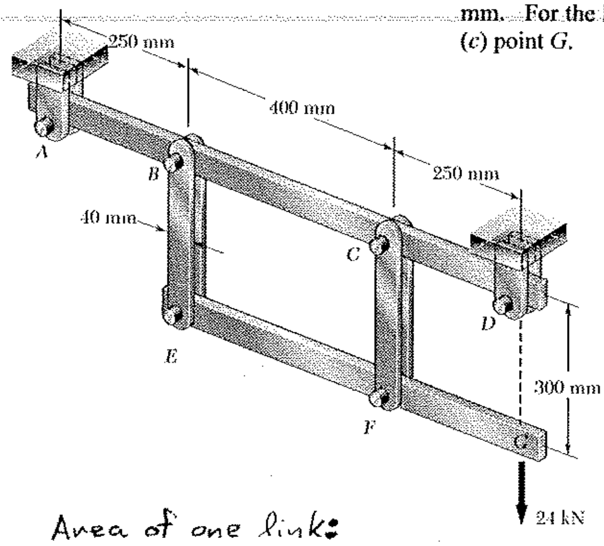
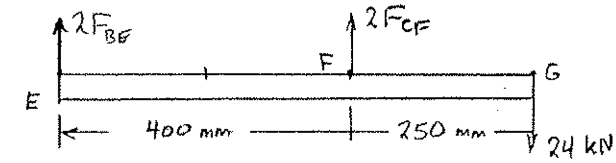
Problem 2.25

2.25 Each of the four vertical links connecting the two horizontal members is made of aluminum ($E = 70$ GPa) and has a uniform rectangular cross section of 10×40 mm. For the loading shown, determine the deflection of (*a*) point *E*, (*b*) point *F*, (*c*) point *G*.



Statics. Free body EFG.



$+\circlearrowleft \Sigma M_F = 0:$

$-(400)(2F_{BE}) - (250)(24) = 0$

$F_{BE} = -7.5 \text{ kN} = -7.5 \times 10^3 \text{ N}$

$+\circlearrowleft \Sigma M_E = 0:$

$(400)(2F_{CF}) - (650)(24) = 0$

$F_{CF} = 19.5 \text{ kN} = 19.5 \times 10^3 \text{ N}$

Area of one link:

$A = (10)(40) = 400 \text{ mm}^2$

$= 400 \times 10^{-6} \text{ m}^2$

Length: $L = 300 \text{ mm} = 0.300 \text{ m}$

Deformations.

$$\delta_{BE} = \frac{F_{BE}L}{EA} = \frac{(-7.5 \times 10^3)(0.300)}{(70 \times 10^9)(400 \times 10^{-6})} = -80.357 \times 10^{-6} \text{ m}$$

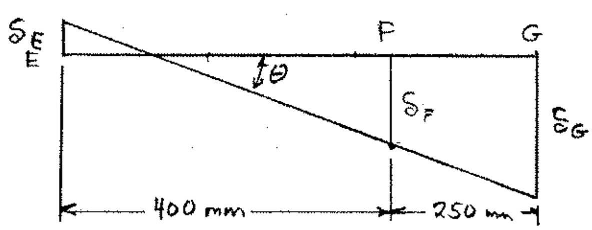
$$\delta_{CF} = \frac{F_{CF}L}{EA} = \frac{(19.5 \times 10^3)(0.300)}{(70 \times 10^9)(400 \times 10^{-6})} = 208.93 \times 10^{-6} \text{ m}$$

(a) Deflection of point E. $\delta_E = |\delta_{BE}|$ $\quad \delta_E = 80.4 \ \mu\text{m} \uparrow$

(b) Deflection of point F. $\delta_F = \delta_{CF}$ $\quad \delta_F = 209 \ \mu\text{m} \downarrow$

Geometry change.

Let θ be the small change in slope angle.



$$\theta = \frac{\delta_E + \delta_F}{L_{EF}} = \frac{80.357 \times 10^{-6} + 208.93 \times 10^{-6}}{0.400} = 723.22 \times 10^{-6} \text{ radians}$$

(c) Deflection of point G. $\delta_G = \delta_F + L_{FG}\theta$

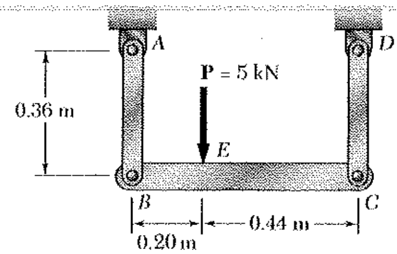
$$\delta_G = \delta_F + L_{FG}\theta = 208.93 \times 10^{-6} + (0.250)(723.22 \times 10^{-6})$$

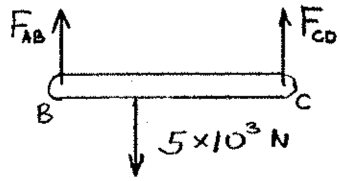
$$= 389.73 \times 10^{-6} \text{ m}$$

$\delta_G = 390 \ \mu\text{m} \downarrow$

Problem 2.27

2.27 Each of the links AB and CD is made of aluminum ($E = 75$ GPa) and has a cross-sectional area of 125 mm^2. Knowing that they support the rigid member BC, determine the deflection of point E.





Use member BC as a free body.

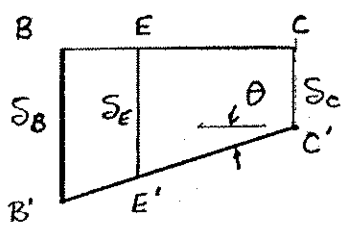
$$\circlearrowleft \Sigma M_C = 0: \quad -(0.64)F_{AB} + (0.44)(5\times10^3) = 0 \qquad F_{AB} = 3.4375\times10^3 \text{ N}$$

$$\circlearrowleft \Sigma M_B = 0: \quad (0.64)F_{CD} - (0.20)(5\times10^3) = 0 \qquad F_{CD} = 1.5625\times10^3 \text{ N}$$

For links AB and CD, $A = 125 \text{ mm}^2 = 125\times10^{-6} \text{ m}^2$

$$\delta_{AB} = \frac{F_{AB}L_{AB}}{EA} = \frac{(3.4375\times10^3)(0.36)}{(75\times10^9)(125\times10^{-6})} = 132.00\times10^{-6} \text{ m} = \delta_B$$

$$\delta_{CD} = \frac{F_{CD}L_{CD}}{EA} = \frac{(1.5625\times10^3)(0.36)}{(75\times10^9)(125\times10^{-6})} = 60.00\times10^{-6} \text{ m} = \delta_C$$



