PHYC10003 Physics 1

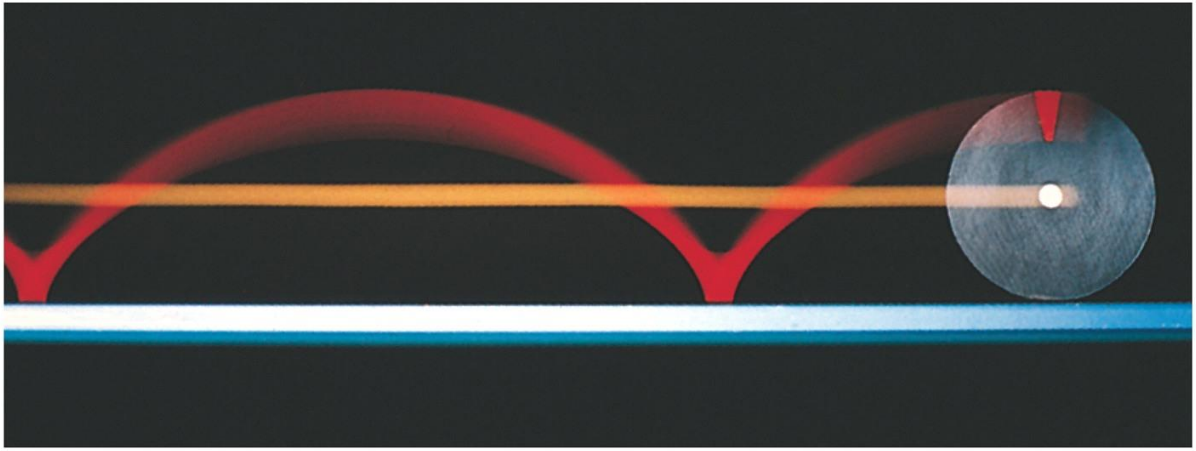
Lecture 15: Rotational Motion

Translation, rotation and rolling

Last Lecture

- Rotational Inertia
- Rotational kinetic energy
- Parallel axis theorem
- Torque
- Newton's Second Law in angular form

Rotation and translation combined



Richard Megna/Fundamental Photographs

Figure 11-2

Rolling objects

- We consider only objects that roll smoothly (no slip)
- The center of mass (com) of the object moves in a straight line parallel to the surface
- The object rotates around the com as it moves
- The rotational motion is defined by:

