Problem Solving and Search

Chapter 3, Sections 1–4

Outline

- ♢ Problem-solving agents
- ♢ Problem types
- ♢ Problem formulation
- ♢ Example problems
- ♢ Basic search algorithms

goal-based agent

Problem-solving agents

Restricted form of general agent:

action or sequence of actions

```
function SIMPLE-PROBLEM-SOLVING-AGENT( p) returns an action
   inputs: p, a percept
   static: s, an action sequence, initially empty
           state, some description of the current world state
           g, a goal, initially null
           problem, a problem formulation

   state ← UPDATE-STATE(state, p)
   if s is empty then
       g ← FORMULATE-GOAL(state)
       problem ← FORMULATE-PROBLEM(state, g)
       s ← SEARCH( problem)
   action ← RECOMMENDATION(s, state)
   s ← REMAINDER(s, state)
   return action
```

→ based on the previous state and percept to find out current state

→ come out sequence of actions

→ find out what action we should take

→ remove action

Note: this is *offline* problem solving.

good solution when we have all the knowledge live in a fairly static environment

Online problem solving involves acting without complete knowledge of the problem and solution. → chaotic world., plan as we go.

Example: Romania

On holiday in Romania; currently in Arad.
Flight leaves tomorrow from Bucharest

Formulate goal:

be in Bucharest

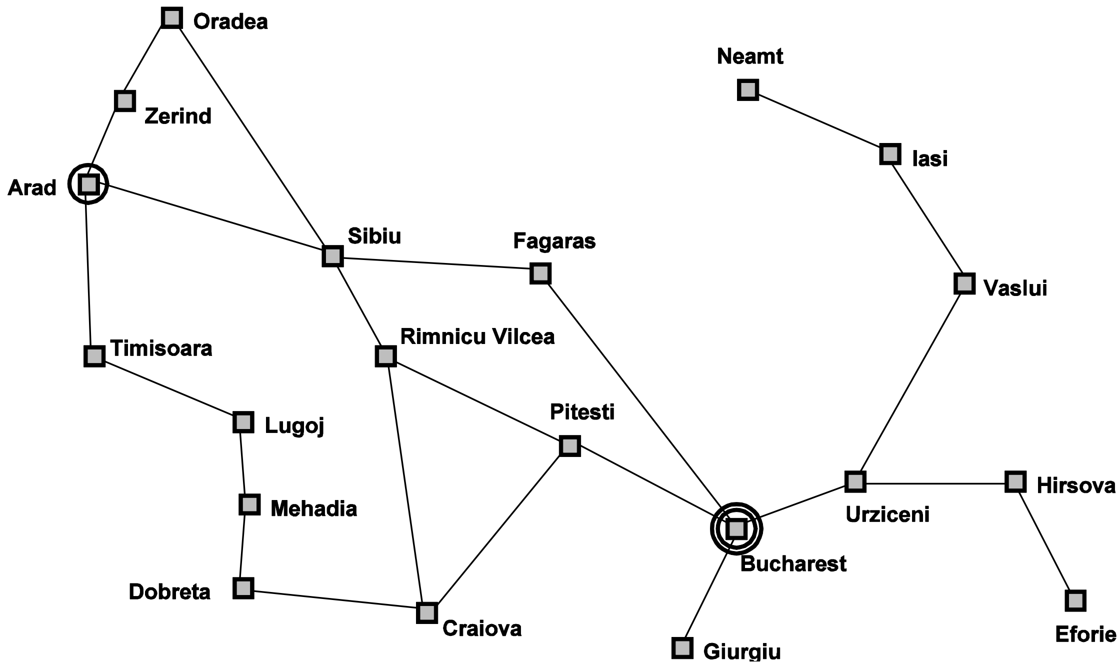
Formulate problem:

states: various cities
operators: drive between cities

Find solution:

sequence of cities, e.g., Arad, Sibiu, Fagaras, Bucharest

Example: Romania



at each state, we exactly know where we are

Single-state problem formulation

A *problem* is defined by four items:

initial state e.g., "at Arad"

actions (or *successor function* $S(x)$)
e.g., Arad $\rightarrow$ Zerind   Arad $\rightarrow$ Sibiu   etc.

goal test, can be
explicit, e.g., $x =$ "at Bucharest" → only one state satisfy the goal
implicit, e.g., $Checkmate$ in chess → several states satisfy the goal.

path cost (additive) additive non-negative function   least cost solution
e.g., sum of distances, number of actions executed, etc.