Deformation diagram.

$$\text{Slope } \theta = \frac{\delta_B - \delta_C}{l_{BC}} = \frac{72.00\times10^{-6}}{0.64}$$

$$= 112.5\times10^{-6} \text{ rad}$$

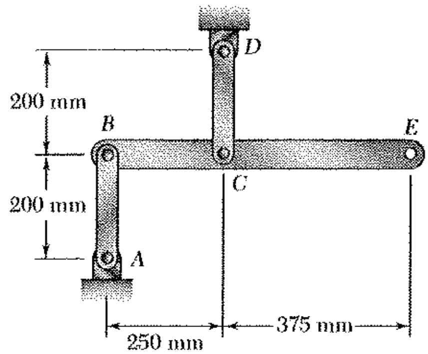
$$\delta_E = \delta_C + l_{EC}\theta$$

$$= 60.00\times10^{-6} + (0.44)(112.5\times10^{-6})$$

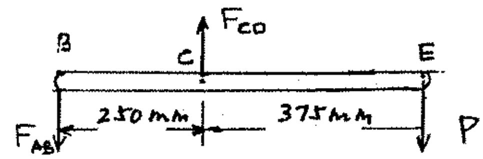
$$= 109.5\times10^{-6} \text{ m} \qquad \delta_E = 0.1095 \text{ mm} \downarrow$$

Problem 2.26

2.26 Each of the links AB and CD is made of steel (E = 200 GPa) and has a uniform rectangular cross section of 6 × 24 mm. Determine the largest load which can be suspended from point E if the deflection of E is not to exceed 0.25 mm.



Statics. Free body BCE.



$$+\circlearrowleft \Sigma M_C = 0: \quad 0.25 F_{AB} - 0.375 P = 0$$
$$F_{AB} = 1.5 P$$
$$+\circlearrowleft \Sigma M_B = 0: \quad 0.25 F_{CD} - 0.625 P = 0$$
$$F_{CD} = 2.5 P$$

Area of link:
$A = (6)(24) = 144 \text{ mm}^2$

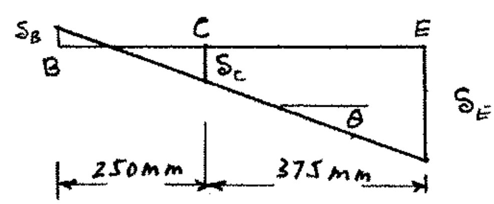
Length: 200 mm
$L =$

Deformations.
$$\delta_{AB} = \frac{F_{AB} L}{EA} = \frac{(1.5P)(0.2)}{(200 \times 10^9)(144 \times 10^{-6})} = 1.0417 \times 10^{-8} P$$
$$\delta_{CD} = \frac{F_{CD} L}{EA} = \frac{(2.5P)(0.2)}{(200 \times 10^9)(144 \times 10^{-6})} = 1.7361 \times 10^{-8} P$$

Deflections. Point B. $\delta_B = \delta_{AB}$ $\quad \delta_B = 1.0417 \times 10^{-8} P \uparrow$

Point C. $\delta_C = \delta_{CD}$ $\quad \delta_C = 1.7361 \times 10^{-8} P \downarrow$

Geometry change.



$$\theta = \frac{\delta_B + \delta_C}{L_{BC}} = \frac{1.0417 \times 10^{-8} P + 1.7361 \times 10^{-8} P}{0.25} = 0.11111 \times 10^{-6} P$$

$$\delta_E = \delta_F + L_{CE}\theta = 1.7361 \times 10^{-8} P + (0.375)(0.11111 \times 10^{-6} P) = 5.9027 \times 10^{-8} P \downarrow$$

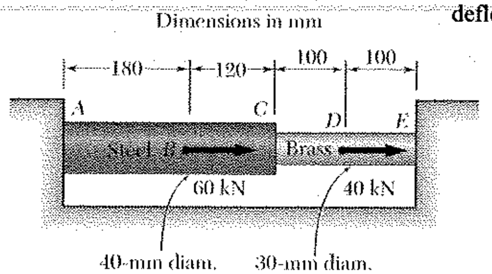
Limiting value of δ_E: $\quad \delta_E = 0.25 \times 10^{-3} \text{ m}$

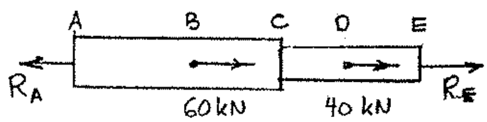
Limiting value of P: $\quad 5.9027 \times 10^{-8} P = 0.25 \times 10^{-3}$

$P = 4235 \text{ N}$ $\qquad P = 4.24 \text{ kN}$

Problem 2.41

2.41 Two cylindrical rods, one of steel and the other of brass, are joined at *C* and restrained by rigid supports at *A* and *E*. For the loading shown and knowing that $E_s = 200$ GPa and $E_b = 105$ GPa, determine (*a*) the reactions at *A* and *E*, (*b*) the deflection of point *C*.





A to C: $E = 200 \times 10^9$ Pa

$A = \frac{\pi}{4}(40)^2 = 1.25664 \times 10^3\ mm^2 = 1.25664 \times 10^{-3}\ m^2$

$EA = 251.327 \times 10^6$ N

C to E: $E = 105 \times 10^9$ Pa

$A = \frac{\pi}{4}(30)^2 = 706.86\ mm^2 = 706.86 \times 10^{-6}\ m^2$

$EA = 74.220 \times 10^6$ N

A to B: $P = R_A$ $\quad L = 180$ mm $= 0.180$ m

$$\delta_{AB} = \frac{PL}{EA} = \frac{R_A(0.180)}{251.327 \times 10^6} = 716.20 \times 10^{-12} R_A$$

B to C: $P = R_A - 60 \times 10^3$ $\quad L = 120$ mm $= 0.120$ m

$$\delta_{BC} = \frac{PL}{EA} = \frac{(R_A - 60 \times 10^3)(0.120)}{251.327 \times 10^6} = 447.47 \times 10^{-12} R_A - 26.848 \times 10^{-6}$$

C to D: $P = R_A - 60 \times 10^3$ $\quad L = 100$ mm $= 0.100$ m

$$\delta_{BC} = \frac{PL}{EA} = \frac{(R_A - 60 \times 10^3)(0.100)}{74.220 \times 10^6} = 1.34735 \times 10^{-9} R_A - 80.841 \times 10^{-6}$$

