Introductory Macroeconomics

Pre-Tutorial #8
Week Starting 3rd May 2021

The Tutorial. This week's tutorial looks at economic growth.

Note that your tutor is under no obligation to go through the answers to the pre-tutorial work in detail. The focus in the tutorial will be on the tutorial work itself – the questions here are preparatory.

Reading Guide. You should look carefully over lectures 15 and 16. You may also find Chapter 14 and 15 of BOFAH useful.

Key Concepts. Growth accounting. Labour/Capital share of income. Solow-Swan model.

Problems.

1. The Republic of Ostralaya has experienced a 3 per cent increase in output per worker this year and a 1 per cent rise in capital per worker. Assuming a Cobb-Douglas production function where capital income accounts for 30 per cent of GDP, calculate how much output per worker growth is explained by capital per worker and total factor productivity.

2. Consider the Solow-Swan model.

   (a) What is meant by the term steady state? Provide an interpretation of the following equation:
   $$\theta \frac{Y_t}{L_t} = \left(\frac{K_{t+1}}{L_{t+1}} - \frac{K_t}{L_t} \right) + (d+n) \frac{K_t}{L_t}$$
   .

   (b) What happens to the capital-labour ratio in steady state? Will K be changing in the steady state? If so, at what rate?

3. What would be the implications for Australia's steady-state per capita capital stock of a rise in the saving rate?

4. The income approach to National Accounts implies:
   $$Y = wL + (r + \delta)K,$$
   where w is the wage, r the interest rate, and δ the capital depreciation rate.

   (a) Provide an economic interpretation to the above equation.

   (b) Assume that the economy is described by a Cobb-Douglas production function. Moreover, assume that the output price (parameter p in the lecture note) is 1. These assumptions imply that the wage is equal to the marginal product of labour and the interest rate plus the depreciation rate is equal to the marginal product of capital. Find expressions for the labour share of income, wL/Y and the capital share of income, $(r + \delta)K/Y$. What determines the labour and capital share in this economy?

Solutions to Pre-Tutorial Work.

1. We apply our standard growth accounting formula:

$$\frac{\frac{Y_t}{L_t} - \frac{Y_{t-1}}{L_{t-1}}}{\frac{Y_{t-1}}{L_{t-1}}} = \frac{A_t - A_{t-1}}{A_{t-1}} + \alpha \frac{\frac{K_t}{L_t} - \frac{K_{t-1}}{L_{t-1}}}{\frac{K_{t-1}}{L_{t-1}}}$$

   The growth rate in output per worker equals the growth rate in technology plus the growth rate in capital per person multiplied by the share of capital income out of total income, α. In this setting, $0.03 = \frac{A_t - A_{t-1}}{A_{t-1}} + \underbrace{0.3 \cdot 0.01}_{0.003}$. Solving implies $\frac{A_t - A_{t-1}}{A_{t-1}} = 0.027$. Our economic interpretation is that capital per worker is responsible for 0.3 percent and technology for 2.7 percent growth in output per worker.

2. (a) The equation implies that the level of savings in per worker terms is equal to changes in the capital per worker (net investment) plus the amount of investment per worker required to keep the level of capital per worker constant (replacement investment).

   (b) In steady state there are no changes in the capital-labour ratio. In the standard model, population size is increasing at a rate given by n. We know that K must grow at the same rate as n otherwise the capital-labour ratio would not be constant.

3. We can answer this by referring to Figure 1. An increase in the saving rate shifts up the saving curve. The level of investment required to maintain the capital-labour ratio, given by the $(n + d)K/L$ line remains unchanged. The effect is that there is an increase in the steady state level of capital per worker from $(K/L)^*$ to $(K/L)^{**}$. Note from the production function we can see that there is a positive relationship between capital per worker and output per worker, so we know that the level of output per worker will also increase.

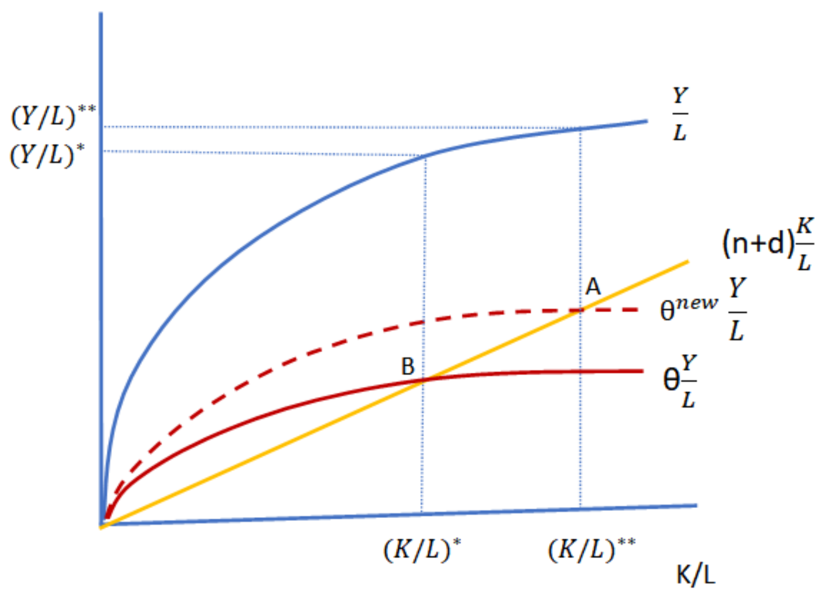

Figure 1: Impact of an increase in the saving rate

4. (a) The National Accounts approach to accounting emphasises that output (GDP), Y, can be measured using a production, an expenditure, or an income approach. Using the income approach, we know that total output will equal the amount paid to labour plus the amount paid to owners of capital. The average wage rate is w and the total labour input is L. Hence, wL is the total payment of income to labour. The interest rate is r, the depreciation rate is δ, and the total capital input is K. Hence, $(r+\delta)K$ is the total payment of income to capital. Noting that income equals output equates the left hand side to the right hand side.

   (b) If $Y = AK^{\alpha}L^{1-\alpha}$ then the marginal product of capital is $\alpha AK^{\alpha-1}L^{1-\alpha}$. The marginal product of labour is $(1-\alpha)AK^{\alpha}L^{-\alpha}$. We can confirm that our National Accounting identity holds with this production function.

$$\begin{aligned} wL + (r+\delta)K &= \underbrace{(1-\alpha)AK^{\alpha}L^{-\alpha}}_{w} L + \underbrace{\alpha AK^{\alpha-1}L^{1-\alpha}}_{r+\delta} K \\ &= (1-\alpha+\alpha)AK^{\alpha}L^{1-\alpha} = Y \end{aligned}$$

   It also implies that

$$\begin{aligned} wL &= \underbrace{(1-\alpha)AK^{\alpha}L^{-\alpha}}_{w} L \\ \rightarrow \frac{wL}{Y} &= \frac{(1-\alpha)AK^{\alpha}L^{1-\alpha}}{AK^{\alpha}L^{1-\alpha}} \\ &= 1-\alpha \end{aligned}$$

   A similar calculation for capital implies

$$\begin{aligned} (r+\delta)K &= \underbrace{\alpha AK^{\alpha-1}L^{1-\alpha}}_{r} K \\ \rightarrow \frac{(r+\delta)K}{Y} &= \frac{\alpha AK^{\alpha}L^{1-\alpha}}{AK^{\alpha}L^{1-\alpha}} \\ &= \alpha \end{aligned}$$

   This means that if the production function is Cobb-Douglas, then the share of income that labour receives is $1-\alpha$ and the share of income that capital receives is α. The key determinant of income shares to factor of productions is the form of the production function.