Problem B

Title: Asteroid Ocean Impact

Team NO. 155

November 6, 2021

Abstract

In this article, in order to get the minimum mass of asteroid hitting on the sea away from the coast 1000 km, which would cause substantial damage to a coastal city, we tackled this problem in three parts:

(1) Establish seabed topographic map and the tsunami propagation model in offshore area to obtain the variation of tsunami height.

To obtain the variation of tsunami height in the sea, we established a simulated seabed topographic map which consists of the offshore area and the deep sea area, by comparing real ocean data. Then, calculate the tsunami height in the offshore area through the tsunami propagation model. The calculation results show that tsunami height at the interface between two area is 0.75 m.

(2) Figure out the needed minimum energy of the asteroid acting on the sea.

Based on Theory of water cavity explosion, and ignored the seismic effect of asteroids on the seafloor (with quantitative analysis), we obtain the energy needed for the asteroid to act on seawater (9.1814×10^{12} kJ).

And then, a relationship with the energy, radius, velocity, and density of the asteroid is established.

(3) Get the minimum mass of the asteroid through finding its constraints.

We assume that, under the condition of the same mass, the asteroid without fragmentation has the maximum effective impact energy, which limits the maximum velocity of the asteroid and determine the density. By modeling the mass loss of asteroids in the atmosphere, we found that it was only a small fraction of the mass of the asteroid. Thus, the mass of the asteroid can be seen as constant. Then, it's easy to obtain the minimum mass of the asteroid according to the kinetic energy theorem.

After completing the above three steps, the result shows that the minimum mass of the asteroid is 1.8574×10^{10} kg.

Keywords: Asteroid, Seabed topography, Deep-water wave, Theory of water cavity explosion, Tsunami, Minimum mass, Error analysis

Contents

1. Introduction

Asteroid hitting the earth will bring great threat to human existence, and impacting on the sea is one of the threats while it causes tsunami disaster. In this paper, we will study the minimum asteroid mass that can pose a substantial damage. We break down this problem into three parts. In the first part, we establish the height variation of tsunami propagation in shallow water by coarse-grilling method fitting by comparing real ocean data. In the second part, we determine the constraint relationship among parameters before asteroid impact according to the theory of water cavity explosion and the height of target tsunami. In the 3rd part, we determine the density and velocity range of the asteroid by considering the fragmentation conditions of the asteroid, and obtain the final minimum asteroid mass by combining the results of the second part.

In the process of building the physical model, we focus on the quantitative analysis and discussion of the errors caused by the mass loss and the impact of asteroids on the undersea earthquakes. The results show that the mass loss and earthquake influence can be ignored.

2. Assumptions and Symbols

2.1 Basic Assumptions for Our Models

- We set the average height of the tsunami near the coastal area as H, and defined the tsunami with H≥30m (according to The Watanabe

Weaven tsunami level) as one that would cause substantial disasters to the city.

- We assume that the asteroid is a rigid sphere of uniformly distributed mass, and that the asteroid collided vertically with the sea
- We consider the collision of an asteroid with the ocean as a cavity explosion
- As the tsunami traveled, we ignored the irregular topography of the ocean floor.
- Assuming that most of the energy from the asteroid impact is in the ocean and very little is on the ocean floor, meaning most of the energy from the asteroid is transferred to the tsunami.

2.2 Notations

Symbol	Definition
H	The average height of the tsunami when it reaches the coast
h_w	Height of tsunami in deep water
Y	Y is the kinetic energy of an asteroid in gigatons (Gt) of TNT equivalent
E_y	the energy of the tsunami
D_c	The depth of the simulated water chamber 5 the radius of the water chamber

R_c	radius of the water chamber
q, α	The parameters determined by the asteroid
ρ_w	the density of water
ρ_0	the density of air
ρ	the density of the asteroid
S	the strength of the material
P	the pressure
v	the speed of the asteroid
Δt	The time asteroid passes through the atmosphere
Δm	the mass loss in the atmosphere
k	the influence parameter
t	the propagation time passes through the surface
M_L	Reese shock series
M_w	the seismic moment
M_t	the tsunami moment
M_s	face wave magnitude

3. Physical Analysis of Model

3.1 The Generation and Spread of Tsunamis

This article uses the Ocean Depth Map (Figure1) on the west coast of Florida as a reference,[1] Figure 2 shows the coarse-graining model of seafloor topography in the selected area.