D to E: $P = R_A - 100 \times 10^3$ $\quad L = 100$ mm $= 0.100$ m

$$\delta_{DE} = \frac{PL}{EA} = \frac{(R_A - 100 \times 10^3)(0.100)}{74.220 \times 10^6} = 1.34735 \times 10^{-9} R_A - 134.735 \times 10^{-6}$$

A to E: $\delta_{AE} = \delta_{AB} + \delta_{BC} + \delta_{CD} + \delta_{DE} = 3.85837 \times 10^{-9} R_A - 242.424 \times 10^{-6}$

Since point E cannot move relative to A, $\delta_{AE} = 0$

(a) $3.85837 \times 10^{-9} R_A - 242.424 \times 10^{-6} = 0$ $\quad R_A = 62.831 \times 10^3$ N $\quad$ 62.8 kN ←

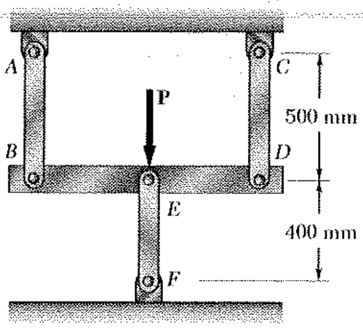
$R_E = R_A - 100 \times 10^3 = 62.8 \times 10^3 - 100 \times 10^3 = -37.2 \times 10^3$ N $\quad$ 37.2 kN ←

(b) $\delta_C = \delta_{AB} + \delta_{BC} = 1.16367 \times 10^{-9} R_A - 26.848 \times 10^{-6}$

$= (1.16369 \times 10^{-9})(62.831 \times 10^3) - 26.848 \times 10^{-6}$

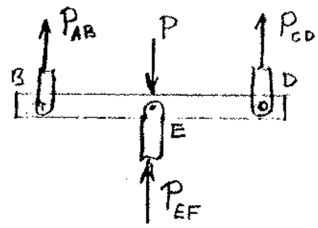
$= 46.3 \times 10^{-6}$ m $\quad \delta_C = 46.3\ \mu m$ →

Problem 2.39



2.39 Three steel rods ($E = 200$ GPa) support a 36-kN load **P**. Each of the rods *AB* and *CD* has a 200-mm^2 cross-sectional area and rod *EF* has a 625-mm^2 cross-sectional area. Neglecting the deformation of rod *BED*, determine (*a*) the change in length of rod *EF*, (*b*) the stress in each rod.

Use member BED as a free body.



By symmetry, or by $\Sigma M_E = 0$

$$P_{CD} = P_{AB}$$

$\Sigma F_y = 0:$

$$P_{AB} + P_{CD} + P_{EF} - P = 0$$

$$P = 2P_{AB} + P_{EF}$$

$$\delta_{AB} = \frac{P_{AB} L_{AB}}{E A_{AB}}, \quad \delta_{CD} = \frac{P_{CD} L_{CD}}{E A_{CD}}, \quad \delta_{EF} = \frac{P_{EF} L_{EF}}{E A_{EF}}$$

Since $L_{AB} = L_{CD}$ and $A_{AB} = A_{CD}$, $\delta_{AB} = \delta_{CD}$

Since points A, C, and E are fixed, $\delta_B = \delta_{AB}$, $\delta_D = \delta_{CD}$, $\delta_E = \delta_{EF}$

Since member BED is rigid, $\delta_E = \delta_B = \delta_C$

$$\frac{P_{AB} L_{AB}}{E A_{AB}} = \frac{P_{EF} L_{EF}}{E A_{EF}} \quad \therefore \quad P_{AB} = \frac{A_{AB}}{A_{EF}} \cdot \frac{L_{EF}}{L_{AB}} P_{EF} = \frac{200}{625} \cdot \frac{400}{500} P_{EF}$$

$$= 0.256\, P_{EF}$$

$$P = 2P_{AB} + P_{EF} = (2)(0.256) P_{EF} + P_{EF} = 1.512\, P_{EF}$$

$$P_{EF} = \frac{P}{1.512} = \frac{36 \times 10^3}{1.512} = 23.810 \times 10^3 \text{ N}$$

$$P_{AB} = P_{CD} = (0.256)(23.810 \times 10^3) = 6.095 \times 10^3 \text{ N}$$

(a) $$\delta = \delta_{EF} = \frac{(23.810 \times 10^3)(400 \times 10^{-3})}{(200 \times 10^9)(625 \times 10^{-6})} = 76.2 \times 10^{-6} \text{ m}$$

$$= 0.0762 \text{ mm}$$

or

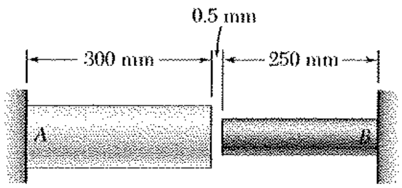
$$\delta = \delta_{AB} = \frac{(6.095 \times 10^3)(500 \times 10^{-3})}{(200 \times 10^9)(200 \times 10^{-6})} = 76.2 \times 10^{-6} \text{ m}$$

(b) $$\sigma_{AB} = \sigma_{CD} = \frac{P_{AB}}{A_{AB}} = \frac{6.095 \times 10^3}{200 \times 10^{-6}} = 30.5 \times 10^6 \text{ Pa} = 30.5 \text{ MPa}$$

$$\sigma_{EF} = -\frac{P_{EF}}{A_{EF}} = -\frac{23.810 \times 10^3}{625 \times 10^{-6}} = -38.1 \times 10^6 \text{ Pa} = 38.1 \text{ MPa}$$

Problem 2.60

2.60 At room temperature (20°C) a 0.5-mm gap exists between the ends of the rods shown. At a later time when the temperature has reached 140°C, determine (*a*) the normal stress in the aluminum rod, (*b*) the change in length of the aluminum rod.



