Spatial Econometrics Models with INLA and MCMC

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Introduction

In this example we compare the implementatin of several Spatial Econometrics models using INLA and MCMC. In particular, the implementation of these methods with INLA is based on the *slm* latent class, whilst the implementation with MCMC has been done with *jags* and it is available in package *SEjags*.

```
library(SEjags)
library(INLA)
```

MCMC models

```
data(columbus)
d <- columbus
W <- nb2mat(col.gal.nb, style = "W")
m.form <-  CRIME ~ INC + HOVAL

#Fit models with SEjags
if(!file.exists("INLAvsMCMC-MCMC.Rdata") ) {
 sem.mcmc <- SEjags(m.form, data = d, W = W, model = "sem",
   n.burnin = 5000, n.iter = 10000, n.thin = 20, linear.predictor = TRUE)
 slm.mcmc <- SEjags(m.form, data = d, W = W, model = "slm",
   n.burnin = 5000, n.iter = 10000, n.thin = 20, linear.predictor = TRUE)
 sdm.mcmc <- SEjags(m.form, data = d, W = W, model = "sdm",
   n.burnin = 5000, n.iter = 10000, n.thin = 20, linear.predictor = TRUE)

 save(file = "INLAvsMCMC-MCMC.Rdata",
   list = c("sem.mcmc", "slm.mcmc", "sdm.mcmc"))
} else {
  load("INLAvsMCMC-MCMC.Rdata")
}
```

INLA models

```
#Area index
columbus$idx <- 1:nrow(columbus)
#Adjacency matrix as sparse matrix
W.inla <- as(W, "CsparseMatrix")

#Model matrix for SLM models
mmatrix <- model.matrix(m.form, columbus)
mmatrix2 <-  cBind(mmatrix, W.inla %*% mmatrix[,-1])
colnames(mmatrix2)[4:5]<- paste("lag", colnames(mmatrix2)[2:3],sep="")

#Zero-variance for error term
```

```
#Zero-variance to remove effect in linear predictor: DOES NOT WORK
zero.variance = list(prec=list(initial = 25, fixed=TRUE))
#Large variance to allow for an uncnstrained estimation of the other parameters
#zero.variance = list(prec=list(initial = 1/100, fixed=TRUE))

#Compute eigenvalues for SLM model (as in Havard's code)
e = eigen(W.inla)$values
re.idx = which(abs(Im(e)) < 1e-6)
rho.max = 1/max(Re(e[re.idx]))
rho.min = 1/min(Re(e[re.idx]))
rho = mean(c(rho.min, rho.max))

#
#Variance-covarinace matrix for beta coeffients' prior
#
betaprec <- .001
#Standard regression model
Q.beta = Diagonal(n = ncol(mmatrix), x = 1)
Q.beta = betaprec * Q.beta
#Regression model with lagged covariates
Q.beta2 = Diagonal(n = ncol(mmatrix2), x = 1)
Q.beta2 = betaprec * Q.beta2


#Arguments for slm latent model
args.slm = list(
   rho.min = rho.min,
   rho.max = rho.max,
   W = W.inla,#as(W.inla,"dgTMatrix"),
   X = matrix(0, nrow(mmatrix),0),
   Q.beta = matrix(1,0,0)
)

#Hyperparameters
hyper.slm = list(
   prec = list(prior = "loggamma", param = c(0.01, 0.01)),
      rho = list(initial = 0, prior = "logitbeta", param = c(1,1))
)


#Control fixed
c.fixed <- list(prec = 0.001, prec.intercept = 0.001)
```

