

On Scene Reconstruction from Spatial Coherence Measurements

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Abstract: We determine the positions and dimensions of obscurants and apertures from coherence measurements of partially coherent light by leveraging the authors' recent closed-form approximation formula for the coherence of propagated field in the Fresnel regime. © 2018 The Author(s)

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1. Problem Description and Methods

We consider the inverse problem of recovering a generalized source of partially coherent light propagated in the Fresnel regime from coherence measurements collected at an unknown distance. A generalized source is defined as a Gauss-Schell beam modulated by a piecewise constant transmission function t : For N fixed, $-\infty < a_1 < \dots < a_N < \infty$, $t(x) = c_j$, for the transverse coordinate $x \in (a_j, a_{j+1})$, where each c_j is a complex valued constant, $j = 0, \dots, N$. Hence, the coherence of a generalized source as function of y_1 (the intensity coordinate) and y_2 (the coherence coordinate) is

$$G(y_1, y_2) = A \exp\{iy_1 y_2 / R^2\} N^w(y_1) N^\sigma(y_2) t(y_1 + y_2) t^*(y_1 - y_2), \quad (1)$$

where $N^\beta(x) = \exp\{-x^2/2\beta^2\}$ denotes the Gaussian of variance β , A the amplitude, w the width of the intensity profile, σ the coherence width, R the radius of curvature of an acquired quadrature phase, and d the propagation distance. Among others, such generalized sources model the field's interaction with obscurants and apertures.

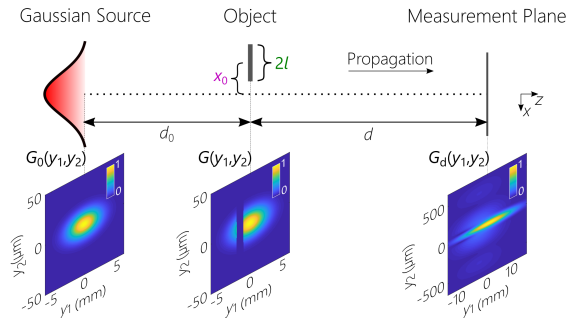


Fig. 1. A single object scene with $x_0 = -1.5$ mm, $l = 0.5$ mm, $d_0 = 10$ cm, and $d = 100$ cm.

Figure 1 above exemplifies an obscurant of half-width l centered at x_0 situated at distance d from the measurement plane. The diagrams at the bottom of the figure show the coherence at different transverse planes. For this example, the modulating transmission function has $N = 2$, $a_1 = x_0 - l$ and $a_2 = x_0 + l$.

The problem of concern here is to determine the generalized source (i.e., location, transmission function) from coherence measurements.

Prior work on the inverse problem has mostly relied on intensity-only measurements (See for example [1] and references therein). The few works using Fourier-based inversion and back-propagation of the field's coherence require measurement of the full coherence function [2–5], thus incur large sampling and computation complexity. Moreover, these methods typically require additional information on the distance, which is difficult to obtain in general.

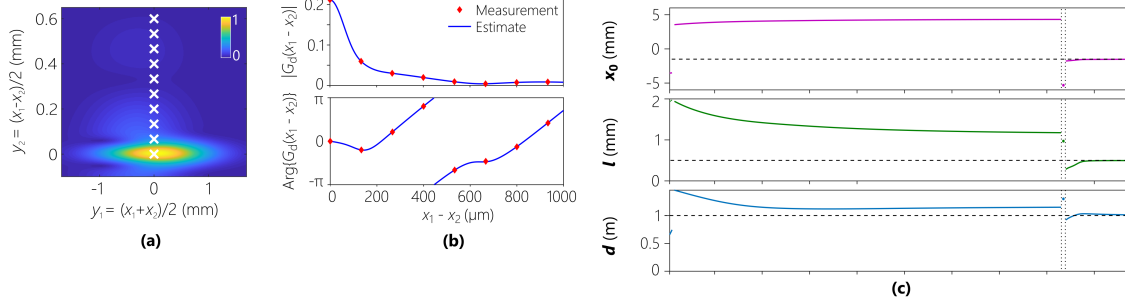


Fig. 2. Minimum residual algorithm recovering three object parameters: x_0 , l , and d .

In the recent work [6], we derived a closed-form approximation formula for the coherence of the field propagated in the Fresnel regime. This formula is in terms of some conjugated Hilbert transforms and explicits the breakpoints in the transmission function, distance to the measurement plane, and statistic parameters.

To solve the inverse source problem, we devise a minimum residual approach based on this explicit closed-form formula. The coherence measurements yield an overdetermined problem. We leverage this extra dimensionality in the data to devise a global method by applying some local minimization schemes to a family of residuals that share a unique minimum. Given a vector of breakpoints $\mathbf{a} = (a_1, \dots, a_N)$ and distance d , we consider the residual, $f(y_1, y_2; \mathbf{a}, d) = \bar{G}_d(y_1, y_2; \mathbf{a}, d) - G_d(y_1, y_2)$, between the measured coherence G_d and the prediction \bar{G}_d given by the closed-formed formula in [6]. Given M sample points (y_1^k, y_2^k) , $k = 1, \dots, M$, we minimize the objective function

$$F(\mathbf{a}, d) = \frac{1}{M} \sum_{k=1}^M |f(y_1^k, y_2^k; \mathbf{a}, d)|^2, \quad (2)$$

with respect to the parameters \mathbf{a}, d using gradient-descent. When the partial derivatives fall below prescribed thresholds, meaning that a local minimum has been found, the algorithm performs an additional check for a global minimum. A characteristic of the global minimizer is that the actual and estimated coherence functions closely match at all sample points, and thus the residual is small at each point.

2. Results

A Gauss-Schell source is propagating a distance $d_0 = 10$ cm in free space, where it is blocked by a single object of width $2l = 1$ mm centered along the transverse axis at $x_0 = -1.5$ mm, and the detector is located at a distance $d = 1$ m from the object plane as shown in Fig. 1. From coherence measurements at the detection plane collected along the y_2 -axis shown in Fig. 2(a), we estimate the parameters of the scene. The results of the minimum residual algorithm are shown in Fig. 2. The estimated values are $x_0 = -1.521$ mm, $l = 496.0$ μ m, and $d = 1.013$ m, i.e., the maximum parameter error is less than 1.5% (and could be reduced by using smaller gradient thresholds).

We also demonstrated the ability of the algorithm to handle more complicated scenes with multiple objects in different planes transverse to the direction of propagation, as well as reconstruction from experimental data. These results are not included here due to space constraints.

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