coupon (or current) yield* is the coupon (or interest) promised by the issuer divided by the current price of the bond
  - Coupon yield = Coupon payment/Price

Characteristics of Debt Securities

Issuer	*Maturity*	*Coupon (%)*	*Rating*	*Issue Size ($m)*	*Month High (%)*	*Month Low (%)*	*Closing Yield (%)*
NAB	24-Mar-22	3.25	AA-	500	2.59	2.48	2.52
NAB	21-Sep-26	4.00	AA-	150	3.48	3.36	3.38
Qantas	27-Apr-20	6.50	BB+	250	2.88	2.78	2.82
Qantas	12-Oct-26	4.75	BB+	175	3.74	3.63	3.67
Telstra	15-Jul-20	7.75	A	500	2.51	2.40	2.44
Telstra	19-Apr-21	2.90	A	300	2.63	2.51	2.56
Telstra	16-Sep-22	4.00	A	500	2.85	2.74	2.78
Telstra	19-Apr-27	4.00	A	550	3.51	3.41	3.45
Wesfarmers	28-Mar-19	6.25	A-	500	2.39	2.35	2.35
Wesfarmers	12-Mar-20	4.75	A-	350	2.40	2.31	2.33
Westpac	25-Feb-19	4.50	AA-	1100	2.02	1.95	1.95
Westpac	11-Feb-20	7.25	AA-	390	2.38	2.28	2.31
Westpac	4-Jun-26	4.13	AA-	425	3.20	3.12	3.15

Source: http://www.yieldreport.com.au/category/corporate/monthly-corporate. Market quote data on selected bonds traded on the ASX sorted by issuer, maturity and coupon rate.

3.5 *Pricing Pure Discount Securities*

- The price of discount securities is calculated as the present value of the face value at a market determined *yield to maturity* (YTM or r_D)
- The *yield to maturity* is the rate of return earned by an investor who holds the security until it matures *assuming* no default occurs on the security
- Market yields to maturity are *always* quoted on an annual basis. The interest rate factor for securities maturing in less than a year is…
  - Interest rate factor = (Time to maturity/365) × r_D = (n/365) × r_D
- The price of a discount security is…
  - $P_0 = F_n/[1 + (n/365) \times r_D]$

Japan. US 360

Pricing Pure Discount Securities

- *Example:* Calculate the price of a Treasury bill maturing in 90 days and earning a yield to maturity of 4% p.a. Assume that the face value of the bill is $1 million. If you purchased the bill for $990,233.32 and held it until maturity (that is, 90 days) what return would you have earned on the bill? If the yield to maturity falls to 3.5% what happens to the bill’s price? If the yield to maturity rises to 4.5% what happens to the bill’s price?

Pricing Pure Discount Securities

- Price of the bill at a yield to maturity of 4.0%
  - $P_0 = 1000000/[1 + (90/365)0.040] = \$990{,}233.32$
- If you purchased the bill for \$990,233.32 and held it to maturity, your return is…
  - 90-day return = (1000000 – 990233.32)/990233.32 = 0.9863%
  - Annualized return = 0.9863(365/90) = 4%
- Price of the bill if the yield to maturity *falls* to 3.5%
  - $P_0 = 1000000/[1 + (90/365)0.035] = \$991{,}443.71$ (+0.122%)
- Price of the bill if the yield to maturity *rises* to 4.5%
  - $P_0 = 1000000/[1 + (90/365)0.045] = \$989{,}025.88$ (–0.122%)
- *Why are prices and market yields inversely related to each other?*

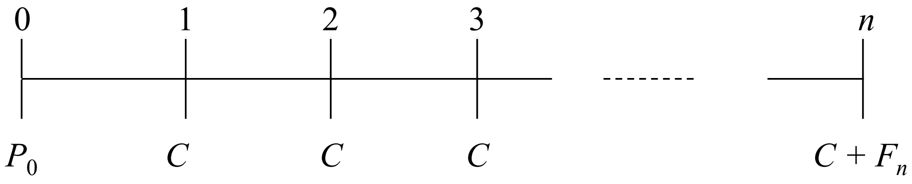
receive – not change face value

interest rate change

↳ price change

3.6 Pricing Coupon Paying Bonds

- Coupon paying bonds pay a fixed coupon payment (typically) every six months with the repayment of face value at maturity



