

MAST10008 Assignment 6

Due Thursday 23 May at 12pm in your tutor's assignment box

1. Let A be an $n \times n$ matrix over \mathbb{F}_2 that is both invertible and diagonalisable. Prove that A is the identity matrix I_n .

2. Consider the quadric surface defined by the equation

$$3x^2 + 3y^2 + 3z^2 - 3xy - 4yz = 66.$$

Use diagonalisation to identify the type of this quadric, according to the list appearing at https://en.wikipedia.org/wiki/Quadric#Euclidean_space

3. Given the following matrix:

$$A = \begin{bmatrix} 2 & 1 + 3i \\ i & -1 \end{bmatrix}$$

- (a) Check that $\lambda = -i$ is an eigenvalue and find the other eigenvalue.
 - (b) Find the linearly independent eigenvectors of A .
 - (c) Find an invertible matrix P and a diagonal matrix D such that $A = PDP^{-1}$.
4. Classify (all) the critical points of the “egg carton surface¹” that is the graph of the function $f: \mathbb{R}^2 \rightarrow \mathbb{R}$ given by

$$f(x, y) = \sin(x) \sin(y).$$

¹Not the technical name—I just made it up. Want to know why? Use your favourite mathematical software (or just visit WolframAlpha) to graph it.