

## Exercise #1

```
clear
```

```
format short  
type ele1
```

```
function E1=ele1(n,r,i,j)  
E1=eye(n);  
E1(j,:) = E1(j,:) + E1(i,:).*r;  
end
```

```
type ele2
```

```
function E2=ele2(n,i,j)  
E2=eye(n);  
E2([i j],:) = E2([j i],:);  
end
```

```
type ele3
```

```
function E3=ele3(n,j,k)  
E3=eye(n);  
E3(j,:)=E3(j,:).*k;  
end
```

### Part 1

```
n=4; r=5; i=1; j=3; k=2
```

```
k = 2
```

(a)

```
I=eye(4)
```

```
I = 4x4  
    1    0    0    0  
    0    1    0    0  
    0    0    1    0  
    0    0    0    1
```

```
E1=ele1(n,r,i,j)
```

```
E1 = 4x4  
    1    0    0    0  
    0    1    0    0  
    5    0    1    0  
    0    0    0    1
```

```
% row 3 is replaced with (row 3) plus 5*(row 1)
```

```
E2=ele2(n,i,j)
```

```
E2 = 4x4
      0      0      1      0
      0      1      0      0
      1      0      0      0
      0      0      0      1
```

```
% rows 1 and 3 are interchanged
```

```
E3=ele3(n,j,k)
```

```
E3 = 4x4
      1      0      0      0
      0      1      0      0
      0      0      2      0
      0      0      0      1
```

```
% row 3 is scaled by k=2
```

(b)

```
detI=det(I)
```

```
detI = 1
```

```
detE1=det(E1)
```

```
detE1 = 1
```

```
% same as detI
```

```
detE2=det(E2)
```

```
detE2 = -1
```

```
% negative of detI
```

```
detE3=det(E3)
```

```
detE3 = 2
```

```
% 2 times detI (based on k, which is 2 here)
```

(c)

```
invE1=inv(E1)
```

```
invE1 = 4x4
    1    0    0    0
    0    1    0    0
   -5    0    1    0
    0    0    0    1
```

```
% the 5 is now a -5
```

```
invE2=inv(E2)
```

```
invE2 = 4x4
    0    0    1    0
    0    1    0    0
    1    0    0    0
    0    0    0    1
```

```
% same as E2
```

```
invE3=inv(E3)
```

```
invE3 = 4x4
   1.0000    0    0    0
    0    1.0000    0    0
    0    0    0.5000    0
    0    0    0    1.0000
```

```
% the 2 is now a 0.5
```

(d)

```
M=[1 1 1 1; 2 2 2 2; 3 3 3 3; 4 4 4 4]
```

```
M = 4x4
    1    1    1    1
    2    2    2    2
    3    3    3    3
    4    4    4    4
```

```
E1*M
```

```
ans = 4x4
    1    1    1    1
    2    2    2    2
    8    8    8    8
    4    4    4    4
```

```
% row 3 replaced by (row 3) plus 5*(row 1)
```

```
E2*M
```

```
ans = 4x4
    3    3    3    3
    2    2    2    2
    1    1    1    1
    4    4    4    4
```

```
% row 1 and 2 interchanged
```

```
E3*M
```

```
ans = 4x4
    1    1    1    1
    2    2    2    2
    6    6    6    6
    4    4    4    4
```

```
% row 3 scaled by 2
```

## Part 2

```
A=eye(6)
```

```
A = 6x6
    1    0    0    0    0    0
    0    1    0    0    0    0
    0    0    1    0    0    0
    0    0    0    1    0    0
    0    0    0    0    1    0
    0    0    0    0    0    1
```

```
E1=ele1(6,3,2,5)
```

```
E1 = 6x6
    1    0    0    0    0    0
    0    1    0    0    0    0
    0    0    1    0    0    0
    0    0    0    1    0    0
    0    3    0    0    1    0
    0    0    0    0    0    1
```

```
E2=ele2(6,2,3)
```

```
E2 = 6x6
    1    0    0    0    0    0
    0    0    1    0    0    0
    0    1    0    0    0    0
    0    0    0    1    0    0
    0    0    0    0    1    0
    0    0    0    0    0    1
```

```
E3=ele3(6,4,5)
```

```
E3 = 6x6
    1    0    0    0    0    0
    0    1    0    0    0    0
    0    0    1    0    0    0
    0    0    0    5    0    0
    0    0    0    0    1    0
```

```
0    0    0    0    0    1
```

```
A=E3*E2*E1*A
```

```
A = 6×6
```

```
1    0    0    0    0    0
0    0    1    0    0    0
0    1    0    0    0    0
0    0    0    5    0    0
0    3    0    0    1    0
0    0    0    0    0    1
```

```
% We know this matrix is invertible because it began as the identity square
% matrix. Only elementary row operations were performed on it, which are
% reversible. This means a reduced form of this matrix is the identity
% matrix that we started with. All identity matrices are linearly
% independent and invertible. Therefore, this matrix is invertible.
```

```
inv1=inv(A)
```

```
inv1 = 6×6
```

```
1.0000    0    0    0    0    0
0    0    1.0000    0    0    0
0    1.0000    0    0    0    0
0    0    0    0.2000    0    0
0    0    -3.0000    0    1.0000    0
0    0    0    0    0    1.0000
```

```
inv2=inv(E1)*inv(E2)*inv(E3)
```

```
inv2 = 6×6
```

```
1.0000    0    0    0    0    0
0    0    1.0000    0    0    0
0    1.0000    0    0    0    0
0    0    0    0.2000    0    0
0    0    -3.0000    0    1.0000    0
0    0    0    0    0    1.0000
```

```
if(isequal(inv1,inv2))
    disp("The inverses match.")
else
    disp("Check the code!")
end
```

```
The inverses match.
```

## Exercise #2

```
type rredef
```

```
function R = rredef(A)
```

```
[m,n] = size(A);
```

```

if (n >= m)
    for p = 1:n
        if any(A(:,p))
            index = p;
            [max_val, max_row] = max(abs(A(:,p)));
            A([1 max_row],:) = A([max_row 1],:);
            A(1,:) = A(1,:) ./ A(1,p);
            if (A(1,p) == -1)
                A(1,p) = A(1,p) * -1;
            end
            break
        end
    end
end

for j = 1:(n-1)
    for p = j:(m-1)
        A(p+1,:) = A(p+1,:) - ( A(j,:) .* A(p+1,index) );
    end
    A = closetozeroroundoff(A,7);
    if (index < m-1)
        index = index + 1;
    else
        if (A(p+1,index+1) == 0)
            for v = (index+1):n
                if (A(p+1,v) ~= 0)
                    A(p+1,:) = A(p+1,:) ./ A(p+1,v);
                end
            end
            break
        else
            A(p+1,:) = A(p+1,:) ./ A(p+1,index+1);
            break
        end
    end
end

for u=1:n
    if (A(j+1,u) ~= 0)
        A(j+1,:) = A(j+1,:) ./ A(j+1, index);
    end
end
end

rows = m;
for j = 1:(n-1)
    for p = 1:(rows-1)
        A(rows - p,:) = A(rows - p, :) - A(rows,:) .* A(rows - p,index + 2 -j);
    end
    A = closetozeroroundoff(A,7);
    rows = m - j;
    if (rows <= 1)
        break
    end
end

else
    for p = 1:n
        if any(A(:,p))
            index = p;
            [max_val, max_row] = max(abs(A(:,p)));
            A([1 max_row],:) = A([max_row 1],:);
            A(1,:) = A(1,:) ./ A(1,p);

