Bonds – Big Picture

 We now move from a portfolio investing and asset pricing focus, which tends to emphasize equity to a focus on bonds.

 Bonds are much easier than asset pricing equity valuation (especially when risk-free debt). It is just some variation of:

$$P_t = \sum_{t=1}^{T} \frac{E[\widetilde{CF}_t]}{(1 + E[\widetilde{r}])^t}$$

 So, we are going to make this more interesting by including a lot of real-world detail.

Bonds: A Refresher on Pricing without Risk

The plan:

- 1. Review basic valuation
- 2. Language of Bonds: Interest Rates

Bonds (Debt)

Bonds are a contractual obligation by a borrower (issuer) to pay the lender

- · Coupons (or interest)
 - fixed or floating interest payments that the issuer promises to pay the bond holder 1, 2 or 4 times per year.
 - Traditionally bonds only paid fixed coupons and hence bonds are often called <u>"fixed income"</u> securities.

- . Face value (or par value) of the bond -> pregment occurred at the end
 - An amount the issuer promises to pay on the maturity date to the lender.

```
"principle" of bond
```

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51/4% REPAYABLE 15TH NOVEMBER, 1987



51/4% DEF 001067

INTEREST FOR SIX MONTHS
ON \$20
REPAYABLE 1987
\$0.52

51/4% DEF 001067

INTEREST FOR SIX MONTHS
ON \$20
REPAYABLE 1987 \$0.53

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51/4% DEF 001067
INTEREST FOR SIX MONTHS 15TH NOV.,1987
ON \$20
REPAYABLE 1987 \$0.53

An example from the ASX:

<u>Code</u>	Coupon	Maturity date	Face value	Bid	Offer	Last	Yield	<u>High</u>	Low	# of Trades	<u>Value</u>	Volume	Valuation Price	Status	Pay. Freq.	Next Ex-date	Next Pay. Date
GSBI21	5.75%	15/5/21	100	109.55	109.68	109.6	0.98%	109.6	109.6	2	118,916.00	1,085	109.6		2	6/11/19	15/11/19
GSBW2																	
<u>1</u>	2.00%	21/12/21	100	100	0	102.8	0.89%	0	0	0	0	0	102.66		2	12/12/19	23/12/19
GSBM22	5.75%	15/7/22	100	114	0	114.294	0.91%	114.327	114.29	5	137,724.29	1,205	114.294	<u>XI</u>	2	4/7/19	15/7/19
GSBK31	1.50%	21/6/31	100	0	0	101.593	1.36%	0	0	0	0	0	100.492		2	12/12/19	23/12/19
GSBG33	4.50%	21/4/33	100	130	0	137.95	1.52%	137.95	137.95	1	11,036.00	80	137.95		2	10/10/19	21/10/19
GSBK35	2.75%	21/6/35	100	0	117.56	116.2	1.61%	0	0	0	0	0	115.13		2	12/12/19	23/12/19

- Face Value is the amount of money a bond holder receives at maturity.
 - For Australian Treasury Bonds (Commonwealth Government Bonds) face values are either \$1,000,000 or \$1000.
- Maturity date is the day the last payment is made on the bond.

will get either final conpon payment & face value payment

 Coupon is the Coupon Rate. The percent of the Face Value paid as interest per year. Each coupon payment is:

=Face Value x Coupon Rate ÷ Payments Per Year = coupon per period

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Example of a bond – quotes:

Code	Coupon	Maturity date	Face value	<u>Bid</u>	<u>Offer</u>	<u>Last</u>	Yield	<u>High</u>	Low	# of Trades	<u>Value</u>	Volume	Valuation Price	Status	Pay. Freq.	Next Ex-date	Next Pay. Date
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GSBK35	2.75%	21/6/35	100	0	117.56	116.2	1.61%	0	0	0	0	0	115.13		2	12/12/19	23/12/19

- Bid price the price at which a bond dealer bids to buy the bond from you
- Offer price the price for which a bond dealer offers to sell you the bond. Also, called the ask price.
- Yield is the Yield to Maturity, this is the return you receive if
 - you buy the bond at the offer,
 - reinvest coupons at the same yield and interest rates stay the same
 - hold the bond to maturity.

unlikely, sine interest change & hard to reinvest est rates stay the same at the same yield

__Source: https://www.asx.com.au/asx/markets/interestRateSecurityPrices.do?type=GOVERNMENT_BOND
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Two Flavors of Bonds

- Coupon-paying bonds
 - They pay fixed coupons as interest.
 - Typically 1, 2, or 4 times per year.