$$s = \theta R,$$ **Eq. (11-1)**

$$v_{com} = \omega R$$ **Eq. (11-2)**

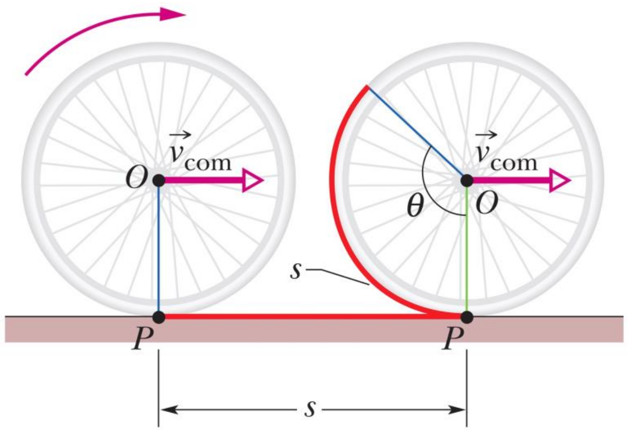


Figure 11-3

Rolling – translation and rotation

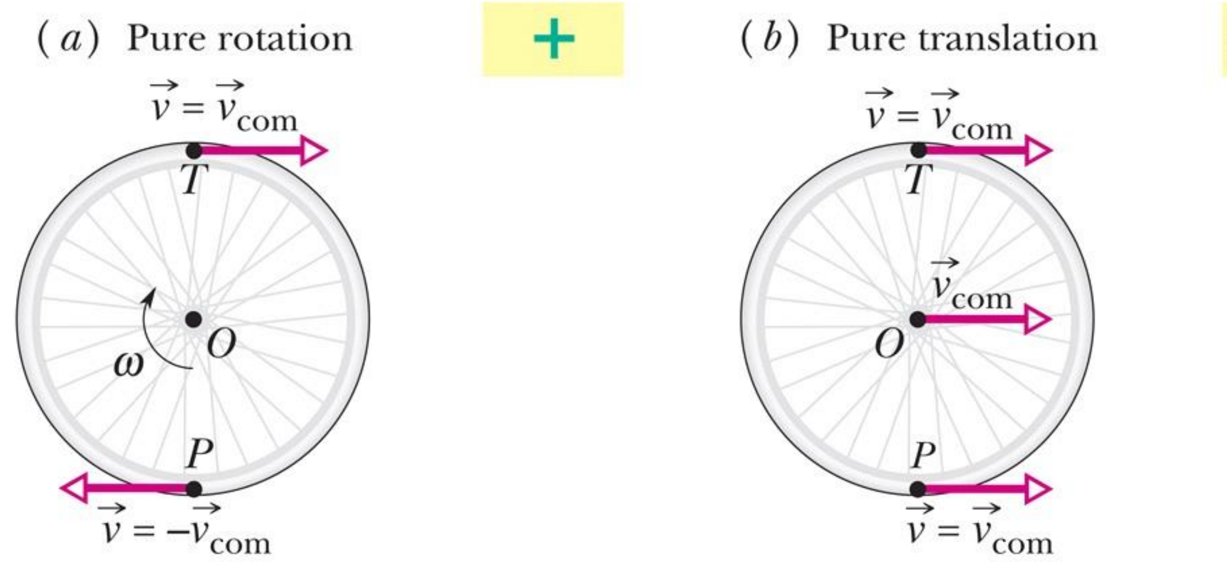
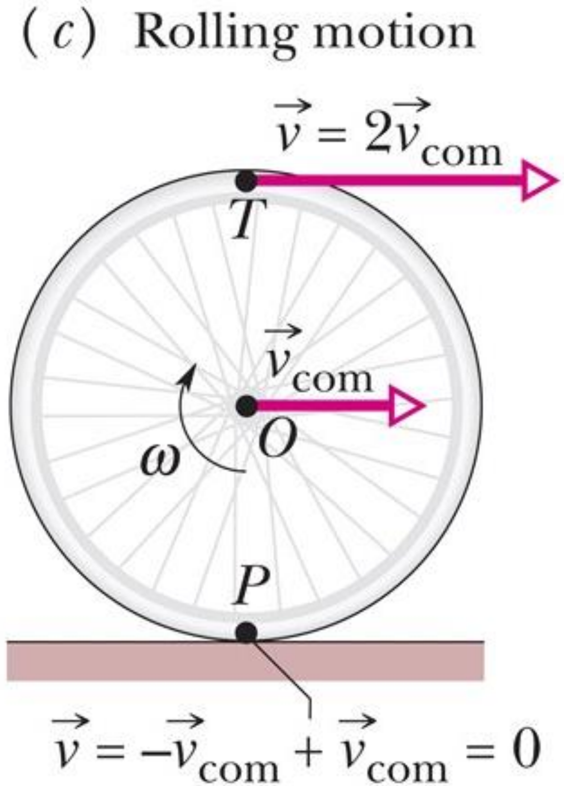


Figure 11-4

If the object is rolling, then the lower point in contact with the surface is stationary and has a velocity equal in magnitude and opposite in direction to the upper point

Rolling-translation and rotation



- The velocity of a point at the top of the disk is the vector sum of the velocity of the surface relative to the centre of mass and the velocity of the centre of mass relative to the observer (us).
- The velocity of the point in contact with the surface must be zero (no slipping).

Translational and rotational energy

- Combine translational and rotational kinetic energy:

$$K = \tfrac{1}{2} I_{com}\omega^2 + \tfrac{1}{2} M v_{com}^2.$$

Eq. (11-5)

A rolling object has two types of kinetic energy: a rotational kinetic energy ($\tfrac{1}{2} I_{com}\omega^2$) due to its rotation about its center of mass and a translational kinetic energy ($\tfrac{1}{2} M v_{com}^2$) due to translation of its center of mass.

- If a wheel accelerates, its angular speed changes
- A force must act to prevent slip

$$a_{com} = \alpha R$$

Eq. (11-6)

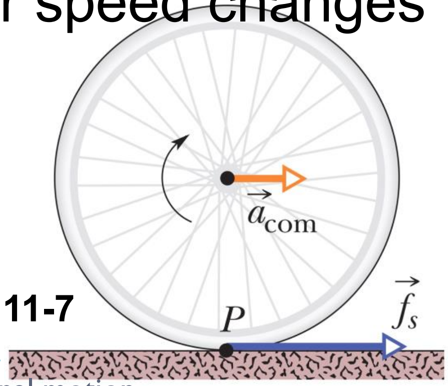


Figure 11-7

Rolling with friction

- If slip occurs, then the motion is *not* smooth rolling!
- For smooth rolling down a ramp:
  1. The gravitational force is vertically down
  2. The normal force is perpendicular to the ramp
  3. The force of friction points up the slope

