

# The CIEDE2000 Color-Difference Formula: Implementation Notes, Supplementary Test Data, and Mathematical Observations

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## ABSTRACT

This paper and the associated data and programs provided with it are intended to assist color engineers and scientists in correctly implementing the recently developed CIEDE2000 color-difference formula. We indicate several potential implementation errors that are not uncovered in tests performed using the original sample data published with the standard. A supplemental set of data is provided for comprehensive testing of implementations. The test data, Microsoft Excel spreadsheets and MATLAB scripts for evaluating the CIEDE2000 color difference are made available at the first author's website. Finally, we also point out small mathematical discontinuities in the formula.

**Keywords:** color-difference metrics; CIE; CIELAB; CIE94; CIEDE200; CMC

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# 1. INTRODUCTION

The CIEDE2000 formula was published by the CIE in 2001.<sup>1</sup> Developed by members of CIE Technical Committee 1-47, the formula provides an improved procedure for the computation of industrial color differences. The methodology used for developing the formula from experimental color difference data was described by Luo, Cui and Rigg<sup>2</sup> in a paper published in this journal. The paper also included the equations for the computation of the CIEDE2000 color difference as an appendix.

The CIEDE2000 formula is considerably more sophisticated and computationally involved than its predecessor color difference equations for CIELAB<sup>3</sup>  $\Delta E_{ab}^*$  and the CIE94<sup>4</sup> color difference  $\Delta E_{94}$ . Therefore it is important to verify that software implementations for computing color differences based on the new formula are extensively tested prior to their deployment. Toward this end, both the CIEDE2000 publications mentioned in the last paragraph included an identical set of worked examples for “confirmation of software implementation of CIEDE2000”.<sup>1</sup> However, these worked examples are a small set and do not adequately test the implementation of the CIEDE2000 formula. We discovered this limitation, in our efforts to implement the formula and validate it against publicly available implementations on the world-wide-web. In this process, we found several independent implementations. *For the worked examples included in the CIE technical report on CIEDE2000<sup>1</sup> all of these implementations provided results that were in agreement with the published data (up to reasonable numerical precision).* However, upon further testing, we discovered that on additional data, the results for the different implementations were different. In fact, several of the implementations distributed on the Internet, including some from reputable sources, were erroneous. Several of our own early implementations of the formula were also in this category of implementations that are erroneous but nevertheless provide correct results over the limited set of worked examples included with the draft standard.

We therefore believe that correct implementation of the CIEDE2000 formula is non-trivial and our goal in this paper is to assist in this process.

This paper has three main contributions. Firstly, we provide a comprehensive set of data for testing software implementations of the CIEDE2000 color-difference equation. The data is specifically designed for exercising components in the CIEDE2000 equations that may be erroneously computed and are not exercised by the original worked examples included with the draft standard. We also point out several of the common errors and indicate parts of the test data that help identify these. Secondly, we provide implementations of the CIEDE2000 color difference formula in Microsoft Excel and MATLAB.<sup>5</sup> These implementations agree with the original worked examples included in the CIE Technical report and with each other on the supplemental data included in this paper. We believe these to be correct implementations of the equations. Thirdly, we highlight and characterize discontinuities in the CIEDE2000 color difference formula that arise from its defining equations.

This paper addresses only the computational and mathematical aspects of the CIEDE2000 equations and does not in any way address psychophysical evaluation of the formula, attempt to improve its uniformity, or define its domain of applicability. Some comments of that nature are included in the notes by Kuehni<sup>6</sup> and Luo et al.<sup>7</sup> published in this journal.

## **2. THE CIEDE2000 COLOR-DIFFERENCE FORMULA**

In this section, we briefly outline the equations for the computation of the CIEDE2000 color-difference. The description follows the presentation of Luo, Cui and Rigg<sup>2</sup> but is slightly modified to make it closer to an algorithmic statement. It is included here in order to relate implementation notes and comments in subsequent sections to the equations presented here.

The CIEDE2000 color difference formula is based on the CIELAB color space. Given a pair of color values in CIELAB space  $L_1^*, a_1^*, b_1^*$  and  $L_2^*, a_2^*, b_2^*$ , we denote the CIEDE2000 color difference between them as \*

$$\Delta E_{00}(L_1^*, a_1^*, b_1^*; L_2^*, a_2^*, b_2^*) = \Delta E_{00}^{12} = \Delta E_{00} \quad (1)$$

Given two CIELAB color values  $\{L_i^*, a_i^*, b_i^*\}_{i=1}^2$  and parametric weighting factors  $k_L$ ,  $k_C$  and  $k_H$ , the process of computation of the color difference is summarized in the following equations, grouped as three main steps.

**1. Calculate  $C'_i, h'_i$ :**

$$C_{i,ab}^* = \sqrt{(a_i^*)^2 + (b_i^*)^2} \quad i = 1, 2 \quad (2)$$

$$\bar{C}_{ab}^* = \frac{C_{1,ab}^* + C_{2,ab}^*}{2} \quad (3)$$

$$G = 0.5 \left( 1 - \sqrt{\frac{\bar{C}_{ab}^{*7}}{\bar{C}_{ab}^{*7} + 25^7}} \right) \quad (4)$$

$$a_i' = (1 + G)a_i^* \quad i = 1, 2 \quad (5)$$

$$C_i' = \sqrt{(a_i')^2 + (b_i^*)^2} \quad i = 1, 2 \quad (6)$$

$$h_i' = \begin{cases} 0 & b_i^* = a_i' = 0 \\ \tan^{-1}(b_i^*, a_i') & \text{otherwise} \end{cases} \quad i = 1, 2 \quad (7)$$

**2. Calculate  $\Delta L', \Delta C', \Delta H'$ :**

$$\Delta L' = L_2^* - L_1^* \quad (8)$$

$$\Delta C' = C_2' - C_1' \quad (9)$$

$$\Delta h' = \begin{cases} 0 & C_1' C_2' = 0 \\ h_2' - h_1' & C_1' C_2' \neq 0; |h_2' - h_1'| \leq 180^\circ \\ (h_2' - h_1') - 360 & C_1' C_2' \neq 0; (h_2' - h_1') > 180^\circ \\ (h_2' - h_1') + 360 & C_1' C_2' \neq 0; (h_2' - h_1') < -180^\circ \end{cases} \quad (10)$$

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\*The commonly accepted notation is simply  $\Delta E_{00}$ , however, in certain cases, we will find it useful to use the longer and more explicit versions.

$$\Delta H' = 2\sqrt{C'_1 C'_2} \sin\left(\frac{\Delta h'}{2}\right) \quad (11)$$

