BE/APh 161: Physical Biology of the Cell, Winter 2025
Homework #9
Due 2:30 PM, March 12, 2025.

Problem 9.1 (Asters and vortices in the Lee-Kardar model, 20 pts).
Recall the dynamical equations for the Lee-Kardar model for active matter consisting of microtubules and motors. (In this problem, only indices i and j are summed over.)

$$\partial_t m = \partial_i \partial_i m - \partial_i (m T_i), \tag{9.1}$$

$$\partial_t T_i = C\, T_i (1 - T_j T_j) + \partial_j (m \partial_j T_i). \tag{9.2}$$

We consider here steady states the two-dimensional case. In working this problem it is useful to know the following expressions in polar coordinates.

$$\partial_i f = \left(\partial_r f, \frac{1}{r}\,\partial_\theta f\right)^{\mathsf{T}} \qquad \text{(gradient of a scalar-valued function)}, \tag{9.3}$$

$$\partial_i \partial_i f = \frac{1}{r}\,\partial_r (r \partial_r f) + \frac{1}{r^2}\,\partial_\theta^2 f \quad \text{(Laplacian of a scalar-valued function)}, \tag{9.4}$$

$$\partial_i u_i = \frac{1}{r}\,\partial_r (r u_r) + \frac{1}{r}\,\partial_\theta u_\theta \qquad \text{(divergence of a vector)}. \tag{9.5}$$

a) An aster has all tubules oriented in the radial direction toward the center of the aster. Derive an expression for $m(r)$. This function shows how the motor concentration decays away from the center of the aster.
b) Conversely, a vortex has all of the tubules oriented in the angular direction. Again, derive an expression demonstrating how the motor density decays away from the vortex.
c) Which has a larger region of high motor density, an aster or a vortex?

Problem 9.2 (Flow patterns in a cavity (10 points)).
A classic problem in beginning fluid mechanics courses features a channel with a lid sliding over it, as depicted below.

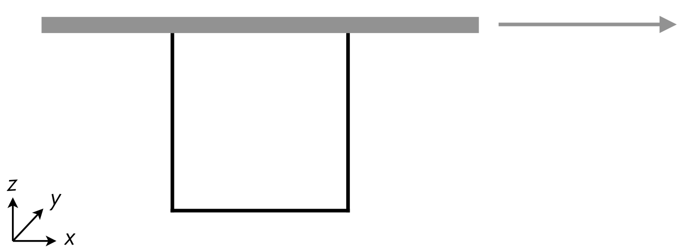

Figure 1: Depiction for a channel with a lid sliding over it. The channel and lid are very long in the y-direction.

a) Assuming the lid is sliding slowly enough such that the Reynolds number is very small, sketch the streamlines of the flow in the channel.

b) Imagine a cross-section of the channel in the y-z plane. What is the *net* flux of fluid through this cross-section as the lid is slid?

Problem 9.3 (Flow past an object (10 points)).
Take a look at the picture below of a cylinder moving in a tank of water. Is the Reynolds number above or below unity? Explain your reasoning.

Figure 2: Photograph by Sadatoshi Taneda a cylinder moving through a tank of water. The flow is visualized using aluminum powder. The image is taken from *An Album of Fluid Motion* by Milton Van Dyke, Parabolic Press, 1982.

Problem 9.4 (Power input for cytoplasmic streaming, 15 pts).
Cytoplasmic streaming in *Drosophila* oocytes is driven by kinesin motors. Figure 3 shows a quantification of cytoplasmic streaming velocities. From this figure, estimate the number of kinesin motors driving streaming. Here are a few hints to help you.

- In this case, the cytoplasm can be modeled as a viscous fluid, so that the forces necessary to drive flow are proportional to the velocity gradient.
- The cytoplasmic viscosity is much bigger than water. An estimate of about $200\times$ that of water is reasonable.
- Using these facts, you should be able to use dimensional analysis to come up with combinations of parameters and flow speed that gives dimension of power per volume.
- Use rules of thumb for kinesin's velocity and force to estimate the power each motor can transmit to the cytoplasm.

After you perform your estimate, ask yourself if the result makes sense. If it does not, speculate on what you may be missing in your estimate.

This is the kind of procedure I have been discussing all term. By performing estimates about a system, you can identify where gaps in your knowledge might exist and and that can help drive what experiments or bigger research questions to tackle next.

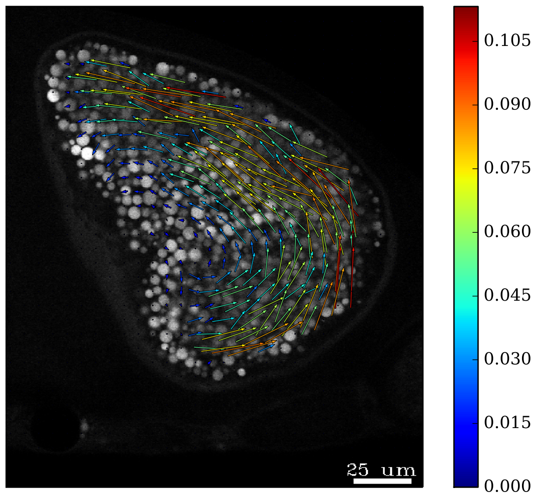

Figure 3: Quantification of cytoplasmic flow velocities in a streaming *Drosophila* oocyte as measured using particle image velocimetry. Autofluorescent granules in a thin focal slice in the center of the oocyte was imaged using a fluorescent confocal microscope. The arrows indicate the local flow velocity. They are color-coded according to speed. The units on the colorbar are µm/s. Scale bar, 25 µm.