Problem Chosen	2022 MCM/ICM Summary Sheet	Team Control Number
C		2218394

Seize the Opportunity: Price Prediction and Trading Strategy Optimization for Gold and Bitcoin

In terms of the high volatility of financial assets, great profiting opportunities and high risks exists at the same time for making transactions. It is challenging to perfectly capture the optimal time and with strategies for decision making. In this work, we are striving to accurately detect the price fluctuations of gold and bitcoin, identify opportunities and come up with an optimal strategy to obtain as much returns as we can from the price differences, while taking the impacts of transaction costs into consideration.

Our first task is to forecast the price fluctuations of gold and bitcoin. When exploring the time-series data property, it suggests that our data set lacks fitness for popular financial data analysis model such as ARIMA. Hence, we build a GRU model to analyze the data series. In addition, we include essential typical technical indicators to construct a gradient boosted tree model, which takes market stochastic forces into account. As this compound method captures both time factor and financial factors in the market, it highly improves the accuracy and flexibility of the predictions.

Taking the data obtained from above, we attempt and analyze two asset management strategies–hedging portfolio strategy and pure profit optimization strategy–under the given conditions. After conducting the CCD-MGARCH model and co-integration test of gold and bitcoin, we find out that the difference in volatility and the frequent changing correlation between gold and bitcoin suggest that this pairing strategy is difficult to determine and unlikely to generate reliable results. Then, we construct another strategy which treats gold and bitcoin as independent options. We define a optimization method to determine most profitable strategies for trading the two assets. Because of the high "all-weight" transaction rates from the simulation result, we adapt our previous optimization method into an "all-weight" transaction strategy in the single Bitcoin market. Additionally, we evaluate the potential returns of different-time-period-transaction on daily bases, in order to find out the best trading opportunities. Regarding to the high commissions, our strategy manages to avoid losses from frequent transactions and unnecessary transactions of the assets with low volatility, which is gold in our scenario.

Finally, we make an cost sensitivity analysis, which indicates high effectiveness of our model. The implementation of our price prediction model and selected trading strategy shows strong accuracy and distinguished performance.

Keywords: Gated Recurrent Unit Model, Gradient Boosted Tree Model, DCC-MGARCH Model, Simplex Model, Linear Optimization

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1 Introduction

1.1 Problem Restatement

Regarding to the volatility of financial assets, especially bitcoin, the difference between current value and future value of these assets within a certain period provides potential opportunities traders to earn profit[1]. However, the randomness and uncertainty of the markets, the frequent fluctuation of asset prices combined with corresponding transaction costs make it difficult for traders to capture the exact future price change and set perfect transaction decisions to earn profits. Based on the historical prices of gold and bitcoin, we aim to make effective predictions to upcoming price changes of gold and bitcoin and generate a daily trading strategy to optimize total returns; and assess the transaction cost sensitivity of our determined strategy.

In order to achieve our goals, we need to first explore the patterns and measures which can indicate future price fluctuations of gold and bitcoin respectively, based on time series and regression model analysis. Then, we generate portfolio strategies to maximize potential profits, evaluate returns and transaction cost sensitivity, and determine the superior strategy as our solution.

1.2 Model Framework

The brief structure of our work is illustrated in Figure 1.

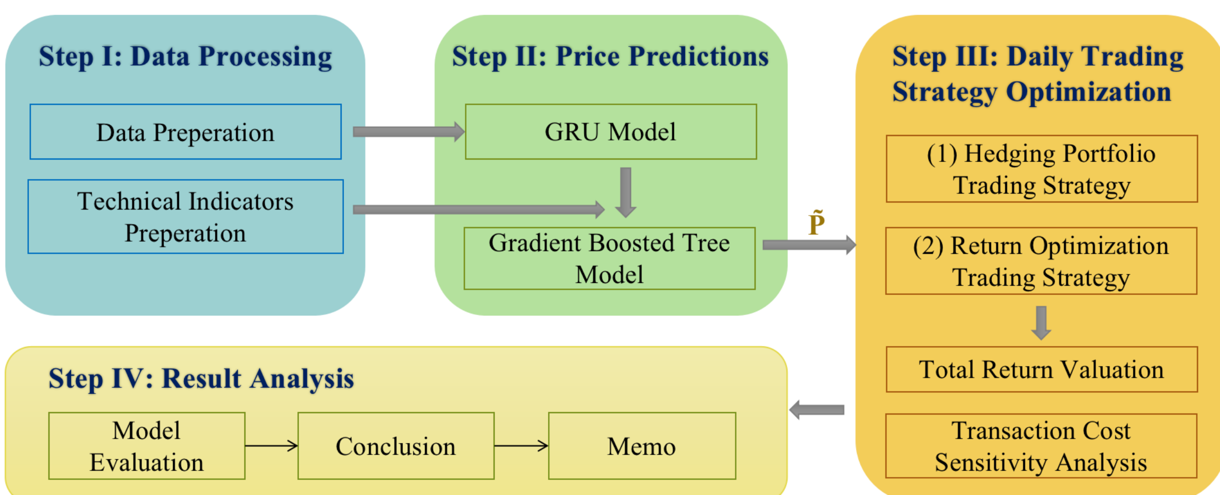

Figure 1: Overview Flowchart

2 Assumptions and Notations

2.1 Assumptions

To simplify our model, we make the following assumptions:

- *Short sale* is *not* allowed in the trading market, which means that investors are not allowed to borrow any cash or assets from the third party.
- Investors can only trade gold or bitcoin with *cash*; investors cannot trade gold with bitcoin or the other way directly.
- Investors are allowed to purchase *any quantity* of gold and bitcoin within their purchasing power.
- The *future* asset price trends are connected to the *historical developments*.
- The impacts of *external* factors in the market are recognized but not explicitly analyzed in our model.
- There are potential *connections* exist between the changes of gold price and bitcoin price.
- The updated movements of asset prices will have impacts on the investors' psychological *expectations* of the future price trends.

Table 1: Notations and their Descriptions

Symbol	Description
d	dth day (a stock variable)
x	x days (a flow variable)
n	length of a fixed time period
d_0	closest date before d
d_1	closest date after d
N	number of periods that satisfy certain conditions
K	a set of time period lengths with different values of n eg. $\mathbb{K} = \{1, 3\}$
r	daily price return
q	accumulative price return
p	price of single financial product
$\widehat{p}$	forecasted price
α	commission rate for some financial product

2.2 Notations

The nomenclature involved in our model construction is presented above in Table 1.

3 Price Forecasting

3.1 Data Preparation

3.1.1 Data Imputation

The two data files **LBMA-GOLD.csv** and **BCHAIN-MKPRU.csv** that will be used to generate the input of our model are fairly simple. Nevertheless, one key issue that will need to be addressed is that bitcoin and gold can be traded on different sets of days. To better train our model, we simply fill the missing gold price on days that the market is not open using linear interpolation:

$$p_d = p_{d_0}\left(1 - \frac{d - d_0}{d_1 - d_0}\right) + p_{d_1}\left(1 - \frac{d_1 - d}{d_1 - d_0}\right) = p_{d_0}\left(1 - \frac{d - d_0}{d_1 - d_0}\right) + p_{d_1}\left(\frac{d - d_0}{d_1 - d_0}\right) \tag{1}$$

In the above formula, p_d denotes the price data we are trying to impute, d denotes the date of this price data, d_0, d_1 are the dates that are closest dates before/after d, and p_{d_0}, p_{d_1} are the gold prices at day d_0, d_1, respectively.

3.1.2 Data Transformation

After imputation, we now have the price of bitcoin and gold on each day. Nevertheless, after using the price data to train and validate our time-series model, we found the result very unsatisfactory. After investigation, we found that it might be caused by the significant discrepancies between the training data and validation data. Due to the nature of time-series models, we have to use the data from first 80 percent of the trading days to train the model and use data from last 20 percent of the trading days to validate the model. However, the bitcoin prices in last 20 percent of the trading days are much higher than bitcoin prices in the first 80 percent of the trading days, resulting in poor performance and generalization ability of the model. On the other hand, to predict the future price trends, what we care about most is not the absolute bitcoin and gold prices, but the **patterns** in historical data. Considering that similar patterns and trends could occur at any price level, we decide to use relative price changes in percentages instead of daily prices as input data, and the daily price changes in percentage is calculated by the formula below:

$$r_d = \frac{p_d - p_{d-1}}{p_{d-1}} \tag{2}$$

Accordingly, the output of our model will be a prediction of price rate-of-change between the current date and x days from the current date. Let this accumulated rate-of-change be $q_{d,x}$, then $q_{d,x}$ is given by

$$q_{d,x} = \left(\prod_{i=d+1}^{d+x} (1 + r_i) \right) - 1 \tag{3}$$

Note that when $x = 1$, $q_{d,x} = r_{d+1}$.

3.2 GRU Model Implementation

Since the price data we used is inherently sequential, it is intuitive to choose time-series based model that is designed to process sequential data. In order to determine the suitable model for the data set, we firstly conduct the **stationarity and stability test** with the use of ADF and PP test, observing the lack of stability of original pricing data compared to the stability of first-order differential pricing data. In addition, by plotting two ACF figures just as shown below, we also deduce a lack of **autocorrelation**[2] for the original differential pricing data because of the near-zero **ACF** values for both products, driving our focus on traditional time series models such as ARIMA[3] and ARMA to more accurate machine learning models.