```

```

        if (A(1,p) == -1)
            A(1,p) = A(1,p) * -1;
        end
        break
    end
end

for j = 1:(n-1)
    for p = j:(m-1)
        A(p+1,:) = A(p+1,:) - ( A(j,:) .* A(p+1,index) );
    end
    A = closetozeroroundoff(A,7);
    if (index < n-1)
        index = index + 1;
    else
        if (A(p+1,index+1) == 0)
            for v = (index+1):n
                if (A(p+1,v) ~= 0)
                    A(p+1,:) = A(p+1,:) ./ A(p+1,v);
                end
            end
            break
        else
            A(p+1,:) = A(p+1,:) ./ A(p+1,index+1);
            break
        end
    end
end

for u=1:n
    if (A(j+1,u) ~= 0)
        A(j+1,:) = A(j+1,:) ./ A(j+1, index);
    end
end

index = 0;
for p=m:-1:1
    for j=1:n
        if (A(p,j) ~= 0)
            index = j;
            rows = p;
            break
        end
    end
    if (index ~= 0)
        break
    end
end

for j = 1:(n-1)
    for p = 1:(rows-1)
        A(rows - p,:) = A(rows - p, :) - A(rows,:) .* A(rows - p,index + 1 -j);
    end
    A = closetozeroroundoff(A,7);
    rows = rows - j;
    if (rows <= 1)
        break
    end
end

end

for p=1:(m-1)
    if (A(p,:) == 0)

```

```

        A([p m], :) = A([m p], :);
    end
end

R=[A];
r=rref(A);
if (closetozeroroundoff(R-r,7) == 0)
    disp('the reduced echelon form of A is')
    R
else
    disp('Something is wrong!')
end

```

type `closetozeroroundoff`

```

function B=closetozeroroundoff(A,p)
A(abs(A)<10^-p)=0;
B=A;
end

```

%(a)

```
A=[2 1 1;1 2 3;1 1 1]
```

```

A = 3x3
     2     1     1
     1     2     3
     1     1     1

```

```
R=rref(A);
```

the reduced echelon form of A is

```

R = 3x3
     1     0     0
     0     1     0
     0     0     1

```

%(b)

```
A=[zeros(3),randi(10,3,2)]
```

```

A = 3x5
     0     0     0     1    10
     0     0     0     3    10
     0     0     0     6     2

```

```
R=rref(A);
```

the reduced echelon form of A is

```

R = 3x5
     0     0     0     1     0
     0     0     0     0     1
     0     0     0     0     0

```

%(c)

```
A=magic(4)
```

```

A = 4x4
    16     2     3    13
     5    11    10     8
     9     7     6    12
     4    14    15     1

```



```
R=rredef(A);
```

the reduced echelon form of A is

```
R = 4x4
    1.0000    0    0    1.0000
         0    1.0000    0    3.0000
         0    0    1.0000 -3.0000
         0    0    0    0
```

```
%(d)
```

```
A=magic(5)
```

```
A = 5x5
    17    24     1     8    15
    23     5     7    14    16
     4     6    13    20    22
    10    12    19    21     3
    11    18    25     2     9
```

```
R=rredef(A);
```

the reduced echelon form of A is

```
R = 5x5
     1     0     0     0     0
     0     1     0     0     0
     0     0     1     0     0
     0     0     0     1     0
     0     0     0     0     1
```

```
%(e)
```

```
A=ones(3)
```

```
A = 3x3
     1     1     1
     1     1     1
     1     1     1
```

```
R=rredef(A);
```

the reduced echelon form of A is

```
R = 3x3
     1     1     1
     0     0     0
     0     0     0
```

```
%(f)
```

```
A=rand(3,4)
```

```
A = 3x4
    0.9706    0.8003    0.9157    0.6557
    0.9572    0.1419    0.7922    0.0357
    0.4854    0.4218    0.9595    0.8491
```

```
R=rredef(A);
```

the reduced echelon form of A is

```
R = 3x4
    1.0000     0     0 -0.9097
         0    1.0000     0    0.7715
         0     0    1.0000    1.0060
```

```
%(g)
```

```
A=randi(10,5,3);A=[A,A(:,3)]
```

```
A = 5x4
```

```
10    7    1    1
 7    2    1    1
 8    8    9    9
 8    1    7    7
 4    3    4    4
```

```
R=rref(A);
```

the reduced echelon form of A is

```
R = 5x4
```

```
1    0    0    0
0    1    0    0
0    0    1    1
0    0    0    0
0    0    0    0
```

## Exercise #3

```
format compact
type nonhomogen.m
```

```
function x=nonhomogen(A,b)
format
[~,n]=size(A);
fprintf('reduced echelon form of [A b] is ')
R=rref([A,b])
[nRow,nCol] = size(A);
if(rank(R) ~= rank(A))
    disp('The system is inconsistent')
elseif(nRow == nCol && det(A) ~= 0)
    disp('The system has a unique solution')
    A\b
else
    disp('There are infinitely many solutions')
    homobasis_b(A, b);
end
syms Col(C), syms p
fprintf('the general solution of the non-homogeneous system is\n')
fprintf('the column space of the matrix C translated by the vector p')
x=Col(C)+p
x=[];
```

```
type homobasis_b
```

```
function [C,p] = homobasis_b(A, b)
format
[m,n]=size(A);
red_ech_form=rats(rref(A));
num_rref_A = rref(A);
C=[];
disp('the homogeneous system has non-trivial solutions')
[~,pivot_c]=rref(A);
S=1:n;
nonpivot_c=setdiff(S,pivot_c);
q=numel(nonpivot_c);
```

```

j=1:q;
fprintf('a free variable is x%i\n',nonpivot_c(j))
C=zeros(n,q);
% Creating necessary intermediate variable
identity_for_C = eye(size(nonpivot_c,2));
% Code to RECALCULATE C's rows
for i=1: numel(pivot_c)
    C(pivot_c(1,i),:)=num_rref_A(i,nonpivot_c)*-1;
end
for i=1:size(nonpivot_c,2)
    C(nonpivot_c(1,i),:)= identity_for_C(i,:);
end
fprintf('a basis for the solution set of the homogeneous system\n')
fprintf('is formed by the columns of the matrix')
C

R = rref([A,b]);
[m,n] = size(A);
p=zeros(n,1);
j = 1;
for i=1:n
    if(ismember(i, pivot_c))
        p(i) = R(j, n+1);
        j = j+1;
    end
end
disp('particular solution of the non-homogeneous system is the vector')
p

```

```

%(a)
A=[1 -2 3], b=randi(10,1,1)

```

```

A = 1×3
    1    -2     3
b = 10

```

```

x=nonhomogen(A,b);

```

```

reduced echelon form of [A b] is
R = 1×4
    1    -2     3    10
There are infinitely many solutions
the homogeneous system has non-trivial solutions
a free variable is x2
a free variable is x3
a basis for the solution set of the homogeneous system
is formed by the columns of the matrix
C = 3×2
    2    -3
    1     0
    0     1
particular solution of the non-homogeneous system is the vector
p = 3×1
    10
     0
     0
the general solution of the non-homogeneous system is
the column space of the matrix C translated by the vector p
x = p + Col(C)

