PHYC10003 Physics 1

Lecture 14: Rotational Motion

Rotational energy, torque and work

Last lecture

- Angular variables
- Angular displacement
- Angular velocity
- Radial and tangential angular acceleration
- Analogies with linear motion

Rotational energy of rigid bodies

Foreverspin^{TM} Titanium Spinning Top (Image: Foreverspin ^{TM})

Rotational kinetic energy

- Apply the kinetic energy formula for a point particle and sum over all the particles $K = \Sigma \frac{1}{2} m_i v_i^2$
- Different linear velocities (same angular velocity for all particles but possibly different radii)
- Then write velocity in terms of angular velocity:

$$K = \sum \tfrac{1}{2} m_i (\omega r_i)^2 = \tfrac{1}{2} \left(\sum m_i r_i^2 \right) \omega^2, \qquad \textbf{Eq. (10-32)}$$

  We call the quantity in parentheses on the right side the **rotational inertia**, or **moment of inertia**, *I*
- This is a constant for a ***rigid*** object and given rotational axis
- Caution: the ***axis*** for *I* must always be specified

Moment of inertia - rotational inertia

- We can write:

$$I = \sum m_i r_i^2 \qquad \text{(rotational inertia)}$$

Eq. (10-33)

- And rewrite the kinetic energy as:

$$K = \tfrac{1}{2} I \omega^2 \qquad \text{(radian measure)}$$

Eq. (10-34)

- Use these equations for a finite set of rotating particles
- Rotational inertia corresponds to how difficult it is to change the state of rotation (speed up, slow down or change the axis of rotation)

Rotating rod- dependence on axis

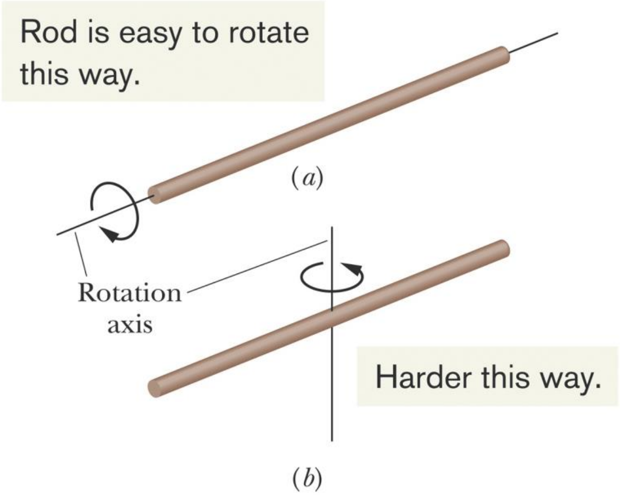
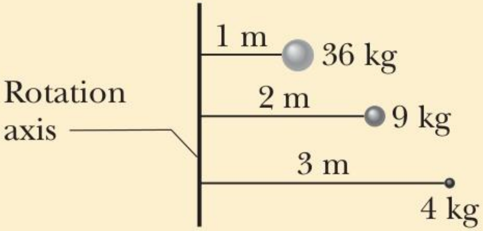


Figure 10-11

Checkpoint 4

The figure shows three small spheres that rotate about a vertical axis. The perpendicular distance between the axis and the center of each sphere is given. Rank the three spheres according to their rotational inertia about that axis, greatest first.



Answer: They are all equal!

Moment of inertia – continuous bodies

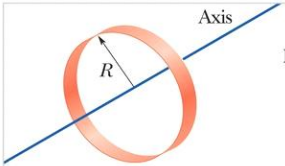
- Integrating Eq. 10-33 over a continuous body:

$$I = \int r^2\, dm \quad \text{(rotational inertia, continuous body).}$$ **Eq. (10-35)**

- In principle we can always use this equation
- But there is a set of common shapes for which values have already been calculated (Table 10-2) for common axes

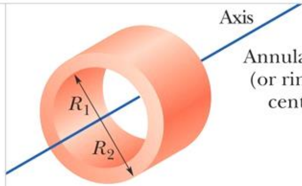
Moments of inertia - examples

Table 10-2 **Some Rotational Inertias**



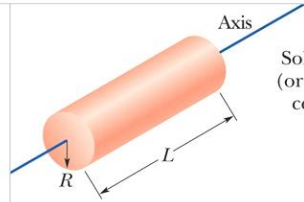
Hoop about central axis

$I = MR^2$ (a)



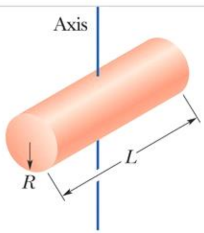
Annular cylinder (or ring) about central axis

