Lösungen Klasse 5 (Grundlagenfach)

Rechenregeln mit dem Summenzeichen

1. $\displaystyle\sum_{i=1}^{n}(3i-1)^2 = \sum_{i=1}^{n}(9i^2-6i+1) = 9\sum_{i=1}^{n}i^2 - 6\sum_{i=1}^{n}i + \sum_{i=1}^{n}1 = 9\,\frac{n(n+1)(2n+1)}{6} - 6\frac{n(n+1)}{2} + n =$

$$= \frac{n}{2}(6n^2+9n+3-6n-6+2) = \underline{\underline{\frac{n}{2}(6n^2+3n-1)}}$$

z.B. für n = 4: $\displaystyle\sum_{i=1}^{4}(3i-1)^2 = 2^2+5^2+8^2+11^2 = 214 = 2(6\cdot 16+12-1)$

2. $\displaystyle\sum_{i=1}^{n}(4i-n)^2 = \sum_{i=1}^{n}(16i^2-8i\cdot n+n^2) = 16\sum_{i=1}^{n}i^2 - 8n\sum_{i=1}^{n}i + n^2\sum_{i=1}^{n}1 =$

$$= 16\,\frac{n(n+1)(2n+1)}{6} - 8n\frac{n(n+1)}{2} + n^2\cdot n = \frac{n}{3}(16n^2+24n+8-12n^2-12n+3n^2) =$$

$$= \underline{\underline{\frac{n}{3}(7n^2+12n+8)}}$$

z.B. für n = 3: $\displaystyle\sum_{i=1}^{3}(4i-3)^2 = 1^2+5^2+9^2 = 107 = 7\cdot 9+36+8$

3. $\displaystyle\sum_{i=1}^{n+1}(4i^2-3i) = 4\sum_{i=1}^{n+1}i^2 - 3\sum_{i=1}^{n+1}i = 4\,\frac{(n+1)(n+2)(2(n+1)+1)}{6} - 3\frac{(n+1)(n+2)}{2} =$

$$= \frac{(n+1)(n+2)}{6}(8n+12-9) = \underline{\underline{\frac{(n+1)(n+2)(8n+3)}{6}}}$$

z.B. für n = 2: $\displaystyle\sum_{i=1}^{3}(4i^2-3i) = 4-3+16-6+36-9 = 38 = \frac{3\cdot 4\cdot 19}{6}$