```

```

%(b)
A=magic(3), b=randi(10,3,1)

```

```
A = 3x3
    8    1    6
    3    5    7
    4    9    2
b = 3x1
    1
    5
    4
```

```
x=nonhomogen(A,b);
```

reduced echelon form of [A b] is

```
R = 3x4
    1.0000    0    0   -0.3194
    0    1.0000    0    0.4722
    0    0    1.0000    0.5139
```

The system has a unique solution

```
ans = 3x1
   -0.3194
    0.4722
    0.5139
```

the general solution of the non-homogeneous system is  
the column space of the matrix C translated by the vector p

$x = p + \text{Col}(C)$

```
%(c)
```

```
A=magic(4), b=randi(10,4,1)
```

```
A = 4x4
    16     2     3    13
     5    11    10     8
     9     7     6    12
     4    14    15     1
b = 4x1
     8
     8
     2
     5
```

```
x=nonhomogen(A,b);
```

reduced echelon form of [A b] is

```
R = 4x5
     1     0     0     1     0
     0     1     0     3     0
     0     0     1    -3     0
     0     0     0     0     1
```

The system is inconsistent

the general solution of the non-homogeneous system is  
the column space of the matrix C translated by the vector p

$x = p + \text{Col}(C)$

```
%(d)
```

```
B=[0 1 2 3;0 2 4 6]; A=[B; eye(4)], b=sum(A,2)
```

```
A = 6x4
     0     1     2     3
     0     2     4     6
     1     0     0     0
     0     1     0     0
     0     0     1     0
     0     0     0     1
```

```
b = 6x1
    6
   12
    1
    1
    1
    1
```

```
x=nonhomogen(A,b);
```

reduced echelon form of [A b] is

```
R = 6x5
    1     0     0     0     1
    0     1     0     0     1
    0     0     1     0     1
    0     0     0     1     1
    0     0     0     0     0
    0     0     0     0     0
```

There are infinitely many solutions

the homogeneous system has non-trivial solutions

a free variable is x

a basis for the solution set of the homogeneous system

is formed by the columns of the matrix

C =

4x0 empty double matrix

particular solution of the non-homogeneous system is the vector

```
p = 4x1
    1
    1
    1
    1
```

the general solution of the non-homogeneous system is

the column space of the matrix C translated by the vector p

$x = p + \text{Col}(C)$

```
%(e)
```

```
A=[0 1 0 2 0 3; 0 2 0 4 0 6; 0 4 0 8 0 6], b=ones(3,1)
```

```
A = 3x6
    0     1     0     2     0     3
    0     2     0     4     0     6
    0     4     0     8     0     6
```

```
b = 3x1
    1
    1
    1
```

```
x=nonhomogen(A,b);
```

reduced echelon form of [A b] is

```
R = 3x7
    0     1     0     2     0     0     0
    0     0     0     0     0     1     0
    0     0     0     0     0     0     1
```

The system is inconsistent

the general solution of the non-homogeneous system is

the column space of the matrix C translated by the vector p

$x = p + \text{Col}(C)$

```
%(f)
```

```
A=[0 1 0 2 0 3; 0 2 0 4 0 6; 0 4 0 8 0 6], b=sum(A,2)
```

```
A = 3x6
    0     1     0     2     0     3
    0     2     0     4     0     6
    0     4     0     8     0     6
b = 3x1
     6
    12
    18
```

```
x=nonhomogen(A,b);
```

reduced echelon form of [A b] is

```
R = 3x7
    0     1     0     2     0     0     3
    0     0     0     0     0     1     1
    0     0     0     0     0     0     0
```

There are infinitely many solutions

the homogeneous system has non-trivial solutions

a free variable is x1

a free variable is x3

a free variable is x4

a free variable is x5

a basis for the solution set of the homogeneous system  
is formed by the columns of the matrix

```
C = 6x4
    1     0     0     0
    0     0    -2     0
    0     1     0     0
    0     0     1     0
    0     0     0     1
    0     0     0     0
```

particular solution of the non-homogeneous system is the vector

```
p = 6x1
     0
     3
     0
     0
     0
     1
```

the general solution of the non-homogeneous system is

the column space of the matrix C translated by the vector p

$x = p + \text{Col}(C)$

```
%(g)
```

```
A=[0 0 1 2 3;0 0 2 4 5], b=A(:,end)
```

```
A = 2x5
    0     0     1     2     3
    0     0     2     4     5
b = 2x1
     3
     5
```

```
x=nonhomogen(A,b);
```

reduced echelon form of [A b] is

```
R = 2x6
    0     0     1     2     0     0
    0     0     0     0     1     1
```

There are infinitely many solutions

the homogeneous system has non-trivial solutions

a free variable is  $x_1$   
a free variable is  $x_2$   
a free variable is  $x_4$   
a basis for the solution set of the homogeneous system  
is formed by the columns of the matrix

$C = 5 \times 3$

```
1  0  0
0  1  0
0  0 -2
0  0  1
0  0  0
```

particular solution of the non-homogeneous system is the vector

$p = 5 \times 1$

```
0
0
0
0
1
```

the general solution of the non-homogeneous system is  
the column space of the matrix  $C$  translated by the vector  $p$

$x = p + \text{Col}(C)$

```
%(h)
```

```
A=[0 0 1 2 3;0 0 2 4 6], b=A(:,end)
```

$A = 2 \times 5$

```
0  0  1  2  3
0  0  2  4  6
```

$b = 2 \times 1$

```
3
6
```

```
x=nonhomogen(A,b);
```

reduced echelon form of  $[A \ b]$  is

$R = 2 \times 6$

```
0  0  1  2  3  3
0  0  0  0  0  0
```

There are infinitely many solutions

the homogeneous system has non-trivial solutions

a free variable is  $x_1$

a free variable is  $x_2$

a free variable is  $x_4$

a free variable is  $x_5$

a basis for the solution set of the homogeneous system

is formed by the columns of the matrix

$C = 5 \times 4$

```
1  0  0  0
0  1  0  0
0  0 -2 -3
0  0  1  0
0  0  0  1
```

particular solution of the non-homogeneous system is the vector

$p = 5 \times 1$

```
0
0
3
0
0
```

the general solution of the non-homogeneous system is

the column space of the matrix  $C$  translated by the vector  $p$

$x = p + \text{Col}(C)$

## Exercise #4

```
type areavol
```

```
function D=areavol(A)
format
D=0;

%find (rows x col) = (m,n)
[m,n]=size(A);

%parallelogram (2x2)
if m == 2 && n==2
    %check if linearly independent
    if rank(A) == 2
        %Area = b*h
        D = A(1,1) * A(2,2);
        fprintf('The area of the parallelogram is\n')
        D
    else
        fprintf('The parallelogram cannot be built')
    end
end

%parallelepiped (3x3)
else
    %check if linearly independent
    if rank(A) == 3
        %Area = b*h*w
        D = A(1,1) * A(2,2) * A(3,3);
        fprintf('The area of the parallelepiped is\n')
        D
    else
        fprintf('The parallelepiped cannot be built')
    end
end
end

end
```

```
%(a)
A=eye(2)
```

```
A = 2x2
    1    0
    0    1
```

```
D=areavol(A);
```

```
The area of the parallelogram is
D = 1
```

```
%(b)
A=magic(3)
```

```
A = 3x3
    8    1    6
    3    5    7
    4    9    2
```

```
D=areavol(A);
```