Fitting models with INLA:

```
#SEM model
hyper.sem <- hyper.slm
hyper.sem$rho$initial <- 0.85 #Fixed to posterior mode from MCMC
hyper.sem$rho$fixed <- TRUE

#Change zero variance
zero.variance$prec$initial <- 1/100

#Control inla
```

```
c.inla <- list(strategy = "laplace", fast = FALSE,
  tolerance = 0.001,
     int.strategy = "ccd", h = 0.001, dz = 0.05, stencil = 9)
#  int.strategy = 'grid', diff.logdens = 0.1, h = 0.001, dz = 0.01, stencil = 9)

# Create linear combinations on the covariates to estimate
# linear predictor (and fitted values).

n <- nrow(columbus)

#Test how the structure of the linear combinatios should be
lc1 <- inla.make.lincomb(list("(Intercept)" = 1,
  INC = columbus$INC[1], HOVAL = columbus$HOVAL[1],
  idx = c(1, rep(NA, n-1))
))

lc.linpred <- lapply(1:n, function(X) {
  idx.lc <- rep(NA, 49)
  idx.lc[X] <- 1
  aux <- as.list(mmatrix[X, ])
  aux$idx = idx.lc
  inla.make.lincomb(aux)
})


lc.linpred <- do.call(c, lc.linpred)
names(lc.linpred) <- paste("lc.linpred", 1:n, sep = "")

#Linear combination of the fixed effects
lc.fixed <- inla.make.lincombs(INLA:::inla.uncbind(mmatrix))
names(lc.fixed) <- paste("lc.fixed", 1:n, sep = "")


#Linear combinations for SLM and SDM
lc.linpred2 <- lapply(1:n, function(X) {
  idx.lc <- rep(NA, 49)
  idx.lc[X] <- 1
  inla.make.lincomb(list(idx = idx.lc))
})
lc.linpred2 <- do.call(c, lc.linpred2)
names(lc.linpred2) <- paste("lc", 1:n, sep = "")

sem.inla<-inla(CRIME ~ INC + HOVAL +
   f(idx, model = "slm", args.slm = args.slm, hyper = hyper.slm),
   data = as.data.frame(columbus), family = "gaussian",
   lincomb = c(lc.linpred, lc.fixed), control.predictor = list(compute=TRUE),
   control.fixed = c.fixed,
   control.inla = c.inla,
   control.family = list(hyper = zero.variance),
   control.compute = list(dic = TRUE, cpo = TRUE)
)
```

