# OPEN SOURCE APPLICATION FOR THE UNDERSTANDING OF POLARIZATION


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## Abstract

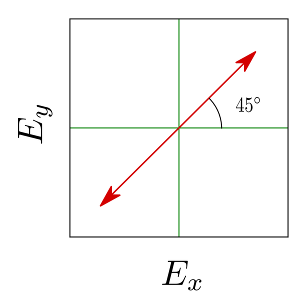
- In certain polarization experiments such as characterization of birrefringent optical elements (our current goal), one must be able to generate and analyze large sets of elliptical polarization states in the laboratory. In practice, this is done using optical elements such as polarizers and retarders acting upon a known state of polarization.
- When dealing with linear or circular polarization it is quite easy to discern how to use retarders and polarizers. There is however a big chance that even experts find some difficulties at generating non standard polarization states parting from their mathematical representation.
- While performing ellipsometry measurements we identified the need to easily translate book definitions of polarization states into positions of the elements that generate states.
- The translation is achieved by means of a minimization procedure based on Jones calculus.
- An intuitive and straightforward python-based, graphical application was built in order to quickly translate polarization states into angles of physical elements for use in teaching and research environments.


The following QR code contains a link to a GIT repository where this poster and the application are hosted. Feel free to scan the code, visit the repository and download both the poster an the code for future reference.

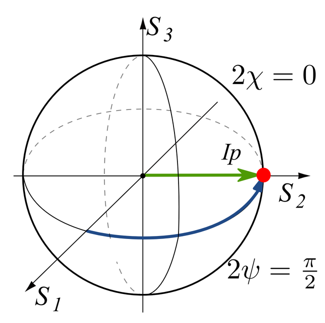
https://github.com/bebopsan/Ellipsometry_for_dummies

## Motivation

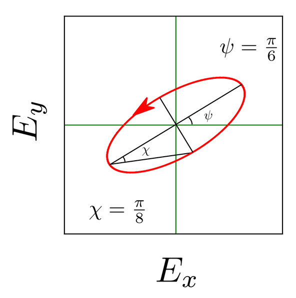
Suppose you are given the task to experimentally generate a linear state of polarization defined by any of the following descriptions:



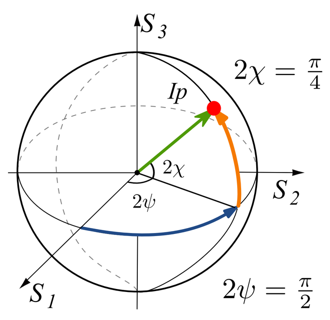

$$\mathbf{J} = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix} \qquad |J\rangle = \left|\tfrac{\pi}{4}, 0\right\rangle$$




This is an easy task because it is one of the most basic states of polarization. We could either: a) Use a $\lambda/2$ retarder at an angle of 22,5° with respect to horizontal polarization. Or b) locate a polarizer at an angle of 45° along a circularily polarized beam.
If on the other hand, you are supposed to generate uncommon states of polarization such as the one defined below, you may encounter great difficulties no matter which definition you decide to follow.