3. Calculate CIEDE2000 Color Difference  $\Delta E_{00}$ :

$$\bar{L}' = (L_1^* + L_2^*)/2 \quad (12)$$

$$\bar{C}' = (C'_1 + C'_2)/2 \quad (13)$$

$$\bar{h}' = \begin{cases} \frac{h'_1 + h'_2}{2} & |h'_1 - h'_2| \leq 180^\circ; C'_1 C'_2 \neq 0 \\ \frac{h'_1 + h'_2 + 360^\circ}{2} & |h'_1 - h'_2| > 180^\circ; (h'_1 + h'_2) < 360^\circ; C'_1 C'_2 \neq 0 \\ \frac{h'_1 + h'_2 - 360^\circ}{2} & |h'_1 - h'_2| > 180^\circ; (h'_1 + h'_2) \geq 360^\circ; C'_1 C'_2 \neq 0 \\ (h'_1 + h'_2) & C'_1 C'_2 = 0 \end{cases} \quad (14)$$

$$T = 1 - 0.17 \cos(\bar{h}' - 30^\circ) + 0.24 \cos(2\bar{h}') \\ + 0.32 \cos(3\bar{h}' + 6^\circ) - 0.20 \cos(4\bar{h}' - 63^\circ) \quad (15)$$

$$\Delta\theta = 30 \exp\left\{-\left[\frac{\bar{h}' - 275^\circ}{25}\right]^2\right\} \quad (16)$$

$$R_C = 2\sqrt{\frac{\bar{C}'^7}{\bar{C}'^7 + 25^7}} \quad (17)$$

$$S_L = 1 + \frac{0.015(\bar{L}' - 50)^2}{\sqrt{20 + (\bar{L}' - 50)^2}} \quad (18)$$

$$S_C = 1 + 0.045\bar{C}' \quad (19)$$

$$S_H = 1 + 0.015\bar{C}'T \quad (20)$$

$$R_T = -\sin(2\Delta\theta)R_C \quad (21)$$

$$\Delta E_{00}^{12} = \Delta E_{00}(L_1^*, a_1^*, b_1^*; L_2^*, a_2^*, b_2^*) \\ = \sqrt{\left(\frac{\Delta L'}{k_L S_L}\right)^2 + \left(\frac{\Delta C'}{k_C S_C}\right)^2 + \left(\frac{\Delta H'}{k_H S_H}\right)^2 + R_T \left(\frac{\Delta C'}{k_C S_C}\right) \left(\frac{\Delta H'}{k_H S_H}\right)} \quad (22)$$

Several notes clarifying the implementation of the above equations are helpful:

1. The definition of the modified hue  $h'_i$  in Eqn. (7) uses a four-quadrant arctangent. The modified hue  $h'_i$  has the geometric interpretation in the two dimensional  $X-Y$  plane as the

angular position of the point  $(a'_i, b_i^*)$  measured from the positive  $X$  axis. To emphasize this we use the notation  $\tan^{-1}(b_i^*, a'_i)$  as opposed to the more common notation  $\tan^{-1}\left(\frac{b_i^*}{a'_i}\right)$ . The four-quadrant arctangent may be readily obtained using the two argument inverse tangent function available in several programming environments. Typically, these functions return an angular value in radians ranging from  $-\pi$  to  $\pi$ . This must be converted to a hue angle in degrees between  $0^\circ$  and  $360^\circ$  by addition of  $2\pi$  to negative hue angles, followed by a multiplication by  $180/\pi$  to convert from radians to degrees. This will provide a result consistent with subsequent equations. For the case where  $b_i^* = a'_i = 0$ , the modified hue is indeterminate and we have defined it as zero, a fact that we exploit in equation (14).

2. The chroma difference  $\Delta C'$  in Eqn. (9) and the hue difference  $\Delta H'$  in Eqn. (11) should be computed as signed differences as indicated in the equations and not as absolute values of the differences. While the equations defining the CIEDE2000 formula<sup>1,2</sup> are unambiguous in this respect, implementers may be susceptible to making this error<sup>†</sup>. The CIELAB<sup>3</sup>  $\Delta E_{ab}^*$  and the CIE94<sup>4</sup>  $\Delta E_{94}$  color differences used only terms in  $\Delta H^2$  and  $\Delta C^2$  hence required only the absolute value of the hue difference. This we conjecture might lead software implementers to use the absolute values for the hue and chroma difference when they begin with existing implementations of earlier color difference equations. This implementation error is hard to detect, because the sign of the hue and chroma differences plays a role only in the cross term

$$R_T \left( \frac{\Delta C'}{k_C S_C} \right) \left( \frac{\Delta H'}{k_H S_H} \right)$$

in Eqn. (22). Its impact is localized in the blue hue region (around  $275^\circ$ ), outside of which  $\Delta\theta$  and therefore the  $R_T$  term is rather small. Therefore the error of ignoring the sign of the hue and chroma difference is not readily diagnosed unless specifically tested for.

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<sup>†</sup>Some of the implementations of the CIEDE2000 color difference formula found on the Internet, were seen to have this error.

3. The computation of the hue angle difference in equation (10) is based on the description in the CIE technical report.<sup>1</sup> In addition, we use the reasonable interpretation that the hue difference between the origin in the  $a'$ ,  $b^*$  plane and any point in the plane is zero. The paper by Luo, Cui and Rigg<sup>2</sup> does not fully describe the hue difference computation. Using the arithmetic difference directly can result in an incorrect sign for the metric hue difference  $\Delta H'$  in (11). Additional details may also be found in Sève.<sup>8</sup>
4. The computation of the mean hue  $\bar{h}'$  used in equation (14) is not defined unambiguously in either the CIE technical report<sup>1</sup> or the paper by Luo, Cui and Rigg.<sup>2</sup> In most computations of angle, any ambiguities of  $360^\circ$  may be ignored. Due to the  $\Delta\theta$  term in Eqn. (16), this fact is, however, not true of the CIEDE2000 color difference equations. A verbatim interpretation of the note in section 2.6.1 of the CIE Technical report,<sup>1</sup> can lead to mean hue angles over  $360^\circ$ , which conflicts with the sentence at the end of section 2.7 of the same document. Likewise, a verbatim interpretation of the (slightly different) description for the mean hue computation in the appendix of the paper by Luo, Cui and Rigg<sup>2</sup> can lead to negative angles. In this paper, the definition of Eqn. (14) is therefore selected to provide an angle between  $0^\circ$  and  $360^\circ$  that agrees with the CIE technical report<sup>1</sup> up to addition and subtraction of multiples of  $360^\circ$ . Most existing implementations of the CIEDE2000 color difference formula found on the Internet at the time of this writing, were seen to have an error in the computation of average hue. The impact of the error may however be quite small in some cases.
5. The defined “boundary cases” for the computation of the mean hue in Eqn. (14) are classified differently in the CIE technical report<sup>1</sup> and in the paper by Luo, Cui and Rigg.<sup>2</sup> Their descriptions do not agree for the (rather rare) case when the absolute difference between the modified hue angles is exactly  $180^\circ$ . Our equations and implementations follow the CIE technical report<sup>1</sup> to the extent it is unambiguous. In the absence of a clear directive, we have (arbitrarily but within reason) defined the computation of mean hue

for the case when  $|h'_1 - h'_2| > 180^\circ$  and  $(h'_1 + h'_2) = 360^\circ$ . Our computation for this case results in a mean hue of  $0^\circ$  instead of the other reasonable choice of  $360^\circ$ .

6. The CIEDE2000 color difference is symmetric. However, the signed differences in lightness, hue and chroma in equations (8)-(11) are clearly not symmetric. Our notation assumes that the subscript 1 represents the reference color from which these signed differences are computed to the sample 2.

### 3. SUPPLEMENTAL TEST DATA FOR CIEDE2000 EQUATIONS

Table 1 provides a set of supplemental test data against which implementations of the CIEDE2000 color difference formula may be tested. The data are also available electronically at the first author's website<sup>5</sup> along with Microsoft Excel and MATLAB implementations of what we believe are correct implementations of the formula.