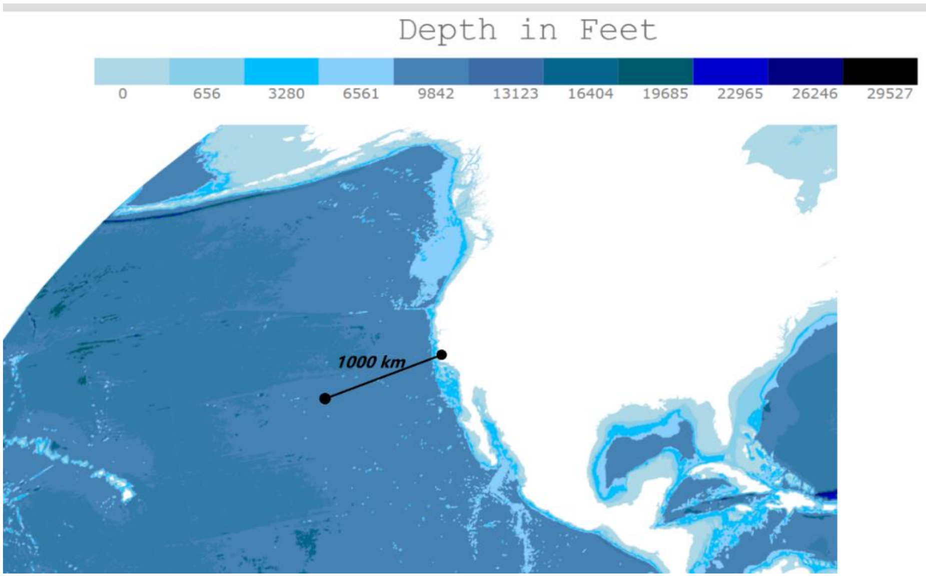

Figure 1 Ocean Depth Map of America

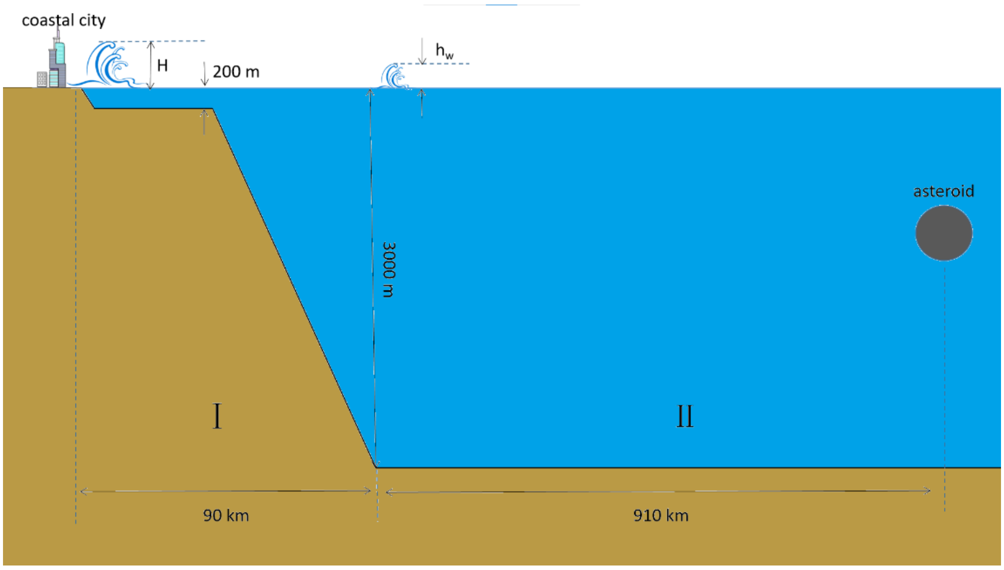

Figure 2 the coarse-graining model of seafloor topography

We tackle the transmission into two stages as the Figure 2, Which is called area Ⅰ (90km) and area II (910km).

