Rough analysis of a hooded shooter

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Abstract

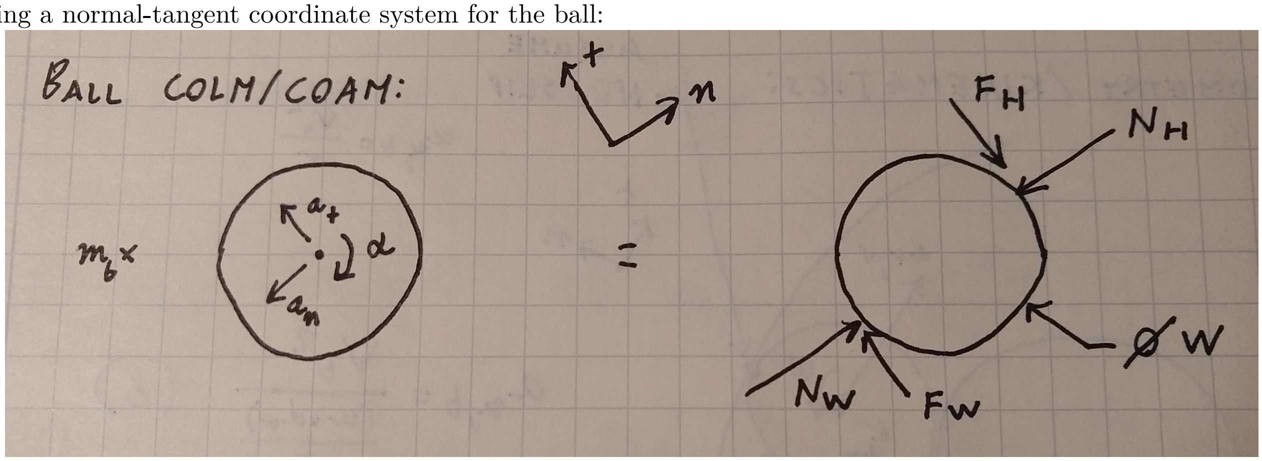
We noticed inconsistency in our hooded shooter as balls deteriorated throughout our week 1 event. To help redesign efforts, I decided to go back to the blackboard to gain a better understanding of the underlying physics so we can make educated improvements that will help consistency in exit velocity.

Parameters

- m_b, the mass of the ball
- I_b, the moment of inertia of the ball
- d_b, the uncompressed diameter of the ball
- m_b, the mass of the flywheel
- I_b, the moment of inertia of the flywheel
- d_b, the diameter of the flywheel

Conservation of Linear/Angular momentum on the ball

Using a normal-tangent coordinate system for the ball:



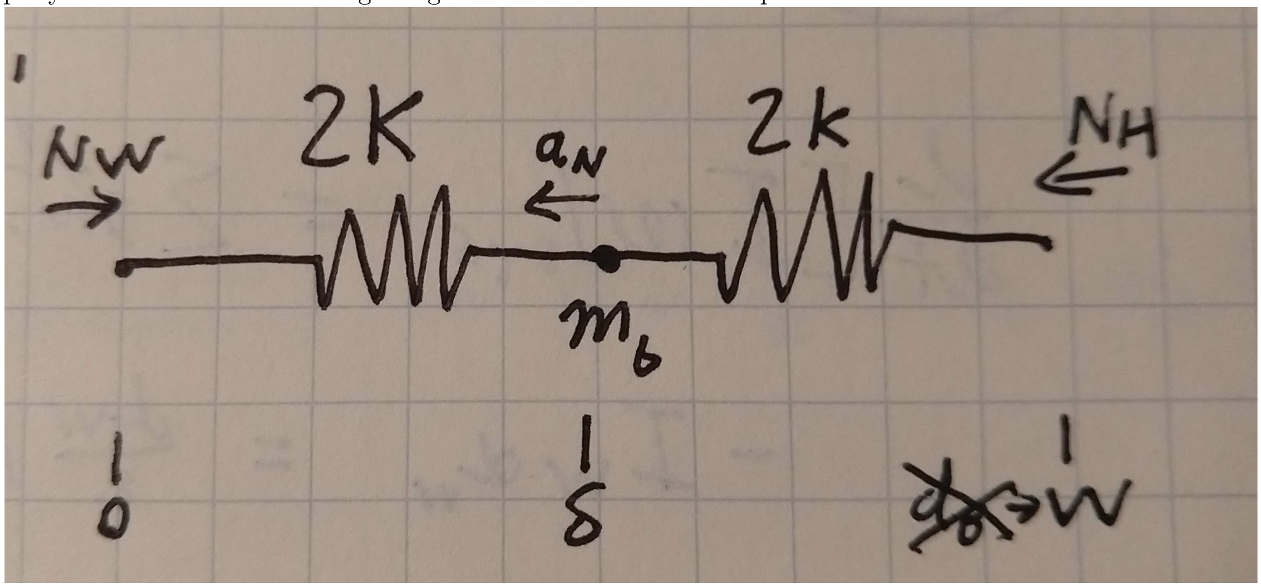
$$m_b a_{t,b} = F_w - F_h \tag{1}$$

$$m_b a_{n,b} = N_h - N_w \tag{2}$$

$$I_b \alpha_b = (F_h + F_w)\frac{w}{2} \tag{3}$$

Modeling ball stiffness and weight

We'll treat the ball as a sort of massed spring. This isn't perfect and I'm not remembering the way to do it properly but should be close enough to get a feel for what the centripetal effects are like.