1. The CIELAB pairs labeled 1-3 test that the true chroma difference  $\Delta H$  (a signed quantity) is used in computing  $\Delta E_{00}$  instead of using just the absolute value.
2. The CIELAB pairs labeled 4-6 test that the true hue difference  $\Delta C$  (a signed quantity) is used in computing  $\Delta E_{00}$  instead of using just the absolute value.
3. The CIELAB pairs labeled 7-16 test that the arctangent computation for the determination of the hue in Eqn.(7) and the computation of mean hue in Eqn.(14) are performed correctly.
4. Note that the CIEDE2000 color difference formula is designed to be symmetric with respect to the two samples between which the color difference is computed. In the notation of Eqn. (1),

$$\Delta E_{00}^{12} = \Delta E_{00}(L_1^*, a_1^*, b_1^*, L_2^*, a_2^*, b_2^*) = \Delta E_{00}^{21} = \Delta E_{00}(L_2^*, a_2^*, b_2^*, L_1^*, a_1^*, b_1^*)$$



Pair	$i$	$L_i$	$a_i$	$b_i$	$a'_i$	$C'_i$	$h'_i$	$\bar{h}'$	$G$	$T$	$S_L$	$S_C$	$S_H$	$R_T$	$\Delta E_{00}$
1	1	50.0000	2.6772	-79.7751	2.6774	79.8200	271.9222	270.9611	0.0001	0.6907	1.0000	4.6578	1.8421	-1.7042	2.0425
	2	50.0000	0.0000	-82.7485	0.0000	82.7485	270.0000								
2	1	50.0000	3.1571	-77.2803	3.1573	77.3448	270.0000	271.1698	0.0001	0.6843	1.0000	4.6021	1.8216	-1.7070	2.8615
	2	50.0000	0.0000	-82.7485	0.0000	82.7485	270.0000								
3	1	50.0000	2.8361	-74.0200	2.8363	74.0743	272.1944	271.0972	0.0001	0.6865	1.0000	4.5285	1.8074	-1.7060	3.4412
	2	50.0000	0.0000	-82.7485	0.0000	82.7485	270.0000								
4	1	50.0000	-1.3802	-84.2814	-1.3803	84.2927	269.0618	269.5309	0.0001	0.7357	1.0000	4.7584	1.9217	-1.6809	1.0000
	2	50.0000	0.0000	-82.7485	0.0000	82.7485	270.0000								
5	1	50.0000	-1.1848	-84.8006	-1.1849	84.8089	269.1995	269.5997	0.0001	0.7335	1.0000	4.7700	1.9218	-1.6822	1.0000
	2	50.0000	0.0000	-82.7485	0.0000	82.7485	270.0000								
6	1	50.0000	-0.9009	-85.5211	-0.9009	85.5258	269.3964	269.6982	0.0001	0.7303	1.0000	4.7862	1.9217	-1.6840	1.0000
	2	50.0000	0.0000	-82.7485	0.0000	82.7485	270.0000								
7	1	50.0000	0.0000	0.0000	0.0000	0.0000	0.0000	126.8697	0.5000	1.2200	1.0000	1.0562	1.0229	0.0000	2.3669
	2	50.0000	-1.0000	2.0000	-1.5000	2.5000	126.8697								
8	1	50.0000	-1.0000	2.0000	-1.5000	2.5000	126.8697	126.8697	0.5000	1.2200	1.0000	1.0562	1.0229	0.0000	2.3669
	2	50.0000	0.0000	0.0000	0.0000	0.0000	0.0000								
9	1	50.0000	2.4900	-0.0010	3.7346	3.7346	359.9847	269.9854	0.4998	0.7212	1.0000	1.1681	1.0404	-0.0022	7.1792
	2	50.0000	-2.4900	-0.0009	-3.7346	3.7346	179.9862								
10	1	50.0000	2.4900	-0.0010	3.7346	3.7346	359.9847	269.9847	0.4998	0.7212	1.0000	1.1681	1.0404	-0.0022	7.1792
	2	50.0000	-2.4900	0.0010	-3.7346	3.7346	179.9847								
11	1	50.0000	2.4900	-0.0010	3.7346	3.7346	359.9847	89.9839	0.4998	0.6175	1.0000	1.1681	1.0346	0.0000	7.2195
	2	50.0000	-2.4900	0.0011	-3.7346	3.7346	179.9831								
12	1	50.0000	2.4900	-0.0010	3.7346	3.7346	359.9847	89.9831	0.4998	0.6175	1.0000	1.1681	1.0346	0.0000	7.2195
	2	50.0000	-2.4900	0.0012	-3.7346	3.7346	179.9816								
13	1	50.0000	-0.0010	2.4900	-0.0015	2.4900	90.0345	180.0328	0.4998	0.9779	1.0000	1.1121	1.0365	0.0000	4.8045
	2	50.0000	0.0009	-2.4900	0.0013	2.4900	270.0311								
14	1	50.0000	-0.0010	2.4900	-0.0015	2.4900	90.0345	180.0345	0.4998	0.9779	1.0000	1.1121	1.0365	0.0000	4.8045
	2	50.0000	0.0010	-2.4900	0.0015	2.4900	270.0345								
15	1	50.0000	-0.0010	2.4900	-0.0015	2.4900	90.0345	0.0362	0.4998	1.3107	1.0000	1.1121	1.0493	0.0000	4.7461
	2	50.0000	0.0011	-2.4900	0.0016	2.4900	270.0380								
16	1	50.0000	2.5000	0.0000	3.7496	3.7496	0.0000	315.0000	0.4998	0.8454	1.0000	1.1406	1.0396	-0.0001	4.3065
	2	50.0000	0.0000	-2.5000	0.0000	2.5000	270.0000								
17	1	50.0000	2.5000	0.0000	3.4569	3.4569	0.0000	346.2470	0.3827	1.4453	1.1608	1.9547	1.4599	-0.0003	27.1492
	2	73.0000	25.0000	-18.0000	34.5687	38.9743	332.4939								
18	1	50.0000	2.5000	0.0000	3.4954	3.4954	0.0000	51.7766	0.3981	0.6447	1.0640	1.7498	1.1612	0.0000	22.8977
	2	61.0000	-5.0000	29.0000	-6.9907	29.8307	103.5532								
19	1	50.0000	2.5000	0.0000	3.5514	3.5514	0.0000	272.2362	0.4206	0.6521	1.0251	1.9455	1.2055	-0.8219	31.9030
	2	56.0000	-27.0000	-3.0000	-38.3556	38.4723	184.4723								
20	1	50.0000	2.5000	0.0000	3.5244	3.5244	0.0000	11.9548	0.4098	1.1031	1.0400	1.9120	1.3353	0.0000	19.4535
	2	58.0000	24.0000	15.0000	33.8342	37.0102	23.9095								
21	1	50.0000	2.5000	0.0000	3.7494	3.7494	0.0000	3.5056	0.4997	1.2616	1.0000	1.1923	1.0808	0.0000	1.0000
	2	50.0000	3.1736	0.5854	4.7596	4.7594	7.0113								
22	1	50.0000	2.5000	0.0000	3.7493	3.7493	0.0000	0.0000	0.4997	1.3202	1.0000	1.1956	1.0861	0.0000	1.0000
	2	50.0000	3.2972	0.0000	4.9450	4.9450	0.0000								
23	1	50.0000	2.5000	0.0000	3.7497	3.7497	0.0000	5.8190	0.4999	1.2197	1.0000	1.1486	1.0604	0.0000	1.0000
	2	50.0000	1.8634	0.5757	2.7949	2.8536	11.6380								
24	1	50.0000	2.5000	0.0000	3.7493	3.7493	0.0000	1.9603	0.4997	1.2883	1.0000	1.1946	1.0836	0.0000	1.0000
	2	50.0000	3.2592	0.3350	4.8879	4.8994	3.9206								
25	1	60.2574	-34.0099	36.2677	-34.0678	49.7590	133.2085	132.0835	0.0017	1.3010	1.1427	3.2946	1.9951	0.0000	1.2644
	2	60.4626	-34.1751	39.4387	-34.2333	52.2238	130.9584								
26	1	63.0109	-31.0961	-5.8663	-32.6194	33.1427	190.1951	188.8221	0.0490	0.9402	1.1831	2.4549	1.4560	0.0000	1.2630
	2	62.8187	-29.7946	-4.0864	-31.2542	31.5202	187.4490								
27	1	61.2901	3.7196	-5.3901	5.5668	7.7487	315.9240	310.0313	0.4966	0.6952	1.1586	1.3092	1.0717	-0.0032	1.8731
	2	61.4292	2.2480	-4.9620	3.3644	5.9950	304.1385								
28	1	35.0831	-44.1164	3.7933	-44.3939	44.5557	175.1161	176.4290	0.0063	1.0168	1.2148	2.9105	1.6476	0.0000	1.8645
	2	35.0232	-40.0716	1.5901	-40.3237	40.3550	177.7418								
29	1	22.7233	20.0904	-46.6940	20.1424	50.8532	293.3339	291.3809	0.0026	0.3636	1.4014	3.1597	1.2617	-1.2537	2.0373
	2	23.0331	14.9730	-42.5619	15.0118	45.1317	289.4279								
30	1	36.4612	47.8580	18.3852	47.9197	51.3256	20.9901	21.8781	0.0013	0.9239	1.1943	3.3888	1.7357	0.0000	1.4146
	2	36.2715	50.5065	21.2231	50.5716	54.8444	22.7660								
31	1	90.8027	-2.0831	1.4410	-3.1245	3.4408	155.2410	167.1011	0.4999	1.1546	1.6110	1.1329	1.0511	0.0000	1.4441
	2	91.1528	-1.6435	0.0447	-2.4651	2.4655	178.9612								
32	1	90.9257	-0.5406	-0.9208	-0.8109	1.2270	228.6315	218.4363	0.5000	1.3916	1.5930	1.0620	1.0288	0.0000	1.5381
	2	88.6381	-0.8985	-0.7239	-1.3477	1.5298	208.2412								
33	1	6.7747	-0.2908	-2.4247	-0.4362	2.4636	259.8025	263.0049	0.4999	0.9556	1.6517	1.1057	1.0337	-0.0004	0.6377
	2	5.8714	-0.0985	-2.2286	-0.1477	2.2335	266.2073								
34	1	2.0776	0.0795	-1.1350	0.1192	1.1412	275.9978	268.0910	0.5000	0.7826	1.7246	1.0383	1.0100	0.0000	0.9082
	2	0.9033	-0.0636	-0.5514	-0.0954	0.5596	260.1842								