3.1.1 Tsunami Propagation In Area Ⅰ

Waves caused by ocean impacts may be the most serious problem produced by impacting asteroids short of the massive killers such as the Cretaceous-Tertiary impactor. Just as on land, most of the kinetic energy of a large asteroid that impacts into the ocean goes into the formation of a crater, but the crater is not stable. The filling in of the crater and its subsequent rebound produces a series of waves that propagate radially away from the impact. Impacts into deep water produce surface waves that do not dampen significantly until they run into shallows where they steepen into breakers and increase in height to form tsunami. [2] Boussinesq equation model and FUNWAVE models are generally used for numerical simulation of tsunami propagation in area Ⅰ for specific seabed conditions. In this paper, we ignore the details of complex seabed topography in Ⅰ area to avoid complex simulation calculation process. Through fuzzy calculation and reference to relevant literature, H is about 40 times h_w. [3]

$$h_w = \frac{H}{40} \tag{1}$$

3.1.2 Tsunami Generation And Propagation in area Ⅱ

The formation of tsunami after asteroid impact and the propagation process of tsunami in deep water area is a complex water movement process caused by asteroid impact, but we can equate this process to the problem of water chamber explosion, thus using the following formula:

$$D_c = qR_c^2 \tag{2}$$

$$E_y \approx \frac{\rho_w g \pi}{3}(R_c D_c)^2 \tag{3}$$

Then we figure out

$$R_c D_c = 2.9515$$

Experiments with underwater nuclear explosives show that the full height of a deep-water wave at a distance r from the underwater detonation of energy Y is given by [4]

$$h_w = 22.7\left(\frac{Y}{10}\right)^{0.54}\frac{1000}{r} \tag{4}$$

In fact, formula 4 is obtained by the following equation system [5]:

$$\begin{cases} R_c D_c = 2.9515 \\ \mathrm{R_c} = \mathrm{R_c(R, P, V)} = \mathrm{R}\left(\frac{2\varepsilon \mathrm{v}^2}{\mathrm{gR}}\right)\left(\frac{\rho}{\rho_\mathrm{w}}\right)^{\delta}\left(\frac{1}{\mathrm{qR}^{\alpha-1}}\right)^{2\delta} \\ D_c = D_c(R, \rho, v) = \sqrt{\frac{2\rho\varepsilon R^3 v^2}{\rho_w g R_c^2}} \end{cases}$$

In order to minimize the mass of the asteroid, we take the H=30 m, $h_w = \frac{H}{40}$=0.75 m. Then we can calculate

$$Y = 6.3656 \times 10^{10}\ KJ$$

3.2 Height of Initial Breakup

Common asteroids are mainly divided into dustballs, stony meteorites, nickel-iron meteoroids, and their corresponding parameters are shown in the following table

	S（dynes/cm^3）	ρ (g/cm^3)
dustballs	1×10^7	0.5
stony meteorites	1×10^8~5×10^8	3
nickel-iron meteoroids	2×10^9	8

We know meteoroid passing through the atmosphere begins to fragment when the aerodynamic pressure, P, exceeds its material strength. The corresponding formula is

$$P_{ram} = \rho_0 v^2 \tag{5}$$

$$\rho = \rho_0 e^{-h/H_0} \tag{6}$$

where ρ_0=0.001293 g/cm^3 is the atmospheric density at h=0 and the scale height H ≈ 8km

Fragmentation begins when the ram pressure, P_{ram}, at the stagnation point in front of the meteoroid reaches the material strength S of the meteoroid. It means that $S \leq P_{max}$.

From Eqs (5) and (6), we find that the critical height at which the ram pressure equals the material strength occurs at:

$$h_{break} = \ln\left(\frac{\rho_0 v^2}{S}\right) H_0 = \ln\left(\frac{P_{max}}{s}\right) H_0 \tag{7}$$

We know that the possible value of v_0, the value of V at the top of the atmosphere is v_0= 11.2km /s, which corresponds to a parabolic encounter with the Earth. For V = v_0, this gives i > max = 1.62 ×10^9 dynes/cm^2. At this velocity, the initial breakup height is h_{break}=41 km for dustballs and comets and 22 to 9.4 km for stony meteorites, but at this velocity, nickel-iron meteoroids can reach sea level without breakup. That's why we select iron at last. And according to the Kinetic energy formula:

$$E = \frac{1}{2}mv^2 \tag{8}$$

We can find that we get the minimum mass of the asteroid when we get the maximum value of V, the value of V is limited by fragmentation (when V is too large, the p_{ram} will increase sharply, and when the p_{ram} exceeds S, the asteroid will breakup).