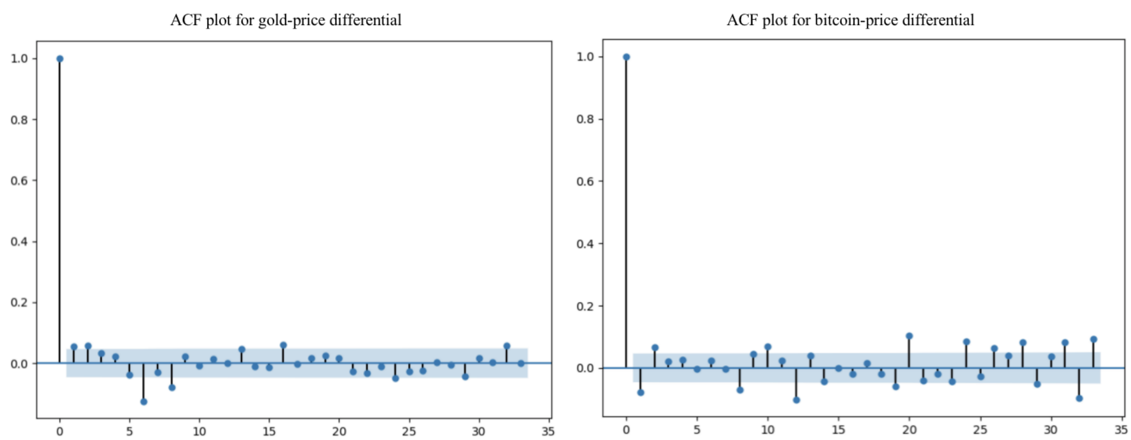

Figure 2: ACF plot Comparison between Gold and Bitcoin

After evaluating several models including RNN, LSTM, and GRU, we found that GRU would be the most suitable choice: vanilla RNN suffers from vanishing gradient and exploding gradient problems [4], and compared with LSTM, GRU has the advantage [5] of achieving shorter training time on small datasets without increase in training loss.

We use a many-to-one method to predict the future price change: we use the sliding window approach to generate multiple price rate-of-change sequences of a fixed length n, then the GRU modle takes this fixed length sequence as input and output a prediction $\widehat{q_{d,x}}$. For example, to predict the price rate-of-change between day 100 day $100 + x$ and $n = 30$, we input the price change rate vector $(r_{71}, r_{72}, \cdots, r_{100})$ and expect the GRU module to output a prediction to indicate the price rate-of-change between day 30 and day $30 + x$, which is $\widehat{q_{100,x}}$. Note that x is not necessarily 1. In our case, we run four different GRU models, using price change rates in past 30 days to predict $\widehat{q_{d,1}}$, $\widehat{q_{d,3}}$, $\widehat{q_{d,5}}$, and $\widehat{q_{d,10}}$ respectively. We could then easily recover the predicted price $\widehat{p_{d+x}}$ using the formula below: Although we are able to generate the predictions of the price rate-of-changes, this preliminary result is far from perfect: the GRU model of a specific asset only focuses on the past trends of that particular asset, but the price change of the alternative asset might also provide some valuable information. Besides, the predictions given by GRU model is inexplicable, whereas we aim to increase the interpretability of the model. Therefore, in a more advanced strategy, these preliminary predictions will be serve as inputs to a gradient boosted tree model along with the price change of another asset as well as several technical indicators to generate stronger predictions.

$$\widehat{p_{d+x}} = (1 + \widehat{q_{d,x}})p_d \tag{4}$$

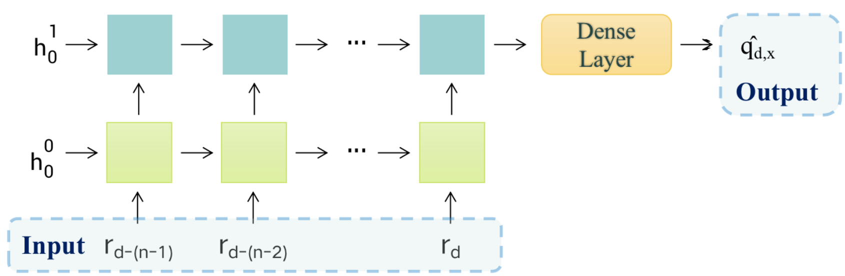

Figure 3: Structure of the Double-Layer GRU Model

3.3 Technical Indicator Preparation

Considering about some potential stochastic forces that will influence the product market price, we determine to add extra financial indicators based on pricing data as input variables to improve forecasting accuracy. Meanwhile, with the aim to avoid problems of overfitting and facilitate the measurement the relative price change, we select 3 typical uncorrelated indicators in the below table that are independent of the absolute price change value.