Table 1. CIEDE2000 total color difference test data

It is recommended that implementations of the formula should explicitly test this by interchanging the two sets of data between which color differences are computed and verifying that color differences are unchanged in this process. In particular, the use of an absolute value for either but not both of the chroma and hue differences will lead to an asymmetric color difference formula. The use of an absolute value for both, will produce a symmetric but incorrect formula.

5. Though the CIEDE2000 color difference equations are only applicable for small color differences, it is preferable that data for testing software implementations should include at least a few large color differences, since larger differences are not easily confused with variation in numerical precision and can therefore more easily distinguish differences or errors in implementations. The CIELAB pairs labeled 17-20 in the table are included for this reason.
6. The CIELAB pairs labeled 25-34 in the table correspond to data published in CIE technical report<sup>1</sup> and in the paper by Luo, Cui and Rigg.<sup>2</sup>

Note that the original set of worked examples included in the CIE Technical report on CIEDE2000<sup>1</sup> do not test any features of the equations mentioned in items 1-4. Implementations having one or more of these errors are therefore not identified by the original worked examples but readily identified using the test data provided here.

## **4. DISCONTINUITIES IN THE CIEDE2000 COLOR DIFFERENCE FORMULA**

From a quick review of equations defining the CIEDE2000 color difference formula published in the CIE Technical report,<sup>1</sup> it appears that all the terms involved are well behaved. One

would therefore expect the  $\Delta E_{00}$  difference to be a continuous and differentiable function of the input CIELAB color pairs. In fact, however, the function has three independent sources of mathematical discontinuities that can be seen by examining the equations presented in this paper. In the following, we describe and characterize these discontinuities in order of decreasing discontinuity magnitude.

### 1. Discontinuity due to Mean-hue computation:

A discontinuity arises in the  $\Delta E_{00}$  difference due to the process used in Eqn. (14) for the computation of mean hue  $\bar{h}'$  from the modified hue angles  $h'_1$  and  $h'_2$  of the two color samples. As indicated in the equation, the mathematical average is used in situations where the absolute difference between the modified hue angles is less than or equal to  $180^\circ$  and in situations where the absolute difference between the modified hue angles is greater than  $180^\circ$ , we add/subtract  $360^\circ$  to/from the sum of the angles and divide by two to get the mean hue. The latter computation is equivalent to adding/subtracting an angle of  $180^\circ$  to/from the mathematical average. This leads to a discontinuity of  $180^\circ$  for the mean hue angle as elaborated in greater detail below.

Consider two modified hue angles  $\phi$  and  $\phi + 180^\circ - \epsilon/2$ , where  $\epsilon$  is a small positive quantity, since the absolute value of difference between these two angles is  $180 - \epsilon/2$ , which is under  $180^\circ$ , the average hue corresponding to these is given by

$$\bar{h}'(\phi, \phi + 180^\circ - \epsilon/2) = (\phi + \phi + 180^\circ - \epsilon/2)/2 = \phi + 90^\circ - \epsilon/4 \quad (23)$$

Next, consider the hue angles  $\phi$  and  $\phi + 180^\circ + \epsilon/2$ . Since the absolute value of difference between these two angles is  $180 + \epsilon/2$ , which is over  $180^\circ$ , the mean hue corresponding to these is

$$\bar{h}'(\phi, \phi + 180^\circ + \epsilon/2) = \begin{cases} (\phi + \phi + 180^\circ + \epsilon/2 + 360^\circ)/2 = \phi + 270^\circ + \epsilon/4 & h'_1 + h'_2 < 360 \\ (\phi + \phi + 180^\circ + \epsilon/2 - 360^\circ)/2 = \phi - 90^\circ + \epsilon/4 & h'_1 + h'_2 \geq 360^\circ \end{cases} \quad (24)$$

Subtracting Eqn. (24) from Eqn. (23), we get

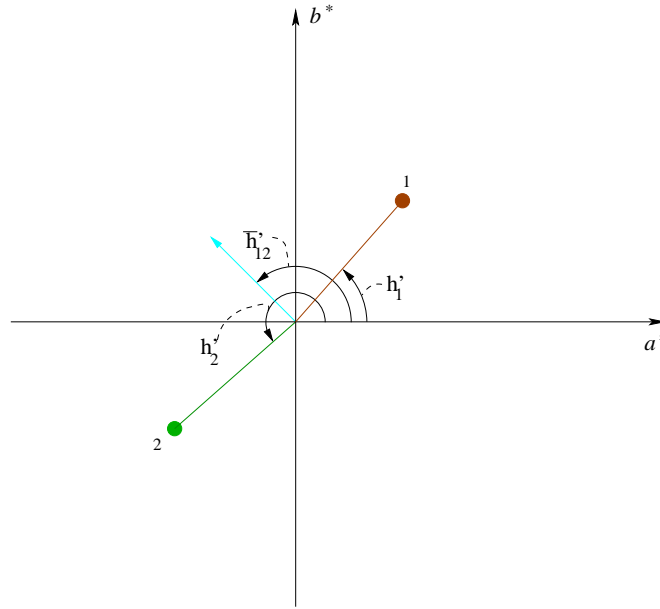
$$\bar{h}'(\phi, \phi + 180^\circ + \epsilon/2) - \bar{h}'(\phi, \phi + 180^\circ - \epsilon/2) = \begin{cases} 180^\circ + \epsilon/2 & h'_1 + h'_2 < 360 \\ -180^\circ + \epsilon/2 & h'_1 + h'_2 \geq 360^\circ \end{cases} \quad (25)$$

Thus a small perturbation  $\epsilon$  in one of the hue angles produces a change of approximately  $180^\circ$  in the mean hue. The mean hue is therefore a discontinuous function.