A *solution* is a sequence of actions
leading from the initial state to a goal state

Note: we sometimes refer to actions as "operators"

Selecting a state space

Real world is absurdly complex

⇒ state space must be *abstracted* for problem solving

(Abstract) state = set of real states

(Abstract) action = complex combination of real actions

e.g., "Arad → Zerind" represents a complex set

of possible routes, detours, rest stops, etc.

For guaranteed realizability, any real state "in Arad"

must get to *some* real state "in Zerind"

? why not some real state "in Arad" can go to some state "in Zerind"

(Abstract) solution =

set of real paths that are solutions in the real world

Each abstract action should be "easier" than the original problem!

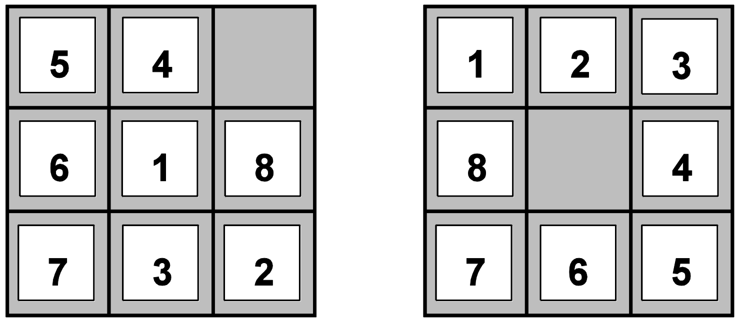
granularity of representation

level of abstraction

airport coffee Arad

. Arad.

Example: The 8-puzzle

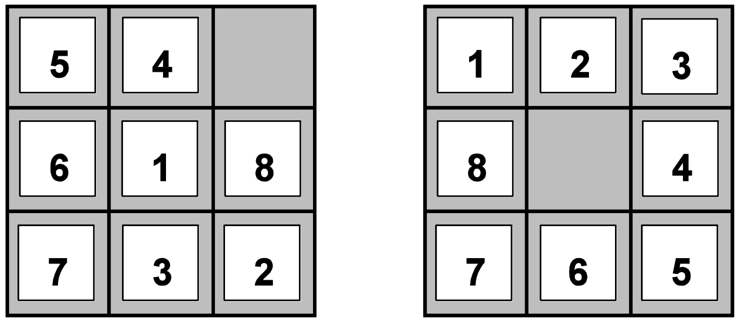


Start State Goal State

states??
actions??
goal test??
path cost??

[Note: optimal solution of n-Puzzle family is NP-hard]

Example: The 8-puzzle



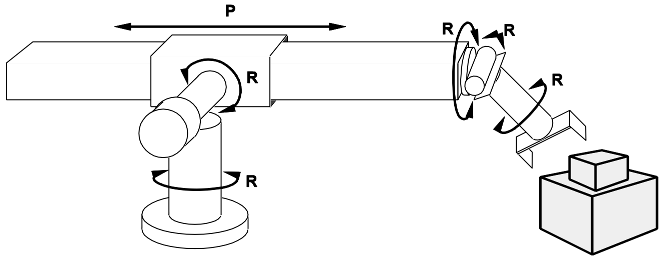
Start State **Goal State**

abstraction

states??: integer locations of tiles (ignore intermediate positions)
actions??: move blank left, right, up, down (ignore unjamming etc.)
goal test??: = goal state (given)
path cost??: 1 per move

[Note: optimal solution of n-Puzzle family is NP-hard]

Example: robotic assembly



states??: real-valued coordinates of
robot joint angles
parts of the object to be assembled

actions??: continuous motions of robot joints

goal test??: complete assembly *with no robot included!*

path cost??: time to execute

Search algorithms

Basic idea: (can look ahead and find impact of each action)

