## MAST30001 Stochastic Modelling - 2016

## Assignment 2

If you didn't already hand in a completed and signed Plagiarism Declaration Form (available from the LMS or the department's webpage), please do so and attach it to the front of this assignment.

**Don't forget** to staple your solutions and to print your name, student ID, and the subject name and code on the first page (not doing so will forfeit marks). The submission deadline is **Friday**, **21 October by 4pm** in the appropriate assignment box at the north end of Richard Berry Building (near Wilson Lab).

There are 2 questions, both of which will be marked. No marks will be given for answers without clear and concise explanations. Clarity, neatness and style count.

- 1. Assume that rainstorms arrive in Melbourne according to a Poisson process with rate 1/2 per day, the amount of rain that falls during each storm is exponentially distributed with mean 4 mm, and the amount of rain that falls in separate storms are independent of each other and the times of arrivals of the storms. Let  $(N_t)_{t\geq 0}$  be the number of storms between now and t days from now and t days from now.
  - (a) What is the expected amount of rainfall in Melbourne over the 5 days?
  - (b) What is the probability that over the next 5 days, there are exactly 3 storms, 2 of which have rainfall amounts less than 4 mm, and 1 with a rainfall amount more than 4 mm?
  - (c) What is the probability that over the next 5 days, there are exactly 3 storms, 2 of which have rainfall amounts less than 4 mm, and 1 with a rainfall amount more than 6 mm?
  - (d) Given that there is exactly 1 storm with a rainfall amount more than 6 mm in the next 5 days, what is the expected number of storms with a rainfall amount less than 7 mm over this same time period?
- 2. Customers arrive to a queuing system according to a Poisson process with rate 4 per hour. If there are fewer than 3 people in the queue, then an arriving customer will join the queue, and otherwise will leave the system. At exponential rate 2 (per hour) times, a server arrives and instantaneously serves all customers in the queue (if there is no one in the system, the server does nothing and exits the system).
  - (a) What is the long run proportion of time there is no one in the queue?
  - (b) What is the average number of customers in the system?
  - (c) What is the expected amount of time that customers that enter the system have to wait for service?
  - (d) What is the long run proportion of arriving customers that enter the system?
  - (e) Given an arriving server finds customers in the system, what is the expected number of customers served?
  - (f) If  $X_t$  denotes the number of customers in the system at time  $t \geq 0$ , find  $P(X_t = 0|X_0 = 0)$ .