

A Portfolio of Nobel Laureates: Markowitz, Miller and Sharpe

Hal Varian

Finance is one of the great success stories of quantitative economics. A recent ad in *The Economist* for a “mathematical economist” described an “excellent opportunity for numerate individual with background in capital markets.” In today’s market, numeracy pays!

But it was not always so. According to Robert Merton (1990):

As recently as a generation ago, finance theory was still little more than a collection of anecdotes, rules of thumb, and manipulations of accounting data. The most sophisticated tool of analysis was discounted value and the central intellectual controversy centered on whether to use present value or internal rate of return to rank corporate investments. The subsequent evolution from this conceptual potpourri to a rigorous economic theory subjected to scientific empirical examination was, of course, the work of many, but most observers would agree that Arrow, Debreu, Lintner, Markowitz, Miller, Modigliani, Samuelson, Sharpe, and Tobin were the early pioneers in this transformation.

Three of these pioneers of quantitative finance have now been justly honored: Harry Markowitz, Merton Miller and William Sharpe received the Nobel Prize in Economic Science in 1990.

From today’s perspective it is hard to understand what finance was like before portfolio theory. Risk and return are such fundamental concepts of finance courses that it is hard to realize that these were once a novelty. But

■ *Hal Varian is Reuben Kempf Professor of Economics and Professor of Finance, University of Michigan, Ann Arbor, Michigan.*

these esoteric theories of the last generation form the basic content of MBA courses today.

The history of the quantitative revolution in finance has recently been summarized in Bernstein (1992). Here I attempt to provide a very brief history of this enterprise, drawing upon the work of Bernstein and the accounts of the Nobel laureates in Markowitz (1991), Miller (1991) and Sharpe (1991). Readers interested in more detailed accounts of the development of modern financial theory should consult these works.

Harry Markowitz

Harry Markowitz was born in 1927 in Chicago. He attended the University of Chicago and majored in economics. He found the subject appealing enough to go on to graduate school and eventually arrived at the thesis stage. While waiting to see Jacob Marschak he struck up a conversation with a stockbroker who suggested that he might write a thesis about the stock market. Markowitz was excited by this idea and started to read in the area.

One of his first books was *The Theory of Investment Value* by John Burr Williams, (1938). Williams argued that the value of a stock should be the present value of its dividends—which was then a novel theory. Markowitz quickly recognized the problem with this theory: future dividends are not known for certain—they are random variables. This observation led Markowitz to make the natural extension of the Williams' theory: the value of a stock should be the *expected* present value of its dividend stream.

But if an investor wants to maximize the expected value of portfolio of stocks he owns, then it is obvious that he should buy only one stock—the one that has the highest expected return. To Markowitz, this was patently unrealistic. It was clear to him that investors must care not only about the expected return of their wealth, but also about the risk. He was then naturally led to examine the problem of finding the portfolio with the maximum expected return for a given level of risk.

The fact that investors should care about both the risk and the return of their investments is so commonplace today that it is hard to believe that this view was not appreciated in 1952. Even Keynes (1939) said, “To suppose that safety-first consists in having a small gamble in a large number of different [companies] . . . strikes me as a travesty of investment policy.” Luckily, Keynes was not held in high repute in Chicago, even in those days, and Markowitz was not deterred from his investigations.

Markowitz posed the problem of minimizing the variance of a portfolio taking as a constraint a required expected return. This way of posing the problem contained two significant insights. First, Markowitz realized that the mathematics could not pick out a single optimal portfolio, but rather could only identify a set of *efficient* portfolios—the set of portfolios that had the lowest



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possible risk for each possible expected return. Secondly, Markowitz recognized that the appropriate risk facing an investor was *portfolio* risk—how much his entire portfolio of risky assets would fluctuate.

Today, we pose the problem of portfolio selection as a quadratic programming problem. The choice variables are the fractions of wealth invested in each of the available risky assets, the quadratic objective function is the variance of return on the resulting portfolio, and the linear constraint is that the expected return of the portfolio achieve some target value. Variables may be subjected to nonnegativity constraints or not, depending on whether short sales are feasible.