The area of the parallelepiped is  
D = 80

```
%(c)
A=randi(10,2)
```

```
A = 2x2
     5     8
     7     8
```

```
D=areavol(A);
```

The area of the parallelogram is  
D = 40

```
%(d)
A=fix(10*rand(3))
```

```
A = 3x3
     2     1     9
     6     1     3
     6     4     5
```

```
D=areavol(A);
```

The area of the parallelepiped is  
D = 10

```
%(e)
B=randi([-10,10],2,1);
A = [B,3*B]
```

```
A = 2x2
    -6    -18
     5     15
```

```
D=areavol(A);
```

The parallelogram cannot be built

```
%(f)
X=randi([-10,10],3,1);
Y=randi([-10,10],3,1);
A=[X,Y,X+Y]
```

```
A = 3x3
    -5     8     3
     0    10    10
     4     1     5
```

```
D=areavol(A);
```

The parallelepiped cannot be built

## Exercise #5

RX (reflection across the x1 -axis)

```
clear
```

$$RX = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$RX = 2 \times 2$$

1	0
0	-1

RY (reflection across the x2 -axis)

$$RY = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$RY = 2 \times 2$$

-1	0
0	1

VS (vertical shear with k = 3 )

$$VS = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$$

$$VS = 2 \times 2$$

1	0
3	1

RS (reflection across the line x2 = x1 )

$$RS = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$RS = 2 \times 2$$

0	1
1	0

RA (reflection across the line x2 = -x1 )

$$RA = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$RA = 2 \times 2$$

0	1
-1	0

type **transf**

```
function C=transf(A,E)
C=A*E;
x=C(1,:);y=C(2,:);
plot(x,y)
v=[-5 5 -5 5];
axis(v)
end
```

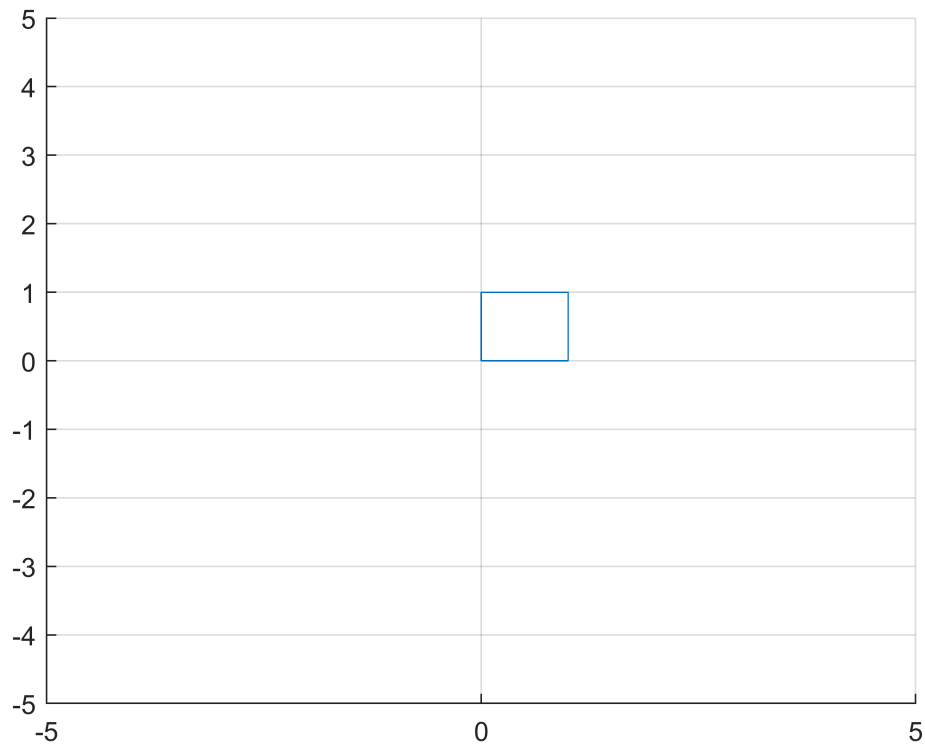
This function receives parameters A and E and first multiplies them together to get C. It then creates 2 sub matrices x and y from C representing the coordinate plane. Then it uses the plot command to display them on the matlab plot. It then draw the x and y axis from -5 to 5.

```
E=[0 1 1 0 0;0 0 1 1 0]; A=eye(2);
hold on
```

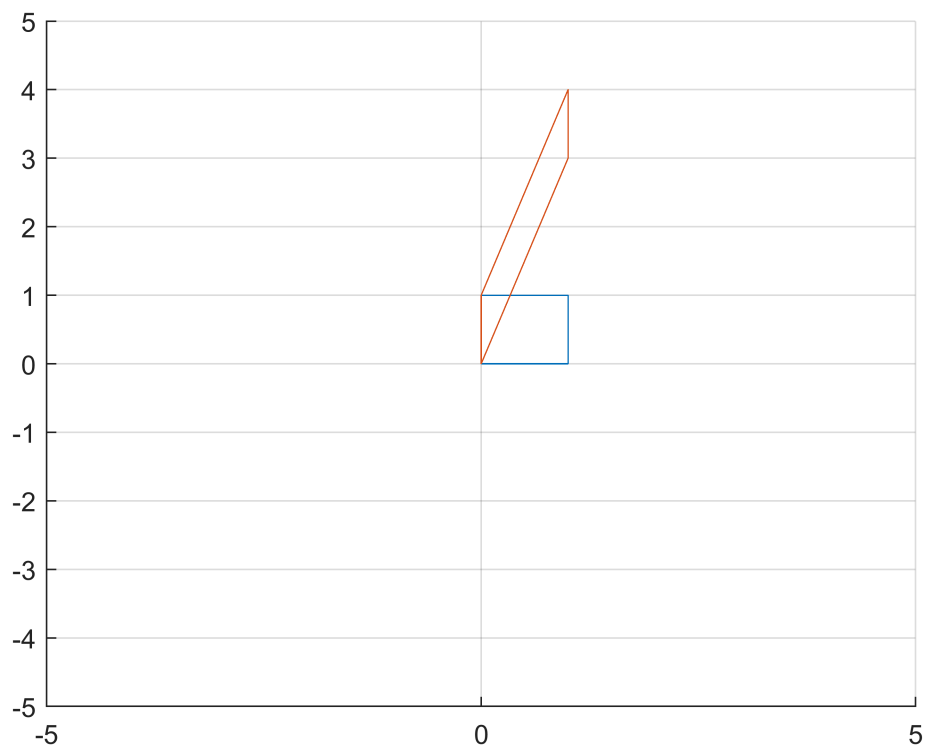
Warning: MATLAB has disabled some advanced graphics rendering features by switching to software OpenGL. For more information, [click here](#).

```
grid on
```