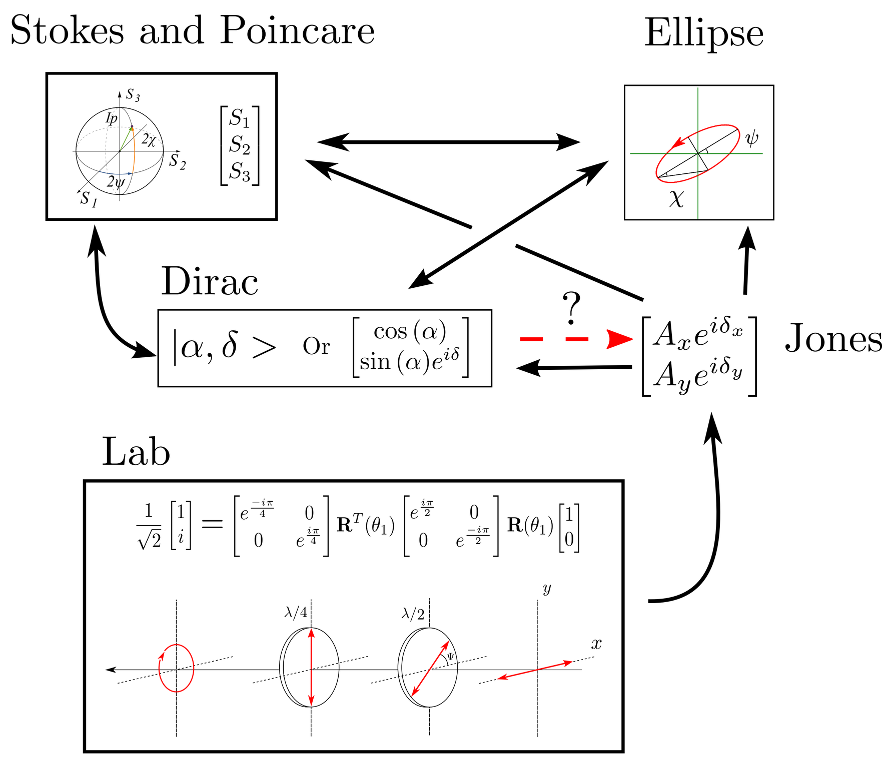

$$\mathbf{J} = \begin{bmatrix}-0.8522 + 0.5232j\\-0.1122 + 0.6819j\end{bmatrix} \quad |J\rangle = \left|\tfrac{7\pi}{24}, -0.857\right\rangle$$




Even if you are an expert in polarimetry you will have a tedious time trying to figure out the configuration in which you have to set $\lambda/4$ and $\lambda/2$ retarders in order to generate this state. We propose an easy to use application to translate polarization states, as given earlier, into angles of laboratory optical elements that generate them. This can be useful for educational and research purposes.

## Transformations

As illustrated in the Motivation, the state of polarization of light can be parametrized in different ways. There are pros and cons on using each of them such as ease of visualization, compactness, and ease on performing calculations. When using coherent light, optical elements in the laboratory are best represented by Jones Matrices acting on Jones vectors. There is however, no direct mathematical relation between representations of polarization states and the position of the retarders that generate a state. What we can do instead is to simulate the elements using Jones matrices, propagate a known Jones vector, and then see if the vector representation is equivalent to the others.




Roughly speaking there are 4 commonly used representations of polarization states. There are analytical transformations between representations and most of them are bidirectional, with the exception of general Jones vectors that have more variables.

Detail on the transformations between representations of polarization [1,2]:

Stokes ⟷ Ellipse

$$S_1 = S_0\cos(2\chi)\cos(2\psi) \qquad \psi = \frac{1}{2}\tan^{-1}\left(\frac{S_2}{S_1}\right)$$
$$S_2 = S_0\cos(2\chi)\sin(2\psi)$$
$$S_3 = S_0\sin(2\chi) \qquad \chi = \frac{1}{2}\tan^{-1}\left(\frac{S_3}{S_0}\right)$$

Stokes ⟷ Dirac

$$S_0 = 1$$
$$S_1 = \cos(2\alpha) \qquad \alpha = \frac{1}{2}\cos^{-1}\left(\frac{S_1}{S_0}\right)$$
$$S_2 = \sin(2\alpha)\cos(\delta)$$
$$S_3 = \sin(2\alpha)\sin(\delta) \qquad \delta = \tan^{-1}\left(\frac{S_3}{S_2}\right)$$

Dirac ⟷ Ellipse

$$\cos(2\alpha) = \cos(2\chi)\cos(2\psi) \qquad \tan(2\psi) = \tan(2\alpha)\cos(\delta)$$
$$\cot(\delta) = \cot(2\chi)\sin(2\psi) \qquad \tan(2\chi) = \sin(2\alpha)\sin(\delta)$$

Ellipse ⟵ Jones

$$\psi = \frac{1}{2}\tan^{-1}\left(\frac{2A_xA_y}{A_x^2 - A_y^2}\cos(\delta_y - \delta_x)\right)$$
$$\chi = \frac{1}{2}\tan^{-1}\left(\frac{2A_xA_y}{A_x^2 - A_y^2}\cos(\delta_y - \delta_x)\right)$$

Stokes ⟵ Jones

$$S_0 = A_x^2 + A_x^2$$
$$S_1 = A_x^2 - A_y^2$$
$$S_2 = 2A_xA_y\cos(\delta)$$
$$S_3 = 2A_xA_y\sin(\delta)$$