Aluminum
$A = 2000\ \text{mm}^2$
$E = 75\ \text{GPa}$
$\alpha = 23 \times 10^{-6}/°\text{C}$

Stainless steel
$A = 800\ \text{mm}^2$
$E = 190\ \text{GPa}$
$\alpha = 17.3 \times 10^{-6}/°\text{C}$

$$\Delta T = 140 - 20 = 120\ °\text{C}$$

Free thermal expansion.

$$\delta_T = L_a\alpha_a(\Delta T) + L_s\alpha_s(\Delta T)$$
$$= (0.300)(23\times10^{-6})(120) + (0.250)(17.3\times10^{-6})(120)$$
$$= 1.347\times10^{-3}\ \text{m}$$

Shortening due to P to meet constraint.

$$\delta_P = 1.347\times10^{-3} - 0.5\times10^{-3} = 0.847\times10^{-3}\ \text{m}$$

$$\delta_P = \frac{PL_a}{E_aA_a} + \frac{PL_s}{E_sA_s} = \left(\frac{L_a}{E_aA_a} + \frac{L_s}{E_sA_s}\right)P$$
$$= \left(\frac{0.300}{(75\times10^9)(2000\times10^{-6})} + \frac{0.250}{(190\times10^9)(800\times10^{-6})}\right)P = 3.6447\times10^{-9}\ P$$

Equating; $\quad 3.6447\times10^{-9}\ P = 0.847\times10^{-3} \qquad P = 232.39\times10^3\ \text{N}$

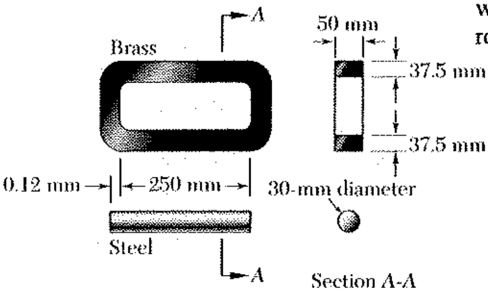
(a) $\sigma_a = -\dfrac{P}{A_a} = -\dfrac{232.39\times10^3}{2000\times10^{-6}} = -116.2\times10^6\ \text{Pa} \qquad -116.2\ \text{MPa}$ ◄

(b) $$\delta_a = L_a\alpha_a(\Delta T) - \frac{PL_a}{E_aA_a}$$
$$= (0.300)(23\times10^{-6})(120) - \frac{(232.39\times10^3)(0.300)}{(75\times10^9)(2000\times10^{-6})} = 363\times10^{-6}\ \text{m}$$

0.363 mm ◄

Problem 2.57

2.57 An brass link (E_b = 105 GPa, α_b = 20.9 × 10^{-6}/°C) and a steel rod (E_s = 200 GPa, α_s = 11.7 × 10^{-6}/°C) have the dimensions shown at a temperature of 20°C. The steel rod is cooled until it fits freely into the link. The temperature of the whole assembly is then raised to 45°C. Determine (*a*) the final stress in the steel rod, (*b*) the final length of the steel rod.



Initial dimensions at T = 20°C.
Final dimensions at T = 45°C.

$\Delta T = 45 - 20 = 25°C$

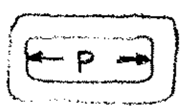
Free thermal expansion of each part.

Brass link: $(\delta_T)_b = \alpha_b(\Delta T)L = (20.9\times10^{-6})(25)(0.250) = 130.625\times10^{-6}$ m

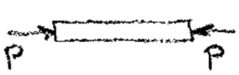
Steel rod: $(\delta_T)_s = \alpha_s(\Delta T)L = (11.7\times10^{-6})(25)(0.250) = 73.125\times10^{-6}$ m

At the final temperature the difference between the free length of the steel rod and the brass link is

$$\delta = 120\times10^{-6} + 73.125\times10^{-6} - 130.625\times10^{-6} = 62.5\times10^{-6}\ \text{m}$$



Add equal but opposite forces P to elongate the brass link and contract the steel rod.



Brass link: $E = 105\times10^{9}$ Pa

$A_b = (2)(50)(37.5) = 3750\ \text{mm}^2 = 3.750\times10^{-3}\ \text{m}^2$

$$(\delta_P) = \frac{PL}{EA} = \frac{P(0.250)}{(105\times10^{9})(3.750\times10^{-3})} = 634.92\times10^{-12}P$$

Steel rod: $E = 200\times10^{9}$ Pa $\qquad A_s = \frac{\pi}{4}(30)^2 = 706.86\ \text{mm}^3 = 706.86\times10^{-6}\ \text{m}^2$

$$(\delta_P)_s = \frac{PL}{E_sA_s} = \frac{P(0.250)}{(200\times10^{9})(706.86\times10^{-6})} = 1.76838\times10^{-9}P$$

$(\delta_P)_b + (\delta_P)_s = \delta \qquad 2.4033\times10^{-9}P = 62.5\times10^{-6}$

$P = 26.006\times10^{3}$ N

(a) Stress in steel rod. $\qquad \sigma_s = -\frac{P}{A_s} = -\frac{(26.006\times10^{3})}{706.86\times10^{-6}}$

$= -36.8\times10^{6}$ Pa $\qquad \sigma_s = -36.8$ MPa ◄

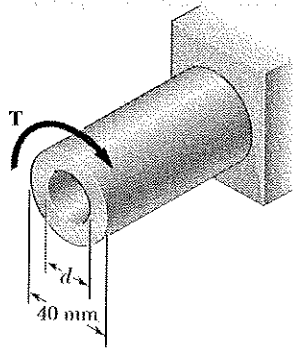
(b) Final length of steel rod. $\qquad L_f = L_o + (\delta_T)_s - (\delta_P)_s$

$L_f = 0.250 + 120\times10^{-6} + 73.125\times10^{-6} - (1.76838\times10^{-9})(26.003\times10^{3})$

$= 0.250147$ m $\qquad L_f = 250.147$ mm ◄

Problem 3.4

3.4 Knowing that $d = 30$ mm, determine the torque **T** that causes a maximum shearing stress of 52 MPa in the hollow shaft shown.



$c_2 = \frac{1}{2} d_2 = (\frac{1}{2})(40) = 20 \text{ mm}$ $\qquad c = 0.02 \text{ m}$

$c_1 = \frac{1}{2} d_1 = (\frac{1}{2})(30) = 15 \text{ mm}$

$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(20^4 - 15^4) = 171806 \text{ mm}^4$

$\tau_{max} = \frac{Tc}{J}$

$T = \frac{J\tau_{max}}{c} = \frac{(171806 \times 10^{-12})(52 \times 10^6)}{0.02} = 446.7 \text{ Nm}$

Problem 3.5

3.5 (a) Determine the torque that can be applied to a solid shaft of 20-mm diameter without exceeding an allowable shearing stress of 80 MPa. (b) Solve part a, assuming that the solid shaft has been replaced by a hollow shaft of the same cross-sectional area and with an inner diameter equal to half of its own outer diameter.

(a) Solid shaft: $c = \frac{1}{2} d = \frac{1}{2}(0.020) = 0.010 \text{ m}$

$J = \frac{\pi}{2} c^4 = \frac{\pi}{2}(0.10)^4 = 15.7080 \times 10^{-9} \text{ m}^4$

$T = \frac{J\tau_{max}}{c} = \frac{(15.7080 \times 10^{-9})(80 \times 10^6)}{0.010} = 125.664$ $\qquad T = 125.7 \text{ N}\cdot\text{m}$

(b) Hollow shaft: $\qquad$ Same area as solid shaft.

