COSC 4820

Closure of FD's

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Why Closure

- Answers the question "Does some FD follow from a set of FD's?"
- Some knowledge about FD's is helpful.
- Basically
  - Compute closure of left side of the FD in question
  - Requires a set of FD's that are satisfied by the relation in question
  - Closure used to answer question "Is right side of the subject in the closure?"
  - If true, then the FD follows from the set.

Algorithm

- INPUT: A set of attributes A and a set of FD's S
- OUTPUT: The closure (X) of the set of attributes
  1. split FD's as necessary
  2. initialize $X = A$
  3. find $(Y \to D) \in S$ where $Y \in X$ and $D \notin X$
  4. add D to X
  5. when no more can be added to X, X is the closure of A

Result

- We make this claim
- $A_1 A_2 \cdots A_n \rightarrow B_1 B_2 \cdots B_m$ follows from a set of FD's S iff all of $B_1, B_2, \ldots, B_m$ are in $\{A_1, A_2, \ldots, A_n\}^+$

Remember from FD Rules

- A set of FD's *follows* from another if
  - every relation satisfying the second
  - also satisfies the first
- If S and T are equivalent then
  - T follows from S and
  - S follows from T

Transitive Rule

- Formalizes what we have already talked about
- If $A_1 A_2 \cdots A_n \to B_1 B_2 \cdots B_m$ and $B_1 B_2 \cdots B_m \to C_1 C_2 \cdots C_k$ hold in relation R then $A_1 A_2 \cdots A_n \to C_1 C_2 \cdots C_k$ also holds in R
- Use closure to see this

Basis

- Which FD's to use to represent a set S?
- *Basis* is any set equivalent to S
- A *minimal basis*, B, meets 3 conditions:
  1. All FD's in B have singleton right sides
  2. If any FD is removed from B it is no longer a basis
  3. For any FD, F, in B, if one or more attributes are removed from the left side of F, the result is no longer a basis

Projecting Functional Dependencies (1)

- I have a relation with a set of FD's
- I project a set of attributes from this relation
- Which, if any FD's from the original set still hold?

Projecting Functional Dependencies (2)

- They are the ones
  - That follow from the original set
  - That involve only attributes in the projected version of the relation
  - Whose number is exponential in the number of attributes in the projected relation

How do we do this?

- Of course there is an algorithm
- Start with
  - S, set of FD's that hold for relation R.
  - Lets call the set of attributes for R, A
  - $R_1 = \pi_L(R)$, and its attributes are A_1
- Output the set, T,of FD's that hold for R_1

Process of the algorithm

1. Initialize $T = \emptyset$
2. For each $X \subseteq A_1$ compute X^+ with respect to S.
   May involve elements of A not in A_1.
   Add to T all non-trivial FD's $X \rightarrow K$ s.t. $K \in X^+$ and $K \in A_1$.
3. T is now a basis for the FD's that hold in R_1, but may not be minimal. Make minimal by ...

Process of the algorithm

- If $F \in T$ and F follows from other FD's in T, remove F from T
- Let $Y \to B$ be in T, with at least two attributes in Y. Let Z be Y with one attribute removed. If $Z \to B$ follows from FD's in T (including $Y \to B$) then replace $Y \to B$ with $Z \to B$.
- Repeat (a) and (b) until no more changes can be made to T