Zero Coupon Bonds

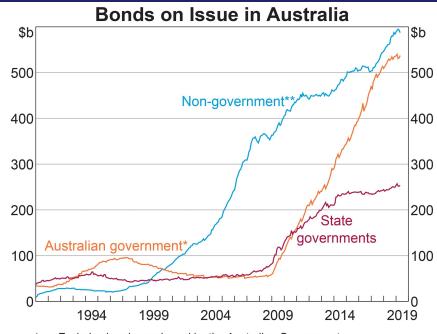
- They don't pay coupons
- All interest comes from selling the bond at a discount to the face value.
- Many short-term (< 1 year) bonds are zeros.
 - Australian Treasury Notes (≤ 6 mo.)
 - U.S. Treasury Bills ($\leq 1 \ yr$.)
 - (Corporate) Commercial Paper ($\leq 270 \ days$)



Bonds on issue in Australia

- ~\$800 Billion in Government Bonds on issue.
- For comparison, the ASX market capitalization is about \$2 trillion
- Non-government is:
 - Financial Institutions
 - Non-residents
 - Kangaroo Bonds
 - Asset-backed Securities
 - Corporations

Source: https://www.rba.gov.au/chart-pack/bond-issuance.html



- * Excludes bonds purchased by the Australian Government
- ** Excludes ADIs' self-securitisations, includes government-guaranteed bonds

Sources: ABS; AOFM; Bloomberg; KangaNews; RBA; State Treasury Corporations

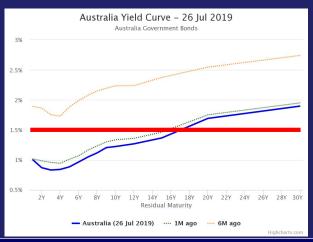
Assumptions

- Perfect markets:
 - No differences in opinion.
 - No taxes.
 - 3. No transaction costs.
 - 4. No big sellers/buyers—we have infinitely many clones that can buy or sell.



- Equal rates of return for all periods
 - Flat term structure

No risk



Risk-Free (Bond) valuation

- No risk and no differences of opinion
- Equal rates of return

$$P_{t} = \sum_{t=1}^{T} \frac{CF_{t}}{\left(1 + r_{f}\right)^{t}} = \sum_{t=1}^{T} \frac{Coupon_{t}}{\left(1 + r_{f}\right)^{t}} + \frac{Face\ Value_{T}}{\left(1 + r_{f}\right)^{T}}$$

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Example 1:

 Suppose a risk-free, 1-year, zero-coupon bond will pay \$1000 next year. The return on risk-free Australian government zeros is 1%.

$$P = \frac{1000}{1.01} = 990.10$$

What is this bond worth today?

- Before we answer let's take a guess.
 - Should the price be higher or lower than \$1000?
 - Why? By about how much?

Example 1 - Solution

$$P_t = \frac{CF_{t+1}}{1 + r_f}$$

$$P_t = \frac{1000}{1 + .01} = 990.10$$

- · This bond is priced at a discount.
- Anytime P<FV ⇒ the bond is priced at a discount.

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In prior subjects instead of...

$$P_t = \sum_{i=1}^T \frac{CF_{t+i}}{\left(1+r_f\right)^i} \quad \text{you might} \quad P_t = \left(\sum_{i=1}^T \frac{C}{1+r_f}\right) + \frac{FV}{1+r_f}$$
 Or
$$S_r = \alpha_1 \cdot \frac{1}{1-\theta} \quad \text{Annuity Factor}$$

$$P_t = C\left(\frac{1}{r_f}\right) \left(1-\frac{1}{\left(1+r_f\right)^T}\right) + FV \cdot \frac{1}{\left(1+r_f\right)^T}$$

Why are we using a more general formula?

There aren't formulas for all types of assets.