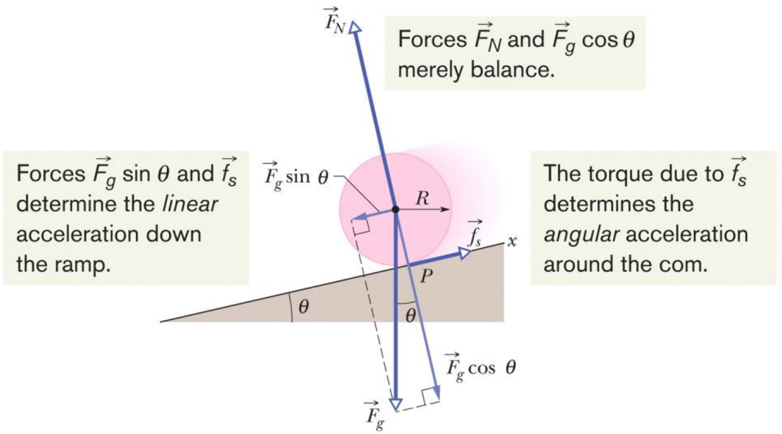


Figure 11-8

Rolling with friction

Write the moment of inertia as $I=cMR^2$, where M is the mass and

- C=0 is a point particle
- C=2/5 is a sphere of radius R
- C=1 is a hoop of radius R

Can accommodate anything else that rolls by adjusting c

A point particle sliding with no friction behaves similarly to a very small particle rolling with friction, because there is no accumulation of rotational motion if the moment of inertia is very small.

Rolling with friction- speed

Conservation of energy. At the bottom of the ramp, vertical height h:

$$\frac{1}{2}I_{cm}\omega^2+\frac{1}{2}Mv_{cm}^2=Mgh$$

Substitute $\mathrm{v}_{cm}=\omega R$

$$\frac{1}{2}cMR^2\left(\frac{v_{cm}}{R}\right)^2+\frac{1}{2}Mv_{cm}^2=\frac{1}{2}M\left(1+c\right)v_{cm}^2=Mgh$$

$$\Rightarrow v_{cm}=\sqrt{\frac{2gh}{1+c}}$$

Result is independent of *M* and *R*, depending only on geometry of object through c

Rolling with friction- acceleration

From linear kinematics with constant acceleration:

$$v_{cm}^2 = 2a_{cm}\Delta x, \quad \text{where } \Delta x = h/\sin\theta$$

Therefore

$$a_{cm} = \frac{v_{cm}^2}{2\Delta x} = \left(\frac{2gh}{1+c}\right)\frac{\sin\theta}{2h} = \frac{g\sin\theta}{1+c}$$

A frictionless sliding point has acceleration $a_{particle}$=g sin θ, so

$$a_{cm} = \frac{a_{particle}}{1+c} \quad \text{for } c > 0$$

Rolling with friction

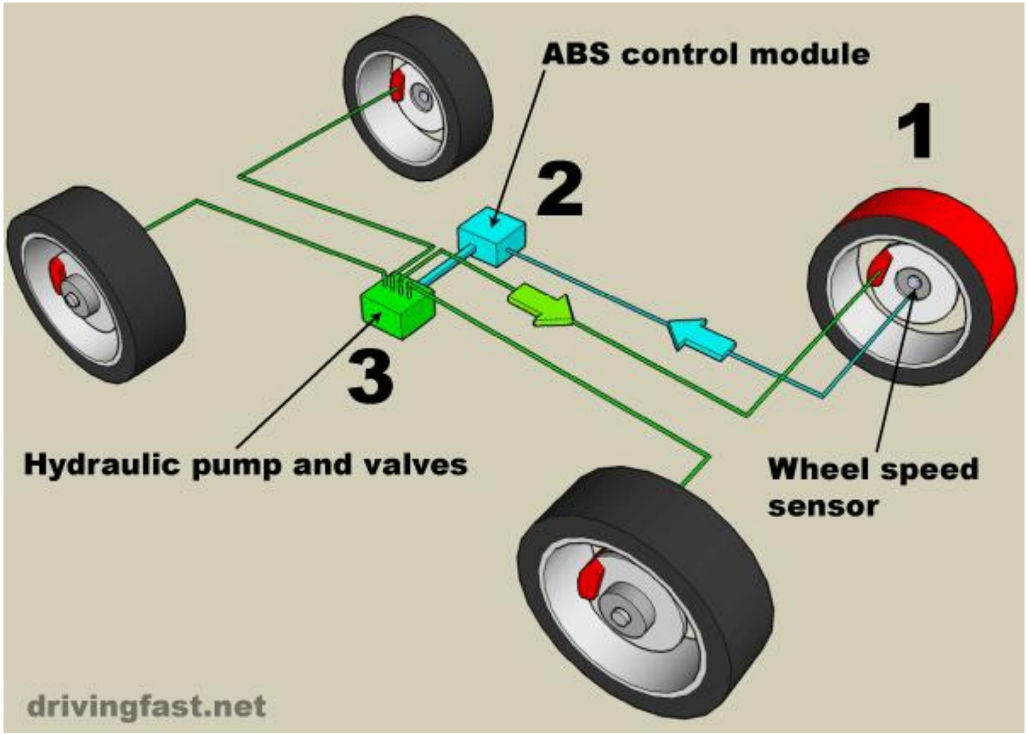
Compare the acceleration of a frictionless particle with that of an object with non-zero moment of inertia