Cash flows occur at the end of each period

- This is an annuity (coupon, C) *plus* a single cash flow (face value, F_n) valuation problem

$\frac{C}{r}\left(1 - \frac{1}{(1+r)^n}\right)$ $\quad \frac{F_n}{(1+r)^n}$

Pricing Coupon Paying Bonds

- The price of a coupon paying bond is…

$$P_0 = \left(\frac{C}{r_D}\right)\left(1 - \frac{1}{(1+r_D)^n}\right) + \frac{F_n}{(1+r_D)^n}$$

- Note that the *current* coupon (C_0) is *not* relevant to our estimate of the current (that is, year 0) price as the estimated price is assumed to be an *ex-coupon* price
  - An ex-coupon price is the price immediately *after* the current period's coupon has been paid
  - *Example:* The price at the end of year 3 would be…

$$P_3 = \left(\frac{C_4}{r_D}\right)\left(1 - \frac{1}{(1+r_D)^{n-3}}\right) + \frac{F_n}{(1+r_D)^{n-3}}$$

Pricing Coupon Paying Bonds

- The *yield to maturity* (YTM or r_D) is the required rate of return earned by an investor who buys the bonds at P_0 and holds them until they mature *assuming* that the issuer does not default on its obligations
- The yield to maturity is the interest rate that discounts the bond's future cash flows to equal the market price today
  - This is why it is also called an *internal rate of return*

Case Study 3: Microsoft's Jumbo Issue

- *Microsoft Sells $19.75 Billion of Bonds in Its Biggest Ever Sale*
- *August 1, 2016:* Microsoft Corp. (*Nasdaq:* MSFT) raised US$19.75 billion in the third-largest US corporate bond sale of the year to help finance its planned purchase of LinkedIn Corp. *Investors put in more than $50 billion of orders for the deal in the software maker's biggest ever sale. The strong demand helped Microsoft to borrow at lower rates than it paid for the $13 billion of bonds it raised previously. It also saved about $40 million in annual interest payments compared with what it was offering to pay initially,* according to people familiar with the matter.
- Microsoft in June announced plans to buy LinkedIn for US$26.2 billion, which it said it would fund primarily through issuing new debt. The software maker said it expects to close that purchase by the end of this year.

Case Study 3: Microsoft's Jumbo Issue

- *S&P Global Ratings assigned the bonds the top AAA grade in a note reviewing the sale on Monday. Moody's Investors Service also gave the bonds its top grade. CreditSights, a research firm, recommended that investors buy the securities, and changed its rating on the company's bonds to the equivalent of a "buy" from the equivalent of a "hold."*
- *Microsoft's debt issuance is tied to avoiding an increase in its tax bill. Companies with cash holdings from overseas profits have to pay a 35 percent tax to repatriate those funds to the US. Rather than use that cash to fund its acquisition and pay the hefty taxes that result, it is far cheaper for large multinationals like Microsoft to borrow the funds.*

Case Study 3: Microsoft's Jumbo Issue

- *According to analysts at S&P Global, Microsoft generates "only a portion" of its cash in the US, and is expected to "continue to access the debt market to help fund its annual dividend payments and share repurchases". Roughly 95 percent of Microsoft's US$123bn of cash was held overseas at the end of 2016, according to its latest filing with US securities regulators.*

Source: https://www.bloomberg.com/news/articles/2016-08-01/microsoft-plans-first-bond-sale-since-october-to-fund-buybacks

Case Study 3: Microsoft's Jumbo Issue

- Microsoft Corp. on Aug 1, 2016 announced the pricing of its offering of $19.75 billion aggregate principal amount of senior unsecured notes. The notes consist of the following tranches
  - $2.50 billion of 1.100 percent notes due Aug 8, 2019
  - $2.75 billion of 1.550 percent notes due Aug 8, 2021
  - $1.50 billion of 2.000 percent notes due Aug 8, 2023
  - $4.00 billion of 2.400 percent notes due Aug 8, 2026
  - $2.25 billion of 3.450 percent notes due Aug 8, 2036
  - $4.50 billion of 3.700 percent notes due Aug 8, 2046
  - $2.25 billion of 3.950 percent notes due Aug 8, 2056