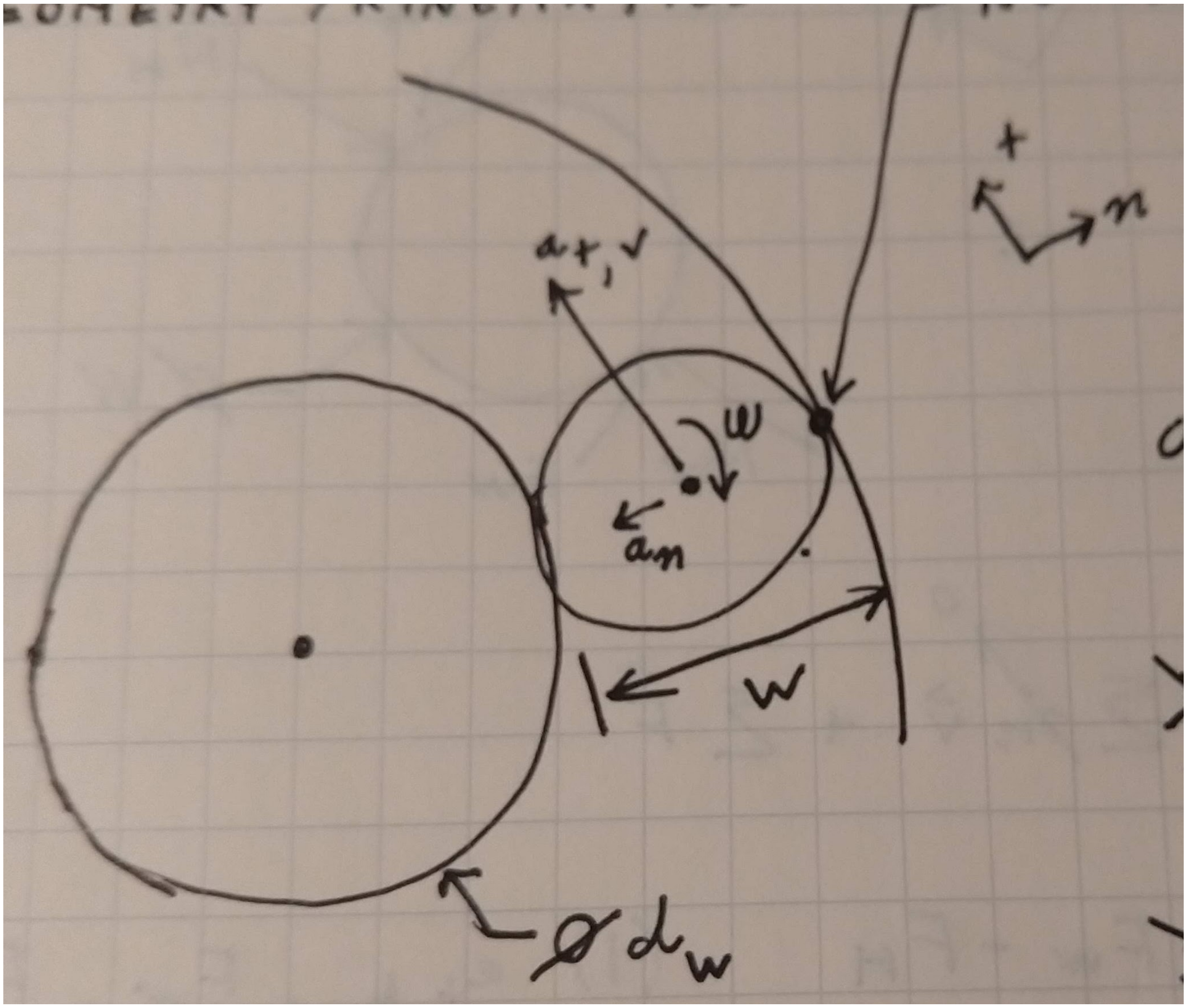
Recalling that springs can be modeled as $F = k\ \Delta l = k(l_{final} - l_{initial})$,

$$N_w = 2k(\frac{d_b}{2} - \delta) \tag{4}$$

$$N_h = 2k(\frac{d_b}{2} - (w - \delta)) \tag{5}$$

Modeling geometry and kinematics

We will assume no slip on the rear hood of the shooter. Definitely not a perfect assumption, especially if you're actually lining your hood with pro-slip material.