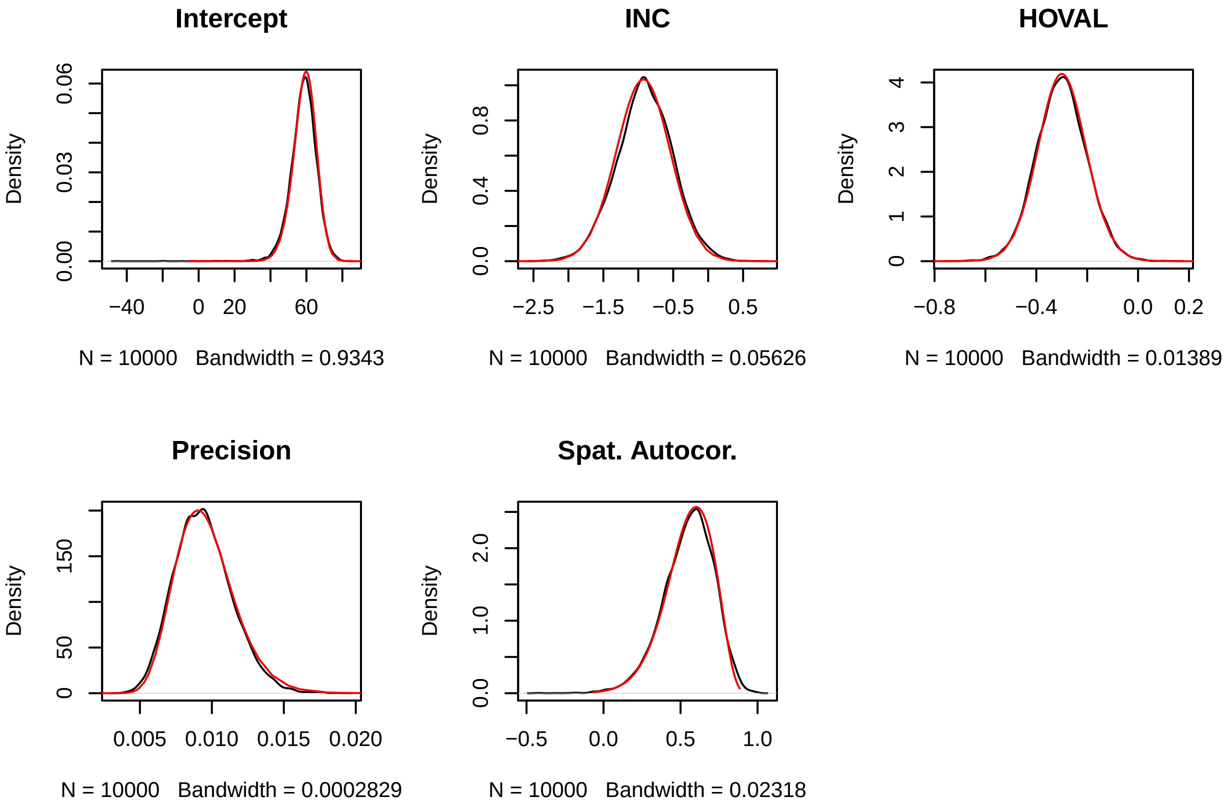
```r
#SLM model
slm.inla<-inla( CRIME ~ -1 +
   f(idx, model="slm",
      args.slm=list(rho.min = rho.min, rho.max = rho.max, W = W.inla, X=mmatrix,
         Q.beta = Q.beta),
      hyper=hyper.slm),
   data=as.data.frame(columbus), family="gaussian",
   lincomb = lc.linpred2, control.predictor = list(compute=TRUE),
   control.fixed = c.fixed,
   control.inla = c.inla,
   control.family = list(hyper=zero.variance),
   control.compute=list(dic=TRUE, cpo=TRUE)
)

#SDM model
hyper.sdm <- hyper.slm

#Fix rho
hyper.sdm$rho$initial <- 0.77 #Fixed to posterior mode from MCMC
#hyper.sdm$rho$fixed <- TRUE

#Use stronger prior
hyper.sdm$rho$param <- c(140, 60)


sdm.inla <- inla( CRIME ~ -1 +
   f(idx, model = "slm",
      args.slm = list(rho.min = rho.min, rho.max = rho.max, W = W.inla,
        X = mmatrix2, Q.beta = Q.beta2),
      hyper = hyper.sdm),
   data = as.data.frame(columbus), family = "gaussian",
   lincomb = lc.linpred2, control.predictor = list(compute=TRUE),
   control.fixed = c.fixed,
   control.inla = c.inla,
   control.family = list(hyper = zero.variance),
   control.compute = list(dic = TRUE, cpo = TRUE)
)
```

Results

Transform estimate of spatial autocorrelation provided by INLA:

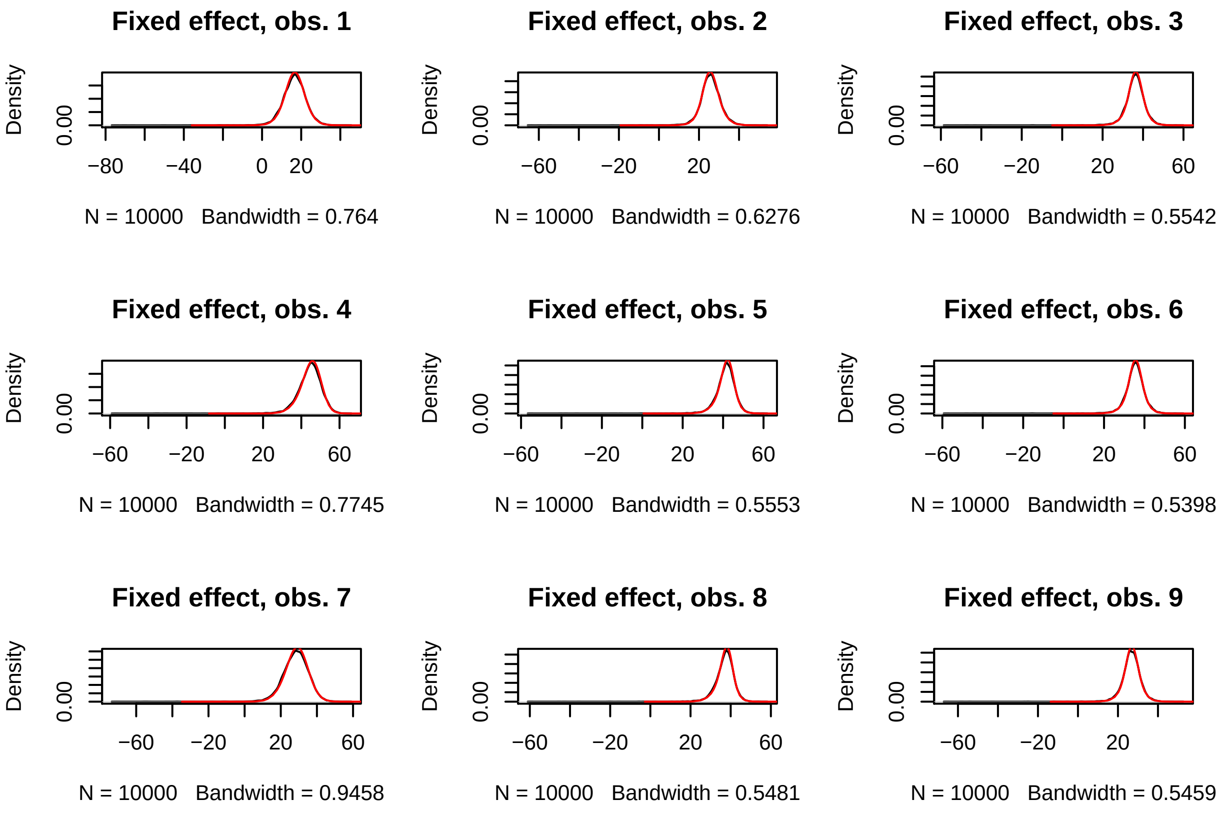
```r
  ff <- function(z){z * (rho.max - rho.min) + rho.min}
  semmarg <- inla.tmarginal(ff, sem.inla$marginals.hyperpar[[2]])
  slmmarg <- inla.tmarginal(ff, slm.inla$marginals.hyperpar[[2]])
  sdmmarg <- inla.tmarginal(ff, sdm.inla$marginals.hyperpar[[2]])
```

Spatial Error Model (SEM)



