Problem Set 5

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## Introduction

As an application of the some of the properties of expected values, problems 1-7 step through a proof that the expected value of the random variable that defines sample variance is the population variance, given that the population variance is defined.

For each of these questions, let be independent, identically distributed random variables with defined mean and variance .

Question 8 gives examples of jointly distributed random variables that are independent and jointly distributed random variables that aren’t independent.

Please complete the following tasks regarding the data in R. Please generate a solution document in R markdown and upload the .Rmd document and a rendered .doc, .docx, or .pdf document. Please turn in your work on Canvas.

These questions were rendered in R markdown through RStudio (<https://www.rstudio.com/wp-content/uploads/2015/02/rmarkdown-cheatsheet.pdf>, <http://rmarkdown.rstudio.com> ).

## Question 1 (5 points)

Let be independent, identically distributed random variables with mean and variance , and define the random variable by . Justify the equality

## Question 2 (5 points)

Let be independent, identically distributed random variables with mean and variance .

In terms of and , what is the value of ? Note that , while and . Please justify your answer.

Confirm numerically that your answer is correct for which has mean equal to and variance equal to .

f2<-function(x){x^2\*dgamma(x,shape=3,scale=2)}  
integrate(f2,0,Inf)$value

## [1] 48

mu<-6  
sigma2<-12  
mu^2 + sigma2 #E(X^2) = (mu)^2 + (sigma)^2

## [1] 48

## Question 3 (5 points)

Assuming that for all , what is . Recall that for any random variables with defined means.

## Question 4 (5 points)

Define the random variable by . What is the value of ? Please justify your answer.

Recall that the mean of equals and the variance equals . The fact that mentioned above may also be useful. Further, is constant with respect to the index in the sum.

## Question 5 (10 points)

Why is

### xbar is a constant so you can pull it out in front of the sum. The sum from i = 1 to n of X\_i is equal to n times the mean

## Question 6 (5 points)

Assuming that , that , and that , please simplify .

## Question 7 (5 points)

If , what is the value of ?

## Question 8

* Consider the probability space defined by where , the set of events is the power set of , and is defined by the density for all . Let be the random variable on this probability space defined by and . Define by , . Are these random variables independent?

fx (0) = 1/2 fx (1) = 1/2

fy (2) = 1/3 fy (3) = 2/3

(0,2) = 1/2 \* 1/3 = 1/6

(0,3) = 1/2 \* 2/3 = 2/6 = 1/3

(1,2) = 1/2 \* 1/3 = 1/6

(1,3) = 1/2 \* 2/3 = 2/6 = 1/3

No, these are not independent.

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(0,2) = 1/2 \* 1/3 = 1/6

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(1,2) = 1/2 \* 1/3 = 1/6

(1,3) = 1/2 \* 2/3 = 2/6 = 1/3

No, these are not independent.

## Question 9

Suppose is a sample from a random variable and is a sample from a random variable with variances and respectively. What weighted average with minimizes the variance of ?