$A = \pi(c_2^2 - c_1^2) = \pi[c_2^2 - (\frac{1}{2}c_2)^2] = \frac{3}{4}\pi c_2^2 = \pi c^2$

$c_2 = \frac{2}{\sqrt{3}} c = \frac{2}{\sqrt{3}}(0.010) = 0.0115470 \text{ m}$

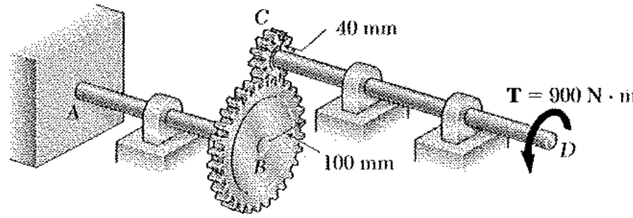
$c_1 = \frac{1}{2} c_2 = 0.0057735 \text{ m}$

$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.0115470^4 - 0.0057735^4) = 26.180 \times 10^{-9} \text{ m}^4$

$T = \frac{\tau_{max} J}{c_2} = \frac{(80 \times 10^6)(26.180 \times 10^{-9})}{0.0115470} = 181.38$ $\qquad T = 181.4 \text{ N}\cdot\text{m}$

Problem 3.21

3.21 A torque of magnitude $T = 900$ N · m is applied at D as shown. Knowing that the allowable shearing stress is 50 MPa in each shaft, determine the required diameter of (*a*) shaft *AB*, (*b*) shaft *CD*.



$T_{CD} = 900\text{ Nm.}$

$T_{AB} = \frac{r_B}{r_C} T_{AB} = \frac{100(900)}{40} = 2250\text{ Nm.}$

$\tau_{max} = 50\text{ MPa.}$

$\tau_{max} = \frac{Tc}{J} = \frac{2T}{\pi c^3}$

$c = \sqrt[3]{\frac{2T}{\pi \tau_{max}}}$

(a) Shaft AB: $c = \sqrt[3]{\frac{(2)(2250)}{\pi(50\times10^6)}} = 0.0306\text{ m} = 30.6\text{ mm}$

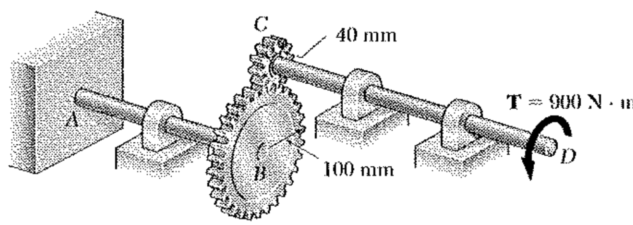
$d_{AB} = 2c = 61.2\text{ mm}$ ◄

(b) Shaft CD: $c = \sqrt[3]{\frac{(2)(900)}{\pi 50\times10^6}} = 0.0225\text{ m} = 22.5\text{ mm}$

$d_{CD} = 2c = 45\text{ mm}$ ◄

Problem 3.22

3.22 A torque of magnitude $T = 900$ N · m is applied at D as shown. Knowing that the diameter of shaft *AB* is 60 mm and that the diameter of shaft *CD* is 45 mm, determine the maximum shearing stress in (*a*) shaft *AB*, (*b*) shaft *CD*.



$T_{CD} = 900\text{ Nm}$

$T_{AB} = \frac{r_B}{r_C} T_{CD} = \frac{100}{40}(900) = 2250\text{ Nm.}$

(a) Shaft AB: $c = \frac{1}{2} d_{AB} = 30\text{ mm}$

$\tau_{max} = \frac{Tc}{J} = \frac{2T}{\pi c^3}$

$\tau_{max} = \frac{(2)(2250)}{\pi(0.03)^3} = 53.05\text{ MPa.}$

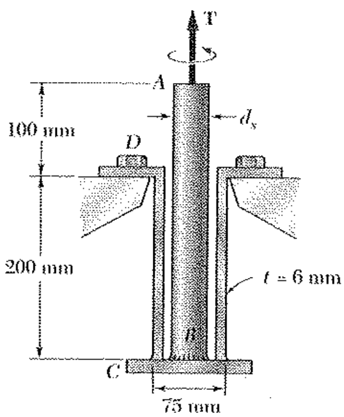
(a) $\tau_{max} = 53.05\text{ MPa}$ ◄

(b) Shaft CD: $c = \frac{1}{2} d_{CD} = 22.5\text{ mm}$

$\tau_{max} = \frac{2T}{\pi c^3} = \frac{(2)(900)}{\pi(0.0225)^3} = 50.3\text{ MPa}$

(b) $\tau_{max} = 50.3\text{ MPa.}$ ◄

Problem 3.7



3.7 The solid spindle AB is made of a steel with an allowable shearing stress of 84 MPa, and sleeve CD is made of a brass with an allowable shearing stress of 50 MPa. Determine (*a*) the largest torque **T** that can be applied at A if the allowable shearing stress is not to be exceeded in sleeve CD, (*b*) the corresponding required value of the diameter d_s of spindle AB.

(a) Sleeve CD: $c_2 = \frac{1}{2} d_2 = \frac{1}{2}(75) = 37.5$ mm.

$c_1 = c_2 - t = 37.5 - 6 = 31.5$ mm

$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.0375^4 - 0.0315^4) = 1.56 \times 10^{-6} m^4$

$\tau_{max} = \frac{Tc_2}{J}$

$T_{CD} = \frac{J \tau_{CD}}{c_2} = \frac{(1.56 \times 10^{-6})(50 \times 10^6)}{0.0375} = 2080$ N.m.

For equilibrium $T = 2.08$ kN

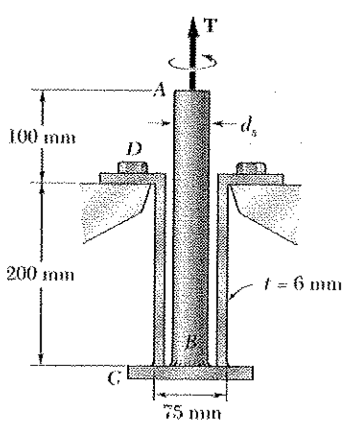
(b) Solid spindle AB: $T = 2080$ N.m.