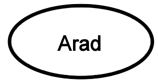
can contemplate how the world would look like. ≡ offline, simulated exploration of state space

by generating successors of already-explored states

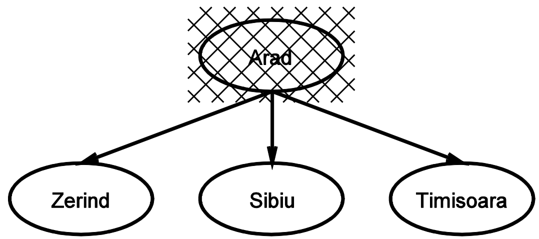
(a.k.a. *expanding* states)

```
function GENERAL-SEARCH( problem, strategy) returns a solution, or failure
    initialize the search tree using the initial state of problem
    loop do
        if there are no candidates for expansion then return failure
        choose a leaf node for expansion according to strategy
        if the node contains a goal state then return the corresponding solution
        else expand the node and add the resulting nodes to the search tree
    end
```

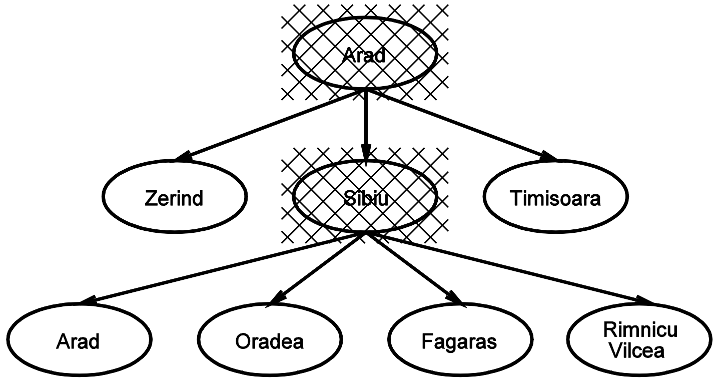
General search example



General search example



General search example

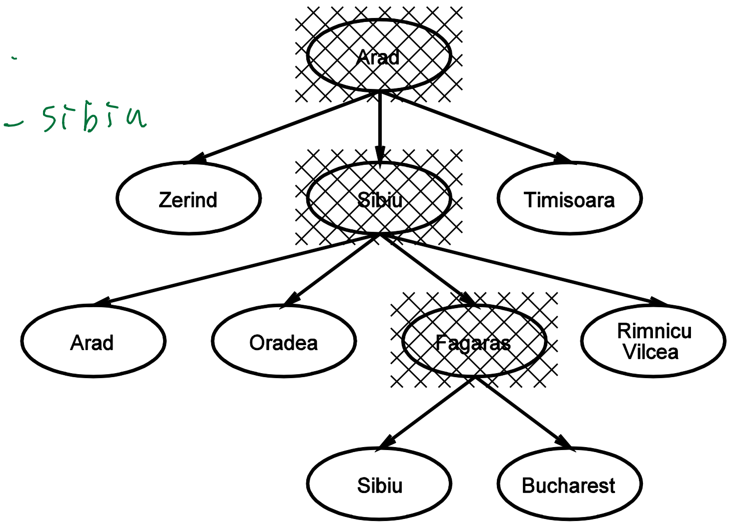


General search example

when the strategy fails.

Arad – sibiu – Arad – sibiu

infinite search space



Implementation of search algorithms

function GENERAL-SEARCH(*problem*, QUEUING-FN) **returns** a solution, or failure