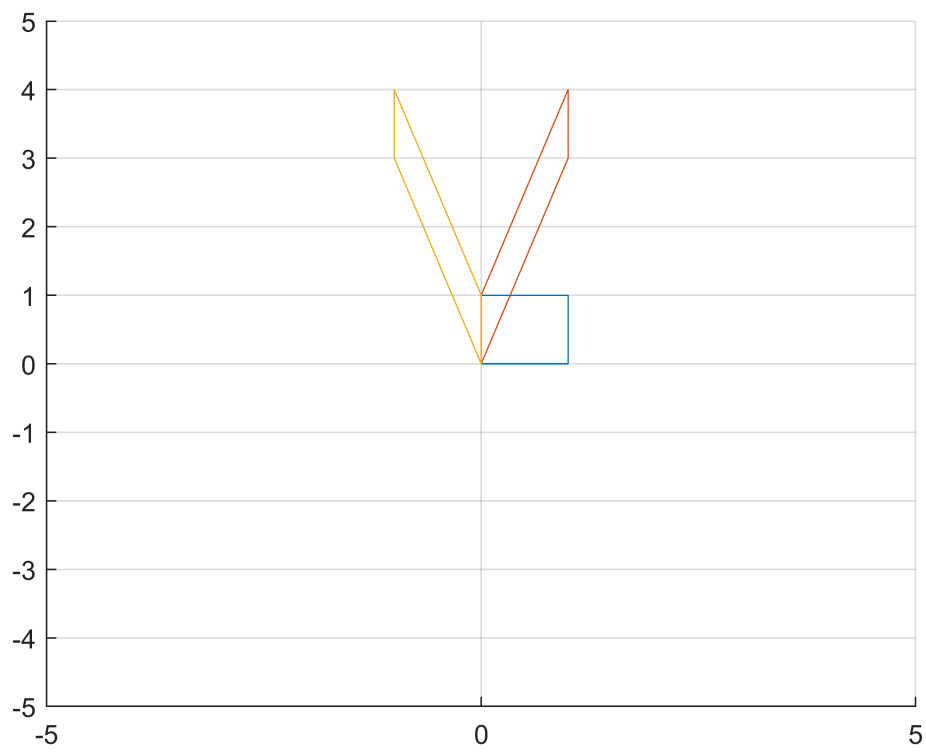
```
E=transf(A,E);
```



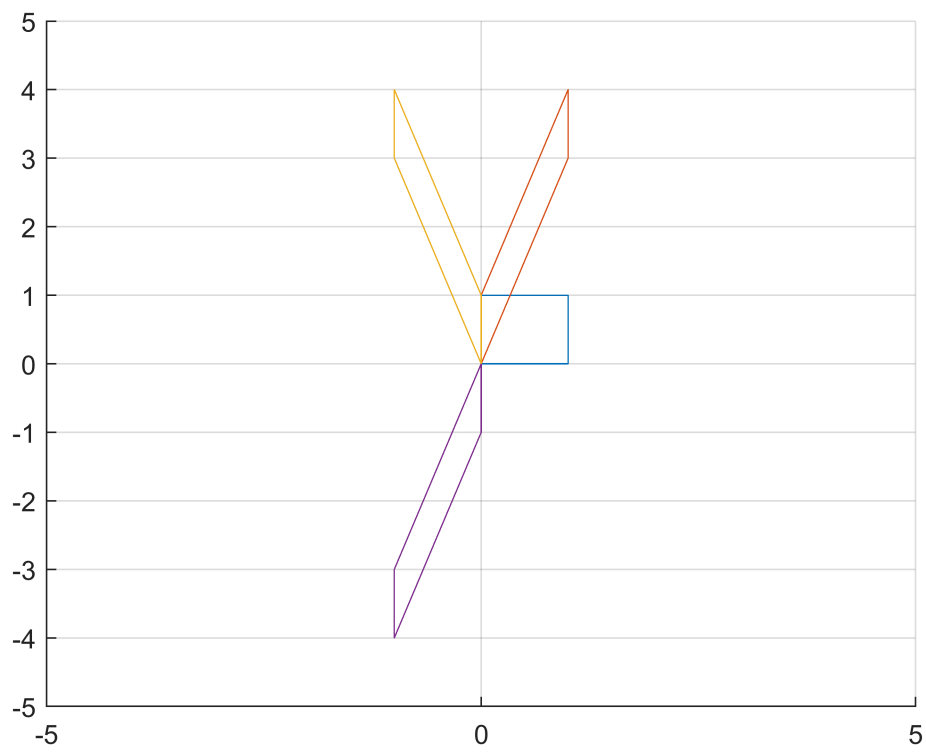
```
A = VS;  
E=transf(A,E);
```



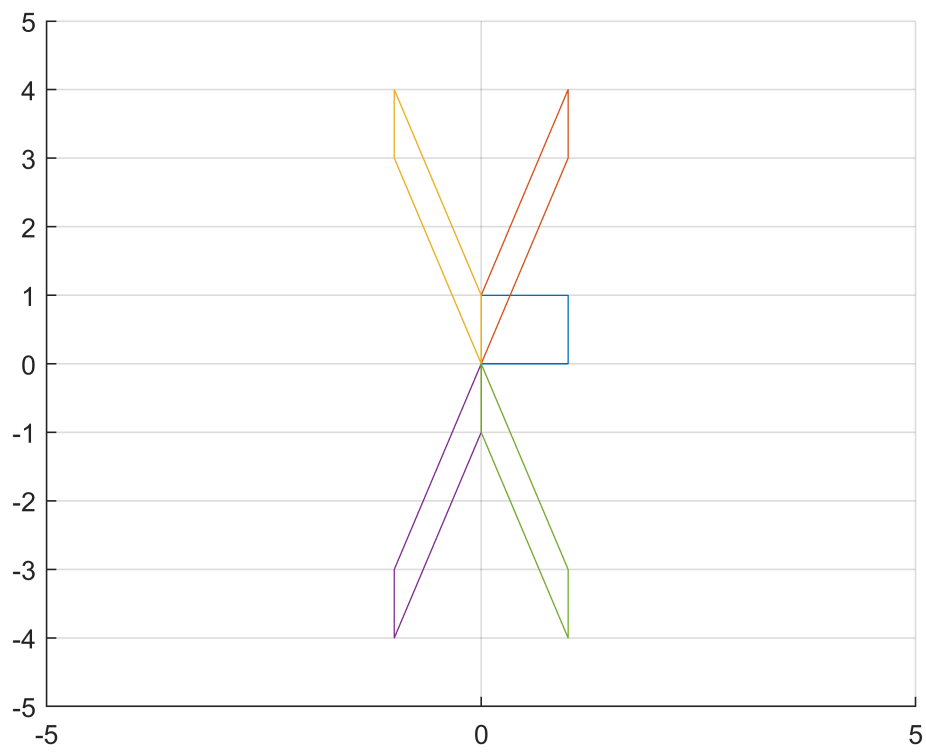
```
A = RY;  
E=transf(A,E);
```



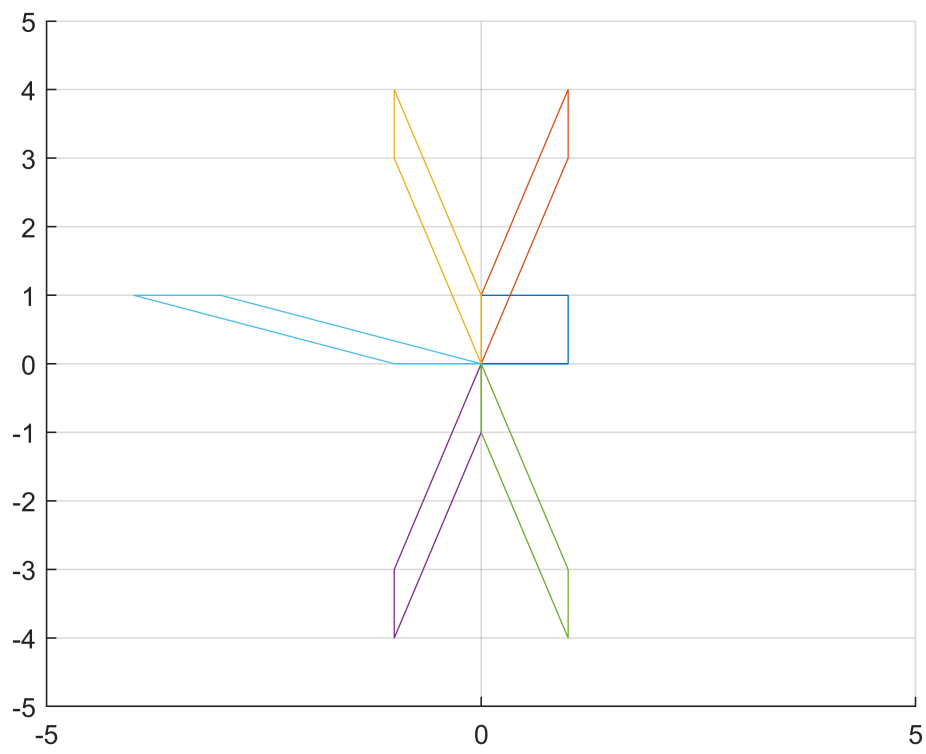
```
A = RX;  
E=transf(A,E);
```



```
A = RY;  
E=transf(A,E);
```

