THE UNIVERSITY OF MELBOURNE

FNCE10002 Principles of Finance

Semester 1, 2019

Introduction to Financial Mathematics II

Tutorial Questions for Week 2

This tutorial is divided into two parts. The answers to the questions in Part I need to be submitted at the ***beginning*** *of your tutorial. All answers must be* ***handwritten*** *and in* ***original*** *(photocopies/emails will not be accepted). Please follow the instructions on the Tutorial Hand-in Sheet available on the LMS via the Tutorials link. The answers to questions in Part II do not need to be submitted and will be discussed in your tutorial. Please make sure that you have worked through these questions and are prepared to discuss them if called upon by your tutor.*

Note that questions flagged as "EXM" are past exam questions that I've used in this subject or subjects similar in scope to this subject, while those flagged as "TXT" are sourced from the textbook. Detailed answers to the questions in Part II will only be provided in tutorials. Brief answers may be provided via the LMS after a time lag. This policy is in place to ensure that you attend your tutorials regularly and receive timely feedback from your tutor. If you are unsure of any answer you should check with your tutor, a pit stop tutor, online tutor or me.

Part I: Answers to be Submitted to Your Tutor

A. Problems

A1.[EXM] You have just won the *First Annual Prime Minister Look-Alike* contest and have been offered the following alternative ways of receiving the prize money. Assume that each alternative is riskfree (that is, the cash flows are certain to occur) and the interest rate is 8% per annum.

a) $140,000 at the end of year 3.
b) $28,000 at the end of each of the next 5 years with the first cash flow occurring at the end of year 1. $PV_0(OA) = \frac{c}{r}[(1+r)^n - 1]$
c) $9,000 at the end of each year in perpetuity with the first cash flow occurring at the end of year 1.
d) $12,000 at the end of each year in perpetuity with the first cash flow occurring at the end of year 4.

Assuming end-of-the-year cash flows, which is the *best* way to receive the prize money? *Show all your calculations*.

A2. Refer to the case study "No Latte for You!" discussed in class. We made the simplifying assumption that the cost of a latte would not increase over time. Now assume that you drink a $4 latte a day and over the month the cost is $80. Assume that the cost of the latte is expected to increase at a rate of 6% per annum, or 0.5% per month forever. As before, you choose to forgo your daily latte and instead invest this (now growing) amount at the end of each month in an investment fund that earns an interest rate of 12% p.a. What is the value of your growing investment at the end of: (a) 10 years and (b) 50 years? What are the present values of your investments? *Round your final answers to the nearest dollar.*

A3. Refer to the case study "To Charge or Not to Charge" discussed in class where at the end of her time horizon your friend owed \$58,126 on her credit card. Calculate the amount that your friend would have to repay every month for her to able to repay this credit card debt by the end of: (a) month 24 and (b) month 36. *Show all calculations.*

Part II: Submission of Answers Not Required

B. Multiple Choice Questions

For each question pick the *most reasonable* response based *only* on the information provided.

B1.[EXM] You friend just seen the following advertisement at her local bank: "Deposit \$40,000 today and receive \$2,000 every year growing at a constant annual rate forever." Assume the bank pays an interest rate of 8% per annum and the first cash flow you receive from the bank is at the end of year 1. The implied annual growth rate in these cash flows in **closest** to:

b

growing perpetuity

0 1 2 3 ...
-40000 C_1 C_2 C_3

$r=8\%$.

a) –3%.
b) 3%.
c) 5%.
d) 8%.

$\frac{2000}{8\%-g} = 40000$

$PV_0 = \frac{C_1}{r-g}$

$40000 = \frac{2000}{8\%-g}$ $8\%-g = \frac{1}{20}$ $g=0.03$.
$0.08-g=0.05$

B2.[EXM] Assume that you are 25 years old and decide to start saving for your retirement. You plan to save \$5,000 at the end of each year (so the first deposit will be made one year from now), and will make the last deposit when you retire at age 65. Suppose you can earn 8% per annum on your retirement savings. The amount you will have saved for your retirement is **closest** to: $r=8\%$.

ordinary Annuity

b.

25 26 65
0 C C C...C.

$FV(OA) = (\frac{C}{r})[(1+r)^n-1]$
$= \frac{5000}{8\%}[(1+8\%)^{40}-1]$

a) \$1,194,706.
b) \$1,295,283.
c) \$1,398,905.
d) \$1,403,905.

$\frac{5000}{8\%}[(1+8\%)^{40}-1]$

B3.[TXT] Consumer Insurance, Inc. sells extended warranties on appliances that provide coverage after the manufacturers' warranties expire. An analyst for the company forecasts that the company will have to pay warranty claims of \$5 million per year for three years, with the first costs expected to occur four years from today. The company wants to set aside a lump sum today to cover these costs and money invested today will earn an interest rate of 10% per annum. Assuming end-of-the-year cash flows, the amount that the company needs to invest today is **closest** to: $r=10\%$

b

Deferred annuity

0 1 2 3 4 5 6 7
\$5 million

a) \$3,756,574.
b) \$9,342,044.
c) \$11,907,834.
d) \$12,434,260.

$5{,}000{,}000[(\frac{1}{1+r})^4+(\frac{1}{1+r})^5+(\frac{1}{1+r})^6]$

B4.[EXM] An investor expects to receive \$20,000 over the next four years where the cash flows are to be received at the beginning of each year. If the interest rate is 6% p.a. compound annually the future value of these cash flows at the end of year 4 is **closest** to: $r=6\%$

d

annuity due

-1 0 1 2 3 4

$FV=[\frac{C}{r}-\frac{C}{r}(\frac{1}{1+r})^4](1+r)$

$= \frac{c}{r}[1-(\frac{1}{1+r})^4](1+r)^5$
$= \frac{c}{r}[(1+r)^5-(1+r)]$

a) $69,302.
b) $73,460.
c) $87,492.
d) $92,742.