$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3}$

$c = \sqrt[3]{\frac{2T}{\pi \tau}} = \sqrt[3]{\frac{(2)(2080)}{\pi (84 \times 10^6)}} = 0.025$ m $= 25$ mm

$d_s = 2c = (2)(25)$ $d_s = 50$ mm

Problem 3.8



3.8 The solid spindle AB has a diameter $d_s = 38$ mm and is made of a steel with an allowable shearing stress of 84 MPa, while sleeve CD is made of a brass with an allowable shearing stress of 50 MPa. Determine the largest torque **T** that can be applied at A.

Solid spindle AB: $c = \frac{1}{2} d_s = \frac{1}{2}(38) = 19$ mm

$J = \frac{\pi}{2} c^4 = \frac{\pi}{2}(0.019)^4 = 204.7 \times 10^{-9}$

$\tau_{max} = \frac{Tc}{J}$

$T_{AB} = \frac{J \tau_{al}}{c} = \frac{(204.7 \times 10^{-9})(84 \times 10^6)}{0.019} = 905$ Nm.

Sleeve CD: $c_2 = \frac{1}{2} d_2 = \frac{1}{2}(75) = 37.5$ mm.

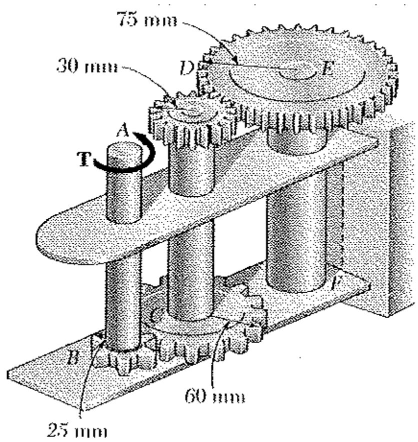
$c_1 = c_2 - t = 37.5 - 6 = 31.5$ mm.

$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.0375^4 - 0.0315^4) = 1.56 \times 10^{-6} m^4$

$T_{CD} = \frac{J \tau_{al}}{c_2} = \frac{(1.56 \times 10^{-6})(50 \times 10^6)}{0.0375} = 2080$ Nm

Allowable value of torque T is the smaller. $T = 2.08$ kN.m

Problem 3.27



3.27 A torque of magnitude $T = 120$ N · m is applied to shaft *AB* of the gear train shown. Knowing that the allowable shearing stress is 75 MPa in each of the three solid shafts, determine the required diameter of (*a*) shaft *AB*, (*b*) shaft *CD*, (*c*) shaft *EF*.

STATICS

Shaft AB. $T_{AB} = T_A = T_B = T$

Gears B and C. $r_B = 25\text{mm},\ r_C = 60\text{ mm}$

Force on gear circles. $F_{BC} = \dfrac{T_B}{r_B} = \dfrac{T_C}{r_C}$

$$T_C = \frac{r_C}{r_B} T_B = \frac{60}{25} T = 2.4\,T$$

Shaft CD. $T_{CD} = T_C = T_D = 2.4\,T$

Gears D and E. $r_D = 30\text{ mm},\ r_E = 75\text{ mm}$

Force on gear circles. $F_{DE} = \dfrac{T_D}{r_D} = \dfrac{T_E}{r_E}$

$$T_E = \frac{r_E}{r_D} T_D = \frac{75}{30}(2.4\,T) = 6T$$

Shaft EF. $T_{EF} = T_E = T_F = 6T$

REQUIRED DIAMETERS

$$\tau_{max} = \frac{Tc}{J} = \frac{2T}{\pi c^3} \qquad c = \sqrt[3]{\frac{2T}{\pi\tau}} \qquad d = 2c = 2\sqrt[3]{\frac{2T}{\pi\tau_{max}}}$$

$$\tau_{max} = 75\times10^6\text{ Pa}$$

(a) Shaft AB. $T_{AB} = T = 120$ N·m

$$d_{AB} = 2\sqrt[3]{\frac{2(120)}{\pi(75\times10^6)}} = 20.1\times10^{-3}\text{ m} \qquad d_{AB} = 20.1\text{ mm}$$

(b) Shaft CD. $T_{CD} = (2.4)(120) = 288$ N·m

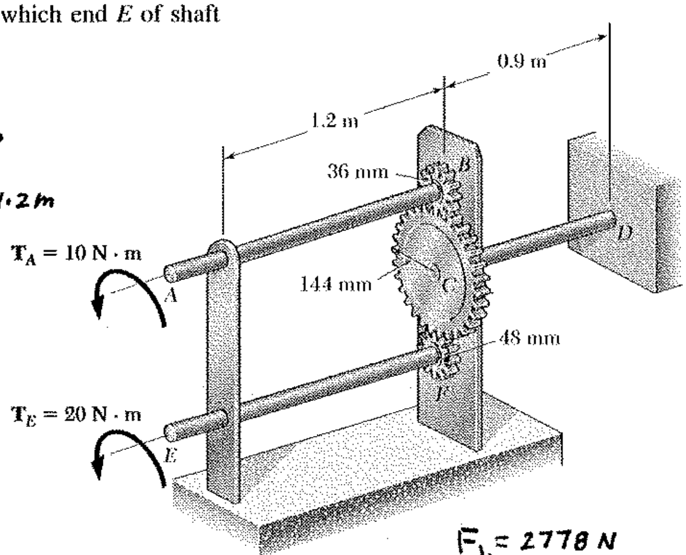
$$d_{CD} = 2\sqrt[3]{\frac{(2)(288)}{\pi(75\times10^6)}} = 26.9\times10^{-3}\text{ m} \qquad d_{CD} = 26.9\text{ mm}$$

(c) Shaft EF. $T_{EF} = (6)(120) = 720$ N·m

$$d_{EF} = 2\sqrt[3]{\frac{(2)(720)}{\pi(75\times10^3)}} = 36.6\times10^{-3}\text{ m} \qquad d_{EF} = 36.6\text{ mm}$$

Problem 3.39

3.39 Three solid shafts, each of 18-mm diameter, are connected by the gears shown. Knowing that $G = 77$ GPa, determine (*a*) the angle through which end *A* of shaft *AB* rotates, (*b*) the angle through which end *E* of shaft *EF* rotates.



