School of Mathematics and Statistics
MAST20009 Vector Calculus, Semester 1 2020
Assignment 2 and Cover Sheet

Student Name	*Student Number*
Tutor's Name	*Tutorial Day/Time*

Submit your assignment via the MAST20009 website before 11am on Monday 27th April.

- This assignment is worth 5% of your final MAST20009 mark.
- Assignments must be neatly handwritten in blue or black pen on A4 paper. Diagrams can be drawn in pencil.
- Full working must be shown in your solutions.
- Marks will be deducted for incomplete working, insufficient justification of steps, incorrect mathematical notation and for messy presentation of solutions.

1. Consider a rectangular box R with faces parallel to the coordinate planes. Find the dimensions of R with the largest volume that can be inscribed in the ellipsoid

$$x^2 + 2y^2 + 4z^2 = 12.$$

In your solution, you must

(a) Define the variables, function and constraint(s) relevant to this problem.

(b) Use Lagrange Multipliers to solve the constrained extrema problem.

(c) Justify why the dimensions you have found give the maximum volume of R.

2. Consider the path

$$\mathbf{c}(t) = (a\sin t,\, -bt,\, a\cos t)\,, \quad t \geq 0$$

where $a > 0$ and $b > 0$.

In this question you will calculate the curvature κ of the path using two different methods.

(a) Describe in words and/or provide a sketch of the curve traced out by the path $\mathbf{c}$.

(b) Find the unit tangent vector $\mathbf{T}(t)$ to $\mathbf{c}$. Hence compute the curvature of $\mathbf{c}$.

(c) Determine the arclength parameter $s(t)$ for $\mathbf{c}$.

(d) Parametrise the path in terms of arclength s.

(e) Using part (d), find the unit tangent vector $\mathbf{T}(s)$ to $\mathbf{c}$. Hence compute the curvature of $\mathbf{c}$.

3. Let $\boldsymbol{F}(x, y, z)$ be a C^2 vector field in $\mathbb{R}^3$. By direct calculation (without using the vector identities), prove that

$$\boldsymbol{\nabla} \times (\boldsymbol{\nabla} \times \boldsymbol{F}) = \boldsymbol{\nabla}(\boldsymbol{\nabla} \cdot \boldsymbol{F}) - \nabla^2 \boldsymbol{F}.$$