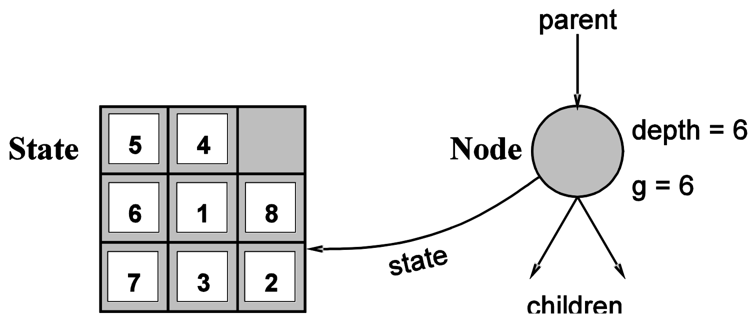
nodes ← MAKE-QUEUE(MAKE-NODE(INITIAL-STATE[*problem*]))
loop do
  if *nodes* is empty **then return** failure
  node ← REMOVE-FRONT(*nodes*)
  if GOAL-TEST[*problem*] applied to STATE(*node*) succeeds **then return** *node*
  nodes ← QUEUING-FN(*nodes*, EXPAND(*node*, OPERATORS[*problem*]))
end

generate states

generate new node

Implementation contd: states vs. nodes

A *state* is a (representation of) a physical configuration
A *node* is a data structure constituting part of a search tree
includes *parent*, *children*, *depth*, *path cost* $g(x)$
States do not have parents, children, depth, or path cost!



The EXPAND function creates new nodes, filling in various fields and using OPERATORS (or ACTIONS) of problem to create the corresponding states.

Search strategies

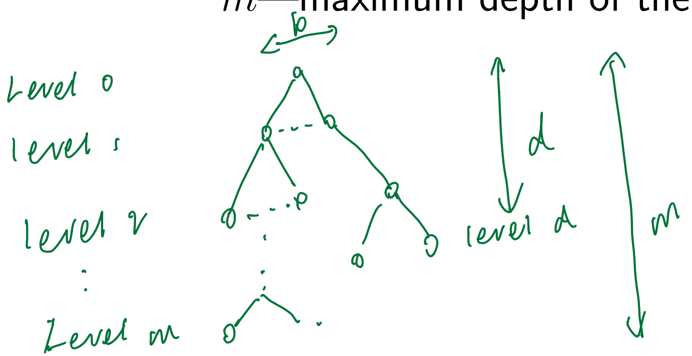
A strategy is defined by picking the *order of node expansion*

Strategies are evaluated along the following dimensions:

- completeness—does it always find a solution if one exists?
- time complexity—number of nodes generated/expanded
- space complexity—maximum number of nodes in memory
- optimality—does it always find a least-cost solution?

Time and space complexity are measured in terms of

- b—maximum branching factor of the search tree
- d—depth of the least-cost solution
- m—maximum depth of the state space (may be ∞)



m: maximum depth

b: maximum branching factor

d: depth of least cost solution

Uninformed search strategies

Uninformed strategies use only the information available in the problem definition

Breadth-first search

Uniform-cost search

Depth-first search

Depth-limited search

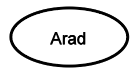
Iterative deepening search

Breadth-first search

Expand shallowest unexpanded node

Implementation:

QueueingFn = put successors at end of queue

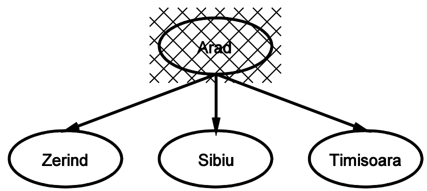


Breadth-first search

Expand shallowest unexpanded node

Implementation:

$\textsc{QueueingFn}$ = put successors at end of queue



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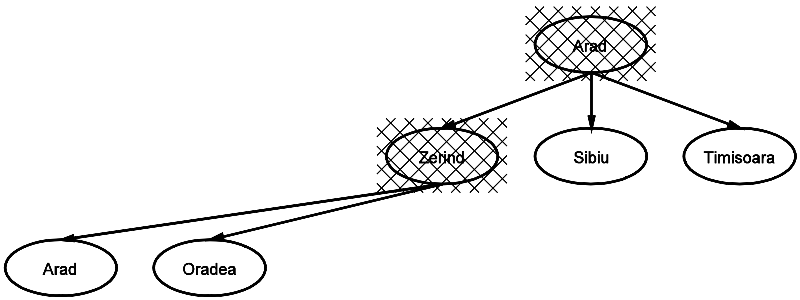