3.3.1 Rolling Standard Deviation (RSD)

The rolling standard deviation (RSD) is a straightforward measure of the price volatility of financial product within a period. To illustrate its importance, this indicator is generally utilized to calculate

Indicator Category	Technical Indicator	Indicator Abbreviation
Volatility Indicator	Rolling Standard Deviation	RSD
Psychological Indicator	Psychological Line	PSY
Momentum Indicator	Relative Strength Index	RSI

Table 2: Technical Indicators and their Descriptions

bollinger bands which works as the signal of overbought and oversold periods for traders. However, since the use of financial indicator here is to serve for a new pricing forecasting algorithm instead of trading time determination, we still determine to use RSD as an indicator instead of the Bollinger Bands.

$$RSD_d = \sqrt{\frac{\sum_{d=1}^{n}(p_d - SMA)^2}{n}} \tag{5}$$

To calculate the RSD, we apply the formula above based on the calculated periodical Simple Moving Average (SMA) value to generate a new attribute column. To elaborate more, as the formula below indicates, SMA is the unweighted mean of the previous n data points, which presents the price of n days in this case so that it can describe the price centrency within continuous time periods.

$$SMA_d = \frac{\sum_{k=d-n}^{i}(p_k)}{n} \tag{6}$$

3.3.2 Psychological Line (PSY)

The psychological line (PSY) is a widely-used index considering about the psychological effects of historical price change on dealers in markets. To be more specific, this index is established on the assumption that market participants will resist paying more for a share than others, unless of course the share continues to move up, and vice versa. Hence, traders purchasing the stock at the top of a trading range will wait until the price comes back to get out.

$$PSY_d = \frac{N_{upward}}{n} \tag{7}$$

Therefore, in order to improve the forecasting accuracy of our algorithm during the continuous price shock periods with different directions, we apply the formula above to generate this new variable for the next-step price forecasting.

3.3.3 Relative Strength Index (RSI)

The relative strength index (RSI) is a prevalent financial technical analysis momentum oscillator which is employed to evaluate the velocity and magnitude of directional price movements through the comparison of bidirectional close-to-close movements.

$$RSI_d = 100 - 100 \times \frac{1}{1 + \frac{MA_{U_d}}{MA_{D_d}}} \tag{8}$$

To compute for the value of RSI_d following the above formula, we firstly manage to calculate the upward change (U) and downward change (D) sequence with the use of formula below:

$$U_d = \begin{cases} p_d - p_{d-1} & if\ p_d > p_{d-1} \\ 0 \quad if\ p_d < p_{d-1} \end{cases} \tag{9}$$

$$D_d = \begin{cases} p_{d-1} - p_d \text{ if } p_{d-1} > p_i \\ 0 \text{ if } p_{d-1} < p_d \end{cases} \tag{10}$$

In addition, with the obtained upward change and downward change series, we then manage to calculate both averages to plug in the RSI_d formula.

$$MA_{U_d} = U_d + MA_{U_{d-1}} \times \frac{period - 1}{period} \tag{11}$$

$$MA_{D_d} = U_d + MA_{D_{d-1}} \times \frac{period - 1}{period} \tag{12}$$

3.4 Gradient Boosted Tree Model Implementation

After computing all technical indicators and preliminary predictions, we put all these data into a Gradient Boosted Tree Model to generate the final predictions. Similar to the preliminary predictions, these final predictions will represent the price rate-of-change between the current date and the predicted date. Among all available gradient boosted tree models, we choose XGBoost because compared with traditional gradient boosted tree models such as AdaBoost, it provides regularization options and provide more flexibility [6]. There is also less data pre-processing required to optimize the performance. Following is a complete list of input features to the XGBoost model to predict rate of change between dth and $d + x$th days:

- The asset price rate-of-change on the dth day.
- The alternative asset price rate-of-change on the dth day.

- The preliminary prediction from GRU model based on the asset price rate-of-change from $d-n$th day to dth day.
- 30-day rolling standard deviation.
- Psychological line
- Relative strength index

The predictions made by the gradient boosted tree model will be used to develop the final trading strategy.

3.5 Results and Analysis

After taking several technical indicators and the price of the alternative asset into account to develop a final model, we found that compared with the naive predictions, final predictions achieve much smaller root mean square error in the cross validation stage.