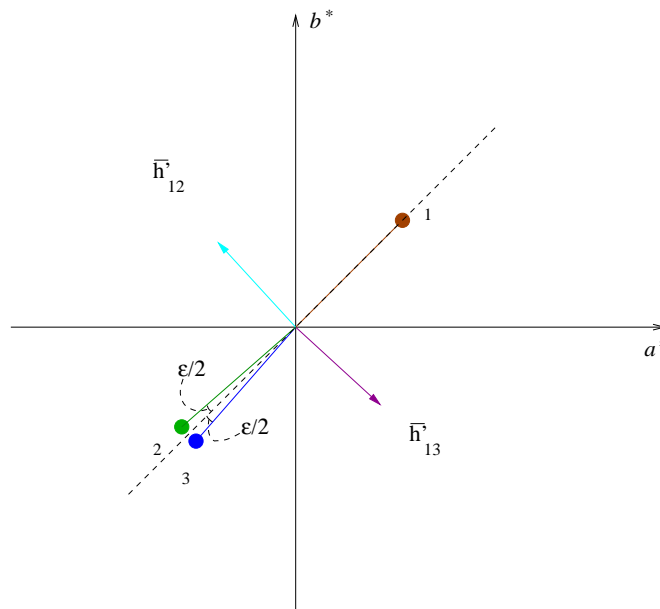
A geometric illustration of the discontinuity provides clearer insight than the equations presented above. The mean hue  $\bar{h}'$  can be geometrically interpreted as the hue angle of the angular bisector of line segments drawn at the modified hue angles  $h'_1$  and  $h'_2$ . This is illustrated in Fig. 1. The two dots in this figure labeled 1, 2 represent sample colors projected onto the  $a'-b^*$  plane. Line segments have been drawn from the origin in the plane to each of the colors' projections. The modified hue angle of each color is the angle that the line segment makes with respect to the  $a'$  axis measured in the counter-clockwise direction as indicated by the arcs labeled  $h'_1$  and  $h'_2$ . The angular bisector of the two line segments is shown as the cyan line segment with a pointed arrow. The mean hue  $\bar{h}'_{12}$  corresponds to the angle that this bisector makes with respect to the  $a'$  axis measured in the counter-clockwise direction as indicated by the arcs labeled  $\bar{h}'_{12}$ .

Using the geometric interpretation of the mean hue, the discontinuity introduced in its computation is readily seen in Fig. 2. In the figure, three sample colors labeled 1, 2, and 3 are plotted on the  $a'-b^*$  plane such that the modified hue angles of 2 and 3 are close to  $180^\circ$  apart from the modified hue angle of 1 with the absolute hue angle difference between 1 and 2 just under  $180^\circ$  and the absolute hue angle difference between 1 and 3 just over  $180^\circ$ . The cyan line segment with the arrow represents the mean hue  $\bar{h}'_{12}$  for samples 1 and 2 and the magenta line segment with the arrow represents the mean hue  $\bar{h}'_{13}$  for samples 1 and 3. From the figure, it is clear that the small perturbation from 2 to 3 produces a change in mean hue of over  $180^\circ$ .

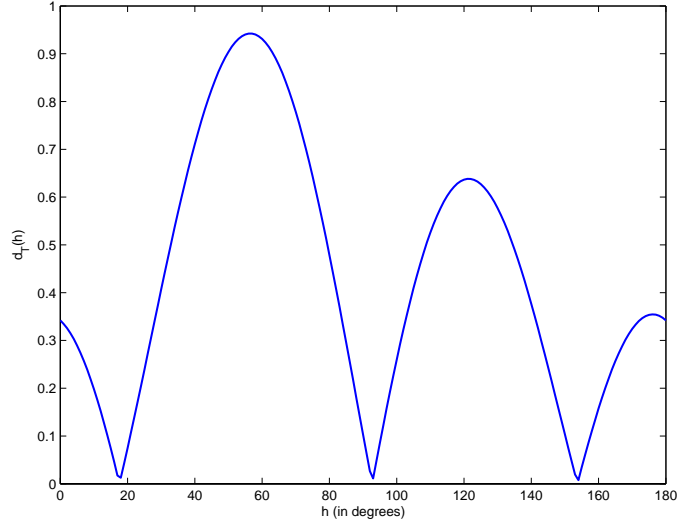
The discontinuity resulting from the mean hue computation impacts the terms in the



**Figure 1.** Geometrical illustration of the computation of mean hue.



**Figure 2.** Geometric illustration of the 180° discontinuity in mean hue computation.



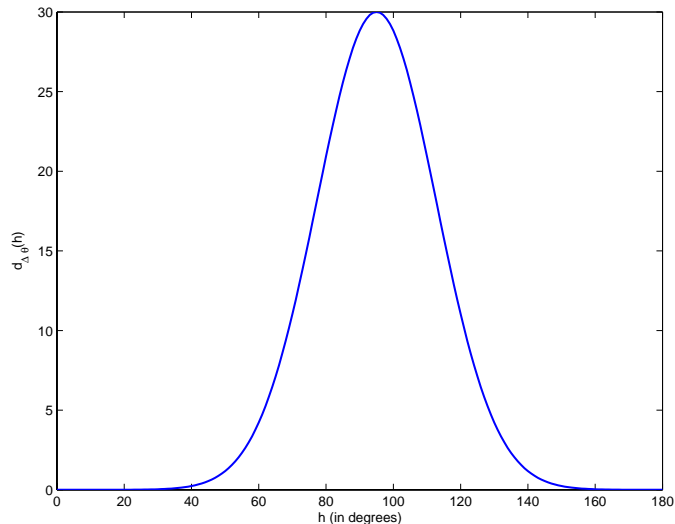
**Figure 3.** The magnitude of discontinuity  $d_T(h)$  in the term  $T$  of the CIEDE2000 equations as a function of mean hue.

CIEDE2000 equations that use this mean hue in further computations. At the lowest level, this occurs in the terms  $T$  and  $\Delta\theta$  in equations (15) and (16), respectively. If  $f(h)$  is any function of the mean hue, the magnitude of the discontinuity introduced in  $f(\bar{h})$  can be readily determined as

$$d_f(h) = |f(h) - f(h + 180)| \quad h < 180^\circ \quad (26)$$

Plots of the discontinuity magnitude in  $T$ ,  $d_T(h)$ , and the discontinuity magnitude in  $\Delta\theta$ ,  $d_{\Delta\theta}(h)$ , are shown in Figs. 3 and 4, respectively.