Morgan Stanley Dean Witter BOXES Pharmaceutical Index due 10/31/2031

Ticker Symbol: RXB CUSIP: 61744Y520 Exchange: NYSEA

Security Type: Special Product - Index Based

QUANTUMONLINE.COM SECURITY DESCRIPTION: Morgan Stanley Dean Witter & Co., now Morgan Stanley, Pharmaceutical Basket Opportunity eXchangeablE Securities (BOXES), due 10/31/2031, price to the public \$16.1093, exchangeable for a cash amount based on the closing prices of the underlying stocks of the AMEX Equal Weighted Pharmaceutical Index (DGE). The BOXES are senior unsecured debt securities of Morgan Stanley Dean Witter & Co. The BOXES pay a quarterly base coupon rate equal to the cash distributions of the underlying stocks of the Index plus an annual supplement (see prospectus fo details). The BOXES are exchangeable at the holder's option on or after 12/26/2001 for cash based on the closing prices of the underlying stocks for the Index. On or after 11/26/2008, the issuer may required holders to exchange the BOXES for cash based on the closing prices of the underlying stocks of the Index. The BOXES are senior unsecured debt securities issued by Morgan Stanley and will rank equally with all of their other unsecured and unsubordinated debt. See the IPO prospectus for further information on the BOXES by using the 'Link to IPO Prospectus' provided below.

Source: http://www.quantumonline.com/search.cfm?tickersymbol=RXB&sopt=symbol

Ex 2: How much you remember from Principles of Finance?

- What is the value of a semi-annual-coupon-paying, risk-free bond with a 3% coupon rate and a face value (par value) of \$1000, if the bond matures in exactly 1 yr. & the bond equivalent yield on 1-year risk-free debt is 1%.
 - Remember: Equal rates of return for all periods

coupon per period =
$$15\%$$
. $\times 1000 = 15$.

$$\frac{15}{(1+0.5\%)} + \frac{1015}{(1+0.5\%)^2} = 10.9.85$$

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Ex. 2: 3% semi-annual coupon bond

• 3% coupon implies \$30 in coupons per year.

Coupon Rate
$$\times$$
Face Value = \$\$ Coupons Paid per Year $0.03 \times 1000 = 30

Semi-annual coupon paying implies it pays 2 times per year or

$$\frac{$30}{2}$$
 = \$15 every 6 months

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Solving Ex. 2

$$P_t = \sum_{t=1}^{T} \frac{CF_{t+i}}{\left(1 + r_f\right)^t}$$

$$P_t = \frac{\$15}{\left(1 + \frac{.01}{2}\right)^1} + \frac{\$1015}{\left(1 + \frac{.01}{2}\right)^2}$$

$$P_t = \$14.925 + \$1004.926 = \$1019.85$$

This bond is selling at a premium.

Selling at ...

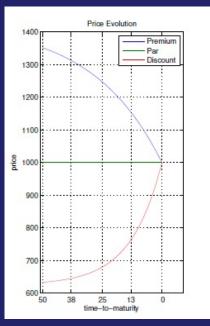
- Working example:
 - suppose the face value (FV) of a bond is \$1000.

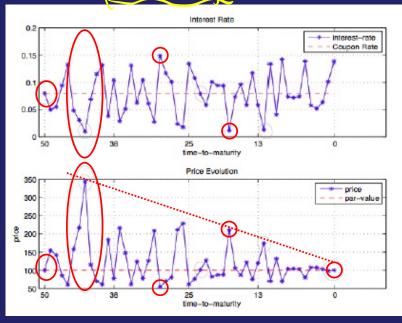


- P=FV ⇒ the bond is priced at par.
 P=1000
- P<FV ⇒ the bond is priced at a discount.
 - For example, P=\$950.
- P>FV ⇒ the bond is priced at a premium.
 - For example, P=\$1020.

Discount and premium price moves

If interest rates are constant and we use flat prices (w/o coupons)





Technical Detail: Flat Price vs. Invoice Price

Bond prices are quoted as <u>flat prices</u>, that is, without <u>accrued</u> interest.

ask price + account interes

The price including accrued interest is called the invoice price.



- When you calculate the present value of a bond's cash flows, you are calculating the invoice price.

to get flat price, more price - accorded interest



- Note: for semiannual coupon bonds there are fewer days between coupons in the first half.
 - There were 182 days from 31-Dec-19 to 30-Jun-20,
 - but 183 days from 1-Jul-20 to 31-Dec-20.