$I = \frac{1}{2}M(R_1^2 + R_2^2)$ (b)



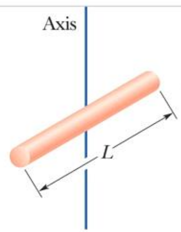
Solid cylinder (or disk) about central axis

$I = \frac{1}{2}MR^2$ (c)



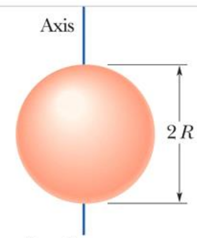
Solid cylinder (or disk) about central diameter

$I = \frac{1}{4}MR^2 + \frac{1}{12}ML^2$ (d)



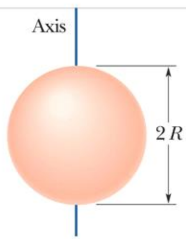
Thin rod about axis through center perpendicular to length

$I = \frac{1}{12}ML^2$ (e)



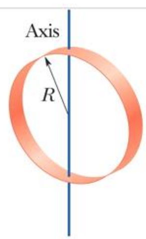
Solid sphere about any diameter

$I = \frac{2}{5}MR^2$ (f)



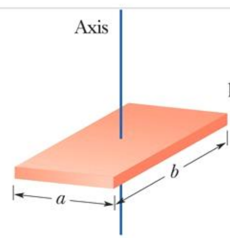
Thin spherical shell about any diameter

$I = \frac{2}{3}MR^2$ (g)



Hoop about any diameter

$I = \frac{1}{2}MR^2$ (h)



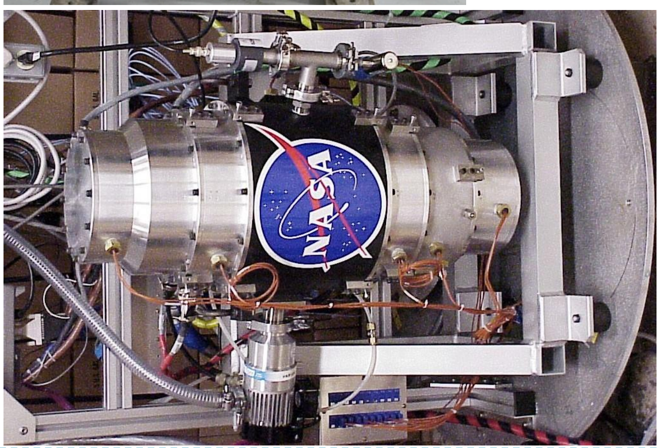
Slab about perpendicular axis through center

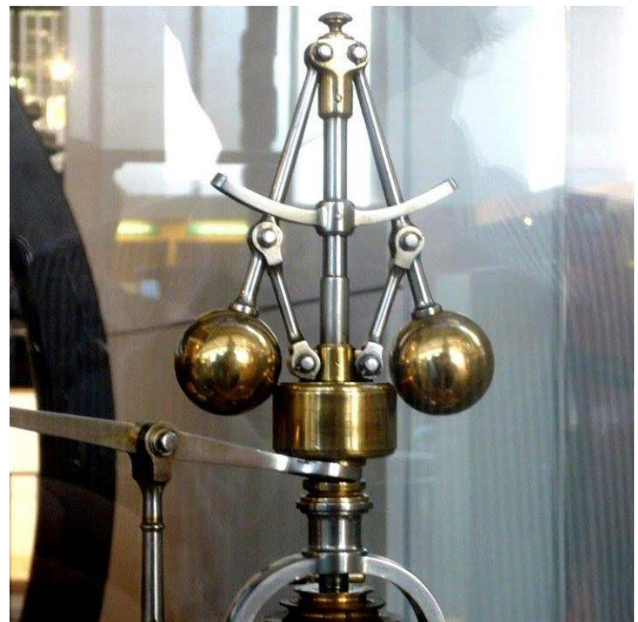
$I = \frac{1}{12}M(a^2 + b^2)$ (i)

Table 10-2

Moment of inertia: machines

The moment of inertia of spinning objects has been used to store kinetic energy in flywheels (left) and energy recovery devices (lower left), or to control the rate of rotation of other machines (steam engine governor (lower right)





Inertia – parallel axis theorem

- If we know the moment of inertia for the center of mass axis, we can find the moment of inertia for a parallel axis with the **parallel-axis theorem**:

$$I = I_{\text{com}} + Mh^2$$ **Eq. (10-36)**

We need to relate the rotational inertia around the axis at *P* to that around the axis at the com.