<u>Geometry:</u>

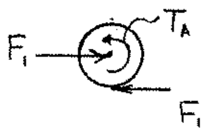
$r_B = 0.036\,m$, $r_c = 0.144\,m$, $r_F = 0.048$

$L_{AB} = 1.2\,m$, $L_{CD} = 0.9\,m$, $L_{EF} = 1.2\,m$

<u>Statics:</u> $T_A = 10$ Nm

$T_E = 20$ Nm

<u>Gear B.</u>



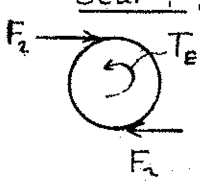
$+\circlearrowleft \sum M_B = 0:$

$-r_B F_1 + T_A = 0$

$-0.036 F_1 + 10 = 0$

$F_1 = 277.8$ N

<u>Gear F.</u>



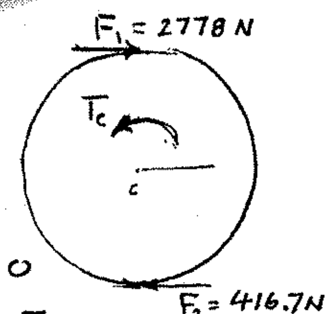
$+\circlearrowleft \sum M_F = 0:$

$-r_F F_2 + T_E = 0$

$-0.048 F_2 + 20 = 0$

$F_2 = 416.7$ N

<u>Gear C.</u>



$+\circlearrowleft \sum M_c = 0$

$-r_c F_1 - r_c F_2 + T_c = 0$

$-(0.144)(2778) - (0.144)(10) + T_c = 0$

$T_c = 41.4$ Nm

<u>Deformations:</u>

For all shafts $c = \frac{1}{2} d = 0.009$ m

$J = \frac{\pi}{2} c^4 = 10.306 \times 10^{-9}\ m^4$

$$\varphi_{A/B} = \frac{T_{AB} L_{AB}}{GJ} = \frac{(10)(1.2)}{(77\times10^9)(10.306\times10^{-9})} = 0.01512 \text{ rad}$$

$$\varphi_{E/F} = \frac{T_{EF} L_{EF}}{GJ} = \frac{(20)(1.2)}{(77\times10^9)(10.306\times10^{-9})} = 0.03024 \text{ rad}$$

$$\varphi_{C/D} = \frac{T_{CD} L_{CD}}{GJ} = \frac{(41.4)(0.9)}{(77\times10^9)(10.306\times10^{-9})} = 0.04695 \text{ rad}$$

<u>Kinematics:</u> $\varphi_c = \varphi_{C/D} = 0.04695$ rad

$r_B \varphi_B = r_c \varphi_c \qquad \varphi_B = \frac{r_c}{r_B}\varphi_c = \frac{0.144}{0.036}(0.04695) = 0.1878$ rad

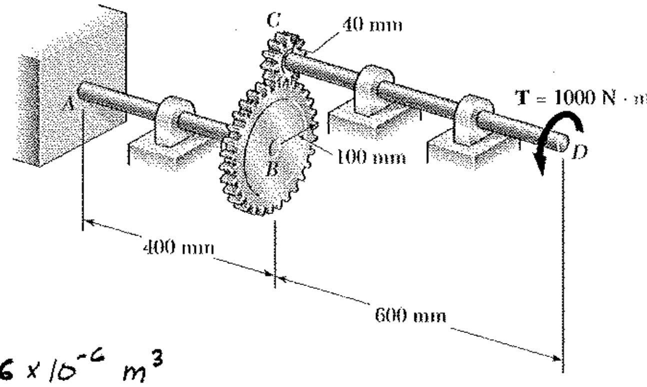
(a) $\varphi_{A/C} = \varphi_B + \varphi_{A/B} = 0.1878 + 0.01512 = 0.20292$ rad $\qquad \varphi_B = 11.6°$

$r_F \varphi_F = r_c \varphi_c \qquad \varphi_F = \frac{r_c}{r_F}\varphi_c = \frac{0.144}{0.048}(0.04695) = 0.14085$ rad

(b) $\varphi_E = \varphi_F + \varphi_{E/F} = 0.14085 + 0.03024 = 0.17109$ rad $\qquad \varphi_F = 9.8°$

Problem 3.49

3.49 The design of the gear-and-shaft system shown requires that steel shafts of the same diameter be used for both AB and CD. It is further required that $\tau_{max} \le 60$ MPa and that the angle ϕ_D through which end D of shaft CD rotates not exceed 1.5°. Knowing that $G = 77$ GPa, determine the required diameter of the shafts.



$T_{CD} = T_D = 1000$ N·m

$T_{AB} = \frac{r_B}{r_C} T_{CD} = \frac{100}{40}(1000) = 2500$ N·m

For design based on stress, use larger torque. $T_{AB} = 2500$ N·m.

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3}$$

$$c^3 = \frac{2T}{\pi \tau} = \frac{(2)(2500)}{\pi(60\times 10^6)} = 26.526\times 10^{-6}\ m^3$$

$c = 29.82\times 10^{-3}$ m $= 29.82$ mm, $\quad d = 2c = 59.6$ mm

Design based on rotation angle $\varphi_D = 1.5° = 26.18\times 10^{-3}$ rad

Shaft AB: $T_{AB} = 2500$ N·m, $L = 0.4$ m

$$\varphi_{AB} = \frac{TL}{GJ} = \frac{(2500)(0.4)}{GJ} = \frac{1000}{GJ}$$

Gears
$$\left\{\begin{array}{l} \varphi_B = \varphi_{AB} = \dfrac{1000}{GJ} \\ \varphi_C = \dfrac{r_B}{r_C}\varphi_B = \dfrac{100}{40}\cdot\dfrac{1000}{GJ} = \dfrac{2500}{GJ} \end{array}\right.$$

Shaft CD $\quad T_{CD} = 1000$ N·m, $L = 0.6$ m

$$\varphi_{CD} = \frac{TL}{GJ} = \frac{(1000)(0.6)}{GJ} = \frac{600}{GJ}$$

$$\varphi_D = \varphi_C + \varphi_{CD} = \frac{2500}{GJ} + \frac{600}{GJ} = \frac{3100}{GJ} = \frac{3100}{G\frac{\pi}{2}c^4}$$

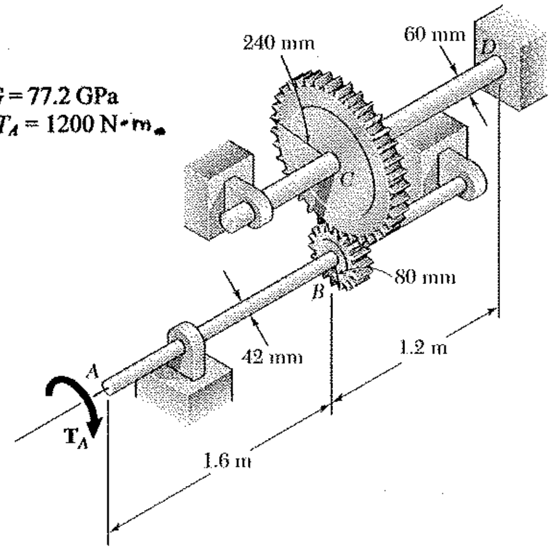
$$c^4 = \frac{(2)(3100)}{\pi G \varphi_D} = \frac{(2)(3100)}{\pi(77\times 10^9)(26.18\times 10^{-3})} = 979.06\times 10^{-9}\ m^4$$

$c = 31.46\times 10^{-3}$ m $= 31.46$ mm, $\quad d = 2c = 62.9$ mm

Design must use larger value for d. $\quad d = 62.9$ mm ◄

Problem 3.41

3.41 Two solid shafts are connected by gears as shown. Knowing that $G = 77.2$ GPa for each shaft, determine the angle through which end A rotates when $T_A = 1200$ N·m.