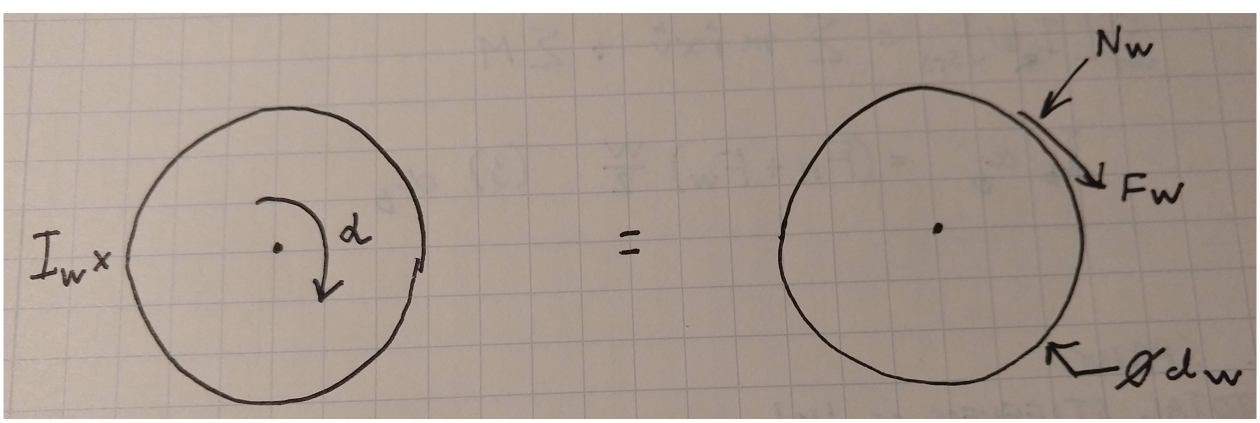


$$a_{n,b} = \frac{v_b^2}{\frac{w+d_w}{2}} \tag{6}$$

$$v_b - \omega_b \frac{w}{2} = 0 \tag{7}$$

$$a_{b,t} - \alpha_b \frac{w}{2} = 0 \tag{8}$$

Conservation of angular momentum on flywheel



$$-I_w \alpha_w = \frac{d_w}{2} F_w \tag{9}$$

Friction model

We will use a basic static friction model that only grips when there is a velocity differential between the surface of the ball and wheel.

$$F_w = \begin{cases} 0 \text{ if } \omega_w \frac{d_w}{2} \leq v_b + \omega_b \frac{w}{2} \\ \mu_w N_w \text{ otherwise} \end{cases} \tag{10}$$

Algebraic solution

Our goal will be to first solve for $\frac{dv_b}{dt}$ and $\frac{d\omega_w}{dt}$.

We'll start by getting N_w and N_h by utilizing the COLM on the ball in the N-direction and the spring equations. Solving these yields:

$$N_w = k(d_b - w) - \frac{m_b \ a_{n,b}}{2} \tag{11}$$

This allows us to rewrite the friction model.

$$F_w = \begin{cases} 0 \text{ if } \omega_w \frac{d_w}{2} \leq v_b + \omega_b \frac{w}{2} \\ \mu_w (k(d_b - w) - \frac{m_b \ a_{n,b}}{2}) \text{ otherwise} \end{cases} \tag{12}$$

Utilizing the kinematics equations allows further refinement.

$$F_w = \begin{cases} 0 \text{ if } \omega_w \frac{d_w}{4} \leq 2v_b \\ \mu_w (k(d_b - w) - \frac{m_b \ v_b^2}{w + d_w}) \text{ otherwise} \end{cases} \tag{13}$$

Solving (1) and (3) gives:

$$\frac{m_b a_{t,b}}{2} = \frac{I_b \alpha_b}{w} F_w \tag{14}$$

Using the kinematics equations allows further refinement.

$$(\frac{m_b}{2} + \frac{2I_b}{2^2}) a_{t,b} = F_w \tag{15}$$

Setting up and solving the ODEs

We can then write the first differential equation.

$$\frac{dv_b}{dt} = \frac{F_w}{\frac{m_b}{2} + \frac{2I_b}{w^2}} \tag{16}$$

We can write the second differential equation from (9).

$$\frac{d\omega_w}{dt} = -\frac{d_w F_w}{2I_w} \tag{17}$$

There is a third subtle ODE.

$$\frac{dx}{dt} = v_b \tag{18}$$

This along with our initial conditions will define the ODEs we must solve.