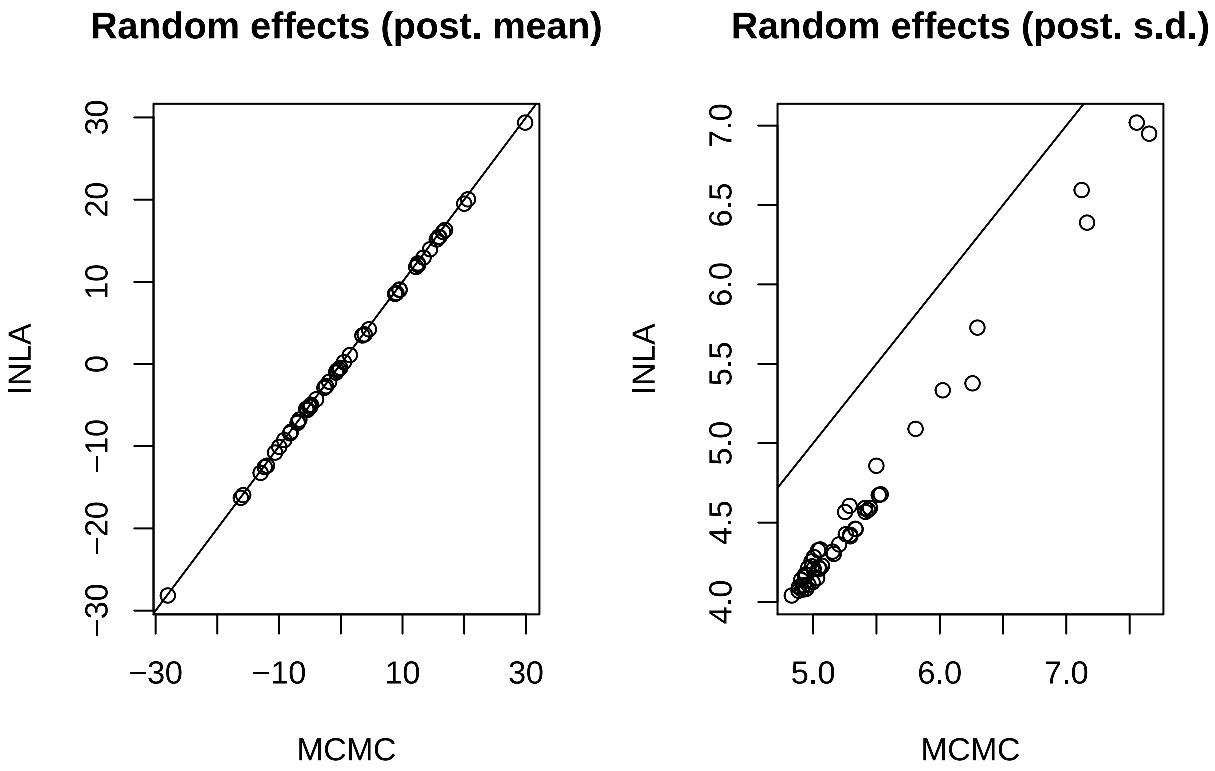
Effect of the covariates

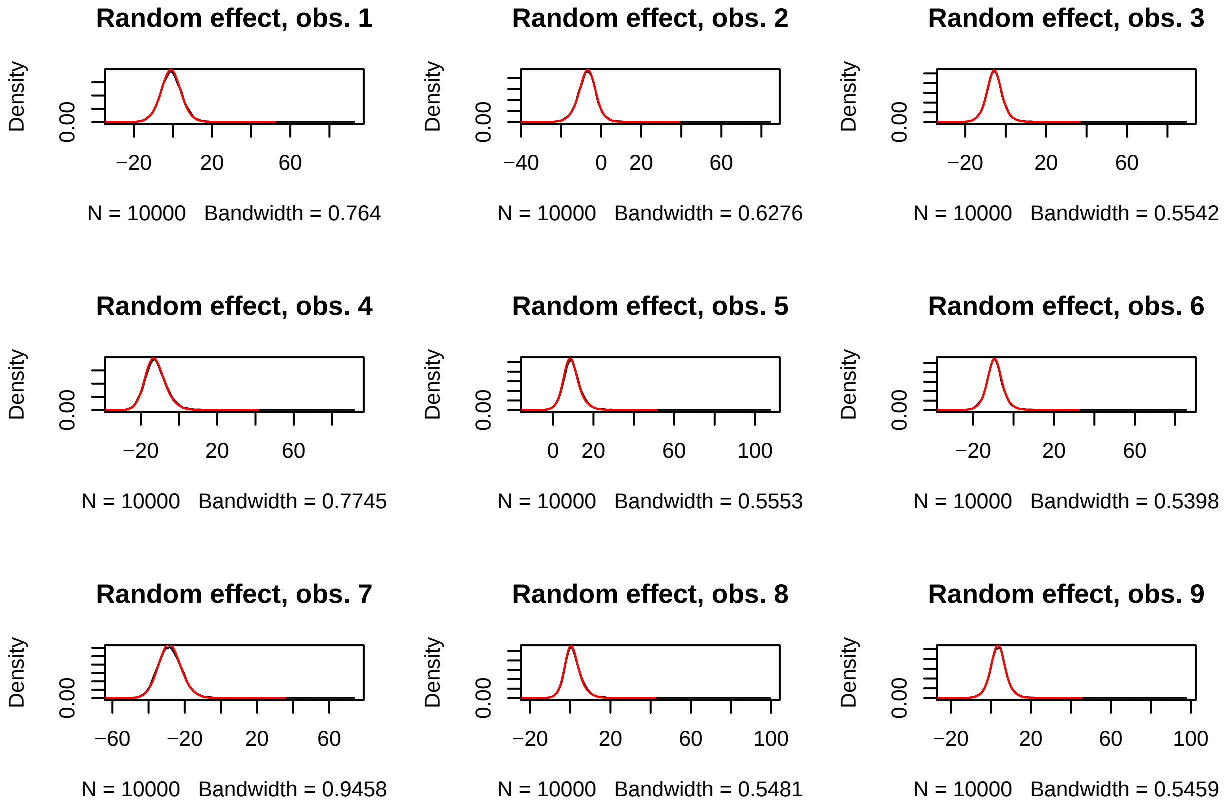
The next plots display the estimated effect of the covariates alone to see if there is agreement.