$$a_{cm} = \frac{a_{particle}}{1+c} \quad \text{for } c > 0$$

Imagine that we release a point particle, a sphere and a hoop (all of the same mass) at the top of an incline plane, height h, with friction.

- For any θ the point particle will reach the bottom of the plane first.
- Of the other objects, the sphere will have the greatest acceleration, and the hoop will have the least acceleration.
- The hoop converts more potential energy into rotational kinetic energy than translational kinetic energy.

(ABS) Anti lock brake systems



Slipping can be dangerous!

ABS senses the wheel speed and releases the brakes (in short bursts) so that the tyre speed matches the road speed.

Experienced drivers are (sometimes) able to affect this by rapidly varying brake pedal pressure.

Rotation and the yo-yo

- As a yo-yo moves down a string, it loses potential energy *mgh* but gains rotational and translational kinetic energy
- To find the linear acceleration of a yo-yo accelerating down its string:
  1. Rolls down a “ramp” of angle 90°
  2. Rolls on an axle instead of its outer surface
  3. Slowed by tension *T* rather than friction

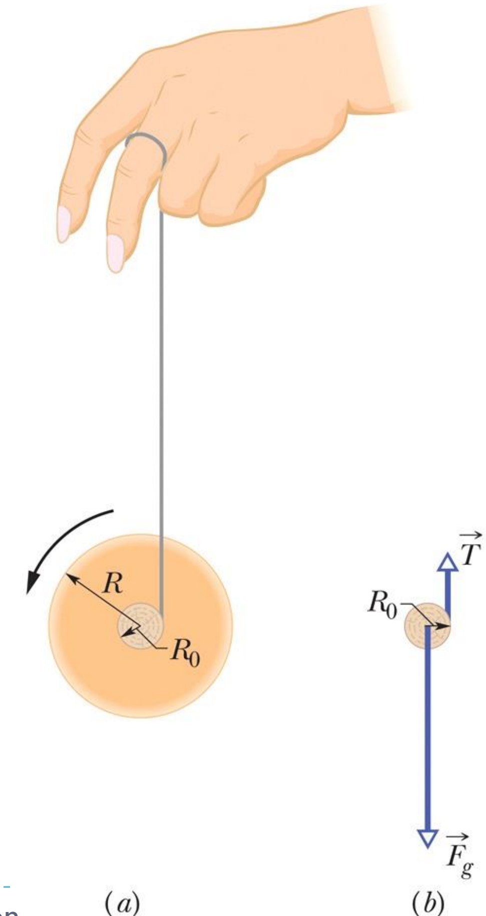


Figure 11-9

Acceleration of the yo-yo

- Replacing the values in 11-10 leads us to:

$$a_{\text{com}} = -\frac{g}{1 + I_{\text{com}}/MR_0^2}, \quad \textbf{Eq. (11-13)}$$

Example Calculate the acceleration of the yo-yo

- M = 150 grams, R_0 = 3 mm, I_{com} = $Mr^2/2$ = 3E-5 kg m^2
- Therefore a_{com} = -9.8 m/s^2 / (1 + 3E-5 / (0.15 * 0.003^2))
  = - .4 m/s^2

Torque – generalization

- Previously, torque was defined only for a rotating body and a fixed axis
- Now we redefine it for an individual particle that moves along any path relative to a fixed point
- The path need not be a circle; torque is now a vector
- Direction determined with right-hand-rule