• What is the value of a semi-annual-coupon-paying, risk-free bond with a 3% coupon rate and a face value (par value) of \$1000 to be settled on 30-Mar-2020, if the bond matures in 276 days on 31-Dec. 2020, the last coupon was paid 90 days ago on 31-Dec. 2019 & the bond equivalent yield on similar risk-free debt that matures in 276 days is 1%?

**The coupon rate and a face value (par value) of \$1000 to be settled on 30-Mar-2020, if the bond matures in 276 days on 31-Dec. 2020, the last coupon was paid 90 days ago on 31-Dec. 2019 & the bond equivalent yield on similar risk-free debt that matures in 276 days is 1%?

**The coupon rate and a face value (par value) of \$1000 to be settled on 30-Mar-2020, if the bond matures in 276 days on 31-Dec. 2019 & the bond equivalent yield on similar risk-free debt that matures in 276 days is 1%?

- Notes: 31-bec-2019 30-Mar 2020 30-Jun-2020 31-Dec-2020
 - 1. A semi-annual bond that pays twice a year and matures on 31-Dec, will have coupon payments on 30-Jun and 31-Dec each year
 - 2. There are 182 days from 31-Dec-2019 to 30-Jun-2020.
 - 3. There are 92 days from 30-Mar-2020 to 30-Jun-2020.

- Calculate accrued interest
- 2. Calculate the invoice price
- 3. Flat Price = Invoice Price Accrued Interest

- $accrued\ interest = \frac{annual\ coupon\ payment}{number\ of\ payments\ per\ year} \times \frac{days\ since\ last\ coupon}{days\ between\ coupons}$
- accrued interest = $\frac{0.03 \times \$1000}{2} \times \frac{90}{182} = \7.4176

Invoice Price

Recall when we had the same bond in Example 2 (7 slides back) with exactly 1 year to maturity that t was from 1 half year to 2 half years. Now we have $92/182^{\text{nds}}$ of a half year remaining in the first half year, plus 1 full half year.

Invoice
$$P_t = \sum_{t=1}^{T} \frac{cF_t}{(1+r_f)^t} = \sum_{t=1}^{1\frac{92}{182}} \frac{cF_t}{(1+r_f)^t}$$

Days remaining in this half year

Days in the current half year

Invoice
$$P_t = \frac{\$15}{\left(1 + \frac{0.01}{2}\right)^{\left(\frac{92}{182}\right)}} + \frac{\$1015}{\left(1 + \frac{0.01}{2}\right)^{\left(1 + \frac{92}{182}\right)}} = \$1022.37$$

• Flat Price = Invoice Price - Accrued Interest

• Flat Price = \$1022.37 - \$7.42

• *Flat Price* = \$1014.95

• The flat price is what will get quoted on many websites and newspapers.

Bonds: Interest Rates

Interest Rates: Conventions and Jargon

- Interest rates (and quotes) are not difficult, but they are tedious and often confusing
 - It's as if everyone computes and quotes them slightly differently.
 - · Sometimes, it is obvious what people mean,
 - sometimes interest rates are intentionally obscure in order to deceive you.
 - There can be a lot of money at stake! Arbitrage desks on Wall Street make most of their money on spreads below 20 basis points!

· You should know what you are talking about. Ask if you are unclear!

Different Definitions of Interest Rates

- Current Yield
- Yield to Maturity (YTM)

- There are two ways to annualize YTM:
 - Bond Equivalent Yield (BEY) ("simple interest methods")
 - Also, Annual Percentage Rate (APR)

Discussed on Day 1

- Effective Annual Rate (EAR)
 - also, Effective Annual Yield and Annual Percentage Yield

Current Yield

Not really an interest rate.



•
$$Current\ Yield = \frac{Coupon}{Price}$$

- Ex. A bond that pays \$5 in coupons per year has a price of \$90.
- What's the current yield?

• Current Yield =
$$\frac{Coupon}{Price} = \frac{\$5}{\$90} = 5.56\%$$

if coupon rate > current yield:

trading at premium

discourt

 The only thing Current Yield might be useful for is see whether a bond is trading at a premium or discount.