Estimate of the random effects

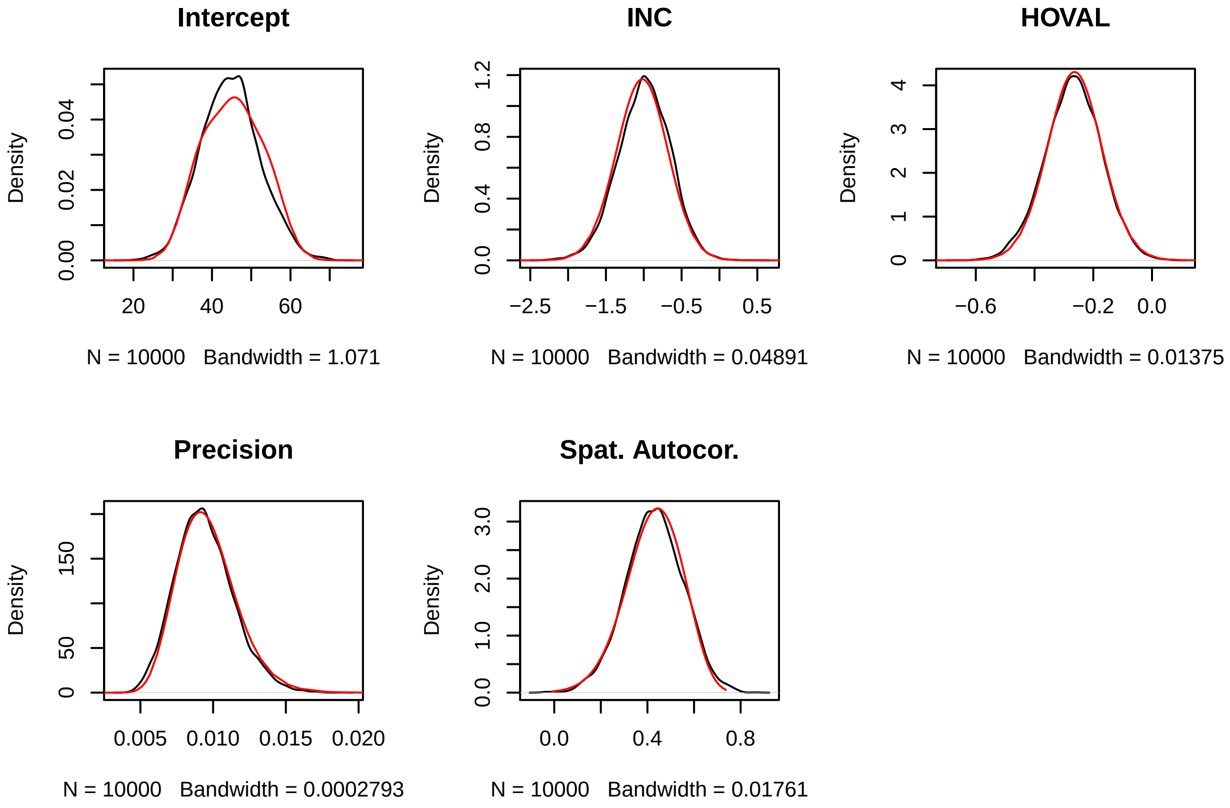
```r
sem.raneff.mcmc <- apply(sem.mcmc$mu, 2, function(X) {columbus$CRIME - X})
sem.raneff.mcmc <- matrix(sem.raneff.mcmc, ncol = 49, byrow = TRUE)

sem.raneff.mean <- apply(sem.raneff.mcmc, 2, mean)
sem.raneff.sd <- apply(sem.raneff.mcmc, 2, sd)
```