Source: https://news.microsoft.com/2016/08/01/microsoft-announces-debt-offerings

Case Study 3: Microsoft's Jumbo Issue

- Consider Microsoft's \$2.25 billion notes paying a coupon of 3.95% and due Aug 8, 2056. Assume that the yield to maturity on these notes is 4.5%. What price should they be selling for today? For simplicity, assume that we're at the end of 2016, the notes mature at the end of 2056 and the coupons are paid annually. How does the price change if the yield to maturity is 3.5%? What is the price if these notes did *not* pay any coupon and the yield to maturity remained at 3.5%? What is the total market value of this issue in each case? Finally, assess the riskiness of Microsoft's issues
- *Note:* The face value *per note* is not given. The market convention is to use a face value of \$100 and do all the calculations based on this *assumed* face value

Case Study 3: Microsoft's Jumbo Issue

- Price with *annual* coupon payments…
  - Coupon payment, $C = 0.0395 \times 100 = \$3.95$ per year
  - Time horizon, $n = 40$ years (we're at the end of 2016)
- At $r_D = 4.5\%$, the issue sells at a *discount* to the face value…

$$P_0 = \left(\frac{3.95}{0.045}\right)\left(1 - \frac{1}{(1+0.045)^{40}}\right) + \frac{100}{(1+0.045)^{40}} = \$89.879 < \$100.00$$

- At $r_D = 3.5\%$, the issue sells at a *premium* to the face value…

$$P_0 = \left(\frac{3.95}{0.035}\right)\left(1 - \frac{1}{(1+0.035)^{40}}\right) + \frac{100}{(1+0.035)^{40}} = \$109.610 > \$100.00$$

Case Study 3: Microsoft's Jumbo Issue

- Price if there were a *no* coupon payments…
  - Coupon payment, $C = \$0$
  - Time periods, $n = 40$
  - Yield to maturity = 3.5%

$$P_0 = \frac{100}{(1+0.035)^{40}} = \$25.257$$

- *Note:* The difference between the price of the bond with and without coupons (at r_D = 3.5%) of \$84.353 (= 109.610 – 25.257) equals the present value of the coupon payments over the 40-year period

$$PV(C) = \left(\frac{3.95}{0.035}\right)\left(1 - \frac{1}{(1+0.035)^{40}}\right) = \$84.353$$

Case Study 3: Microsoft's Jumbo Issue

- The total market value of the issue in each case is the market price per bond multiplied by the market price per bond converted as a percentage…
- Annual coupons; r_D = 4.5%; Price = $89.879
  - Total market value = (89.879/100) × 2.25 = $2.022 billion
- Annual coupons; r_D = 3.5%; Price = $109.610
  - Total market value = (109.610/100) × 2.25 = $2.466 billion
- No coupon and r_D = 3.5%; Price = $25.257
  - Total market value = (25.257/100) × 2.25 = $0.568 billion

Case Study 3: Microsoft's Jumbo Issue

- In assessing the risk of Microsoft's issue we need to estimate the likelihood that the company will default on its debt obligations
  - *Default risk* is also called *credit risk* and is associated with the inability of make the coupon payments and/or maturity (face value) payment when due
- This assessment is typically performed by rating agencies like *S&P Global Ratings* and *Moody's Investors Service*
  - These rating agencies have rated Microsoft's issues as being the highest quality
  - See a summary of rating classifications in the next two slides

Case Study 3: Microsoft's Jumbo Issue

Rating*	Description (Moody's)
Investment Grade Debt	
Aaa/AAA	Judged to be of the best quality. They carry the smallest degree of investment risk and are generally referred to as "gilt edged." Interest payments are protected by a large or an exceptionally stable margin and principal is secure. While the various protective elements are likely to change, such changes as can be visualized are most unlikely to impair the fundamentally strong position of such issues.
Aa/AA	Judged to be of high quality by all standards. Together with the Aaa group, they constitute what are generally known as high-grade bonds. They are rated lower than the best bonds because margins of protection may not be as large as in Aaa securities or fluctuation of protective elements may be of greater amplitude or there may be other elements present that make the long-term risk appear somewhat larger than the Aaa securities.
A/A	Possess many favorable investment attributes and are considered as upper-medium-grade obligations. Factors giving security to principal and interest are considered adequate, but elements may be present that suggest a susceptibility to impairment some time in the future.
Baa/BBB	Are considered as medium-grade obligations (i.e., they are neither highly protected nor poorly secured). Interest payments and principal security appear adequate for the present but certain protective elements may be lacking or may be characteristically unreliable over any great length of time. Such bonds lack outstanding investment characteristics and, in fact, have speculative characteristics as well.