Calculation of torques.

Circumferential contact force between gears B and C

$$F = \frac{T_{AB}}{r_B} = \frac{T_{CD}}{r_C} \qquad T_{CD} = \frac{r_C}{r_B} T_{AB}$$

$$T_{AB} = 1200 \text{ N·m}$$

$$T_{CD} = \frac{240}{80}(1200) = 3600 \text{ N·m}$$

Twist in shaft CD: $c = \frac{1}{2}d = 0.030 \text{ m}, \quad L = 1.2 \text{ m}, \quad G = 77.2 \times 10^9$ Pa

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.030)^4 = 1.27234 \times 10^{-6} \text{ m}^4$$

$$\varphi_{C/D} = \frac{TL}{GJ} = \frac{(3600)(1.2)}{(77.2 \times 10^9)(1.27234 \times 10^{-9})} = 43.981 \times 10^{-3} \text{ rad}$$

Rotation angle at C. $\quad \varphi_C = \varphi_{C/D} = 43.981 \times 10^{-3}$ rad

Circumferential displacement at contact points of gears B an C

$$s = r_C \varphi_C = r_B \varphi_B$$

Rotation angle at B. $\quad \varphi_B = \frac{r_C}{r_B}\varphi_C = \frac{240}{80}(43.981 \times 10^{-3}) = 131.942 \times 10^{-3}$ rad

Twist in shaft AB: $c = \frac{1}{2}d = 0.021 \text{ m}, \quad L = 1.6 \text{ m}, \quad G = 77.2 \times 10^9$ Pa

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.021)^4 = 305.49 \times 10^{-9} \text{ m}^4$$

$$\varphi_{A/B} = \frac{TL}{GJ} = \frac{(1200)(1.6)}{(77.2 \times 10^9)(305.49 \times 10^{-9})} = 81.412 \times 10^{-3} \text{ rad}$$

Rotation angle at A. $\quad \varphi_A = \varphi_B + \varphi_{A/B} = 213.354 \times 10^{-3}$ rad

$$\varphi_A = 12.22^\circ \quad ◄$$

Problem 3.81

3.81 **A steel shaft must transmit 150 kW at speed of 360 rpm. Knowing that G = 77.2 GPa, design a solid shaft so that the maximum shearing stress will not exceed 50 MPa and the angle of twist in a 2.5-m length must not exceed 3°.**

$P = 150\ kW = 150\times10^3\ W \qquad f = \frac{360}{60} = 6\ Hz$

$T = \frac{P}{2\pi f} = \frac{150\times10^3}{2\pi\ (6)} = 3.9789\times10^3\ N\cdot m$

Stress requirement. $\tau = 50\times10^6\ P \qquad \tau = \frac{Tc}{J} = \frac{2T}{\pi c^3}$

$c = \sqrt[3]{\frac{2T}{\pi\tau}} = \sqrt[3]{\frac{(2)(3.9789\times10^3)}{\pi\ (50\times10^6)}} = 37.00\times10^{-3}\ m = 37.00\ mm$

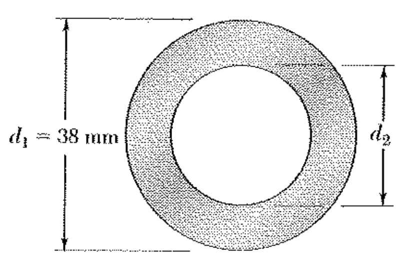
Angle of twist requirement. $\varphi = 3^\circ = 52.36\times10^{-3}\ rad$

$\varphi = \frac{TL}{GJ} = \frac{2TL}{\pi G c^4} \qquad G = 77.2\times10^9\ Pa,\ L = 2.5\ m$

$c = \sqrt[4]{\frac{2TL}{\pi G\varphi}} = \sqrt[4]{\frac{(2)(3.9789\times10^3)(2.5)}{\pi(77.2\times10^9)(52.36\times10^{-3})}} = 35.38\times10^{-3}\ m = 35.38\ mm$

Use larger value. $c = 37.00\ mm \qquad d = 2c = 74.0\ mm$ ◄

Problem 3.82



3.82 **A 1.5-m-long tubular steel shaft of 38-mm outer diameter d_1 and 30-mm inner diameter d_2 is to transmit 100 kW between a turbine and a generator. Determine the minimum frequency at which the shaft can rotate, knowing that G = 77.2 GPa, that the allowable shearing stress is 60 MPa, and that the angle of twist must not exceed 3°.**

$L = 1.5\ m \qquad \varphi = 3^\circ = 52.360\times10^{-3}\ rad$

$c_2 = \frac{1}{2}d_o = 19\ mm = 0.019\ m,\quad c_1 = \frac{1}{2}d_i = 15\ mm = 0.015\ m$

$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.019^4 - 0.015^4) = 125.186\times10^{-9}\ m^4$

Stress requirement. $\tau = 60\times10^6\ Pa \qquad \tau = \frac{Tc_2}{J}$

$T = \frac{J\tau}{c_2} = \frac{(125.186\times10^{-9})(60\times10^6)}{0.019} = 395.32\ N\cdot m$

Twist angle requirement. $\varphi = \frac{TL}{GJ}$

$T = \frac{GJ\varphi}{L} = \frac{(77.2\times10^9)(125.186\times10^{-9})(52.360\times10^{-3})}{1.5} = 337.35\ N\cdot m$

Maximum allowable torque is the smaller value. $T = 337.35\ N\cdot m$

Power transmitted $\quad P = 100\ kW = 100\times10^3\ W \qquad P = 2\pi f T$

Frequency $\quad f = \frac{P}{2\pi T} = \frac{100\times10^3}{2\pi\ (337.35)} = 47.2\ Hz \qquad f = 47.2\ Hz$ ◄