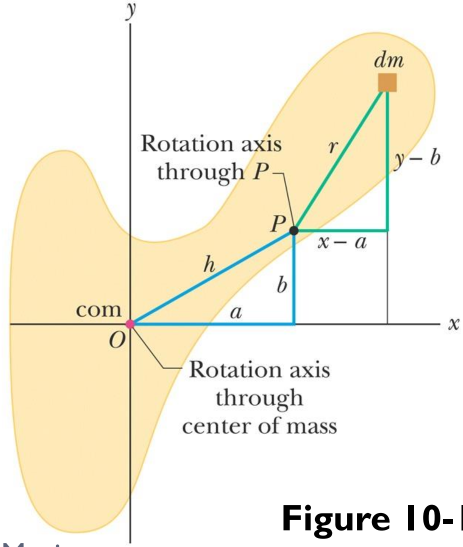


Figure 10-12

- Note the axes *must* be parallel, and the first *must* go through the center of mass
- This does *not* relate the moment of inertia for two arbitrary axes

Moment of inertia - example

Example Calculate the moment of inertia for Fig. 10-13 (b)

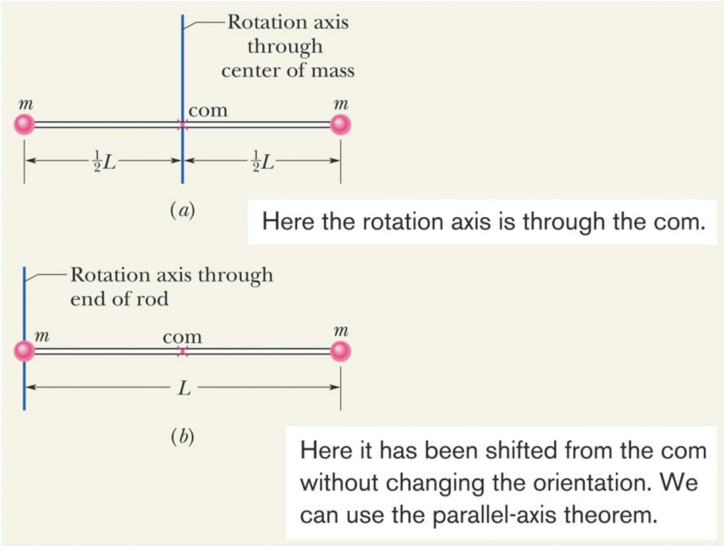
- Summing by particle:

$$I = m(0)^2 + mL^2 = mL^2.$$

- Use the parallel-axis theorem

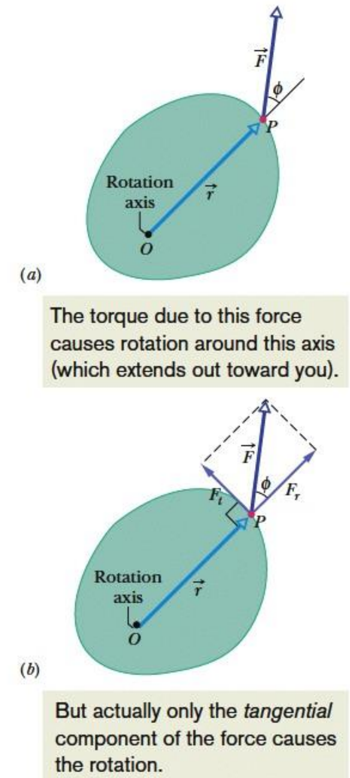
$$I = I_{com} + Mh^2 = \tfrac{1}{2}mL^2 + (2m)(\tfrac{1}{2}L)^2$$
$$= mL^2.$$

Figure 10-1



Rotational motion and force

- The force necessary to rotate an object depends on the angle of the force and where it is applied
- We can resolve the force into components to see how it affects rotation



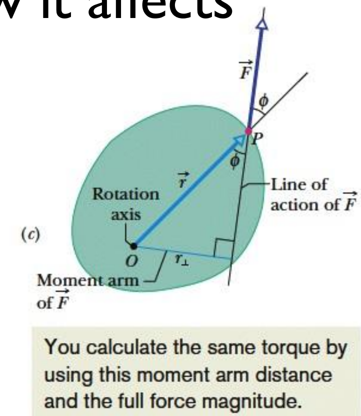


Figure 10-16

Torque

- **Torque** takes these factors into account:

$$\tau = (r)(F \sin \phi). \qquad \textbf{Eq. (10-39)}$$

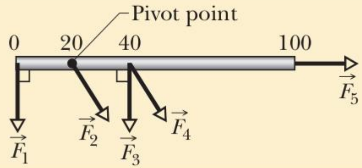
- A line extended through the applied force is called the **line of action** of the force
- The perpendicular distance from the line of action to the axis is called the **moment arm**
- The unit of torque is the newton-meter, N m
- Note that 1 J = 1 N m, but torques are never expressed in joules, torque is not energy

Torque – sign convention, vector addition

- Again, torque is positive if it would cause a counterclockwise rotation, otherwise negative
- For several torques, the **net torque** or **resultant torque** is the sum of individual torques

Checkpoint 6

The figure shows an overhead view of a meter stick that can pivot about the dot at the position marked 20 (for 20 cm). All five forces on the stick are horizontal and have the same magnitude. Rank the forces according to the magnitude of the torque they produce, greatest first.