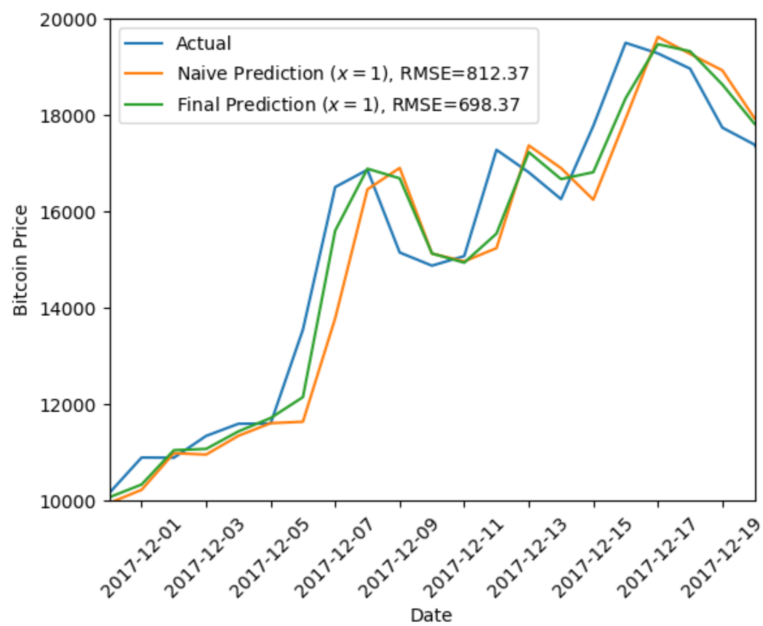

Figure 4: Actual Bitcoin Price, Naive Prediction Price and Final Prediction Price ($n = 30$, $x = 1$)

We derive $\widehat{p_{d+x}}$ for naive strategy (using the GRU model prediction) and final strategy (using the XGBoost model prediction) by equation 4. As shown in figure 4, we could observe that the outcome of the naive prediction (orange line) is largely based on the price change at the current date, and there's significant lagging. The predictions of the final model (green line) is much closer to the actual price, indicating that compared with using only the GRU model, our final model, which incorporates naive

GRU predictions and other indexes into consideration, better learns the patterns and trends and provides more accurate predictions.

4 Trading Strategy Construction

With the support of price forecasting data, we further analyze and improve the fitness of industry-popular statistical arbitrage trading strategies to both Bitcoin and Gold market: **hedging portfolio trading method**, which involves risk evaluation and management; in addition, we also manage to establish an innovative **profit-optimizing trading strategy**, which is more flexible as the potential profits and transaction decisions for each asset are determined independently.

4.1 Hedging Portfolio Trading Strategy

In order to analyze and determine an effective hedging portfolio trading strategy in greater detail, we start with the portfolio design to prepare a single compound product as an input for our profit-optimizing trading volume determination mechanism, which will be introduced in section 4.2.1.

4.1.1 Pair Trading Introduction

Pair trading strategy is a prevalently-used portfolio trading strategy in the market. It is established on the principle to trade by uncovering unusual spreads in two financial products within the portfolio that have a high correlation[7]. This works because when a stock outperforms another, the underperforming one may narrow the spread over the higher valued stock while investors are searching for the lower valued stock. To conclude, this process is profitable through the trading behavior of investors shedding high-valued assets and searching for low-valued assets whereas it can also work to hedge the risk to some extent.

4.1.2 Gold and Bitcoin Price Change Correlation Analysis

Since the two composing financial products within a portfolio under pair trading strategy should be highly correlated, we determine to track the daily price change correlation between two products. Hence, we select the **DCC-MGARCH model**[8]to help use to calculate the use of **time-varying conditional correlation coefficient**[9] as a measure and determine if Bitcoin and Gold are suitable for a portfolio makeup.

For the implementation of the model, since we have already conducted the stationarity and stability test in the section 3.2, ACF test which proves the lack of autocorrelation of change pricing data, we directly execute the **ARCH-LM test**. After observing the **ARCH effect** emerging in the time series data, we establish the GARCH model as a foundation of the DCC-GARCH model, managing to obtain the daily conditional correlation coefficient for the price change of both Bitcoin and Gold, shown in the figure below.

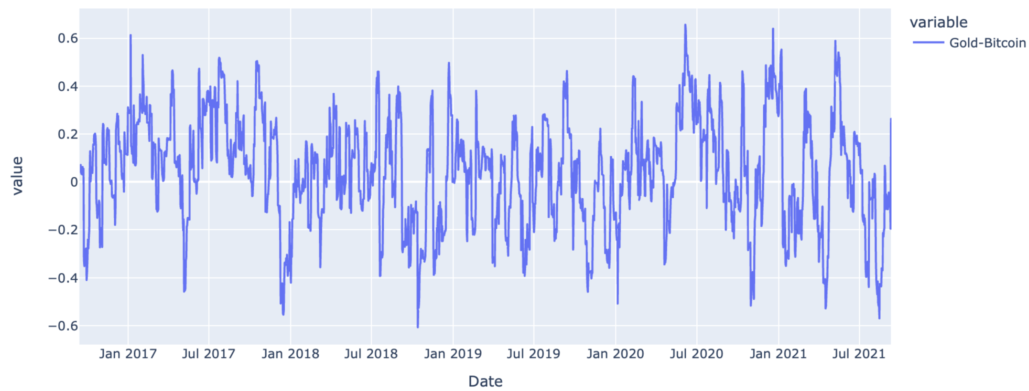

Figure 5: Daily Conditional Correlation Coefficient for Gold and Bitcoin

It can be observed from the above figure that the price change correlation between both financial products is highly bidirectional varying across time. Therefore, at this stage, it's difficult to determine if the correlation satisfies the criteria and if pair strategy is infeasible or not in this case.