The first-order conditions for this quadratic programming problem require that the marginal increase in variance from investing a bit more in a given asset should be proportional to the expected return of that asset. The key insight that arises from this first-order condition is that the marginal increase in variance depends on both the variance of a given asset's return plus the *covariance* of the asset return with all other asset returns in the portfolio.

Markowitz's formulation of portfolio optimization leads quickly to the fundamental point that the riskiness of a stock should not be measured just by the *variance* of the stock, but also by the *covariance*. In fact, if a portfolio is highly diversified, so that the amount invested in any given asset is "small," and the returns on the stocks are highly correlated, then *most* of the marginal risk from increasing the fraction of a given asset in a portfolio is due to this covariance effect.

This was, perhaps, the central insight of Markowitz's contribution to finance. But it is far from the end of the story. As every graduate student knows, the first-order conditions are only the first step in solving an optimization problem. In 1952, linear programming was in its infancy and quadratic programming was not widely known. Nevertheless, Markowitz succeeded in developing practical methods to determine the "critical line" describing

mean-variance efficient portfolios. The initial work in his thesis was described in two papers Markowitz (1952, 1956) and culminated in his classic book (Markowitz, 1959).

When Markowitz defended his dissertation at the University of Chicago, Milton Friedman gave him a hard time, arguing that portfolio theory was not a part of economics, and therefore that Markowitz should not receive a Ph.D. in economics. Markowitz (1991) says, "...this point I am now willing to concede: at the time I defended my dissertation, portfolio theory was not part of Economics. But now it is."

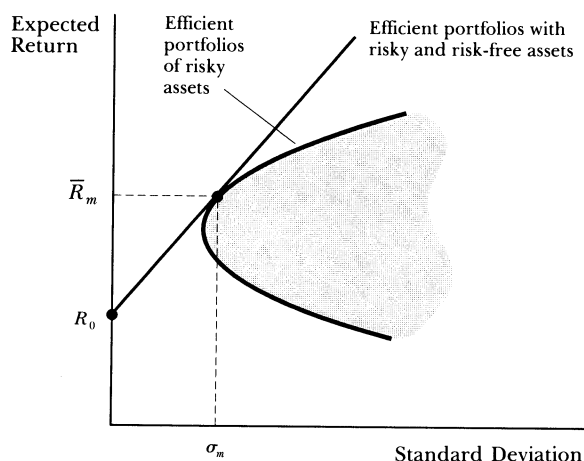
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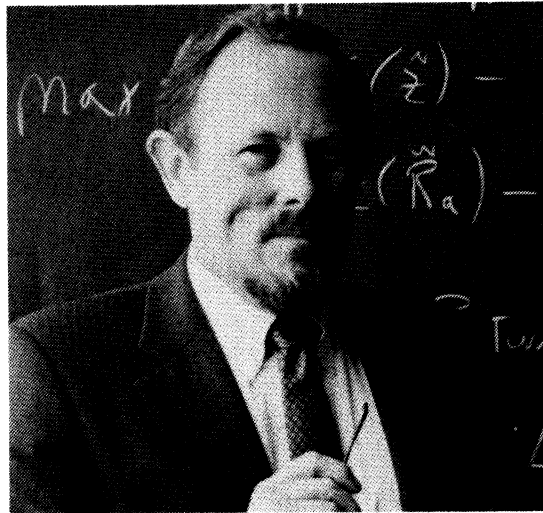
Markowitz's model of portfolio selection focused only on the choice of *risky* assets. Tobin (1958), motivated by Keynes' theory of liquidity preference, extended the model to include a riskless asset. In doing so, he discovered a surprising fact. The set of efficient risk-return combinations turned out to be a straight line!

The logic of Tobin's discovery can be seen with simple geometry. The hyperbola in Figure 1 depicts the combination of mean returns and standard deviation of returns that can be achieved by the various portfolios of risky assets. Each set of risky assets will generate some such hyperbola depicting the feasible combinations of risk and return.