Answer: F_1 & F_3, F_4, F_2 & F_5

Newton's law for rotational motion

- Rewrite *F* = *ma* with rotational variables:

$$\tau_{\text{net}} = I\alpha$$

Eq. (10-42)

- It is torque that causes angular acceleration

The torque due to the tangential component of the force causes an angular acceleration around the rotation axis.

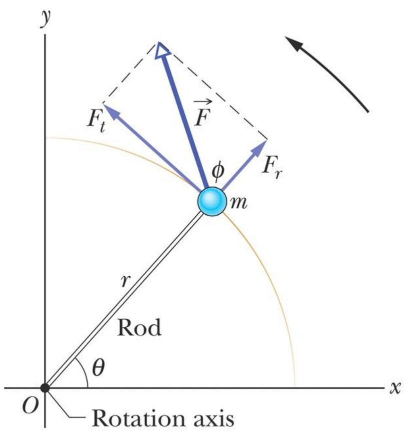


Figure 10-17

Rotational motion and work

- The rotational work-kinetic energy theorem states:

$$\Delta K = K_f - K_i = \tfrac{1}{2}I\omega_f^2 - \tfrac{1}{2}I\omega_i^2 = W$$ **Eq. (10-52)**

- The work done in a rotation about a fixed axis can be calculated by:

$$W = \int_{\theta_i}^{\theta_f} \tau \, d\theta$$ **Eq. (10-53)**

- Which, for a constant torque, reduces to:

$$W = \tau(\theta_f - \theta_i)$$ **Eq. (10-54)**

Analogies – linear and rotational motion

- We can relate work to power with the equation:

$$P = \frac{dW}{dt} = \tau\omega$$

Eq. (10-55)

- Table 10-3 analogies for linear and rotational motion:

Table 10-3 Some Corresponding Relations for Translational and Rotational Motion

Pure Translation (Fixed Direction)		Pure Rotation (Fixed Axis)	
Position	x	Angular position	θ
Velocity	$v = dx/dt$	Angular velocity	$\omega = d\theta/dt$
Acceleration	$a = dv/dt$	Angular acceleration	$\alpha = d\omega/dt$
Mass	m	Rotational inertia	I
Newton's second law	$F_{net} = ma$	Newton's second law	$\tau_{net} = I\alpha$
Work	$W = \int F\,dx$	Work	$W = \int \tau\,d\theta$
Kinetic energy	$K = \frac{1}{2}mv^2$	Kinetic energy	$K = \frac{1}{2}I\omega^2$
Power (constant force)	$P = Fv$	Power (constant torque)	$P = \tau\omega$
Work–kinetic energy theorem	$W = \Delta K$	Work–kinetic energy theorem	$W = \Delta K$

Tab. 10-3

Summary

Kinematic Equations

- Given in Table 10-1 for constant acceleration
- Match the linear case

Linear and Angular Variables Related

- Linear and angular displacement, velocity, and acceleration are related by *r*

Rotational Kinetic Energy and Rotational Inertia

$$K = \frac{1}{2}I\omega^2 \quad \text{(radian measure)}$$

Eq. (10-34)

$$I = \sum m_i r_i^2 \quad \text{(rotational inertia)}$$

Eq. (10-33)

The Parallel-Axis Theorem

- Relate moment of inertia around any parallel axis to value around com axis

$$I = I_{\text{com}} + Mh^2$$

Eq. (10-36)

Summary

Torque

- Force applied at distance from an axis:

$$\tau = (r)(F \sin \phi).$$ **Eq. (10-39)**

- Moment arm: perpendicular distance to the rotation axis

Newton's Second Law in Angular Form

$$\tau_{\text{net}} = I\alpha$$ **Eq. (10-42)**

Work and Rotational Kinetic Energy

$$W = \int_{\theta_i}^{\theta_f} \tau \, d\theta$$ **Eq. (10-53)**

$$P = \frac{dW}{dt} = \tau\omega$$ **Eq. (10-55)**