Yield to Maturity (YTM)

- The Internal Rate of Return of promised cash flows to the bond.
- Equivalently, the discount rate that makes the present value of the promised cash flows (coupon + face value) equal to the price.
 - Assumes the bond does not default.

$$P_t = \sum_{i=1}^{T} \frac{CF_{t+i}}{(1 + r_{YTM})^i}$$

Cash Flows are promised coupon and face value payments

- Often quoted on financial websites (see slides 4 & 5)
- CAUTION! Unless explicitly stated, Yield to Maturity is always quoted in annualized terms as a Bond Equivalent Yield.

Students often forget this on exams

Yield to Maturity (YTM): some notes

- You can calculate the periodic YTM with a simple calculator <u>only</u> if it is a <u>zero coupon</u> bond.
 - Otherwise: Be a good guesser
 - Otherwise: Excel or a Financial calculator

Calculating YTM for a Zero

$$periodic r_{YTM} = \frac{CF_{t+T}}{P_t} - 1$$

Example: YTM for a Zero

- Suppose a zero-coupon bond with a \$1000 face value matures in half a year.
 - Today's price is \$980.39.
 - Let's count T in the number of half-year periods

$$r_{YTM} = \frac{CF_{t+T}}{P_t} - 1$$
 $r_{YTM} = \frac{1000}{980.39} - 1 = 0.02 = r_{period}$

Annualize it as a bond equivalent yield: $0.02 \times 2 = 0.04$ or 4%

YTM, BEY, and EAR

If the bond or loan compounds <u>once</u> per year, then_
 YTM=BEY=EAR

 If the bond or loan compounds more than once per year, then these formulas give you a periodic YTM which needs to be annualized.

$$r_{YTM} = r_{period}$$

- Then there are 2 ways to annualize r_{period} :
 - Bond Equivalent Yield (BEY)
 - Effective Annual Rate (EAR)

Bond Equivalent Yield or Annual Percentage Rate

Repeat from the second lecture.

General formula:

$$r_{BEY} = r_{APR} = r_{period} \times n_{periods \ per \ year}$$

– If the half-year r_{period} =2%, what's the BEY?

- Suppose the interest rate for 1 quarter is 1.5%, what is the bond equivalent yield?

$$r_{BEY} = 1.5\% \times 4 = 6\%$$

- Economically, Bond Equivalent Yields aren't useful.
 - BEY is equivalent to earning interest but not reinvesting the interest.
- To me, the only thing BEY is good for is for getting r_{period}

Bond Equivalent Yield or Annual Percentage Rate

- There are several methods for calculating BEY.
- The most general:

$$r_{BEY} = r_{APR} = r_{period} \times n_{periods \ per \ year}$$

- For Australian Money Market Securities (Zeros with Maturity < 1yr)
 - Are very precise with the number of periods.

$$r_{BEY} = r_{period} \times \frac{365}{n}$$



n is the number of days in the period.

Example: BEY for Australian Money Market securities

 Suppose zero-coupon commercial paper that matures in 90 days has a face value of \$100,000 and quoted interest rate of 4% per year. What is the periodic interest rate (i.e. for 90 days)?

$$r_{BEY} = r_{period} \times \frac{365}{n}$$

$$0.04 = r_{period} \times \frac{365}{90}$$

$$r_{period} = \frac{90}{365} \times 0.04 = 0.0099$$

Example: BEY for Australian Money Market securities

 Suppose zero-coupon commercial paper that matures in 90 days has a face value of \$100,000 and quoted interest rate of 4% per year. What is the price of this bond?

Recall:
$$r_{period} = 0.0099$$

$$P_0 = \frac{100,000}{1 + 0.0099} = \$99,023.33$$

Concept note: This looks like discounting (and the mathematical method is the same), but it is **not** discounting – it is using the <u>definition</u> of YTM from which the quoted interest rate comes.

Effective Annual Rate (EAR)

from the second lecture.

• The Effective Annual Rate (EAR) is the compound periodic rate:

$$r_{EAR} = \left(1 + r_{period}\right)^n - 1$$

n is the number of periods per year.

$$r_{EAR} = \left(1 + \frac{r_{BEY}}{n}\right)^n - 1$$

Example

Repeat from the second lecture.