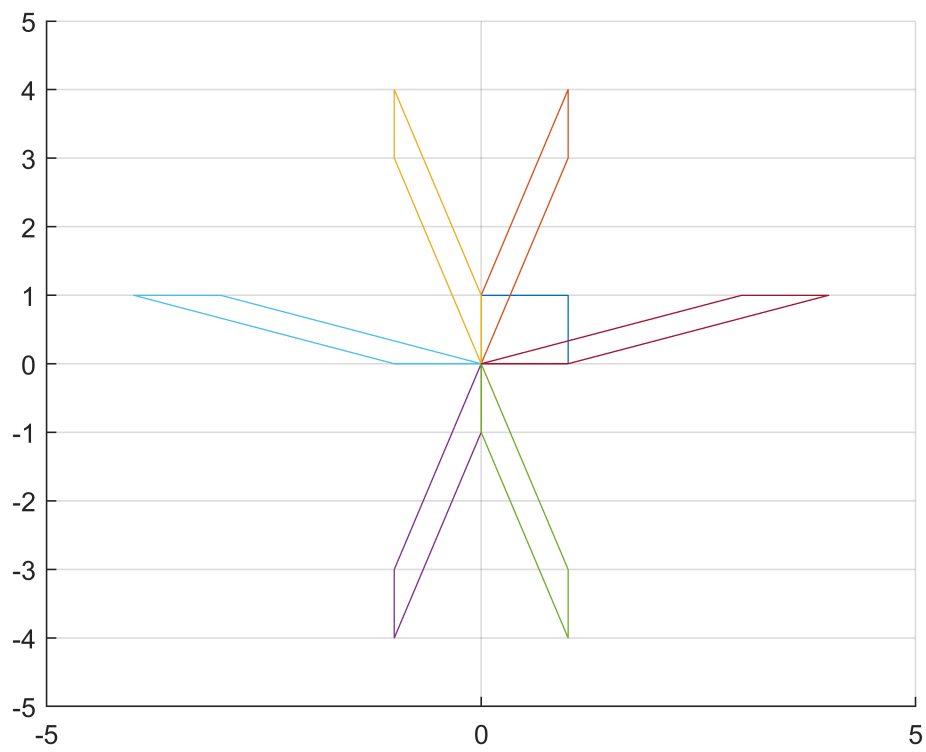


```
A = RS;  
E=transf(A,E);
```

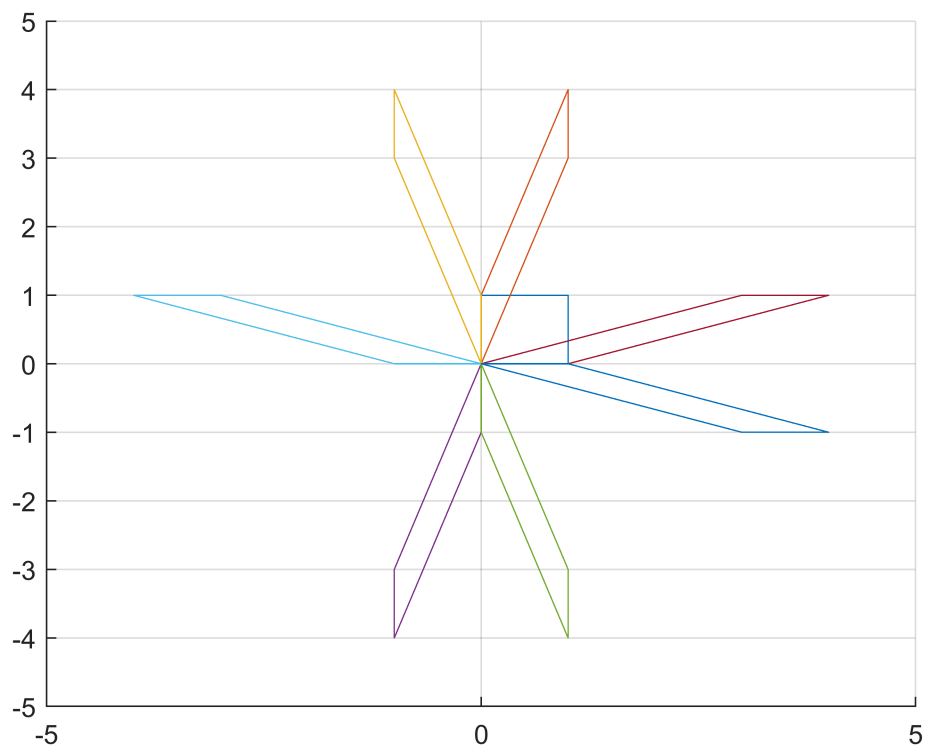


```
A = RY;  
E=transf(A,E);
```

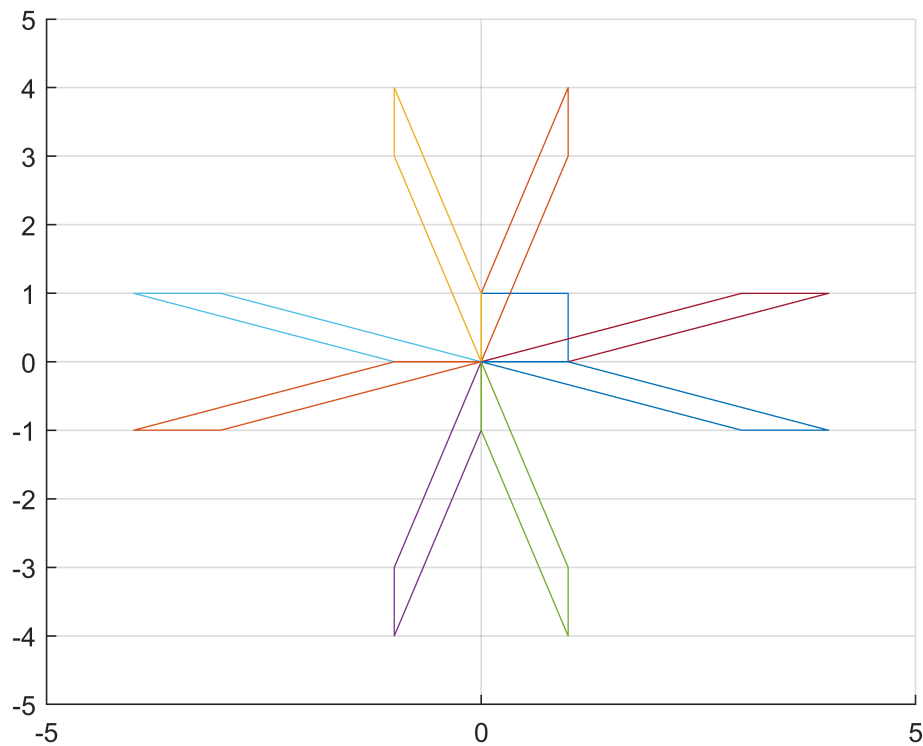




```
A = RX;  
E=transf(A,E);
```



```
A = RY;  
E=transf(A,E)
```



```
E = 2x5
    0    -3    -4    -1     0
    0    -1    -1     0     0
```

## Exercise #6

type `closetozeroroundoff`

```
function B=closetozeroroundoff(A,p)
A(abs(A)<10^-p)=0;
B=A;
end
```

type `cofactor`

```
function C=cofactor(a)
format
format compact
[~,n]=size(a);
%Create the empty variable 'C'
C = zeros(n,n);
%Populating C with the correct values
for i=1:n
    temp1 = a;
    temp1(i,:) = [];
    for j=1:n
        temp2 = temp1;
        temp2(:,j) = [];
        C(i,j) = ((-1)^(i+j))*(det(temp2));
    end
end
end
%Displaying the resulting matrix C
disp('the matrix of cofactors of a is')
```

```
C
end
```

### type **determine**

```
function D=determine(a,C)
D=[];
n=size(a,1);
if (rank(a) < n)
    disp('the determinant of the matrix a is')
    D = 0
    return
end
E = zeros(n,2);
for i=1:n
    currSum = 0;
    for j=1:n
        currSum = currSum + (a(i,j).*C(i,j));
    end
    E(i,1) = currSum;
end

for j=1:n
    currSum = 0;
    for i=1:n
        currSum = currSum + (a(i,j).*C(i,j));
    end
    E(j,2) = currSum;
end

d=det(a);

%Checking to make sure E has the right values
if closetozeroroundoff(abs(E-(d*ones(n,2))),7) == zeros(n,2)
    disp('the determinant of the matrix a is')
    D = E(1,1)
else
    disp('Something went wrong!')
end
end
```

### type **inverse**

```

function B=inverse(a,C,D)
B=[];
if D == 0
    disp('a is not invertible!')
    return
else
    disp('a is invertible!')
    B = (1/D) .* transpose(C)
end

F = inv(a)

if closetozeroroundoff(abs(F-B),7) == zeros(size(a))
    disp('the inverse is calculated correctly and it is matrix')
    B
else
    disp('Something went wrong!')
end
end

```

```

%(a)
a=diag([1,2,3,4,5])

```

```

a = 5x5
    1     0     0     0     0
    0     2     0     0     0
    0     0     3     0     0
    0     0     0     4     0
    0     0     0     0     5

```

```

C=cofactor(a);

```

```

the matrix of cofactors of a is
C = 5x5
   120     0     0     0     0
     0    60     0     0     0
     0     0    40     0     0
     0     0     0    30     0
     0     0     0     0    24

```

```

D=determine(a,C);