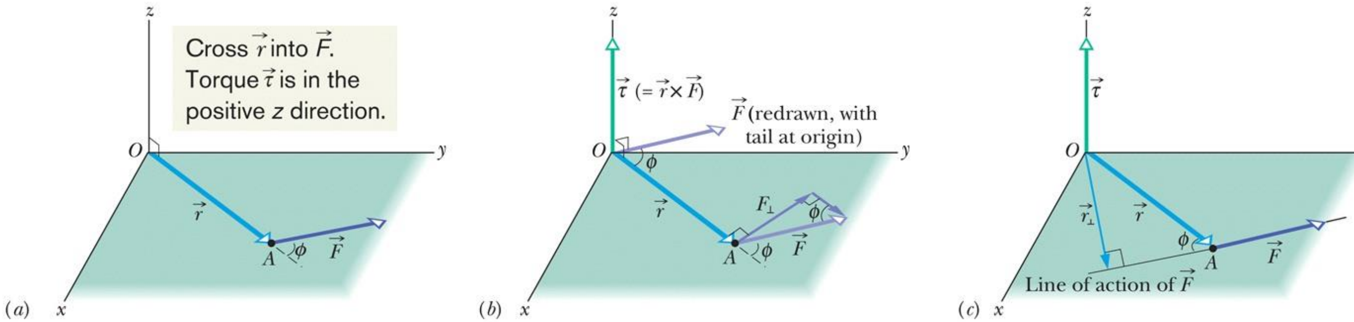


Figure 11-10

Torque - generalization

- The general equation for torque is:

$$\vec{\tau} = \vec{r} \times \vec{F}$$ **Eq. (11-14)**

- We can also write the magnitude as:

$$\tau = rF \sin \phi,$$ **Eq. (11-15)**

- Or, using the perpendicular component of force or the moment arm of *F*:

$$\tau = rF_{\perp},$$ **Eq. (11-16)**

$$\tau = r_{\perp}F,$$ **Eq. (11-17)**

Calculation of the vector product

Example: $\mathbf{A} = 2\mathbf{i} + 3\mathbf{j}$

$$\mathbf{B} = 2\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$$

$$(\mathbf{A}\times\mathbf{B}) = 4(\mathbf{i}\times\mathbf{i}) + 6(\mathbf{i}\times\mathbf{j}) + 4(\mathbf{i}\times\mathbf{k}) + 6(\mathbf{j}\times\mathbf{i}) + 9(\mathbf{j}\times\mathbf{j}) + 6(\mathbf{j}\times\mathbf{k})$$

Unit vector properties

$$\mathbf{i}\times\mathbf{i} = \mathbf{j}\times\mathbf{j} = \mathbf{k}\times\mathbf{k} = 0$$

$$\mathbf{i}\times\mathbf{j} = -\mathbf{j}\times\mathbf{i} = \mathbf{k} \text{ with cyclic permutation}$$

Hence

$$\mathbf{A}\times\mathbf{B} = 6\mathbf{i} - 4\mathbf{j}$$

Torque-vector product (general result)

$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix}$$

$$= (\mathrm{yF}_z - zF_y)\mathbf{i} + (zF_x - xF_z)\mathbf{j} + (xF_y - yF_x)\mathbf{k}$$

- Can always calculate the torque directly from the components of **r** (*x,y,z*) and the components of **F** (F_x, F_y, F_z), in terms of orthogonal unit vectors **i**, **j**, **k**.
- The torque is always a vector that is perpendicular to both r and F, with a direction determined by the right-hand rule.

Summary

Rolling Bodies

$$v_{\text{com}} = \omega R$$ **Eq. (11-2)**

$$K = \tfrac{1}{2} I_{\text{com}} \omega^2 + \tfrac{1}{2} M v_{\text{com}}^2.$$ **Eq. (11-5)**

$$a_{\text{com}} = \alpha R$$ **Eq. (11-6)**

Torque as a Vector

- Direction given by the right-hand rule

$$\vec{\tau} = \vec{r} \times \vec{F}$$ **Eq. (11-14)**

Angular Momentum of a Particle

$$\vec{\ell} = \vec{r} \times \vec{p} = m(\vec{r} \times \vec{v})$$

Eq. (11-18)

Newton's Second Law in Angular Form

$$\vec{\tau}_{\text{net}} = \frac{d\vec{\ell}}{dt}$$ **Eq. (11-23)**