We substitute P_{ram}=S and h=0 into formula (5) to obtain the formula:

$$v_{max} = \left(\frac{S}{\rho_0}\right)^{\frac{1}{2}} \tag{9}$$

This gives V_{max}=2.8 to 6.2 km/s for stones and 12.4 km/s for irons. Irons with initial impact velocities in the range V= 11.2 to 12.4 km/s can never fracture.

The height of tsunami in deep water can be expressed as[4]

$$h_w = 15.6m\left[\left(\frac{R}{203}\right)^3\left(\frac{v}{20}\right)^2\left(\frac{\rho\mu}{3}\right)\right]^{0.54} \times \left(\frac{1000}{r}\right) \tag{10}$$

At this time, we have determined, the value range of V, h_w and R. According to formula (10), we can get the value range of R in line with the meaning of the question, so as to determine the minimum mass of the asteroid. A speed-radius image drawn according to formula (11) at a density of 8g/cm^3 is showed in Figure 3.

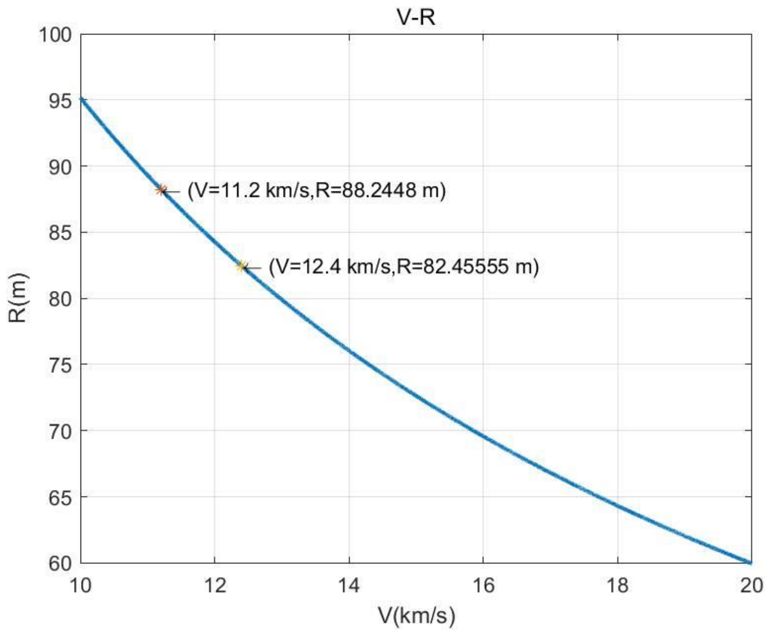

Figure 3 R-V relation

It can also be seen from the Figure 3 that the larger V is, the smaller R is, which is consistent with our previous hypothesis. Therefore, we take the R value when $V=V_{\max}=12.4$ as the final value $R=82.45555$ m. According to $\mathrm{M}=\frac{4}{3}\pi r^3\rho$, the minimum mass of the asteroid before hitting the sea surface is

$$M=1.8786\times10^{10}\ \text{kg}$$

3.3 Atmospheric Loss

Through the mass loss equation [6]:

$$\frac{\mathrm{dm}}{\mathrm{dt}}=-\frac{1}{2}P_{air}v^3A\sigma=-\frac{1}{2}P_{air}v^3\pi r^2\sigma \tag{11}$$

(σ is a small quantity, so the mass loss is small while the density and radius are large, so Δm corresponds to a very small A, so A can be considered as a constant.)

$$\sigma\in\ (3.5\times10^{-10}, 70\times10^{-8})$$

According to the following figure, we can find that atmospheric loss has little influence on the speed of our model and can be almost ignored. The decrease in the velocity of an impactor as a function of height in the atmosphere.

