MAST30027: Modern Applied Statistics

Week 11 Lab Sheet

Suppose that $\mathbf{X} = \begin{pmatrix} X_1 \\ X_2 \end{pmatrix} \sim N(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, with $\boldsymbol{\mu} = \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix}$ and $\boldsymbol{\Sigma} = \begin{pmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{pmatrix}$.

1. Show that the conditional distribution of $X_1 | X_2 = x_2$ is normal with mean $\mu_1 + (x_2 - \mu_2)\sigma_{12}/\sigma_2^2$ and variance $\sigma_1^2 - \sigma_{12}/\sigma_2^2$.

2. Write an R function that uses the Gibbs sampler to generate a sample of size $n = 1000$ from the $N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 4 & 1 \\ 1 & 4 \end{pmatrix}\right)$ distribution. Run at least two Gibbs sampling chains with different initial values. Make trace plots for X_1 and X_2 and see if samples from different chains are mixed well and behave similarly.

3. Use your simulator to estimate $\mathbb{P}(X_1 \geq 0, X_2 \geq 0)$. To get a feel for the convergence rate, calculate the estimate using samples $\{1, \ldots, k\}$, for $k = 1, \ldots, n$, and then plot the estimates against k.

4. Now change $\boldsymbol{\Sigma}$ to $\begin{pmatrix} 4 & 2.8 \\ 2.8 & 4 \end{pmatrix}$ and generate another sample of size 1000.

   What do the traces/estimates look like now?