Note: Investment grade bonds have low default risk while speculative grade bonds have much a higher risk of default.

Case Study 3: Microsoft's Jumbo Issue

Rating*	Description (Moody's)
Speculative Bonds	
Ba/BB	Judged to have speculative elements; their future cannot be considered as well assured. Often the protection of interest and principal payments may be very moderate, and thereby not well safeguarded during both good and bad times over the future. Uncertainty of position characterizes bonds in this class.
B/B	Generally lack characteristics of the desirable investment. Assurance of interest and principal payments of maintenance of other terms of the contract over any long period of time may be small.
Caa/CCC	Are of poor standing. Such issues may be in default or there may be present elements of danger with respect to principal or interest.
Ca/CC	Are speculative in a high degree. Such issues are often in default or have other marked shortcomings.
C/C, D	Lowest-rated class of bonds, and issues so rated can be regarded as having extremely poor prospects of ever attaining any real investment standing.

*Ratings: Moody's/Standard & Poor's

Source: www.moodys.com

Note: Investment grade bonds have low default risk while speculative grade bonds have much a higher risk of default.

Calculating the Yield to Maturity

- *Example 1:* Consider a zero coupon bond with 3 years remaining to maturity and a face value of $1,000. If the bond is selling for $700 what is its yield to maturity?
- *Given:* P_0 = $700, F_n = $1,000 and n = 3 years
- The bond's yield to maturity (r_D) can be calculated using…

$$P_0 = 700 = \frac{1000}{(1+r_D)^3}$$

$$(1+r_D)^3 = \frac{1000}{700}$$

$$r_D = \left(\frac{1000}{700}\right)^{1/3} - 1 = 12.62\%$$

Calculating the Yield to Maturity

- *Example 2:* Consider a bond which pays an annual coupon rate of 10% with 1 year remaining to maturity. If it is selling for $98.214 today what is its yield to maturity?
- The price of the bond is…

$$P_0 = 98.214 = \frac{10}{1+r_D} + \frac{100}{1+r_D} = \frac{110}{1+r_D}$$

- Solving for r_D, we get…

$$r_D = \frac{110}{98.214} - 1 = 12\%$$

Calculating the Yield to Maturity

- *Example 3:* Show that the yield to maturity on a bond which pays an annual coupon rate of 10% and which is selling for \$102.700 with 2 years remaining to maturity lies in the range 8% – 9%. Why would the range for the yield to maturity *not* be 10% – 11%?
- The price of the bond is the present value of the coupons paid over 2 years and the face value paid at the end of year 2

$$P_0 = 102.700 = \frac{10}{(1+r_D)} + \frac{10}{(1+r_D)^2} + \frac{100}{(1+r_D)^2}$$

- *Why would the range for the yield to maturity not be 10% – 11%?*

Pricing Coupon Paying Bonds

- At $r_D = 8\%$, the present value of the right hand side is…

$$\frac{10}{(1+0.08)} + \frac{10}{(1+0.08)^2} + \frac{100}{(1+0.08)^2} = \$103.567$$

- At $r_D = 9\%$, the present value of the right hand side is…

$$\frac{10}{(1+0.09)} + \frac{10}{(1+0.09)^2} + \frac{100}{(1+0.09)^2} = \$101.759$$

- Note that the market price given is \$102.700
- So the yield to maturity *must* lie in the range: $8\% < r_D < 9\%$
  - Note that the exact yield to maturity is 8.48%

3.7 Relating Coupon Rates to the YTM

- *What is the general relation between coupon rates and yields to maturity? What's the economic reasoning behind this relation?*
- When Price = Face value; the bond is selling at *par*
  - YTM = Coupon rate
- When Price > Face value; the bond is selling at a *premium*
  - YTM < Coupon rate
- When Price < Face value; the bond is selling at a *discount*
  - YTM > Coupon rate