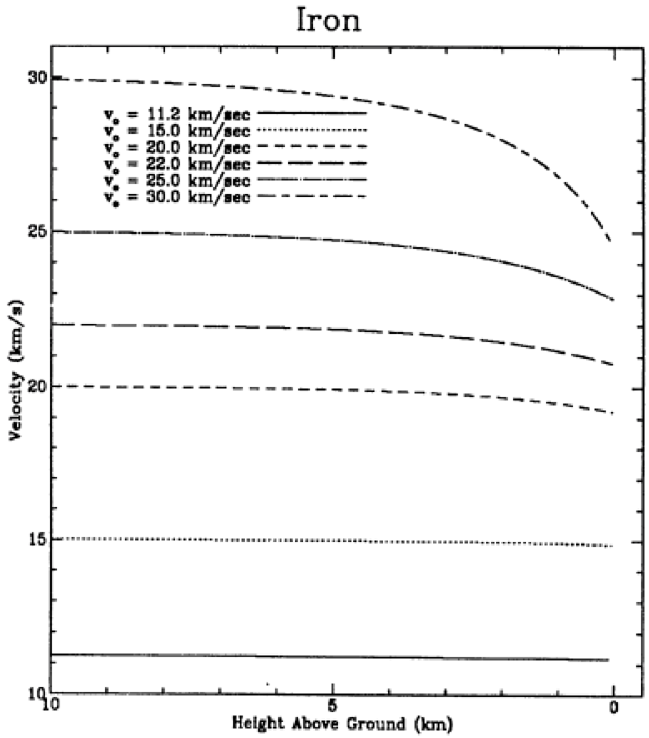

Figure 4.Velocity varies with altitude

Take $v = 12.4km/s$, we can get

$$\Delta t = H/v = 8.065 \text{ s} \tag{12}$$

$$\Delta m = -\frac{1}{2} P_{air} v^3 \pi r^2 \sigma \times \Delta t = -2.1233 \times 10^8 \text{ kg} \tag{13}$$

$$M_0 = 1.8574 \times 10^{10} \text{ kg}$$

Relative mass loss $\frac{\Delta m}{M_0} \times 100\% = 1.14\%$.

Relative mass loss are relatively small and negligible in the calculation process. As the velocity can be seen as constant, the loss of energy is ignored too.

3.4 The Amplification Caused by Seismic Effect

Here we consider the amplification caused by seismic effect when the asteroid reaches the ocean floor. In a tsunami caused by seismic effects in the case of non-asteroid impacts, the tsunami moment M_t equals to the seismic moment M_w [7]. Thus,

$$M_t = \lg h_w + 9.1 \tag{14}$$

When the asteroid directly hits the ground and causes a seismic effect, Reese shock series M_L has a relationship with the energy acting on the ground Y [4] :

$$M_L = 3.9 + 0.7 \lg \frac{Y}{kt} \tag{15}$$

Without a super-massive earthquake (M_L<8), there are Surface wave dispersion:

$$M_s = M_t \text{ [1]}. \tag{16}$$

Empirical relationship between Richter magnitude and face wave magnitude [8]:

$$M_s = 1.13 M_L - 1.08 \tag{17}$$

We can get the relationship between the energy working on the ocean floor *Y'* and the amplification of the waves *H'* from equations (15)(16)(17):

$$H' = (\frac{Y'}{kt})^{0.791} \times 10^{-5.773} \tag{18}$$

Utilizing the study of Hills JG and Goda MP [4], we get the value *k* of iron meteorites: $k = 1.7224 \times 10^{11}$

The propagation velocity of the seismic wave is 5.5~7.0 km/s [9], thus, we get an approximate value t=150 s. Assume that the asteroid exerts most of their energy on sea water, bringing only a fraction of the energy to the seabed. Here we use a relatively large value $Y'=0.1Y$. The wave height increase caused by the seismic effect can be roughly calculated in the substitution formula (18): $H'=0.0016$ m.

This increase is relatively small in the height of the original waves.

4. Results

According to the Watanabe tsunami scale, the minimum height of ocean wave to cause substantial damage to coastal cities is H=30 m. It is then derived from the established tsunami propagation model: h_w=0.75 m. We can get the optimum speed of the asteroid (12.4 km/s), and get the mass of it before the impact. Add the loss of mass when moving in the atmosphere to it, finally we can get the minimum mass of asteroid, which meets the requirement. $\mathrm{M}_0 = 1.8574 \times 10^{10}$ kg. In the later error analysis, through quantitative calculation, we found that the impact of the submarine earthquake caused by the asteroid breaking through the sea water and hitting the seafloor was very small and could be ignored.

