



**FNCE10002 Principles of Finance**  
**Semester 1, 2019**

**Valuation of Debt Securities**  
**Suggested Answers to Tutorial Questions for Week 3**

*Note that detailed answers to tutorial questions from Part II will only be provided in tutorials. The following abridged answers are intended as a guide to those detailed answers. This policy is in place to ensure that you attend your tutorial regularly and receive timely feedback from your tutor. If you are unsure of your answers you should check with your tutor, a pit stop tutor, online tutor or me.*

*While detailed answers to Part I appear below, if you are not sure of the answers to these questions please ask your tutor in the following week's tutorial.*

**Part I – Answers Submitted to Your Tutor**

**A. Problems**

- A1. a) Amount borrowed =  $200000 - 0.20(200000) = \$160,000$ .  
Monthly interest rate =  $9/12 = 0.75\%$ .  
Total number of months =  $25 \times 12 = 300$  months.  
Monthly payment =  $C$ .

We know that the amount borrowed is the present value of the remaining monthly payments:

$$160000 = \left( \frac{C}{0.0075} \right) \left[ 1 - \frac{1}{(1 + 0.0075)^{300}} \right].$$

$$\text{So, } 160000 = C \times (119.1616).$$

$$\text{Monthly payment, } C = 160000/119.1616 = \$1,342.71.$$

- b) Interest paid in month 1 =  $0.0075 \times 160000 = \$1,200$ .

$$\text{Principal repaid in month 1} = 1342.71 - 1200 = \$142.71.$$

$$\text{Principal balance at the end of month 1} = 160000 - 142.71 = \$159,857.29.$$

Similarly, we can get the loan's amortization schedule for the first 4 months as follows:

Month (1)	Monthly Payment (2)	Monthly Interest (3) = (5) <sub>-1</sub> × 0.75%	Principal Repaid (4) = (2) – (3)	Principal Remaining (5) = (5) <sub>-1</sub> – (4)
0	–	–	–	\$160,000.00
1	\$1,342.71	\$1,200.00	\$142.71	\$159,857.29
2	\$1,342.71	\$1,198.93	\$143.78	\$159,713.50
3	\$1,342.71	\$1,197.85	\$144.86	\$159,568.64
4	\$1,342.71	\$1,196.76	\$145.95	\$159,422.69

(i) The total amount owed at the end of month 4 = \$159,422.69.

(ii) The total interest paid in month 2 = \$1,198.93.

(iii) The total principal repaid in month 3 = \$144.86.

- c) Remaining term of the loan =  $300 - 120 = 180$  months. The loan amount outstanding is the present value of the remaining loan payments, that is:

$$\text{Loan amount outstanding} = \left( \frac{1342.71}{0.0075} \right) \left[ 1 - \frac{1}{(1 + 0.0075)^{180}} \right] = \$132,382.36.$$

- d) The effective annual interest rate (*EAR* or  $r_e$ ) is:

$$r_e = (1 + r/m)^m - 1.$$

$$r_e = (1 + 0.09/12)^{12} - 1 = 9.38\%.$$

- A2. a) The annual coupon is \$8. To find the price, solve for  $P_0$  in:

$$\text{Coupon yield} = 0.09 = 8.00/P_0.$$

$$P_0 = 8.00/0.09 = \$88.89.$$

- b) Coupon yield = Coupon/Price =  $5.00/125.00 = 0.04$  or 4%.

- c) To find the yield to maturity, we solve for  $r_D$  in the following expression:

$$P_0 = 78.12 = 100/(1 + r_D)^{10}.$$

$$(1 + r_D)^{10} = 100/78.12.$$

$$r_D = (100/78.12)^{1/10} - 1 = 2.5\%$$

- (i) A rise in the yield to maturity by 0.5% will result in the price falling to:

$$\text{New } P_0 = 100/(1 + 0.03)^{10} = \$74.41.$$

- (ii) A fall in the yield to maturity by 0.5% will result in the price rising to:

$$\text{New } P_0 = 100/(1 + 0.02)^{10} = \$82.03.$$

As the calculations show, when the yield to maturity rises the bond's price falls, and vice versa. That is, prices and yields are inversely related to each other.