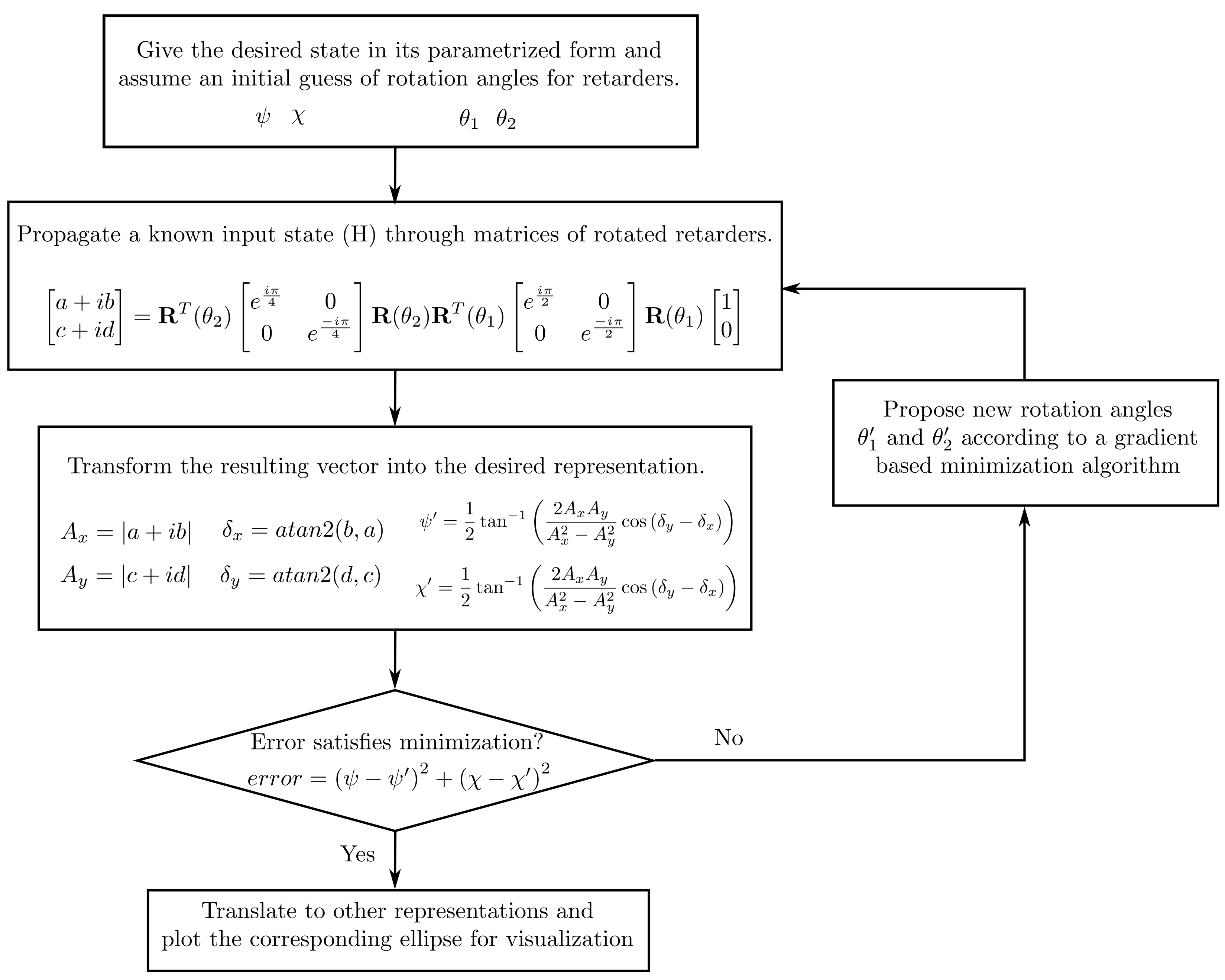
Dirac ⟵ Jones

$$\tan(\alpha) = \frac{A_x}{A_y} \qquad \begin{bmatrix}\cos(\alpha)\\\sin(\alpha)e^{i\delta}\end{bmatrix}$$
$$\delta = \delta_y - \delta_x$$

Some information can be lost!