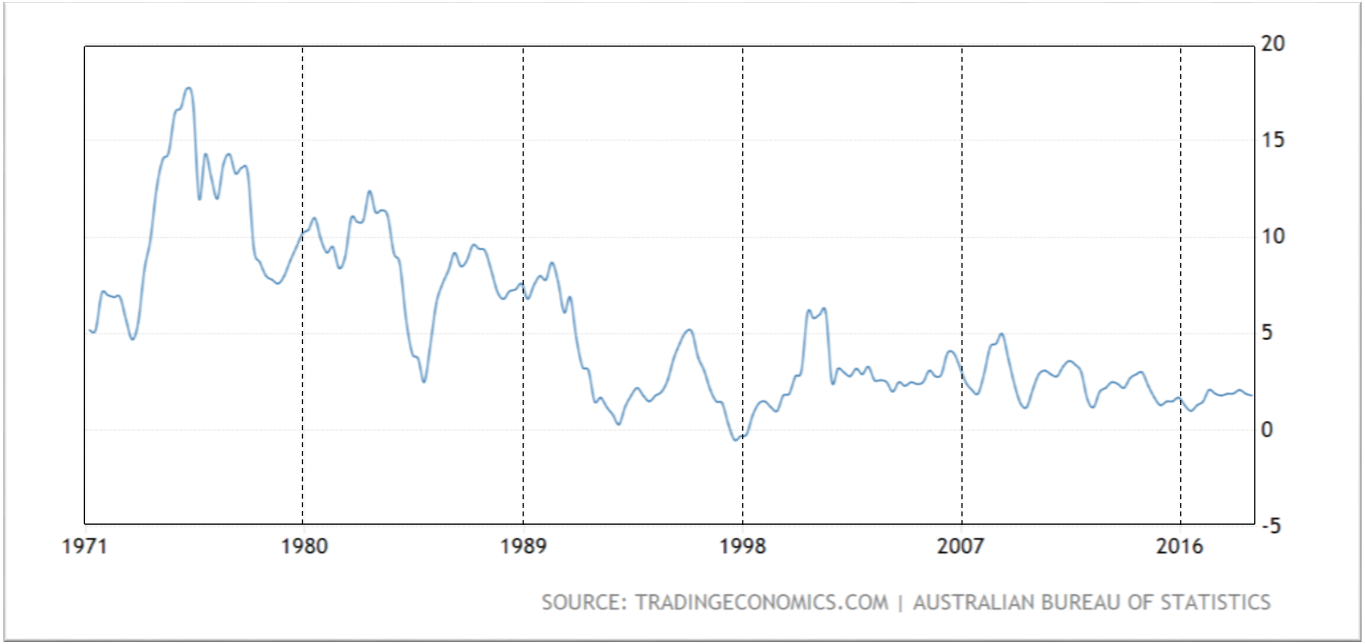
3.8 Price Sensitivity to Interest Rate Changes

- Prices of longer maturity bonds are *more* sensitive to changes in interest rates than are prices of shorter maturity bonds
  - Investors bear *higher* interest rate risk with *longer* maturity bonds
- The probability of interest rates rising over a long time horizon (say, 10 years) is higher than over a short time horizon (say, 1 year)
- A given rise in the interest rate will have a larger effect on the cash flows from a longer term bond than on the cash flows from a shorter term bond
- *What factors drive interest rates?*

Price Sensitivity to Interest Rate Changes

Source: Trading Economics, https://tradingeconomics.com/australia/government-bond-yield. The graph shows the yield to maturity on the 10-year Treasury bond over 1971-2018.

Price Sensitivity to Interest Rate Changes



Source: Trading Economics, https://tradingeconomics.com/australia/government-bond-yield. The graph shows the inflation rate in Australia over 1971-2018.

Price Sensitivity to Interest Rate Changes

- *Example:* Consider two bonds each with an annual coupon rate of 10% and face value of $1,000. Bond A matures in 2 years and Bond B matures in 20 years. Both bonds have just been issued at par implying a yield to maturity of 10% p.a. What will happen to the price of these bonds if market interest rates change unexpectedly to 6%, 8%, 12% and 14%? What is the general relation between prices and yields to maturity?