• Suppose the Bond Equivalent Yield (Annual Percentage Rate) on a quarterly paying corporate bond is 8%, what is the Effective Annual Yield?

n= # of coupon payments per year

$$r_{EAY} = \left(1 + \frac{r_{BEY}}{n}\right)^n - 1$$

$$r_{EAR} = \left(1 + \frac{0.08}{4}\right)^4 - 1$$

$$r_{EAR}=0.0824$$

Example: YTM for a Zero (using EAR to annualise)

- Suppose a zero coupon bond with a \$1000 face value matures in half a year. Today's price is \$980.39.
 - We found that the BEY is 4%

We can calculate the EAR straight from the periodic YTM:

$$r_{EAR} = \left(\frac{CF_{t+T}}{P_t}\right)^{\frac{1}{T}} - 1 = \left(YTM_{period}\right)^{\frac{1}{T}} - 1$$

$$r_{EAR} = \left(\frac{1000}{980.39}\right)^{\frac{1}{.5}} - 1 = 0.0404$$

RBA BEY

- The Reserve Bank of Australia has some very specific variations of the methods we just used for
 - Valuing bonds
 - Calculating bond equivalent yields
 - Calculating prices in between coupon payment dates

- The above will be covered in tutorial and your tutorial assignments

Discount factors

$$P_{t} = \sum_{i=1}^{\infty} \frac{E_{t} \left[\widetilde{CF}_{t+i} \right]}{1 + E_{t} \left[\widetilde{r}_{t \to t+i} \right]}$$

Call
$$d_{0i} = \frac{1}{1 + E_t[\tilde{r}_{t \rightarrow t+i}]}$$
 then

$$P_t = \sum_{i=1}^{\infty} d_{0i} E_t \big[\widetilde{CF}_{t+i} \big]$$

I don't plan on ever using this variation in this subject, I just want you to know it exists.

Other types of bonds

Convertible Bonds

For which of these bonds would investors be willing to accept a lower interest rate?

- The bond holder has the right to convert the bonds to stock under stated conditions. Those conditions could be:
 - The stock rising to a certain price or getting to a certain date (ex. Maturity)

allow them to participate in the increase price of the bond

Callable bonds

- The bond issuer has the right to buy back the bond if the price rises to a call price = call premium low price, high interest rate - straigh - premium par value stated price, i.e. interest rates fall.
- Putable bonds

- if rates go higher (and prices fall) the bond holder can sell back the bonds for the principal PAR+ premium

for the face value of the debt

Inflation Indexed Bond Example

- Inflation indexed bonds or capital index bonds
 - The face value rises with the Consumer Price Index (CPI)
 - Treasury Indexed Bonds (TIBs) Australia
 - Treasury Inflation Protected Securities (TIPS) U.S.

compensation for Change in inflation

- Ex. A TIB with Face Value = \$100 and Annual Coupon Rate = 4%
- If inflation is 2%, what is the *new* face value (=nominal value)? Nominal value = $$100 \times (1 + 0.02) = 102
- What is the new coupon payment? $= $102 \times 0.04 = 4.08

Other types of bonds (continued)

- Floating rate bonds
 - The coupon rate rises or falls with a short-term interest rate
 - Typically the coupon rate is a short-term benchmark rate + a spread
 - The benchmark might be
 - Australian Treasury Notes
 - 90 day bank bill swap rate (BBSW) an average rate from a collection of 90-day bank bills

Floating Rate Notes												
Buy	Sell	Last bend	Coupon Lmark rock	Maturity date	Payment frequency	Next ex-date	Next payment date					
QUBHA - HYBRID 3-BBSW+3.90% 05-10-23 SUB CUM												
108.300	108.800	108.250	5.029%	05/10/2023	4	26/09/2019	08/10/2019					

Question

 For which of these bonds would investors be willing to accept a lower interest rate, for which would they demand a higher interest rate?

- interest rate? consider advantage if better for bond holder

 Convertible Bonds it is an option (valuable) they will accept for interest

 Callable bonds wigher accept lower interest rate
- higher Callable bonds
- Putable bonds Lomer
- Inflation Index Bonds lower -> protect investor from inflation
- Floating Rate Bonds

for fixed rate bond, high inflation win erode the volve

interest flexibility -> bad for bond issuers

-> unstable cashfrow.

-> lower