Random effects (post. mean)
Random effects (post. s.d.)
INLA
30
20
10
0
−10
−20
−30
−30
−10
10
30
MCMC
INLA
7.0
6.5
6.0
5.5
5.0
4.5
4.0
5.0
6.0
7.0
MCMC


Random effect, obs. 1
Density
0.00
−20
20
60
N = 10000 Bandwidth = 0.764
Random effect, obs. 2
Density
0.00
−40
0
20
60
N = 10000 Bandwidth = 0.6276
Random effect, obs. 3
Density
0.00
−20
20
60
N = 10000 Bandwidth = 0.5542
Random effect, obs. 4
Density
0.00
−20
20
60
N = 10000 Bandwidth = 0.7745
Random effect, obs. 5
Density
0.00
0
20
60
100
N = 10000 Bandwidth = 0.5553
Random effect, obs. 6
Density
0.00
−20
20
60
N = 10000 Bandwidth = 0.5398
Random effect, obs. 7
Density
0.00
−60
−20
20
60
N = 10000 Bandwidth = 0.9458
Random effect, obs. 8
Density
0.00
−20
20
60
100
N = 10000 Bandwidth = 0.5481
Random effect, obs. 9
Density
0.00
−20
20
60
100
N = 10000 Bandwidth = 0.5459

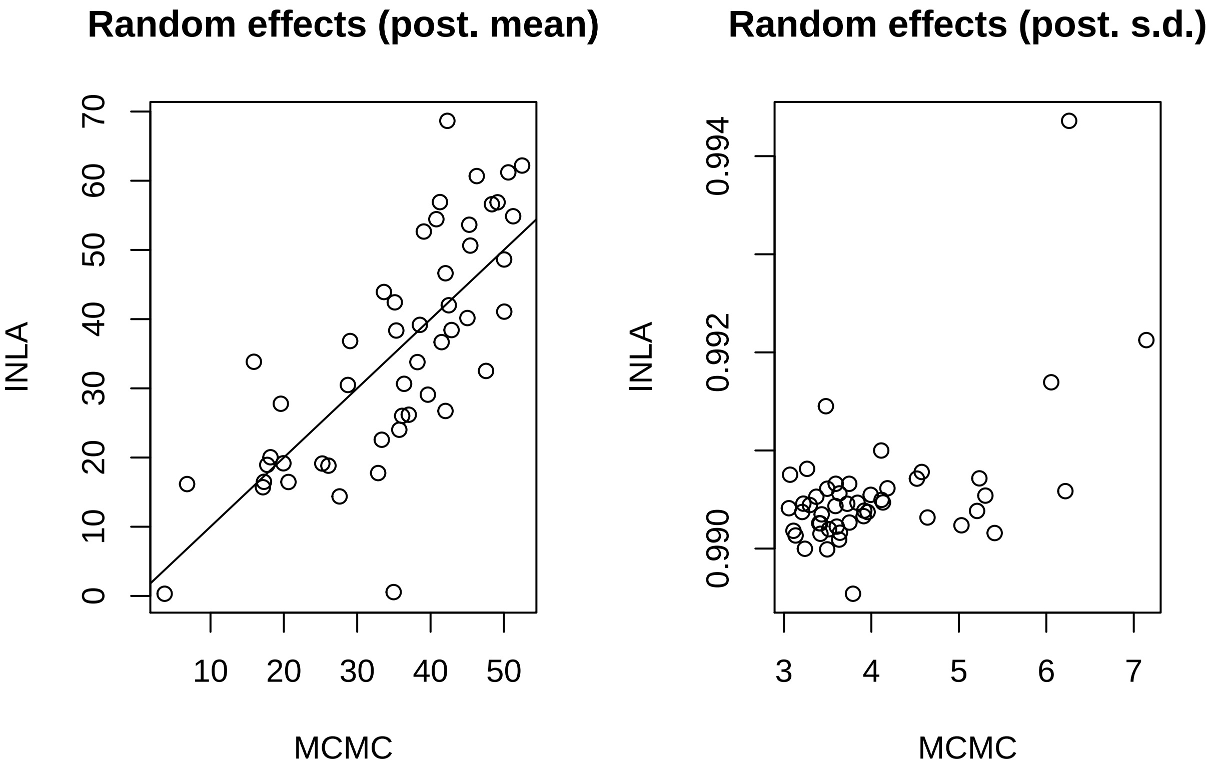
Spatial Lag Model (SLM)