From the plots in Figs. 3 and 4 it is apparent that the discontinuity in the computation of the mean hue causes a significant discontinuity in the  $T$  and  $\Delta\theta$  terms of the CIEDE2000 equations. Since these terms are used in combination with other terms in the computation of the final color difference  $\Delta E_{00}$ , the magnitude of the discontinuity in  $\Delta E_{00}$  cannot be directly inferred based on the discontinuity in these terms. The complicated nature of the  $\Delta E_{00}$  equations makes analytical determination of the discontinuity magnitude rather difficult. Instead, we adopt an empirical approach.



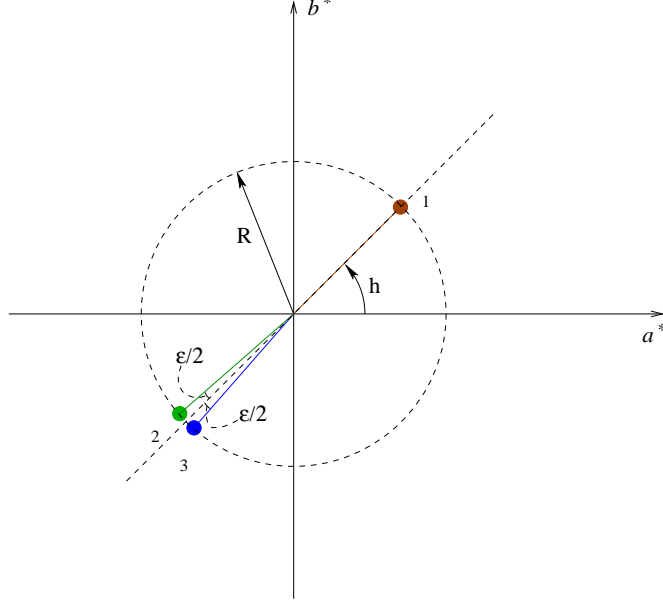
**Figure 4.** The magnitude of discontinuity  $d_{\Delta\theta}(h)$  in the term  $\Delta\theta$  of the CIEDE2000 equations as a function of mean hue.

We begin by selecting a suitable set of colors in CIELAB space that illustrate the discontinuity while minimizing/eliminating the impact of other discontinuities. From Fig. 2, it is apparent that the discontinuity in the computation of the mean hue arises only for colors that are  $180^\circ$  apart in CIELAB hue angle<sup>‡</sup>. From the CIEDE2000 equations it is also clear that the magnitude of the discontinuity in  $\Delta E_{00}$  is not a fixed value but will depend on the choice of the pair of color locations. For estimating the magnitude of discontinuity in the  $\Delta E_{00}$  formula arising due to the mean hue computation, we use the specific configuration shown in the  $a^*-b^*$  plane in Fig 5<sup>§</sup>. As will be illustrated subsequently, the choice of this configuration eliminates any impact of the second discontinuity. The point labeled as 1 represents a reference color is located at CIELAB hue  $h$  and has CIELAB chroma  $R$ . The points labeled 2 and 3 represent two samples, also having CIELAB chroma  $R$ , but located at hue angles  $h + 180^\circ - \epsilon/2$  and  $h + 180^\circ + \epsilon/2$ , respectively. For our computations, we

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<sup>‡</sup>Note that colors are under/over/exactly  $180^\circ$  apart in  $a'-b^*$  hue if and only if they are under/over/exactly  $180^\circ$  apart in CIELAB hue.

<sup>§</sup>Since the  $L^*$  values of the chosen pairs do not contribute to the discontinuity or influence its magnitude, we fix the  $L^*$  values at 50 for our reference and samples (any other value may be equivalently used).



**Figure 5.** Color configuration for empirical evaluation of  $\Delta E_{00}$  discontinuity due to mean hue computation.

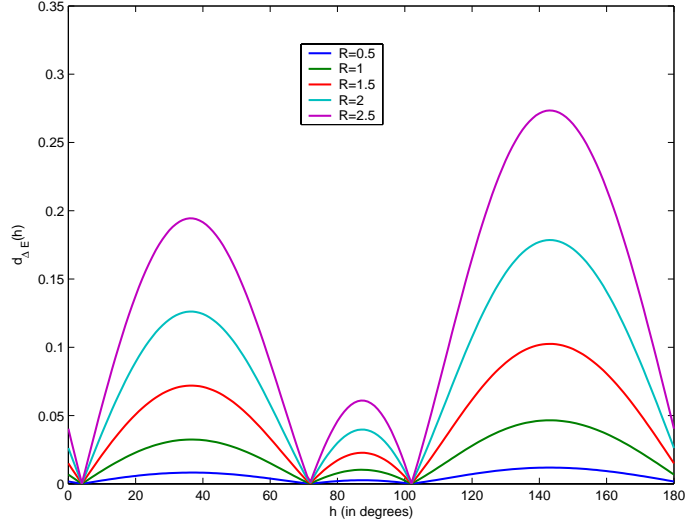
use  $\epsilon/2 = 10^{-6}$  radians. In general,  $\epsilon/2$  should be the smallest possible value whose impact is not masked by the limited precision of computation.

The magnitude of the discontinuity in the CIEDE2000 color difference formula (due to the discontinuity in mean hue computation) is then estimated as the absolute value of the difference between the CIEDE2000 color difference  $\Delta E_{00}^{12}$  between 1 and 2 and the CIEDE2000 color difference  $\Delta E_{00}^{13}$  between 1 and 3

$$d_{\Delta E}(h) = |\Delta E_{00}^{12} - \Delta E_{00}^{13}| \quad (27)$$

Fig. 6 shows plots of the discontinuity magnitude  $d_{\Delta E}(h)$  as a function of the hue  $h$  of the reference sample, for values of the chroma  $R$  ranging from 0.5 to 2.5 in steps of 0.5. For these and all other numerical/graphical  $\Delta E_{00}$  values reported in this paper, we set the parametric weighting factors to unity, i.e.,  $k_L = k_C = k_H = 1$ . From the figure it can be seen that the magnitude of the discontinuity has local maxima at hue values of roughly  $37^\circ$ ,  $87^\circ$  and  $143^\circ$ ; with the highest value at  $143^\circ$ . The magnitude of the





**Figure 6.** The magnitude of discontinuity in  $\Delta E_{00}$  for CIELAB colors as a function of reference color chroma and hue.

discontinuity increases with increase in the chroma value  $R$ . For the chosen chroma ( $R$ ) values of 0.5, 1.0, 1.5, 2.0, 2.5 the maximum values of the discontinuity magnitude (i.e., its value at  $143^\circ$ ) are 0.0119, 0.0465, 0.1025, 0.1786, 0.2734, respectively. The magnitude of the discontinuity is therefore relatively small, though not negligible.

From a computation of the individual terms involved in equation (22), one can also infer that the major contribution to the discontinuity in  $\Delta E_{00}$  is due to the term  $\left(\frac{\Delta H'}{k_H S_H}\right)^2$ . A discontinuity is introduced in this term through  $S_H$  which in turn inherits the discontinuity from  $T$ , which was discussed earlier.