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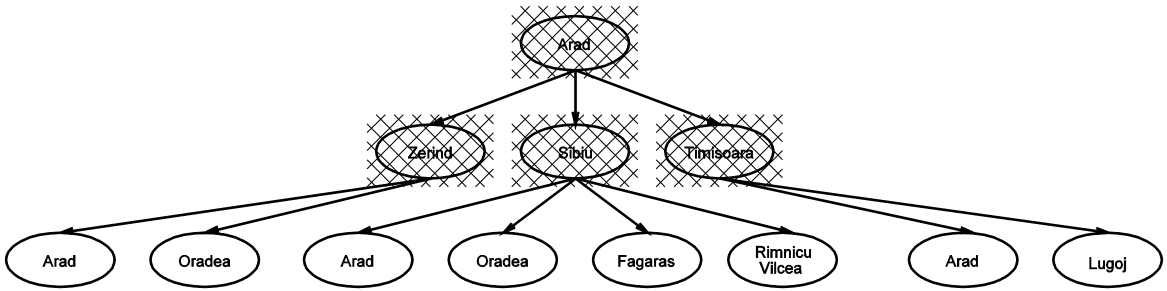


Properties of breadth-first search

Complete??

Time??

Space??

Optimal??

Properties of breadth-first search

Complete?? Yes (if b is finite) search layer by layer

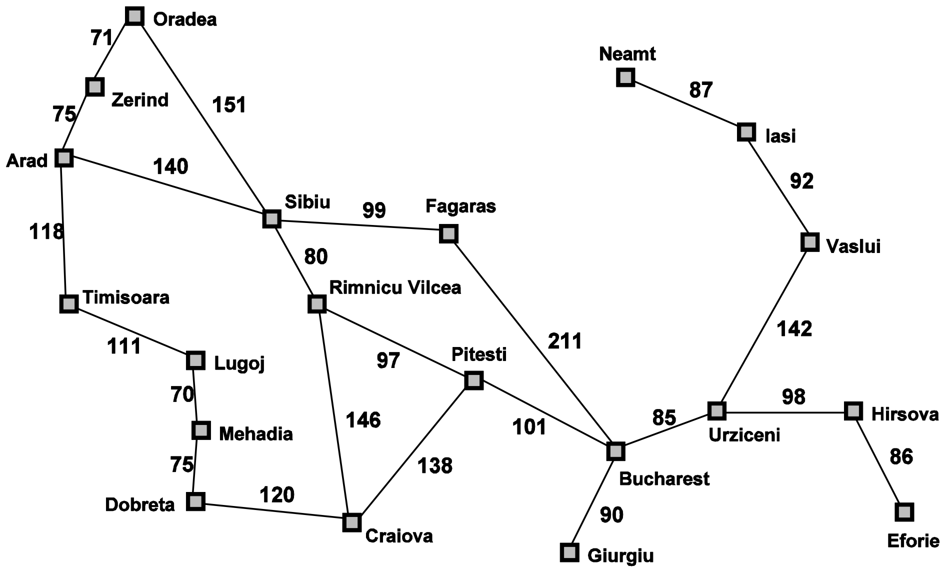
Time?? $1 + b + b^2 + b^3 + \ldots + b^d = O(b^d)$, i.e., exponential in d

Space?? $O(b^d)$ (keeps every node in memory) ⇒ we can delete the explored notes, but we need to keep the unexplored notes

Optimal?? Yes (if cost = 1 per step); not optimal in general

Space is the big problem; can easily generate nodes at 1MB/sec
so 24hrs = 86GB.