The risk-free return has a standard deviation of zero, so it can be represented by a point on the vertical axis, $(0, R_0)$. Now make the following geometric construction: draw a line through the point $(0, R_0)$ and rotate it clockwise until it just touches the set of efficient portfolios. Call the point where

Figure 1





William Sharpe

it touches this line (σ_m, \bar{R}_m) . Now observe that every efficient portfolio consisting of risky assets and the riskless asset can be achieved by combining only two portfolios—one portfolio consisting only of the risk free asset, and one consisting of the portfolio that yields the risk-return combination (σ_m, \bar{R}_m) .

For example, if you want an expected return and standard deviation that is halfway between $(0, R_0)$ and (σ_m, \bar{R}_m) , just put half of your wealth in the risk-free asset and half in the risky portfolio. Points to the right of the risky portfolio can be achieved by leverage: *borrow* money at the rate R_0 and invest it in the risky portfolio.

Tobin's discovery dramatically simplified portfolio selection: his analysis showed the *same* portfolio of risky assets is appropriate for everyone. All that varies is how much money you choose to put in risky assets and how much you choose to put in the riskless asset. Each investor can limit his investment choices to two "mutual funds:" a money market fund that invests only in the riskless asset (e.g., Treasury bills) and another fund that invests only in the magical portfolio that yields (σ_m, \bar{R}_m) .

But one still needs to determine just which stocks, and which proportions of stocks, comprise the magic portfolio m —and that is a difficult and costly computation. The next contribution to portfolio theory was a simplified way to perform this computation. William Sharpe was a doctoral student at UCLA, one of the first students there to take courses in both economics and finance. When it came time to write a thesis, Fred Weston suggested that he talk with Harry Markowitz, who was then at RAND. Markowitz became Sharpe's unofficial thesis advisor and put him to work trying to simplify the computational aspects of portfolio theory.

Sharpe explored an approach now known as the “market model” or the “single factor” model. It assumes that the return on each security is linearly related to a single index, usually taken to be the return on some stock market index such as the S&P500. Thus the (random) return on asset a at time t can be written as

$$R_{at} = c + bR_{mt} + \varepsilon_{at},$$

where R_{mt} is the return on the S&P 500, say, and ε_{at} is an error term with expected value of zero. In this equation c is the expected return of the asset if the market is expected to have a zero return, while the parameter b measures the sensitivity of the asset to “market conditions.” A stock that has $b = 1$ is just as risky as the market index: if the S&P index increases by 10 percent in a given year, we would expect this stock to increase by $c + 10$ percent. A stock that has $b < 1$ is less volatile than the market index, while one with $b > 1$ is more volatile. Sharpe’s motivation in formulating this model was empirical: most stocks move together, most of the time. Hence, it is natural to think that a single factor (or small number of factors) determines most of the cross-sectional variation in returns.

This linear relationship can easily be estimated by ordinary least squares, and the estimated coefficients can be used to construct covariances, which, in turn, can be used to construct optimal portfolios. Sharpe’s approach reduced the dimensionality of the portfolio problem dramatically and made it much simpler to compute efficient portfolios. Problems that took 33 minutes of computer time using the Markowitz model took only 30 seconds with Sharpe’s model. This work led to Sharpe (1963) and a Ph.D. thesis.

Later, while teaching at the University of Washington, Sharpe turned his attention to equilibrium theory in capital markets. Up until this point portfolio theory was a theory of individual behavior—how an individual might choose his investments given the set of available assets.

What would happen, Sharpe asked, if everyone behaved like Markowitz portfolio optimizers? Tobin had shown that everyone would hold the same portfolio of risky assets. If Mr. A had 5 percent of his stock market wealth invested in IBM, then Ms. B should invest 5 percent of *her* stock portfolio in IBM. Of course, they might have different amounts of money invested in the stock market, but each would choose the same portfolio of risky assets. But Sharpe then realized that if everyone held the same portfolio of risky assets, then it would be easy to measure that portfolio: you just need to look at the *total* wealth invested in IBM, say, and divide that by the total wealth in the stock market. The portfolio of risky assets that was optimal for each individual would just be the portfolio of risky assets held by the market.