4.1.3 Co-integration Test

To evaluate the feasibility of pair trading strategy with more solid evidence, we further conduct the co-integration test[10], which works as the essential prerequisite of portfolio design mechanism, on Gold and Bitcoin price separately. Since we have already observed the comparison of lack and unlack of stability for original pricing data and differential pricing data, we directly conduct the co-integration test across the entire processed data set. Just as the price change volatility figure above can illustrate and explain, because of the drastic volatility difference between Bitcoin and Gold price, the current data set doesn't pass the Co-integration test, indicating that a portfolio cannot be established without adjusting the financial product weight within the package frequently.

4.1.4 Pair Trading Strategy Evaluation

In addition to the realization of the fact that a portfolio would not work effectively without adapting two product weights separately with high-frequency, we also observe the extremely high-frequency daily-changing volatility of the Bitcoin price whereas the volatility of Gold price almost stay constant. This can be viewed as a signal of the higher profitability potential and risky level in Bitcoin market and the

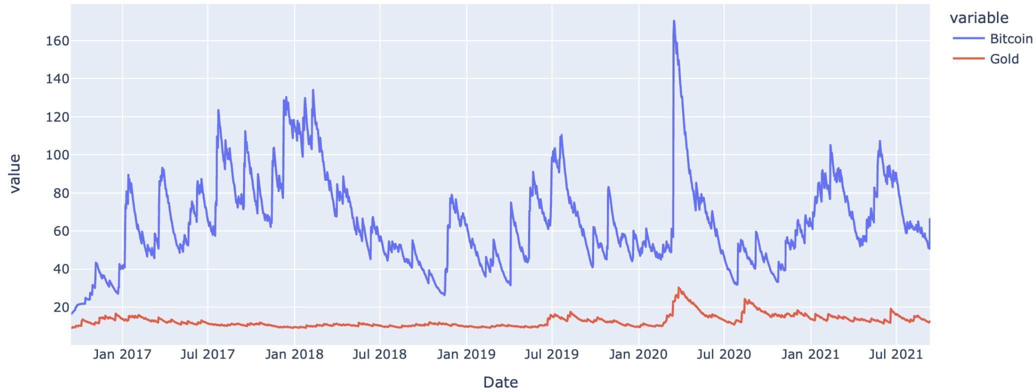

Figure 6: Daily Price Change Volatility of Gold and Bitcoin

lower profitability and neutral-risk nature of Gold market.Considering the relatively high commission rate (1%) and low volatility for gold, we come up with the hypothesis that the there might not be much profit margin for trading gold, and the sole focus on Bitcoin trading will work much more effectively targeting the goal of profit maximization than both the fixed-ratio portfolio trading and flexible-ratio trading on two markets, which we will evaluate in section 4.2.1.

4.2 Return Optimization Trading Strategy

4.2.1 Return Optimization Function

In order to evaluate the validity of our hypothesis in section 4.1.3 and maximize the trader's profit, we design a profit optimization function[11] as an integral part of our daily trading strategy to determine the whether we should trade and the optimal volume for independent Bitcoin market transaction on a day-to-day basis.

This function takes seven inputs: C_d, B_d, G_d, $\widehat{q_{\text{bitcoin } d,x}}$, $\widehat{q_{\text{gold } d,x}}$, α_{gold}, and α_{bitcoin}. C_d, B_d, and G_d are the cash position, bitcoin position and gold position at day d, respectively. $\widehat{q_{\text{bitcoin } d,x}}$ and $\widehat{q_{\text{bitcoin } d,x}}$ are $\widehat{q_{d,x}}$ for bitcoin and gold respectively and are supplied by our price-prediction model. α_{gold}, and α_{bitcoin} are commission fee for the rate of the. The optimization function will output $\widehat{B_{d+x}}$ and $\widehat{G_{d+x}}$, the suggested bitcoin and gold position we should adjust to in day d so that the overall profit would be maximized on day $d+x$ if it turns out that $\widehat{q_{\text{bitcoin } d,x}} = q_{\text{bitcoin } d,x}$ and $\widehat{q_{\text{gold } d,x}} = q_{\text{gold } d,x}$.

Now let T_{d+x} denote the net worth of the portfolio on day $d + x$, the goal is to maximize

$$\begin{aligned}\widehat{T_{d+x}} =& \widehat{B_{d+x}}(1 + \widehat{q_{\text{bitcoin } d,x}}) + \widehat{G_{d+x}}(1 + \widehat{q_{\text{gold } d,x}}) \\ &+ C - \|\widehat{B_{d+x}} - B_d\|\alpha_{\text{bitcoin}} - \|\widehat{G_{d+x}} - G_d\|\alpha_{\text{gold}} - (\widehat{G_{d+x}} - G_d) - (\widehat{B_{d+x}} - B_d)\end{aligned} \quad (13)$$