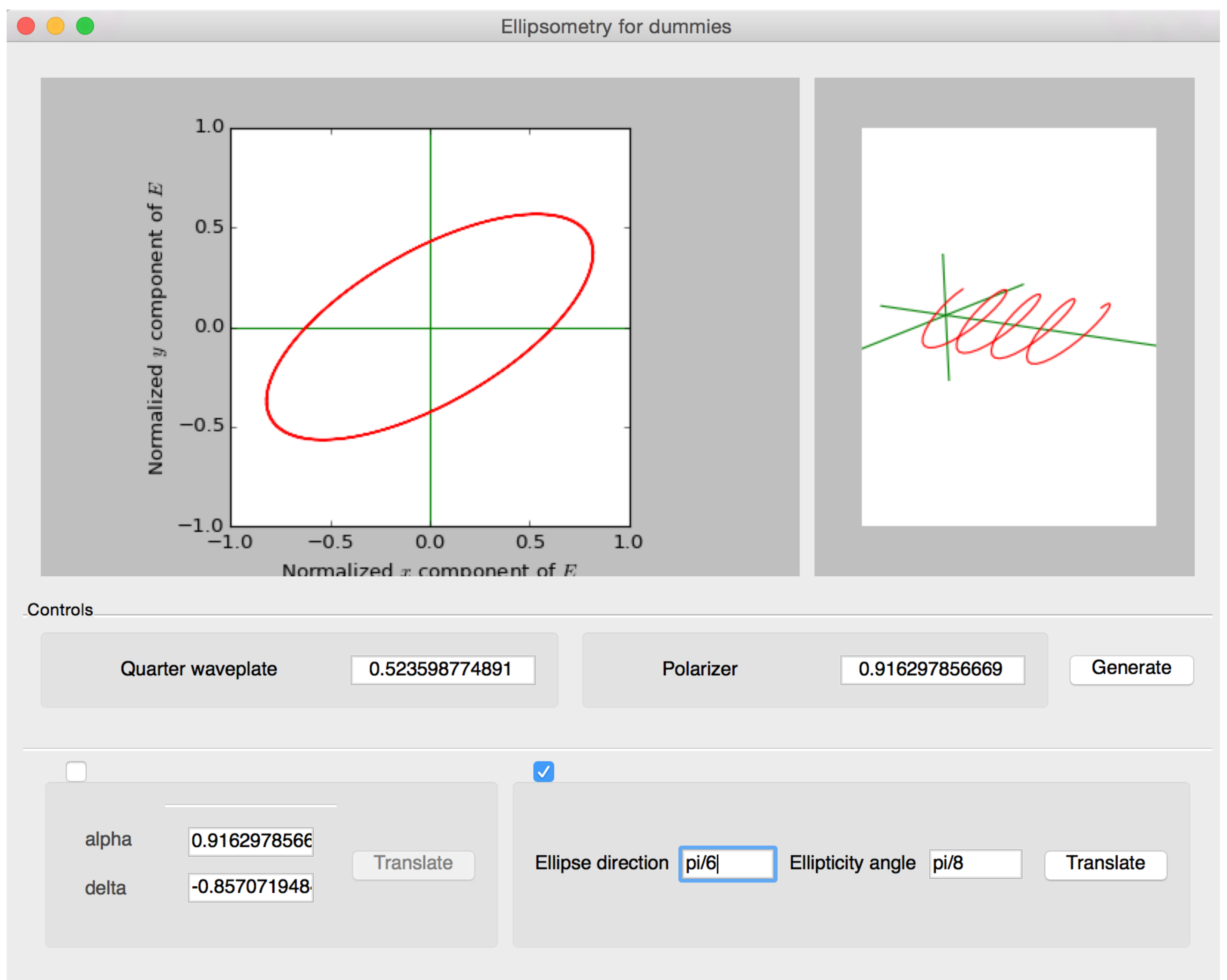
## Minimization Procedure

We implemented a minimization procedure that uses matrix multiplication operations based on Jones vector calculus. The following flow chart illustrates the algorithm:




## Graphical Interfase

The result of this work is a Python based interactive application that runs under the Qt4 graphical interfase. It is a simple but intuitive platform in which you can visualize and translate states of polarization defined from three differen representations. On the bottom right pannel you can set the ellipse direction and ellipticity angle and then translate those parameters to the Dirac and Lab representations. This will automatically plot the trajectory of the pointing vector from a frontal and isometric perspective, allowing the user to see the polarization ellipse and sense of rotation for non linear states. On the bottom left pannel you can give the state in terms of Dirac kets and then translate it to ellipse angles.




## Conclusions

- A comprehensive scheme of the commonly used representations of polarization and their relationships was presented. As well as the challenges we found when trying to translate towards a general Jones vector.
- We implemented a solution for the identification of angles of retarders that consists on a minimization process involving Jones calculus and well known analytical transformations.
- The process of translating polarization states, and finding the angles of optical elements, as well as a graphical representation, was made available to general public through an open source application based on the Python programming language.

## References


[1] Edward Collett. *Field Guide to Polarization.* SPIE Press, 2005.

[2] William H. McMaster. Matrix Representation of Polarization. *Rev. Mod. Phys.*, 33(1):8–28, 1961.