$\frac{c}{r}(1-\frac{1}{(1+r)^4})(1+r)$

B5. You are thinking of building a new machine that will save you $1,000 in the first year. The machine will then begin to wear out so that these savings will decline at a rate of 2% per annum forever. If the interest rate is 5% per annum, the present value of the savings is **closest** to:

b

"growing" $g=-2\%$

$r=5\%$ perpetuity

a) $14,000.
b) $14,286.
c) $20,000.
d) $33,333.

$g=-2\%$
$r=5\%$.

$PV = \frac{C_1}{r-g} = \frac{1000}{7\%} = \$14285.7$

C. Problems

C1.[TXT] Ed Lowman, the 20-year-old star opening batsman of his university cricket team, is approached about skipping his last two years of his four-year university degree and entering the professional cricket sports industry. Ed expects that his cricket career will be over by the time he is 32 years old. Talent scouts for regional cricket teams estimate that Ed could receive a signing bonus of $1 million today, along with a five-year contract for $3 million per year (payable at the end of each year). They further estimate that he could negotiate a contract for $5 million per year for the remaining seven years of his career. The scouts believe, however, that Ed will be a better selection for the Australian Test team if he improves by playing two more years of university cricket. If he stays at university, he is expected to receive a $2 million signing bonus in two years, along with a five-year contract for $5 million per year. After that, the scouts expect Ed to obtain a five-year contract for $6 million per year to take him into retirement. Assume that Ed can earn a 10% per annum return over this time. Should Ed stay or go? (*Hint:* Use a timeline to help you visualize the cash flows.)

5

ordinary

$r=0.1$

专利

C2.[EXM] You work for a pharmaceutical company that has developed a new drug. The patent on the drug will last 17 years. You expect that the drug's profits will be $2 million in its first year (that is, end of year 1) and that this amount will grow at a rate of 5% per annum until the patent expires. Once the patent expires, other pharmaceutical companies will be able to produce the same drug and competition will likely drive profits to zero. Assume that the interest rate is 10% per annum. (*Hint:* Use a timeline to help you visualize the cash flows.)

growing original Annuity

$r=10\%$

0 1 2 3 4 ... 17
2, 2×1.05, 2×1.05^2 ... 2×1.05^{16}

a) Calculate the present value of the profits from the new drug.

$PV_0(GA) = \frac{C_1}{r-g}(1-(\frac{1+g}{1+r})^n) = \frac{2{,}000{,}000}{5\%}(1-(\frac{1.05}{1.1})^{17}) = \$21{,}861{,}455.79$

b) What perpetually growing profits (in year 1) would give you the same present value as that calculated in part (a)?

$\frac{C_1}{r-g} = PV_0(GA)$ $C_1 = PV_0(GA)[r-g] = \$21{,}861{,}455.8 \times (10\% - 5\%) = \$1093{,}072.79$

C3. Refer to the case study "Thanks Aunty!" discussed in class. Suppose you wished to have $3 million by the end of year 50 rather than the amount we calculated in class. What initial contribution would you have wished your aunty had started with, so you had $3 million at the end of 50 years? How does this amount change if your aunty's contributions grow at 3% per annum rather than 5% per annum assumed in the case? *Show all calculations.*

Q1

Q2

$FV_{50} = \$3{,}000{,}000$ $g=5\%$ p.a $r=10\%$ growing ordinary annuity

Q1: $FV(GA) = \frac{C_1}{r-g}[1-(\frac{1+g}{1+r})^n](1+r)^n$

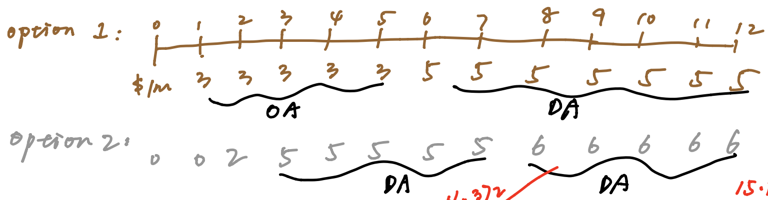
$\$3{,}000{,}000 = \frac{C_1}{5\%}[1-(\frac{1.05}{1.1})^{50}](1.1)^{50}$

$C_1 = \$1416.11$

Q2: $FV(GA) = \frac{C_1'}{r-g'}[1-(\frac{1+g'}{1+r})^n](1+r)^{50}$

$\$3{,}000{,}000 = \frac{C_1'}{7\%}[1-(\frac{1.03}{1.1})^{50}](1.1)^{50} \Rightarrow C_1' = \$1858.29$

Q1



11.372 15.111

for option 1: $PV = 1 + \left(\frac{3}{0.1}\right)\left(1-\left(\frac{1}{1.1}\right)^5\right) + \frac{5}{0.1}\left(1-\left(\frac{1}{1.1}\right)^7\right)\left(\frac{1}{1.1}\right)^5$

$= \$27$ million 27.4864 million

for option 2: $PV = \frac{2}{1.1^2} + \frac{5}{0.1}\left(1-\left(\frac{1}{1.1}\right)^5\right)\left(\frac{1}{1.1}\right)^2 + \frac{6}{0.1}\left(1-\left(\frac{1}{1.1}\right)^5\right)\left(\frac{1}{1.1}\right)^7$

$= \$28.9889$ million

choose option 2 $\boxed{\text{stay}}$.