This insight gave Sharpe an empirical proxy for the risky portfolio in the Tobin analysis: in equilibrium it would simply be the market portfolio. This observation has the important implication that the market portfolio is

mean-variance efficient—that is, it lies on the frontier of the efficient set, and therefore satisfies the first-order conditions for efficiency.

Some simple¹ manipulations of those first-order conditions then yield the celebrated Capital Asset Pricing Model (CAPM):

$$\bar{R}_a = R_0 + \beta_a(\bar{R}_m - R_0).$$

In words, the expected return on any asset a is the risk-free rate plus the risk premium. The risk premium is the “beta” of the asset a times the expected excess return on the market portfolio.

The “beta” of an asset turns out to be the covariance of that asset’s return with the market return divided by the variance of the market return. This is simply the theoretical regression coefficient between the return on asset a and the market return, a result remarkably consistent with the single-factor model proposed in Sharpe’s thesis.

Meanwhile, back on the east coast, Jack Treynor and John Lintner were independently discovering the same fundamental pricing equation of the CAPM. Treynor’s work was never published; Sharpe (1964) and Lintner (1965) remain the classical citations for the CAPM.

The Capital Asset Pricing Model was truly a revolutionary discovery for financial economics. It is a prime example of how to take a theory of individual optimizing behavior and aggregate it to determine equilibrium pricing relationships. Furthermore, since the demand for an asset inevitably depends on the prices of all assets, due to the nature of the portfolio optimization problem, it is inherently a *general* equilibrium theory.

Sharpe’s two major contributions, the single factor model and the CAPM, are often confused. The first is a “supply side” model of how returns are generated; the second is a “demand side” model. The models can hold independently, or separately, and both are used in practice.

Subsequent research has relaxed many of the conditions of the original CAPM (like unlimited short sales) and provided some qualifications about the empirical observables of the model. Sharpe (1991) provides a brief review of these points. Despite these qualifications, the CAPM still reigns as one of the fundamental achievements of financial economics, taught in every finance textbook and intermediate microeconomics texts.²

Merton Miller

In 1990 Merton Miller was named a Distinguished Fellow of the American Economic Association in honor of his many contributions. He has worked on a

¹Sharpe’s proof of the CAPM was given in a footnote.

²Or at least, the good ones.

variety of topics in economics and finance, but the idea singled out by the Nobel Committee was one of his early papers on corporate finance. Portfolio theory and the CAPM focus on the behavior of the demanders of securities—the individual investors. Corporate finance focusses on the suppliers of the securities—the corporations that issue stocks and bonds.

Merton Miller joined Carnegie Tech in 1952 to teach economic history and public finance. In 1956, the dean asked Miller to teach corporate finance in the business school. At first Miller wasn't interested, since finance was then viewed as being a bit too grubby for an economist to dabble in. But after appropriate inducements, Miller sat in on the corporate finance class in the fall and started to teach it the following term.

One of the major issues in corporate finance, then and now, was how to raise capital in the best way. Broadly speaking a firm can issue new equity or new debt to raise money. Each has its advantages and disadvantages: issuing debt increases the fixed costs of the firm, while issuing equity dilutes the shares of the existing shareholders. There were lots of rules of thumb about when to do one and when to do the other. Miller started to look at some data to see if he could determine how corporate financial structure affected firms' values.

He found, much to his surprise, that there was no particular relationship between financial structure and firm value. Some firms had a lot of debt; some had a lot of equity, but there didn't appear to be much of a pattern in terms of how the debt-equity ratio affected market value.

It has been seriously suggested that there should be a *Journal of Negative Results* which could contain reports of all those regressions with insignificant regression coefficients and abysmal R-squares. If such a journal had existed, Miller might well have published his findings there. But there was no such journal, so Miller had to think about *why* there might be no relationship between capital structure and firm value.

Franco Modigliani, whose office was next to Miller's, had been working on some of the same issues from the theoretical side. He was concerned with providing microeconomic foundations for Keynesian models of investment. Building on previous work by Durand (1952), Modigliani had sketched out some models of financial structure that seemed to imply that there was no preferred capital structure. Miller and Modigliani joined forces, and the world of corporate finance has never been the same.