## 2. Discontinuity in Hue Difference Computation:

Similar to the discontinuity in the computation of mean hue, a discontinuity also arises in the computation of hue difference in equation (10). The source of the discontinuity is the inherent ambiguity in the sign of the hue difference between two modified hues that are exactly  $180^\circ$  apart. The sign may be arbitrarily chosen as positive or negative. Though we specify the sign as positive in equation (10), this does not eliminate the problem of

change in sign of the hue difference as one goes from modified hue angles whose arithmetic difference is just under  $180^\circ$  to hue angles whose arithmetic difference is just over  $180^\circ$ . Again, consider three modified hue angles  $h'_1 = \phi$ ,  $h'_2 = \phi + 180^\circ - \epsilon/2$ , and  $h'_3 = \phi + 180^\circ + \epsilon/2$ . From equation (10) the hue differences between the pairs  $h'_1, h'_2$  and  $h'_1, h'_3$  are

$$\Delta h'(h'_1, h'_2) = 180^\circ - \epsilon/2 \quad (28)$$

$$\Delta h'(h'_1, h'_3) = (180^\circ + \epsilon/2) - 360^\circ = -180^\circ + \epsilon/2 \quad (29)$$

and

$$\Delta h'(h'_1, h'_2) - \Delta h'(h'_1, h'_3) = 360^\circ - \epsilon$$

illustrating the discontinuity of  $360^\circ$ . This in turn implies a discontinuity of  $180^\circ$  in  $\Delta h'/2$  which corresponds to a sign reversal in  $\sin\left(\frac{\Delta h'}{2}\right)$  and thus in  $\Delta H'$  in equation (11). Thus the impact of the discontinuity in computation of hue difference is a sign reversal in  $\Delta H'$  ¶.

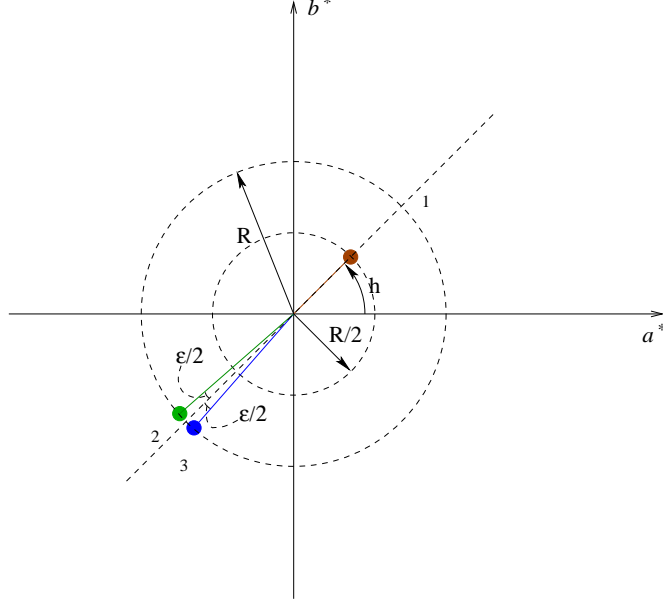
The final  $\Delta E_{00}$  computation in equation (22) has only one term that is not invariant to a sign reversal in  $\Delta H'$ . This is the rotation term

$$\tau = R_T \left( \frac{\Delta C'}{k_C S_C} \right) \left( \frac{\Delta H'}{k_H S_H} \right) \quad (30)$$

It is clear that this term is zero when  $\Delta C' = 0$ . Thus the rotation term  $\tau$  is uniformly zero for color pairs located at the same chroma radius. This is true for the configuration of colors chosen for the illustration of the discontinuity due to mean hue (discussed earlier and presented in Fig. 6). As a result, even though both the discontinuities – due to mean-hue computation and due to hue-difference computation – occur for colors placed  $180^\circ$  apart in hue, the latter made no contribution in our empirical evaluation of the former that was presented in Fig. 6. One cannot, however, isolate the impact of the discontinuity due to hue-difference computation in a similar fashion. We therefore illustrate the impact

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¶The CIE 1976 and 1994 color difference formulae can also be expressed in terms of a metric hue difference  $\Delta H'$ . However, they do not have the discontinuity because they depend only on  $|\Delta H'|^2$  and are therefore independent of the sign of  $\Delta H'$ .



**Figure 7.** Color configuration for empirical evaluation of  $\Delta E_{00}$  discontinuity due to hue-difference computation.

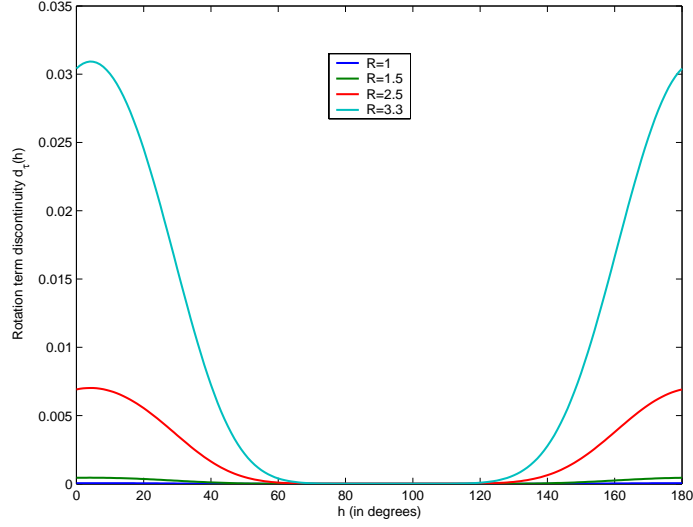
of this discontinuity on  $\tau$  – where it is localized – and on the overall  $\Delta E_{00}$ , where as we illustrate it is dominated by the discontinuity due to mean-hue computation.

From the rotation term in Eqn. (30) and the equations for its components, one can readily see that in order for this term to be non-zero both  $\Delta C'$  and  $\sqrt{C'_1 C'_2}$  must be non-zero. We therefore select the configuration of colors shown in Fig. 7 in the  $a^*-b^*$  plane to empirically evaluate the discontinuity due to hue-difference computation. The reference 1 is located at a hue angle  $h$  at chroma radius  $R/2$  and the two almost identical samples 2 and 3 are located at a chroma radius of  $R$  with hue angles just above and just below  $h + 180^\circ$ . We estimate the magnitude of discontinuity in  $\tau$  as

$$d_\tau(h) = |\tau^{12} - \tau^{13}|$$

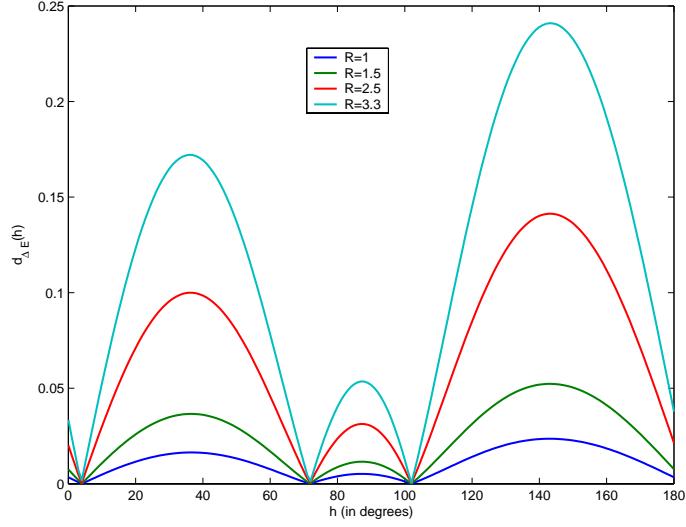
and the magnitude of the overall discontinuity in  $\Delta E_{00}$  as in Eqn.( 27).