Romania with step costs in km



Straight-line distance
to Bucharest

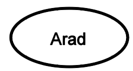
Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	178
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	98
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

Uniform-cost search

Expand least-cost unexpanded node

Implementation:

$\textsc{QueueingFn}$ = insert in order of increasing path cost



Uniform-cost search

Expand least-cost unexpanded node

Implementation:

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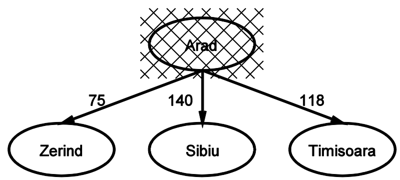
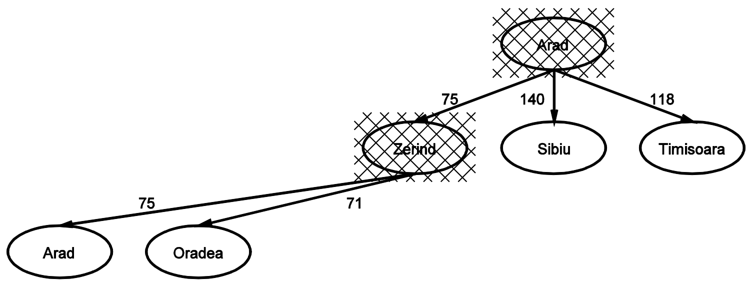


Uniform-cost search

Expand least-cost unexpanded node

Implementation:

$$\textsc{QueueingFn} = \text{insert in order of increasing path cost}$$

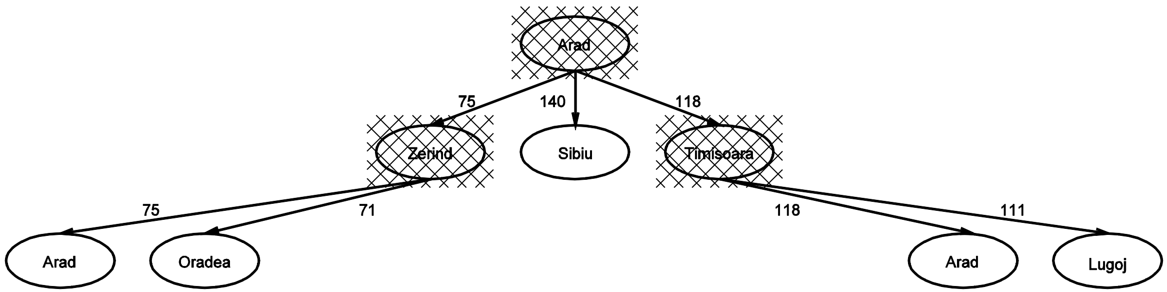


Uniform-cost search

Expand least-cost unexpanded node

Implementation:

$\textsc{QueueingFn}$ = insert in order of increasing path cost



Properties of uniform-cost search

uniform-cost $\xrightarrow{\text{cost } \triangle \text{ or cost} = 1}$ bfs

Complete?? Yes, if step cost $\geq \epsilon$

path cost

Time?? # of nodes with $g \leq$ cost of optimal solution

Space?? # of nodes with $g \leq$ cost of optimal solution

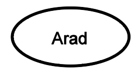
Optimal?? Yes

Depth-first search

Expand deepest unexpanded node

Implementation:

$\textsc{QueueingFn} =$ insert successors at front of queue

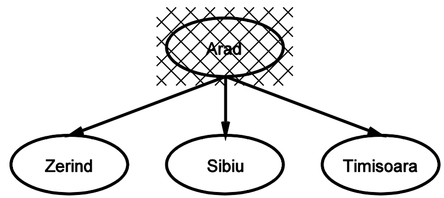


Depth-first search

Expand deepest unexpanded node

Implementation:

$\textsc{QueueingFn}$ = insert successors at front of queue



Depth-first search

Expand deepest unexpanded node

Implementation:

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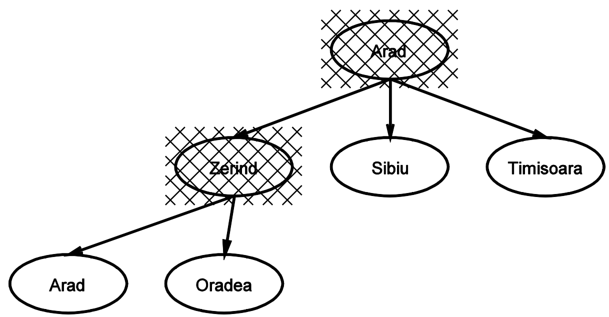


