

What you need to know:

Principle of Superposition:

For two or more travelling waves in a medium:

Total displacement = algebraic sum of displacements of individual waves:

$$D(x, t) = D_1(x, t) + D_2(x, t) + \dots$$

(This is true provided that net displacement does not exceed elastic limit of the medium.)

Standing Waves:

Two waves same amplitude, wavelength and frequency travelling in opposite directions:

$$D = D_1 + D_2 = A \sin(kx - \omega t) + A \sin(kx + \omega t)$$

No how to use trig identity:

$$\sin \alpha + \sin \beta = 2 \sin \frac{1}{2}(\alpha + \beta) \cos \frac{1}{2}(\alpha - \beta)$$

to get resultant standing wave formula:

$$\Rightarrow D = 2A \sin(kx) \cos(\omega t)$$

and know how to interpret this to find nodes and antinodes.

Nodes = points where amplitude is always 0:

Occur when $\sin(kx) = 0$

Antinodes = points where amplitude varies between maximum values i.e. $\pm 2A$

Occur when $\sin(kx) = \pm 1$

Transverse Waves: Reflection at Boundaries:

Fixed boundary: reflected pulse has opposite sign to incident pulse.

Free boundary: reflected pulse has same sign as incident pulse.

Transverse Standing Waves:

Nodes at fixed ends. Know how to draw the diagrams for the 1st, 2nd and 3rd harmonic standing waves and find wavelengths and frequencies.

For the exam:

- You may be asked to add travelling waves together using trigonometric identities and discuss whether or not the resultant wave represents a travelling wave or a standing wave.
- You may be asked to find node/antinode positions for a standing wave.
- You may be asked to draw diagrams of standing waves and resonance conditions for transverse waves on strings and sound waves in pipes. You may be asked to find wavelengths and frequencies for the first few harmonics.
- You may be asked to identify whether two 1D waves interfere constructively or destructively.
- You may be asked whether sound waves from two different sources produce constructive or destructive interference at some location based on the path difference between the sources and the location.
- You may be asked to find the beat frequency for two waves with different frequencies.

Training:

Homework Problems and Problem Class Sheets.

What you need to know:

And then try an exam question:

2008, Exam, Question 9 (a)

[3 + 2 + 3 = 8 marks]

(a) The displacements of two travelling waves are given by:

$$D_1(x,t) = 0.01(m) \sin \left[10(rad.m^{-1})x + 400(rad.s^{-1})t + \frac{\pi}{4}(rad) \right]$$

$$D_2(x,t) = 0.01(m) \sin \left[10(rad.m^{-1})x - 400(rad.s^{-1})t + \frac{\pi}{4}(rad) \right]$$

- (i) Use the appropriate trigonometric identity to find the displacement resulting from the superposition of these two waves.
- (ii) Is the wave resulting from this superposition a travelling wave? Briefly explain your answer.
- (iii) Find a value for the separation between adjacent maxima (antinodes) in the resultant wave.

Some Formulae from Exam:

$$\sin \alpha + \sin \beta = 2 \sin \frac{1}{2}(\alpha + \beta) \cos \frac{1}{2}(\alpha - \beta)$$

$f = \frac{1}{T}, \quad \omega = 2\pi f$	$x(t) = A \cos(\omega t)$	$\omega = \sqrt{\frac{k}{m}}$
$\omega = \sqrt{\frac{g}{l}}$	$\omega = \sqrt{\frac{Mgl}{I}}$	$\beta = 10 \log_{10} \left(\frac{I}{I_0} \right), \quad I_0 = 10^{-12} W / m^2$
$D(x,t) = A \sin(kx \pm \omega t + \phi_0)$	$f_D = \left(\frac{v \pm v_D}{v \mp v_S} \right) f_S$	$v = \sqrt{\frac{T_s}{\mu}}, \quad v = \sqrt{\frac{B}{\rho}}$

Answers: (ii) Not a travelling wave (but be sure to say why in your answer) (iii) $\pi/10$ m