Price Sensitivity to Interest Rate Changes

- Both bonds are currently selling at par
- Price of bonds A and B…

$$\textit{Bond A}: P_0 = \left(\frac{100}{r_D}\right)\left(1 - \frac{1}{(1+r_D)^2}\right) + \frac{1000}{(1+r_D)^2}$$

$$\textit{Bond B}: P_0 = \left(\frac{100}{r_D}\right)\left(1 - \frac{1}{(1+r_D)^{20}}\right) + \frac{1000}{(1+r_D)^{20}}$$

- We can substitute the market interest rates of 6%, 8%, 12% and 14% for r_D in the above expressions to get the prices, as shown in the next slide
  - *Note:* The percentage changes in prices are relative to the original price of $1,000 for each bond

Price Sensitivity to Interest Rate Changes

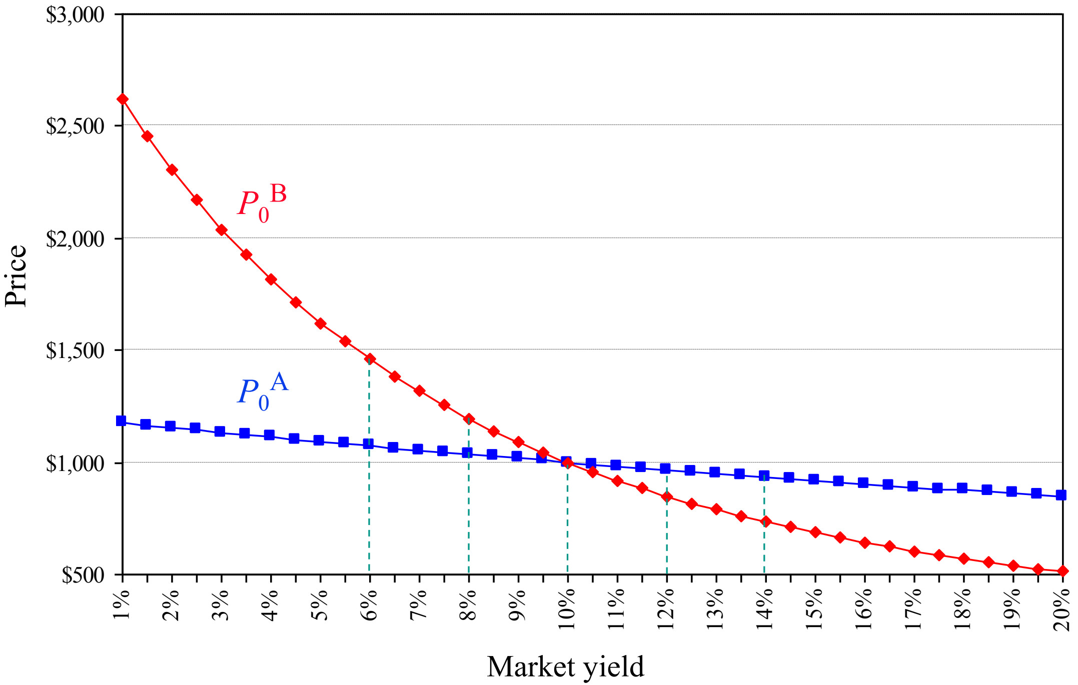
Market Interest Rate	*Price of Bond A*	*Price Change*	*Price of Bond B*	*Price Change*
6%	$1,073.34	+7.33%	$1,458.80	+45.88%
8%	$1,035.67	+3.57%	$1,196.36	+19.63%
10%	$1,000.00	--	$1,000.00	--
12%	$966.20	-3.38%	$850.61	-14.94%
14%	$934.13	-6.59%	$735.07	-26.49%

sensitive

Note: The percentage changes in prices are calculated relative to the original price of $1,000 for each bond

Example: (1073.34 – 1000.00)/1000.00 = +7.33%

Price Sensitivity to Interest Rate Changes



Key Concepts

- The effective interest rate is the annualized rate that takes account of compounding within the year
- The interest rate used to compound or discount cash flows must match the periodicity of those cash flows
- The price of a security today equals the present value of all future expected cash flows discounted at the "appropriate" required rate of return (or discount rate)
- A bond's yield to maturity is the (internal) rate of return that an investor would earn if the bond was held until it matured
- Prices and yields to maturity are inversely related
- Prices of longer maturity bonds are more sensitive to changes in interest rates than are prices of shorter maturity bonds

Formula Sheet

- Effective interest rates
  - $r_e = (1 + r/m)^m - 1$
  - $r_e = e^r - 1$
- The price of a discount security
  - $P_0 = F_n/[1 + (n/365) \times r_D]$
- Price of coupon paying bond

$$P_0 = \left(\frac{C}{r_D}\right)\left(1 - \frac{1}{(1 + r_D)^n}\right) + \frac{F_n}{(1 + r_D)^n}$$

(*Note:* The formula sheets on the mid semester and final exams will contain all the formulas covered in lectures but *without* the descriptions)