Here

$$\widehat{B_{d+x}}(1+\widehat{q_{\text{bitcoin } d,x}}) + \widehat{G_{d+x}}(1+\widehat{q_{\text{gold } d,x}}) \tag{14}$$

is the predicted worth of the bitcoin and gold on day $d+x$ and

$$C - \|\widehat{B_{d+x}} - B_d\|\alpha_{\text{bitcoin}} - \|\widehat{G_{d+x}} - G_d\|\alpha_{\text{gold}} - (\widehat{G_{d+x}} - G_d) - (\widehat{B_{d+x}} - B_d) \tag{15}$$

is the remaining cash based on the assumption that the transaction fee is paid by cash in addition to the transactions instead of subtracting the transaction fee by reducing the amount of assets purchased. Based on our no-borrowing and no-short-sell assumptions in section 2.1, this optimization has following constraints:

$$C - \|\widehat{B_{d+x}} - B_d\|\alpha_{\text{bitcoin}} - \|\widehat{G_{d+x}} - G_d\|\alpha_{\text{gold}} - (\widehat{G_{d+x}} - G_d) - (\widehat{B_{d+x}} - B_d) \geq 0 \tag{16}$$

$$\widehat{B_{d+x}} \geq 0 \tag{17}$$

$$\widehat{G_{d+x}} \geq 0 \tag{18}$$

With the function to maximize and constraints, we are able to employ the **Simplex model** under the **linear programming method**[12] to calculate the position of Bitcoin and Gold expected to hold separately after day d transaction to optimize the profits at day $d+x$ by considering about the forecast price at day $d+x$. In practice, since there are two absolute values in the function and constraints, we split the problem in to four linear programming sub-problems and get four pairs of $(\widehat{B_{d+x}}, \widehat{G_{d+x}})$ and then calculate $\widehat{T_{d+x}}$ for each pair to select the pair that could maximize $\widehat{T_{d+x}}$ as our final result for $\widehat{B_{d+x}}$ and $\widehat{G_{d+x}}$. With the expected position, we are able to follow a simplistic approach by adjusting our bitcoin and gold position to $\widehat{B_{d+x}}$ and $\widehat{G_{d+x}}$. However, the implementation of the profit-optimization strategy presents a tendency of frequent Bitcoin "all-in" and "all-sell". Referring back to our analysis in the section 4.1.4, this observation is reasonable as it can be supported by the high volatility of Bitcoins compared to the relatively fixed volatility of Gold, which further solidify our hypothesis that focus on Bitcoin market should be a more cost-effective trading strategy for the goal of profit maximization potential than combined market transaction considering about the high commission rate in this case. Hence, we determine to employ an independent market trading strategy following the basic principles below:

$$\begin{cases} \text{if } \widehat{q_{d,1}} > \alpha_{\text{bitcoin}}, & \text{use all cash to purchase bitcoin} \\ \text{if } \widehat{q_{d,1}} < -\alpha_{\text{bitcoin}}, & \text{sell all bitcoin positions} \\ \text{otherwise}, & \text{hold} \end{cases} \tag{19}$$

Using this simplistic approach we could trade based on our expectation of the bitcoin price on the next day. However, we expect our trading strategy could take the predicted furture trends in longer time-spans into account. Therefore, we manage to continue the strategy improvement by elaborating more on different time-length possibilities in the next section to help us found better buying and selling points.

4.2.2 Consideration of Daily Fluctuations

To further improve the daily trading strategy by exploring the impact of transaction period lengths as well as daily fluctuations, we include an extra testing step based on the daily profit-optimization determination mechanism.

