BE/APh 161: Physical Biology of the Cell, Winter 2023
Homework #2
Due at the start of lecture, 2:30 PM, January 25, 2023.

Problem 2.1 (How many polymerases?, 5 pts)**.**
On time I was lecturing my BE 150/Bi 250b, which I co-teach with Michael Elowitz. We were talking about strong versus weak promoters, and based on a student question in lecture, we had to make an impromptu estimate of the number of polymerases in a bacterial cell. I started doing some street-fighting estimations (meaning an estimation with no help from any references or calculators), but I could not get an estimate faster than Prof. Elowitz could look it up on BioNumbers. Nonetheless, I think it is a fun and instructive estimate to make. Street-fight your way to that estimate.

Problem 2.2 (Mathematizing a cartoon for ciliar growth, 50 pts)**.**
We considered a model for flagellar growth in lecture. Another model for flagellar or ciliar growth was proposed in Howard, et al., *Nat. Rev. Mol. Biol.*, **12**, 393–398, 2011. The cartoon is shown in Fig. 1, along with the text from the caption in the paper.

Let $c(x,t)$ be the concentration of active growth factors in the cilium and let $\ell(t)$ be the length of the cilium.

a) Write down a set of differential equations to describe the dynamics of c and ℓ. If you like, you may assume a constant number of cargo-carrying motors as we did in lecture for the *Chlamydomonas* flagella, or you may assume that the density of motors is constant. Be sure to state any other assumptions or decisions you made in mathematizing the cartoon.

b) Nondimensionalize your dynamical equation(s) and comment on any physical insight this procedure provides.

c) If you can, solve for $\ell(t)$ analytically. If you cannot, solve it numerically. Also plot the growth rate, $\mathrm{d}\ell/\mathrm{d}t$, over time.

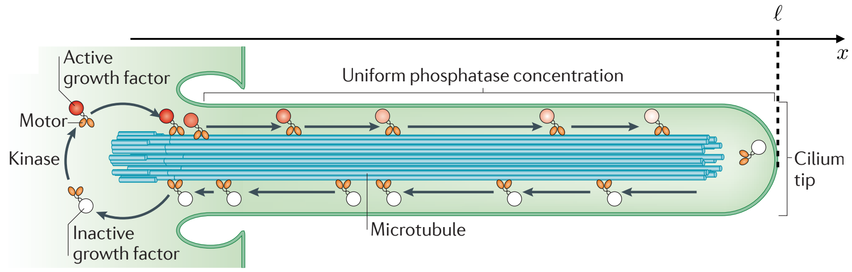

Figure 1: A cartoon describing a possible mechanism for ciliar growth adapted from Howard, et al., *Nat. Rev. Mol. Biol.*, **12**, 393–398, 2011. The text from the caption in the paper reads as follows. "Schematic of an advection-reaction model, a hypothetical mechanism for the length control of cilia and microvilli. Cargoes, for example growth factors, carried along cilia and microvilli are inactivated over time by phosphatases, which may provide a length-dependent signal to the growing tip."

Problem 2.3 (Growth curves, based loosely on page 103 of *PBoC2*, 40 pts)**.**
In Homework 1, we wrote the logistic equation for bacterial growth as

$$\frac{\mathrm{d}c}{\mathrm{d}t} = rc\left(1 - \frac{c}{K}\right), \tag{2.1}$$

where c is the concentration of bacteria, r is the growth rate (we are using r here to avoid the confusion of having upper and lower case K's flying around), and K is the carrying capacity, or the maximum concentration of bacteria that can be present and still have growth. For bacteria growing in media, r and K could also be functions of the the concentration of food in the media, which we will call $F(c,t)$.

a) Write down an expression for $\mathrm{d}F/\mathrm{d}t$. You should try to keep your expression simple. Give your reasoning for how you chose this expression.

b) Sketch functional forms that you think are reasonable for $r(F)$ and $K(F)$. (*Sketch*; do not use plotting software.) Again, try to keep them simple.

c) Based on what you know about bacterial growth, give reasonable values of the parameters you defined in your expressions for $\mathrm{d}F/\mathrm{d}t$, $r(F)$ and $K(F)$. Also give reasonable values for the initial bacteria concentration, c_0, and the initial food concentration, F_0. Explain how you came up with these values; you may use whatever references you like. *Hint*: Working through problem 2.5 of *PBoC2* will help you.

d) Solve the differential equations (numerically or analytically) and plot the results. You can use whatever numerical integration software you like. If you would like to use Python with NumPy/SciPy, the Jupyter notebook accompanying lecture 3 might serve as a useful reference.

e) Explain the shape of the curves.

f) Comment on any enhancements you would propose to this model for bacterial growth.

Problem 2.4 (Boltzmann's grave, 5 pts).
Boltzmann's tomb is in Zentralfriedhoff in Vienna, a beautiful cemetery that also contains the graves of some of the world's greatest composers, including Beethoven, Brahms, Schubert, Strauss, Ligeti, and Falco. Boltzmann's tomb is shown in Fig. 2. Not the equations, $S = k \log W$, at the top of the stone.

Figure 2: Boltzmann's tomb in Zentralfriedhoff in Vienna. Photo from Daderot, licensed under CC-BY-SA-3.0.

Here, S is entropy, k is the Boltzmann constant, $\log$ refers to the natural logarithm, and W is the number of microstates. In class, we derived the famous Boltzmann distribution by maximizing the Shannon entropy, given that we knew an average energy of our system of interest. Derive the equation on Boltzmann's grave using the same technique. To do so, assume we do not know anything about the energy of the system.