$$\omega_w(0) = \omega_{w,i} \tag{19}$$

$$v_b(0) = 0 \tag{20}$$

$$x(0) = 0 \tag{21}$$

To help us out in the solution process we'll define the following constants:

$$A = \frac{d_w}{2I_w} \tag{22}$$

$$B = (\frac{m_b}{2} + \frac{2I_b}{w^2})^{-1} \tag{23}$$

$$C = \mu_w k(d_b - w) \tag{24}$$

$$D = \mu_w \frac{m_b}{w + d_w} \tag{25}$$

This gives us these ODEs:

$$\frac{d\omega_w}{dt} = -A[C - Dv_b^2] \text{ while } \omega_w \frac{d_w}{4} > v_b \tag{26}$$

$$\frac{dv_b}{dt} = B[C - Dv_b^2] \text{ while } \omega_w \frac{d_w}{4} > v_b \tag{27}$$

Solving these (with WolframAlpha) yields:

$$v_b = \sqrt{C/D}\ tanh(B\sqrt{CD}\ t) \tag{28}$$

$$omega_w = \omega_{w,i} - A/B\ \sqrt{C/D}\ tanh(B\sqrt{CD}\ t) \tag{29}$$

$$x = \frac{ln(cosh(B\sqrt{CD}\ t))}{B\ D} \tag{30}$$

Now, again, these are not with the piecewise consideration. Let's re-examine that. Analyzing the DE solutions above, we find that

$$\omega_w = \omega_{w,i} - A/Bv_b \tag{31}$$

We can then combine this with the case where the piecewise equations switch.

$$omega_w d_w/4 = v_b \tag{32}$$

Solving these two yields the steady state velocity $v_{b,steady}$ where the velocity settles to if given enough distance to.

$$v_{b,steady} = \frac{\omega_{w,i}}{\frac{4}{d_w} + \frac{A}{B}} = \frac{\omega_{w,i}}{\frac{4}{d_w} + \frac{d_w}{2I_w}(\frac{m_b}{2} + \frac{2I_b}{w^2})} \tag{33}$$

However, we also need to look at the velocity of the ball at the point in time which it leaves the hood. First we find that time, by finding the inverse function of $x(t)$.

$$t(x) = \frac{cosh^{-1}(e^{BDx})}{B\sqrt{CD}} \tag{34}$$

Substituting this into the solution for $v_b(t)$ yields the velocity of the ball at which it leaves the hood.

$$v_b(x) = e^{-BDx}\sqrt{\frac{C(e^{2BDx} - 1)}{D}} \tag{35}$$

If we expand the solution it becomes

$$v_b(x) = e^{-(\frac{m_b}{2} + \frac{2I_b}{w^2})^{-1}\mu_w \frac{m_b}{w+d_w} x}\sqrt{\frac{\mu_w k(d_b - w)(e^{2(\frac{m_b}{2} + \frac{2I_b}{w^2})^{-1}\mu_w \frac{m_b}{w+d_w} x} - 1)}{\mu_w \frac{m_b}{w+d_w}}} \tag{36}$$

... which is an excruciatingly ugly equation. We can try and simplify it a bit but... it's not getting prettier.

$$v_b(x) = e^{-(\frac{m_b}{2}+\frac{2I_b}{w^2})^{-1}\mu_w \frac{m_b}{w+d_w} x} \sqrt{\frac{km_b(d_b-w)(e^{2(\frac{m_b}{2}+\frac{2I_b}{w^2})^{-1}\mu_w \frac{m_b}{w+d_w} x}-1)}{w+d_w}} \tag{37}$$

We'll button it back up with the big-letter constants and wrap it up into a piecewise equation.

$$v_{b,f} = min \begin{cases} v_b(x) \\ v_{b,steady} \end{cases} \tag{38}$$

$$v_{b,f} = min \begin{cases} e^{-BDx}\sqrt{\frac{C(e^{2BDx}-1)}{D}} \\ \frac{\omega_{w,i}}{\frac{4}{d_w}+\frac{A}{B}} \end{cases} \tag{39}$$

where

$$A = \frac{d_w}{2I_w} \tag{40}$$

$$B = (\frac{m_b}{2}+\frac{2I_b}{w^2})^{-1} \tag{41}$$

$$C = \mu_w k(d_b - w) \tag{42}$$

$$D = \mu_w \frac{m_b}{w+d_w} \tag{43}$$

Conclusions

We can still draw some conclusions from this, though.

- $v_{b,steady}$ does not rely on μ or k; parameters which ball variability would hugely impact.
- $v_{b,steady}$ is reliant upon m_b, but increasing I_b (and perhaps decreasing w?) could 'drown out' the variability of this term.
- In the equation for $v_b(x)$, μ_w is always found alongside x. Both of these terms will help increase the rate at which the ball gains velocity up to the steady-state condition.

Other effects are not so clean-cut mathematically, so numeric analysis based on your situation is probably a good idea.