```

```

the determinant of the matrix a is
D = 120

```

```

B=inverse(a,C,D);

```

```

a is invertible!
B = 5x5
   1.0000     0     0     0     0
     0    0.5000     0     0     0
     0     0    0.3333     0     0
     0     0     0    0.2500     0
     0     0     0     0    0.2000

F = 5x5
   1.0000     0     0     0     0
     0    0.5000     0     0     0
     0     0    0.3333     0     0
     0     0     0    0.2500     0
     0     0     0     0    0.2000

the inverse is calculated correctly and it is matrix

```

```
B = 5×5
    1.0000    0    0    0    0
    0    0.5000    0    0    0
    0    0    0.3333    0    0
    0    0    0    0.2500    0
    0    0    0    0    0.2000
```

```
%(b)
a=ones(4)
```

```
a = 4×4
    1    1    1    1
    1    1    1    1
    1    1    1    1
    1    1    1    1
```

```
C=cofactor(a);
```

the matrix of cofactors of a is

```
C = 4×4
    0    0    0    0
    0    0    0    0
    0    0    0    0
    0    0    0    0
```

```
D=determine(a,C);
```

the determinant of the matrix a is  
D = 0

```
B=inverse(a,C,D); %(c)
```

a is not invertible!

```
a=magic(3)
```

```
a = 3×3
    8    1    6
    3    5    7
    4    9    2
```

```
C=cofactor(a);
```

the matrix of cofactors of a is

```
C = 3×3
   -53    22     7
    52    -8   -68
   -23   -38    37
```

```
D=determine(a,C);
```

the determinant of the matrix a is  
D = -360

```
B=inverse(a,C,D);
```

a is invertible!

```
B = 3×3
    0.1472   -0.1444    0.0639
   -0.0611    0.0222    0.1056
   -0.0194    0.1889   -0.1028
F = 3×3
```

```

    0.1472    -0.1444     0.0639
   -0.0611     0.0222     0.1056
   -0.0194     0.1889    -0.1028
the inverse is calculated correctly and it is matrix
B = 3x3
    0.1472    -0.1444     0.0639
   -0.0611     0.0222     0.1056
   -0.0194     0.1889    -0.1028

```

```

%(d)
a=magic(4)

```

```

a = 4x4
    16     2     3    13
     5    11    10     8
     9     7     6    12
     4    14    15     1

```

```

C=cofactor(a);

```

```

the matrix of cofactors of a is
C = 4x4
   -0.1360   -0.4080    0.4080    0.1360
   -0.4080   -1.2240    1.2240    0.4080
    0.4080    1.2240   -1.2240   -0.4080
    0.1360    0.4080   -0.4080   -0.1360

```

```

D=determine(a,C);

```

```

the determinant of the matrix a is
D = 0

```

```

B=inverse(a,C,D); %(e)

```

```

a is not invertible!