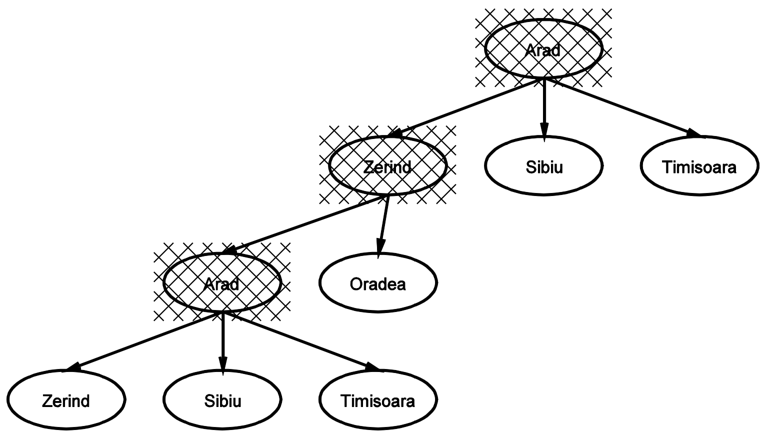
Depth-first search

Expand deepest unexpanded node

Implementation:

QueueingFn = insert successors at front of queue

downside: loop



I.e., depth-first search can perform infinite cyclic excursions

Need a finite, non-cyclic search space (or repeated-state checking)

Properties of depth-first search

Complete??

Time??

Space??

Optimal??

Properties of depth-first search

Complete?? No: fails in infinite-depth spaces, spaces with loops
Modify to avoid repeated states along path
$\Rightarrow$ complete in finite spaces

in worst case, if the solution was in the last branch

Time?? $O(b^m)$: terrible if m is much larger than d
but if solutions are dense, may be much faster than breadth-first

?

Space?? $O(bm)$, i.e., linear space! → keep the information of current branch

Optimal?? No → it retains what it finds first!

Depth-limited search

= depth-first search with depth limit l

Implementation:

Nodes at depth l have no successors

Iterative deepening search

function ITERATIVE-DEEPENING-SEARCH(*problem*) **returns** a solution sequence
    inputs: *problem*, a problem

    for $depth \leftarrow 0$ **to** ∞ **do**
        $result \leftarrow$ DEPTH-LIMITED-SEARCH(*problem*, *depth*)
        if $result \neq$ cutoff **then return** *result*
    end

Properties of iterative deepening search

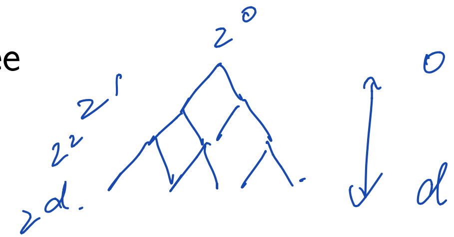
Complete?? Yes

1

Time?? $(d+1)b^0 + db^1 + (d-1)b^2 + \ldots + b^d = O(b^d)$

Space?? $O(bd)$?

Optimal?? Yes, if step cost = 1
Can be modified to explore uniform-cost tree



$d-(d-1)$

Bidirectional Search

Search simultaneously forwards from the start point, and backwards from the goal, and stop when the two searches meet in the middle.

Problems: generate predecessors; many goal states; efficient check for node already visited by other half of the search; and, what kind of search.

work in explicit goal

framework, we can use different search strategy as base

Properties of Bidirectional Search

Complete?? Yes

Time?? $O(b^{\frac{d}{2}})$

Space?? $O(b^{\frac{d}{2}})$

Optimal?? Yes (if done with correct strategy - e.g. breadth first).

Summary

Problem formulation usually requires abstracting away real-world details to define a state space that can feasibly be explored

Variety of uninformed search strategies

Iterative deepening search uses only linear space
and not much more time than other uninformed algorithms

Examples of skills expected:

◇ Formulate single-state search problem

◇ Apply a search strategy to solve problem

◇ Analyse complexity of a search strategy