Effect of the covariates

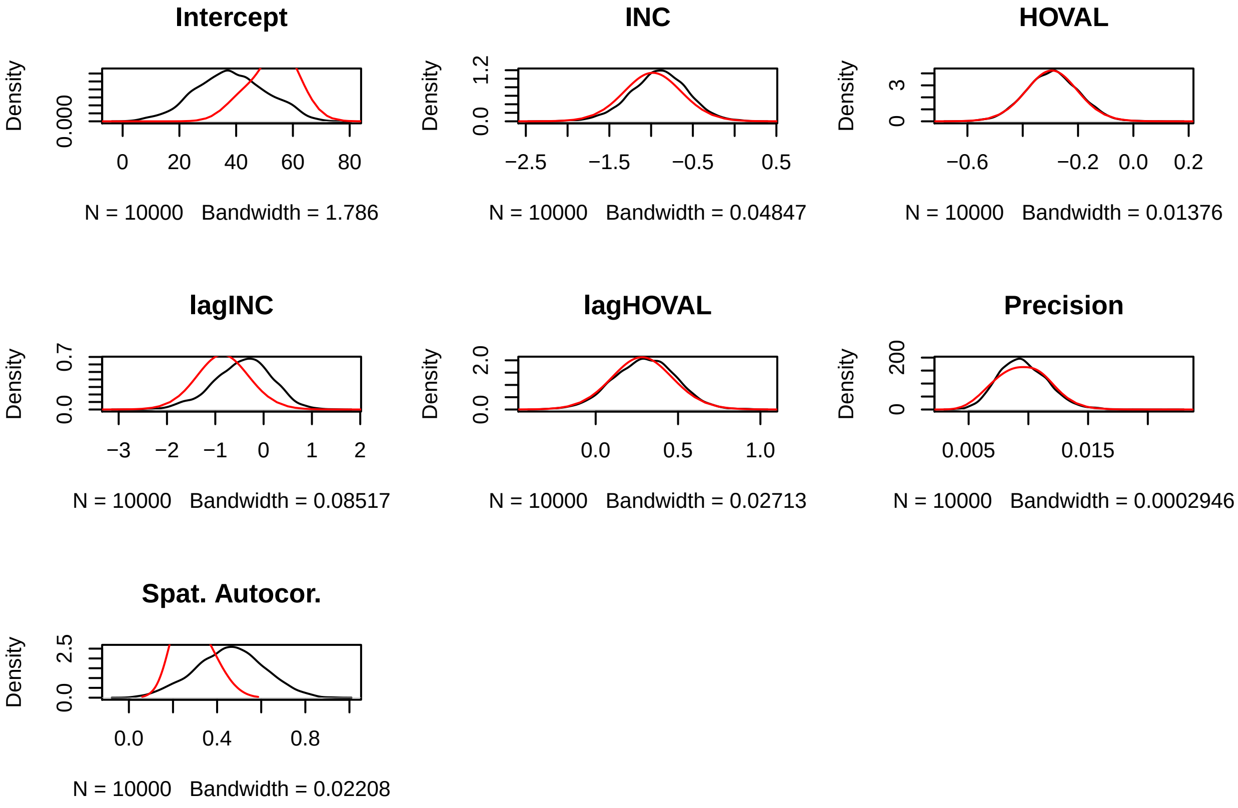


Estimate of the linear predictor

```r
slm.raneff.mcmc <- apply(slm.mcmc$mu, 2, function(X) {columbus$CRIME - X})
slm.raneff.mcmc <- matrix(slm.raneff.mcmc, ncol = 49, byrow = TRUE)

slm.raneff.mean <- apply(slm.raneff.mcmc, 2, mean)
slm.raneff.sd <- apply(slm.raneff.mcmc, 2, sd)
```



Spatial Durbin Model (SDM)



Discussion