BE/APh 161: Physical Biology of the Cell, Winter 2025
Homework #7
Due at the start of lecture, 2:30 PM, February 26, 2025.

Problem 7.1 (Genomes in cells, 10 pts).
In this problem we consider how genomes take up space in cells.

a) If your genome were a single strand of DNA, what would its approximate radius of gyration be if it were free in solution? What implications does this have for the design of a cell?

b) Estimate the radius of gyration of the *E. coli* genome if it were not confined in a cell. How does that compare to the size of an *E. coli* cell?

Problem 7.2 (Viral packaging, 25 pts).
In this problem, we explore estimates of the energetics of viral packaging of $\phi 29$, which we introduced in lecture.

a) (Based on problem 10.6 of *PBoC2*) Estimate the entropy penalty for packing the genome in the viral capsid. You can assume that the entropy of the packed state is nearly zero, since it features almost crystalline packaging. Compare this entropy contribution to the free energy (equal to the entropy times the temperature) to the total bending free energy of packing given by equation 10.42 of *PBoC2*. *Hint:* In computing the entropy of the unpacked state, remember that each configuration of the unpacked state has the same energy, since it is a flexible chain on the length scale of the entire genome.

b) Compare the force required to pack the last bits of genome into the capsid as given by equation 10.43 of *PBoC2* and by the experimental result of $\approx$ 50 pN discussed in lecture and in Figure 10.19(B) of *PBoC2*. What factors might account for any discrepancy you may notice? Make sure you know how equation 10.43 of *PBoC2* is derived.

Problem 7.3 (Flexural rigidity of biopolymers, adapted from problem 10.2 of *PBoC2*, 35 points).

a) Recall that the flexural rigidity of a filament is $K_{\text{eff}} = EI$, where E is the Young's modulus and I is the geometric moment of inertia defined in lecture. We also saw that the persistence length is given by $\xi_p = EI/k_BT$. Given the persistence lengths of DNA, actin filaments, and microtubules (check your lecture notes or BioNumbers), estimate their respective Young's moduli by computing the moment of inertia. You can look up geometric information about the filaments in *PBoC2* sections 2.2.3 and 10.5.1.

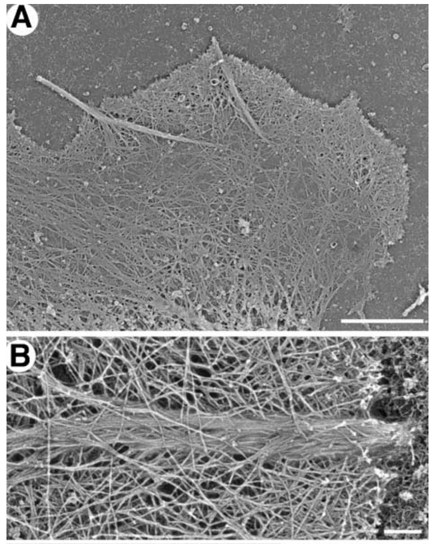

Figure 1: A) Electron micrograph of a B16F1 cell with a few peripherally located filopodia. Scale bar, 5 µm. B) A close-up of one of the filopodia. Scale bar, 1 µm. Image taken from Mejillano, et al., *Cell*, **118**, 363–373, 2004.

b) Filopodia are protrusions of bundled actin filaments often found in adherent cells. They push against the cell membrane. The membrane pushes back on the filopodium with a force of

$$F = 2\pi r\gamma, \tag{7.1}$$

where r is the radius of the end of the filopodium and γ is the surface tension of the membrane. If we have time, we will discuss membrane tensions later in the course, but for this problem, we will take $\gamma \approx 0.035$ pN/nm. We will assume that the filopodium consists of approximately 30 filaments. We will now investigate how long the filopodium can protrude before it buckles, considering two limits.

i) First, we assume that the filaments in the filopodium are not crosslinked. Find the length L that the filopodium can protrude before buckling.

ii) Now, consider the limit where the filaments in the filopodium are very tightly crosslinked, so tightly crosslinked that the filopodium can be considered a solid rod. Find the length L_{cl} that the crosslinked filopodium can protrude before buckling.

iii) In general, what is L_{cl}/L as a function of N, the number of filaments in the filopodium?