Fig. 8 shows plots of the discontinuity magnitude  $d_\tau(h)$  as a function of the hue  $h$  of the reference sample, for values of the chroma  $R$  ranging from 1.0 to 3.3. The sign reversal



**Figure 8.** The magnitude of discontinuity in the rotation term  $\tau$  for CIELAB colors in the configuration of Fig. 7 as a function of hue for various chroma values  $R$ .

in  $\Delta H'$  which causes a sign reversal in  $\tau$  and therefore contributes to this discontinuity (though it also includes the impact of the discontinuity due to mean-hue computation through  $S_H$ ). From the figure, it can be seen that the discontinuity magnitude is significantly smaller than the discontinuity magnitude in  $\Delta E_{00}$  due to mean-hue computation. The discontinuity magnitude has a unique maximum at approximately  $4^\circ$  and increases with increase in  $R$  (chroma of sample). For  $R = 3.3$  the maximum value of the discontinuity magnitude in the rotation term  $\tau$  for this configuration is 0.0309. The location of the maxima for the discontinuity is in agreement with the functional form for  $\Delta\theta$  in Eqn. (16) which indicates that the impact of the rotation term  $\tau$  will be localized around a mean hue of  $275^\circ$ . The hue locations  $h'_1 = 4^\circ$  and  $h'_2 = 184^\circ + \epsilon/2$  result in a mean hue of  $274^\circ + \epsilon/4$  maximizing  $\Delta\theta$ , whereas for hue locations of  $h'_1 = 4^\circ$  and  $h'_2 = 184^\circ - \epsilon/2$  the mean hue is  $94 - \epsilon/4$  for which  $\Delta\theta$  is negligible. Thus even though the discontinuity due to hue-difference computation reverses the sign of  $\tau$ , the simultaneous interaction with the mean-hue discontinuity actually ensures that at least one of the terms becomes quite small.



**Figure 9.** The magnitude of discontinuity in  $\Delta E_{00}$  for CIELAB colors in the configuration of Fig. 7 as a function of hue for various chroma values  $R$ .

Fig. 9 shows plots of the overall discontinuity magnitude  $d_{\Delta E}(h)$  in  $\Delta E_{00}$  as a function of the hue  $h$  of the reference sample, for colors in the configuration of Fig. 7, for values of the chroma  $R$  ranging from 1.0 to 3.3. Upon comparing the plots to those in Fig. 6, one sees a very strong similarity in the shapes of these plots. This indicates that the discontinuity due to mean-hue computation is predominant and the impact of the discontinuity due to hue difference computation is negligible. Overall the discontinuity magnitude is smaller than that encountered due to the mean-hue computation. Once again the magnitude of discontinuity increases with increase in  $R$ .

### 3. Discontinuity due to Hue Roll-over at 360°:

From a mathematical perspective, there is also another discontinuity in  $\Delta\theta$  term in Eqn. (16) at a mean hue of 0/360 degrees because this term is not invariant to shifts of 360° unlike typical trigonometric functions. The break at this discontinuity is however extremely small, the change in  $\Delta\theta$  is only

$$30 \left( \exp \left( - \left[ \frac{85}{25} \right]^2 \right) - \exp \left( - \left[ \frac{275}{25} \right]^2 \right) \right) \approx 2.8620 \times 10^{-4}$$

This discontinuity may therefore be disregarded in most practical applications.

For general applications of the  $\Delta E_{00}$  formula, it is also helpful if we can bound the **maximum discontinuity** magnitude that may be encountered. An empirical evaluation was therefore performed, where for each selected hue angle, we determined the points with the maximal discontinuity in  $\Delta E_{00}$  that were within a CIELAB  $\Delta E_{ab}^*$  of 5 units from each other (equivalently, within 5 CIELAB chroma units from each other). In each of the cases tested, the maximal discontinuity occurred for a configuration of colors that was very close to that used in Fig. 5, with  $R = 2.5$ . Thus the plot corresponding to  $R = 2.5$  in Fig. 6 also represents the maximal discontinuity as a function of hue. Globally, for colors that are within 5 CIELAB  $\Delta E_{ab}^*$  units from each other, the discontinuity in CIEDE2000 color difference  $\Delta E_{00}$  is under 0.2734.

## 5. IMPLICATIONS OF DISCONTINUITIES IN $\Delta E_{00}$

As outlined in the previous section, the significant discontinuity in the computation of the CIEDE2000 color difference manifests itself only for samples that are  $180^\circ$  apart in hue. Since the CIEDE2000 formula is applicable primarily for small color differences, both samples will typically be close together. Therefore, the only situation under which they may lie in opposite quadrants is for the case of colors close to gray. These have a low value of chroma and therefore the magnitude of the discontinuity will be small in practical applications. As illustrated in the previous section, if the samples are under  $5\Delta E_{ab}^*$  units apart, the discontinuity in CIEDE2000 color difference  $\Delta E_{00}$  is under 0.2734, which is small in comparison to color differences encountered in a number of applications, but not negligible. If the samples are  $1\Delta E_{ab}^*$  unit apart, the discontinuity magnitude is smaller than 0.0119, which is negligible in most practical situations.

Because of their small magnitude, the discontinuities in the CIEDE2000 color difference

computation may not be a major concern in most industrial applications, where other sources of experimental variation are much larger. However, the discontinuities do preclude the use of the formula in some situations, for instance in design techniques using gradient based optimization, that not only require continuity of the function but also continuity of the first derivative. For accommodating these applications, it would be desirable to eliminate the discontinuities all together. Since the source of the main discontinuity is the inherent uncertainty of  $180^\circ$  in computing the “mean” of two angles through the geometric process illustrated in Fig. 2, a suitable modification of this process can eliminate this specific discontinuity <sup>||</sup>. One option would be to sacrifice the symmetry of the formula and use only the hue angle of the reference sample in the terms  $T$  and  $\Delta\theta$  instead of the mean.

The discontinuity due to the computation of hue-difference and the (extremely small) discontinuity at mean hue of  $0/360$  degrees are not eliminated even when the computation of mean hue is modified. In order to remove the small discontinuity at a mean hue of  $0/360$  one may choose a different functional form for  $\Delta\theta$  that decays to zero for  $\theta$  in the neighborhood of  $0^\circ$  and  $360^\circ$ . The discontinuity in the computation of hue difference is inherent in the process of computing differences between angles that are  $180^\circ$  apart, the difference may be assumed to be  $180^\circ$  or  $-180^\circ$ . This causes the sign of  $\Delta H'$  in equation (11) to flip. The discontinuity is however eliminated if only the absolute value of  $\Delta H'$  is used in subsequent computations. This will however require additional changes to formula in order to ensure that the rotation term is appropriate.

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<sup>||</sup>From a mathematical standpoint, the discontinuity can be eliminated even if the computation of average hue is unchanged by imposing a constraint of  $180^\circ$  symmetry on terms involving  $\bar{h}'$ . This is immediately apparent from Eqn. (26). However, such constraints are not really meaningful and therefore likely to hurt the performance of the formula with regard to perceptual uniformity.

## 6. CONCLUSION

In order to validate software implementations of the CIEDE2000 color difference formula, additional testing is required beyond the worked examples included in the draft standard.<sup>1</sup> In this note, we provided implementation guidelines and supplementary test data to enable this additional testing. We also provide the test data in electronic format and sample implementations in Microsoft Excel and MATLAB at the first author's website.<sup>5</sup> We also indicate the presence of three discontinuities in the CIEDE2000 formula and characterize these discontinuities in terms of their magnitudes.

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