1 Brief description of the algorithm

In this task, we generate a total number of 2^{2^n} for all possible binary combination of dimensions n ranged from 2 to 5. As we obtain the randomly generated Boolean functions, train the perceptron throughout 20 epochs where both weights $\mathbf{W}$ and threshold $\boldsymbol{\theta}$ are iteratively tuned for the aid of classification. Absolute distance of the output and target is calculated to check whether the output of the perceptron exactly matches the Boolean function of the input with the latest $\mathbf{W}$ and $\boldsymbol{\theta}$. Count as linearly separable if match and iterate the process for 10^4 times. Noted that, duplicated Boolean function are avoided by checking the used sample set.

2 Results

Here is a counting of linearly separable for dimensions 2,3,4,5 under different total number of sampling. Theoretical total numbers of linear separable functions given in the third row, quoted from OEIS.

Number of Dimensions (n)	2	3	4	5
Boolean Functions (2^{2^n})	16	256	65536	4294967296
Linearly Separable Boolean functions	14	104	1882	94572
Fraction of Linearly Separable Functions	87.500%	40.625%	2.872%	0.002%
Linearly Separable Functions Found (10^4 runs)	14	104	252	0
Linearly Separable Functions Found (10^5 runs)	14	104	1337	1
Linearly Separable Functions Found (10^6 runs)	14	104	1882	2

3 Discussion

Shown in results, for 10^4 sampling, all Boolean functions can be found for low dimensions n=2 and n=3 while the counter fluctuates between 200 to 300, account for around 14% in most cases for dimension of 4. For 5-d Boolean functions, none is found separable due to the upper bound of the iteration times.

When we increase the iterations by power, the linear separable functions found increase sharply to 1337 and 100% for 10^6 runs since $10^5 \leq 2^{2^4} \leq 10^6$. This trend can be predicted for 5-d Boolean functions as well.

However, the proportion of linearly separable functions out of the total possible binary combinations continuously decrease as dimensions increase. Here, fraction of linearly separable functions drops from 2.872% (4-d) to 0.002% (5-d), which explains the little growth of functions found. I conjecture that iterations times of 10^{10} could find all linearly separable functions since $10^9 \leq 2^{2^5} \leq 10^{10}$.

A minor inaccuracy within the iteration of my program is that the duplicated randomly generated Boolean function are abandoned directly, in another word, waste a single iteration out of the total runs. Since the chance of repetition in random generation is relatively very low and has little impact compared to the total number of iterations, it will not cause deviations in the final conclusion.