- A3. Annual coupon payment (paid over years 6 – 10) =  $0.10 \times 1000 = \$100$ .  
 Total deferred coupon payments (to be paid at the end of year 10) =  $5 \times 100 = \$500$ .  
 Face value (to be paid at the end of year 10) = \$1,000.

We have a deferred five-year annuity of \$100 and a single cash flow of \$1,500 at the end of year 10. The price today is the present value of the deferred five-year annuity and the present value of the single cash flow in year 10, which is:

$$P_0 = \left[ \left( \frac{100}{0.15} \right) \left( 1 - \frac{1}{(1.15)^5} \right) \right] / 1.15^5 + \frac{1500}{(1.15)^{10}} = \$537.44.$$

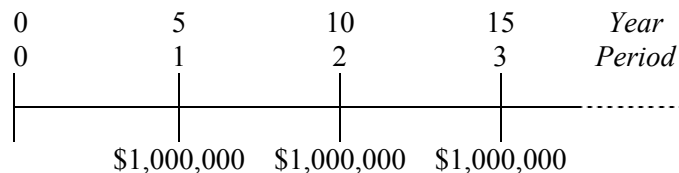
Note that in the above expression,  $\left( \frac{100}{0.15} \right) \left( 1 - \frac{1}{(1.15)^5} \right)$  is the present value of the deferred coupon payment over years 6 – 10 at the end of year 5. This is then discounted over 5 years to year 0.

(Note that in such questions using a time line is very helpful.)

## Part II – Submission of Answers Not Required

### B. Multiple Choice Questions

- B1. B is correct. The effective annual interest rate ( $r_e$ ) is given and we need to solve for the annual percentage rate (that is, the stated interest rate,  $r$ ) using  $1 + r_e = (1 + r/m)^m$ .
- B2. B is correct. The timeline in this case is as follows:



We're given the annual rate, but the cash flows occur every *five* years. So, we first need the effective *five-year* interest rate, which is: 46.933%. As the five-year cash flow is a perpetuity, the present value is \$2,130.697.

- B3. D is correct. The yield to maturity ( $r_D$ ) can be calculated using  $P_0 = \frac{C_1 + F_1}{(1 + r_D)}$ .
- B4. C is correct. Since the bond is selling at the discount, we know that its yield to maturity must be greater than the coupon rate of 12% (why?). So, the only two possible answers are 15% and 16% and all you need to do is plug in one of these in the bond pricing expression to verify whether or not that rate is the yield to maturity.
- B5. D is correct. See your lecture notes or tutorial notes for the explanation.

### C. Problems and Case Studies

- C1. The present value of each alternative is as follows.

*Alternative 2:* As the \$1,850 is paid at the end of every year this is an ordinary annuity. Because the cash flows are annual, and interest is calculated on a monthly basis we first need to calculate the *effective* annual interest rate and then use this rate to obtain the present value of the annual annuity. The effective annual interest rate is 10.47%. The present value is,  $PV_0 = \$9,702.98$ .

*Alternative 3:* Here the monthly interest rate is 0.8333% and the time horizon is 96 months giving a present value of \$9,885.22.

*Alternative 4:* The present value of this amount is \$9,467.24.

So, choose alternative 3 as it has the highest present value.

C2. With semi-annual coupon payments we have the following new information:

Coupon payment,  $C = 0.0395/2 \times 100 = \$1.975$  every six months.

Time periods,  $n = 40 \times 2 = 80$  six-month periods.

- a) The yield to maturity is 2.25% per six months, resulting in a price today of \$89.839.
- b) The yield to maturity is 1.75% per six months, resulting in a price today of \$109.648.
- c) Refer to your tutorial notes.
- d) In one year's time (end of 2018), the time horizon of the issue is 38 years and the coupon payment remains the same.

At  $r_D = 4.5\%$ , the notes continue to trade at a discount to their face value and the price rises to \$90.073.

At  $r_D = 3.5\%$ , the notes continue to trade at a premium to their face value and the price falls to \$109.378.