5. Strengths and Weaknesses

5.1 Strengths

- In this paper, we suppose that the asteroid hits on the sea within 1,000 km on the West coast of Florida. We have ensured the results with the coarse-grained method while effectively reducing the computational amount required for the tsunami propagation simulation.
- When ignoring parameters impact the process (mass loss in the atmosphere, the seismic effect of asteroids hitting the seabed for example), we have not taken approximations blindly. Instead, we establish the theoretical model and quantitatively analyze the effects of these phenomena, effectively explaining the accuracy and feasibility of this approximation.
- The theoretical basis of the explosive cavity theory of asteroid collision is systematically discussed and analyzed, calculating the parameters of the asteroid needed to achieve the target, which is more convincing.

5.2 Weaknesses

- The angle of the asteroid to the sea surface is generally not exactly 90 degrees, and for convenient calculation, the impact is not considered.
- In this paper, the same mass asteroid has the maximum effective impact kinetic energy without fragmentation, for the case of asteroid fragmentation needs further investigation.

6. Conclusions

In this paper, we firstly confirm a minimum height of ocean wave which cause substantial damage to coastal cities according to the Watanabe tsunami scale. Then we get a equation in relation to R, V and ρ by Deep-water wave theoretical model and the Theory of water cavity explosion. Considering the breaking conditions of asteroid, we can figure out the minimum mass of the asteroid $M_0 = 1.8574 \times 10^{10}$ kg

Furthermore, we have also quantified the mass loss of asteroids as they passed through the atmosphere and the impact of earthquakes from asteroid impacts on the sea floor. We found that the mass loss of asteroids is about 1.114%, and the earthquake's impact on tsunami height is about 6 cm, both of which are negligible.

7. References

[1] https://databayou.com/ocean/depth.html

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[6] Dang Leining, Liu Sen, Bai Zhiyong, et al. Study on the sensitivity of asteroid entry and im pact effect evaluation model [J]. Journal of mechanics, 2021, 53 (1): 278-292

[7] Abu Shengzheng,Chen Hongde.The magnitude of the massive earthquakes and tsunami [J].Earthquake geological translation, 1982 (6): 43-48.

[8] http://blog.sciencenet.cn/blog-575926-686661.html

[9] https://zhidao.baidu.com/question/745939677614355052.html

8. Appendix

Code written in matlab

```
syms y;
hw=30/40;r=910;
eqn = hw==22.7*((y/10)^0.54)*(1000/r);
Y=solve(eqn,y)
Ey=1e9*Y*4.19e3
%%
    h0=30;

    r=910;ro=8;hw=h0/40;RR=[];

    v=10:0.001:20; n=length(v);V=[];

    for i=1:n
        Ri=(203*39^(2/3)*hw^(1/3)*r^(1/3))/(39*v(i)^(2/3)*(ro/3)^(9/50));
        RR=[RR,Ri];
        V=[V,v(i)];
    end
    vv1=11.2;
    y1=(203*39^(2/3)*hw^(1/3)*r^(1/3))/(39*vv1^(2/3)*(ro/3)^(9/50))
    vv2=12.4;
    y2=(203*39^(2/3)*hw^(1/3)*r^(1/3))/(39*vv2^(2/3)*(ro/3)^(9/50))
    figure
    plot(V,RR,'LineWidth',2)
    grid on
    title('V-R')
    xlabel('V(km/s)')
    ylabel('R(m)')
    hold on
    plot(vv1,y1,'-*')
    plot(vv2,y2,'-*')
    hold on
    text(vv1,y1,'\leftarrow (V=11.2 km/s,R=88.2448 m)');
    text(vv2,y2,'\leftarrow (V=12.4 km/s,R=82.45555 m)');
%%
r=y2;
ro=8e3;
M=ro*((4/3)*pi*r^3);
M
%%
    m=M;g=9.8;

    p_air=1.293;v=12.4e+03;delta=1e-09;dt=100/12.4;
```

```
    dm=-0.5*p_air*v^3*pi*r^2*delta*dt                %Mass loss of an asteroid
through the atmosphere
    M0=M+dm
    Beta=-dm/M0                                       %The ratio of mass loss to
total mass
```