Miller and Modigliani considered a simple world without taxes or transactions costs and showed that in such a world, the value of a firm would be independent of its capital structure. Their argument was a novel application of the arbitrage principle, or the law of one price. Since the MM theorem has been described at least twice in this journal (Miller, 1988; Varian, 1987), I will give only a very brief outline of the theorem.

The easiest way to think about the MM theorem, in my view, is that it is a consequence of value additivity. Consider any portfolio of assets. Then value additivity says that the value of the portfolio must be the sum of the values of



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the assets that make it up. At first, this principal seems to contradict the insights of Markowitz about portfolio diversification: certainly an asset should be worth more combined in a portfolio with other assets than it is standing alone due to the benefits of diversification.

But the point is that asset values in a well-functioning securities market *already* reflect the value achievable by portfolio optimization. This is the chief insight of the Capital Asset Pricing Model: the equilibrium value of an asset depends on how it covaries with other assets, not on its risk as a stand-alone investment.

In any event, the principal of value additivity is even more fundamental than the Capital Asset Pricing Model, since it rests solely on arbitrage considerations. If a slice of bread and a piece of ham were worth more together as a sandwich than separately, everyone would buy bread and ham and make sandwiches—for a free lunch! The excess demand for bread and ham would push up the price of each, restoring the equilibrium relationship that the value of the whole has to be the sum of the value of the parts.

From this observation, the MM theorem follows quickly. The value of the firm is defined to be the sum of the values of its debt and its equity. If the firm could increase its value by changing how much of its cash flow is paid to bondholders and how much to stockholders, any individual investor could construct a free lunch. The investor would buy a fraction f of the outstanding stocks and the same fraction f of the outstanding bonds, which would give him a fraction f of the total cash flow. He could then repackage this cash flow in the same way as the firm could, thereby increasing the value of the total portfolio—and violating value additivity.

This sort of “home-made leverage” argument is one way to prove the MM theorem. But it is a particularly powerful way since it doesn’t appeal to any

particular model of consumer or firm behavior. It rests solely on the principle of arbitrage—there can be no free lunches in equilibrium.

The theory of the MM proposition is solidly established. The controversies all arise from the assumption of a frictionless world: in particular, no costs to bankruptcy, no asymmetric information, and no taxes. The latter is probably more important than the former. In the United States, at least, interest payments on debt are tax deductible while dividends to shareholders are taxed at both the corporate and individual level.

Since the MM proposition showed that debt and equity were perfect substitutes in the absence of taxes, the favorable tax treatment given to debt should imply that all firms are 100 percent debt-financed. This is contrary to fact—although for a while in the 1980s it looked as though it might come true. Miller (1988), and the comments on this article by Bhattacharya, Modigliani, Ross, and Stiglitz, describe the current state of research on the MM theorem. Suffice it to say that there is still doubt about exactly which frictions are the most relevant ones.

I happened to have lunch with Merton Miller in October 1990, the weekend before the Nobel prize winners were announced. Part of the lunchtime conversation was devoted to speculation about who might win the Nobel Prize in Economics that year. Mert thought that someone from Chicago might well receive the prize that year, and he suggested a few worthy possibilities—his own name not among them, of course. He was awarded the Nobel prize two days later. His 1990 forecast was a bit like the MM theorem itself—it was right in principle, but the details were a little off!

Summary

In reviewing the work of these three economists, we see a common thread of theory and empiricism running through their research. It isn't enough just to formulate a theory of portfolio choice—you've got to find a feasible way to compute optimal portfolios as well. It isn't enough to formulate a theory of capital market equilibrium—the theory should be estimated and tested. It isn't enough just to look at a scatterplot of firm values and debt-equity ratios—we need a theory for why there should or should not be a relationship among these variables.

Financial economics has been so successful because of this fruitful relationship between theory and data. Many of the same people who formulated the theories also collected and analyzed the data. This is a model that the rest of the economics profession would do well to emulate.

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