```

```

a=hilb(4)

```

```

a = 4x4
    1.0000    0.5000    0.3333    0.2500
    0.5000    0.3333    0.2500    0.2000
    0.3333    0.2500    0.2000    0.1667
    0.2500    0.2000    0.1667    0.1429

```

```

C=cofactor(a);

```

```

the matrix of cofactors of a is
C = 4x4
    0.0000   -0.0000    0.0000   -0.0000
   -0.0000    0.0002   -0.0004    0.0003
    0.0000   -0.0004    0.0011   -0.0007
   -0.0000    0.0003   -0.0007    0.0005

```

```

D=determine(a,C);

```

```

the determinant of the matrix a is
D = 1.6534e-07

```

```

B=inverse(a,C,D);

```

```

a is invertible!
B = 4x4

```

0.0160	-0.1200	0.2400	-0.1400
-0.1200	1.2000	-2.7000	1.6800
0.2400	-2.7000	6.4800	-4.2000
-0.1400	1.6800	-4.2000	2.8000

F = 4x4

0.0160	-0.1200	0.2400	-0.1400
-0.1200	1.2000	-2.7000	1.6800
0.2400	-2.7000	6.4800	-4.2000
-0.1400	1.6800	-4.2000	2.8000

the inverse is calculated correctly and it is matrix

B = 4x4

0.0160	-0.1200	0.2400	-0.1400
-0.1200	1.2000	-2.7000	1.6800
0.2400	-2.7000	6.4800	-4.2000
-0.1400	1.6800	-4.2000	2.8000

## Exercise #7

type **production**

```
function x = production(C,d)
n=size(C,2);
x=[];

%check if inputs are valid
%check if C has all nonnegative entries
if all(C >= 0)
    %valid
else
    disp('Consumption matrix contains negative values')
    return
end
%check if d has all nonnegative entries
if all(d >= 0)
    %valid
else
    disp('Final demand vector d contains negative values')
    return
end
%check if C's column sums are all less than 1
if all(sum(C) < 1)
    %valid
else
    disp('Each column sum of C should be less than 1')
    return
end

%should be valid inputs
I = eye(n);

%Aug = [A d]
x = (I-C)\d;
if all(x > 0)
    disp('the unique production vector is')
    x
else
    disp('check the code!')
end

x0=d;
x1=x0;
k=0;
%closetozeroorroundoff(x-x1, 1)
```



```

while all(closetozeroroundoff(x-x1, 1) ~= 0)
    k = k + 1;
    I = I + C^k;
    x1=inv(I)\d;
end

disp('the production vector calculated by recurrence relation is')
x1
fprintf('the number of iteration to match the output x is %i\n',k)

end

```

%(a)

```
C = [.5 .4 .2; .2 .3 .1; .1 .1 .3]
```

```

C = 3×3
    0.5000    0.4000    0.2000
    0.2000    0.3000    0.1000
    0.1000    0.1000    0.3000

```

```
d = [50; 30; 20]
```

```

d = 3×1
    50
    30
    20

```

```
x = production(C, d);
```

```

the unique production vector is
x = 3×1
    225.9259
    118.5185
    77.7778
the production vector calculated by recurrence relation is
x1 = 3×1
    225.6447
    118.3779
    77.6870
the number of iteration to match the output x is 24

```

%(b)

```
C = importdata('consumption.csv')
```

```

C = 7×7
    0.1588    0.0064    0.0025    0.0304    0.0014    0.0083    0.1504
    0.0057    0.2645    0.0436    0.0099    0.0083    0.0201    0.3413
    0.0264    0.1506    0.3557    0.0139    0.0142    0.0070    0.0236
    0.3299    0.0565    0.0495    0.3636    0.0204    0.0483    0.0649
    0.0089    0.0081    0.0333    0.0295    0.3412    0.0237    0.0020
    0.1190    0.0901    0.0996    0.1260    0.1722    0.2368    0.3369
    0.0063    0.0126    0.0196    0.0098    0.0064    0.0132    0.0012

```

```
d = importdata('demand.csv')
```

```

d = 7×1
    74000
    56000
    10500
    25000
    17500

```

```
196000
5000
```

```
x = production(C,d);
```

the unique production vector is

```
x = 7×1
    0.9942
    0.9770
    0.5122
    1.3149
    0.4948
    3.2951
    0.1383
```

the production vector calculated by recurrence relation is

```
x1 = 7×1
    0.9942
    0.9770
    0.5122
    1.3149
    0.4948
    3.2951
    0.1383
```

the number of iteration to match the output x is 18

% Questions

% 1. (3, 4) of C represents inputs consumed per unit of output by basic  
% nonmetal products and agriculture that were purchased from basic metal  
% products and mining

% 2. Sector 7 needs to produce 13,830 units

%(c)

```
C = importdata('consumption.csv')
```

```
C = 7×7
    0.1588    0.0064    0.0025    0.0304    0.0014    0.0083    0.1504
    0.0057    0.2645    0.0436    0.0099    0.0083    0.0201    0.3413
    0.0264    0.1506    0.3557    0.0139    0.0142    0.0070    0.0236
    0.3299    0.0565    0.0495    0.3636    0.0204    0.0483    0.0649
    0.0089    0.0081    0.0333    0.0295    0.3412    0.0237    0.0020
    0.1190    0.0901    0.0996    0.1260    0.1722    0.2368    0.3369
    0.0063    0.0126    0.0196    0.0098    0.0064    0.0132    0.0012
```

```
d = importdata('demand_1.csv')
```

```
d = 7×1
    99640
    75548
    14444
    33501
    23527
    263985
    6526
```

```
x = production(C,d);
```

the unique production vector is

```
x = 7×1
    1.3383
    1.3168
    0.6946
    1.7680
```

```

0.6659
4.4372
0.1843
the production vector calculated by recurrence relation is
x1 = 7×1
1.3383
1.3168
0.6946
1.7680
0.6658
4.4372
0.1843
the number of iteration to match the output x is 19

```

```

%(d)
C = importdata('consumption_1.csv')

```

```

C = 7×7
1.1588    0.0064    0.0025    0.0304    0.0014    0.0083    0.1504
0.0057    0.2645    0.0436    0.0099    0.0083    0.0201    0.3413
0.0264    0.1506    0.3557    0.0139    0.0142    0.0070    0.0236
0.3299    0.0565    0.0495    0.3636    0.0204    0.0483    0.0649
0.0089    0.0081    0.0333    0.0295    0.3412    0.0237    0.0020
0.1190    0.0901    0.0996    0.1260    0.1722    0.2368    0.3369
0.0063    0.0126    0.0196    0.0098    0.0064    0.0132    0.0012

```

```

d = importdata('demand_1.csv')

```

```

d = 7×1
99640
75548
14444
33501
23527
263985
6526

```

```

x = production(C,d);

```

Each column sum of C should be less than 1

```

%(e)
C = importdata('consumption_1.csv')

```

```

C = 7×7
1.1588    0.0064    0.0025    0.0304    0.0014    0.0083    0.1504
0.0057    0.2645    0.0436    0.0099    0.0083    0.0201    0.3413
0.0264    0.1506    0.3557    0.0139    0.0142    0.0070    0.0236
0.3299    0.0565    0.0495    0.3636    0.0204    0.0483    0.0649
0.0089    0.0081    0.0333    0.0295    0.3412    0.0237    0.0020
0.1190    0.0901    0.0996    0.1260    0.1722    0.2368    0.3369
0.0063    0.0126    0.0196    0.0098    0.0064    0.0132    0.0012

```

```

d = importdata('demand_2.csv')

```

```

d = 7×1
99640
75548
14444
33501
23527
263985

```

```
x = production(C,d);
```

Final demand vector d contains negative values