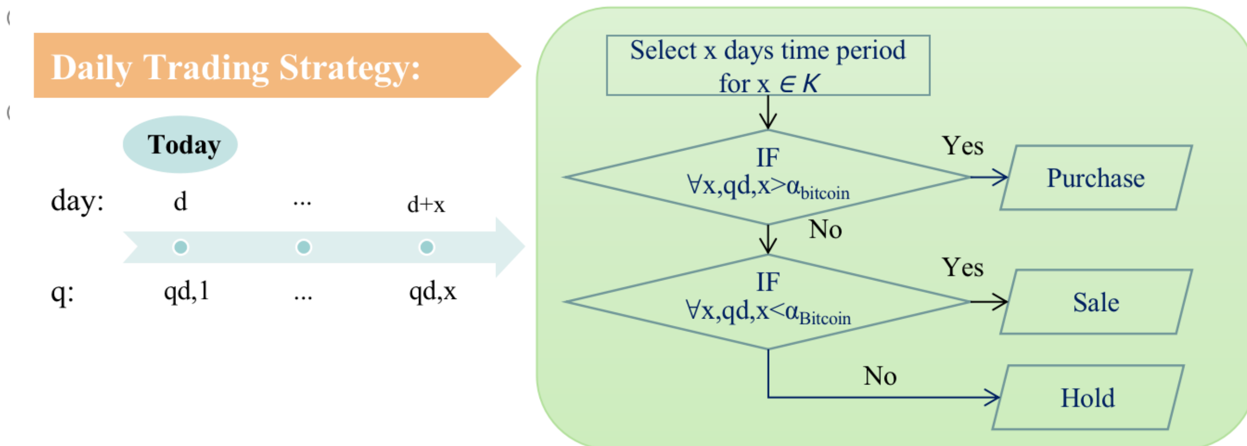

Figure 7: Daily Trading Strategy Flow Chart

As the Step I of above figure illustrates, instead of conducting daily trading test solely dependent on a single predictor $\widehat{q_{d,1}}$, we improve the algorithm by considering a set of possible x values, denoted as $\mathbb{K}$ to determine the best buying/selling date considering both the short-term and mid-term trends. In our case, $\mathbb{K}$ could be $\{1\}$, $\{1,3\}$,]$\{1,3,5\}$ and $\{1,3,5,10\}$ based on the trading frequency preferences, and different choice of K result in varying performance depending on the level of α_{bitcoin}:

$$\begin{cases} \text{if} \quad \forall k \in \mathbb{K}, \widehat{q_{d,x}} > \alpha_{\text{bitcoin}}, & \text{use all cash to purchase bitcoin} \\ \text{if} \quad \forall k \in \mathbb{K}, \widehat{q_{d,x}} < -\alpha_{\text{bitcoin}}, & \text{sell all bitcoin positions} \\ \text{otherwise}, & \text{hold} \end{cases} \tag{20}$$

With the properly determined forecasting periods on each day, we will be able to test the rationality of transaction making on that particular day by including the impact of stochastic daily fluctuations in the short term. More specifically, as the principles above illustrate, if a predictor with smaller x, say $\widehat{q_{d,3}}$ is smaller than α_{bitcoin}, while a predictor with larger x, say $\widehat{q_{d,5}}$, gives a prediction higher than α_{bitcoin}, the trader will, according to the suggestion, decide not to purchase bitcoin immediately. Assume the our predictions are accurate, the trader might be able to purchase bitcoins at a lower price between d and $d+3$. Even if the trader fails to find a lower price, they still have the opportunity to purchase the bitcoin at day $d+3$ with a similar price. Similarly, this decision mechanism gives the trader opportunity to sell bitcoins at a higher price with a "guarantee" to sell the bitcoins at a similar price in a few days later.

4.2.3 Return Optimization Strategy Result Analysis

With the improved algorithm, we are able to take profit optimization factors, transaction period differentiation, and daily position fluctuations into account, improving the accuracy of our daily return-optimization transaction determination model. With the use of model, our predicted worth of the initial \$1000 of investment principal is \$533,323. The plot below presents the fitness of our transaction decision with the market trend, proving the improved precision of this new strategy.

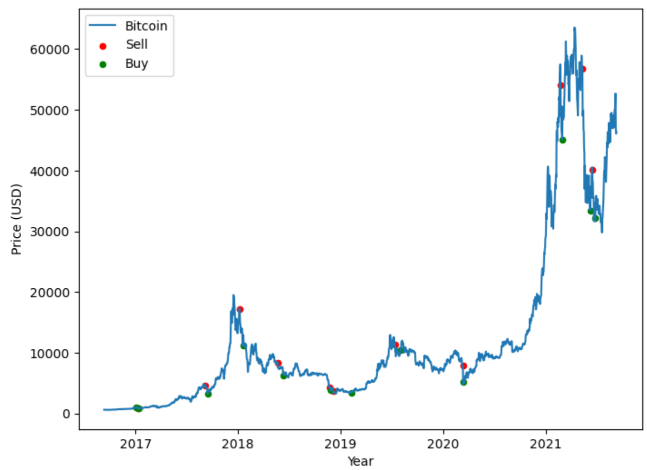

Figure 8: Buying and Selling Dates, $\mathbb{K} = \{1, 3\}$, $\alpha_{\text{bitcoin}} = 0.02$

4.3 Strategy Evaluation and Recommendations

Through the analysis in this section, we manage to prove the relatively low fitness of industry-popular hedging portfolio trading strategy to Bitcoin and Gold market together and establish a new profit-optimizing single market trading strategies. By applying the DCC-MGARCH model, we observe the highly volatile correlation between the two products across time and prove the lack of feasibility for portfolio construction with the conduction of co-integration test.

Based on the realization of ineffectiveness of pair trading, we then establish a new profit-optimizing trading strategy to maximize the total return obtained from each individual market with the constrained cash principal. However, the simulation result generated from the profit-optimization strategy design on both markets signals high recommended "all-in" and "all-sell" rates. This phenomena solidifies the hypothesis we have raised in section 4.1.4 and hence stimulate us to focus on the "all-weight" transactions in single Bitcoin market. To strengthen the accuracy, through the inclusion of different transaction period influences as well as daily fluctuations, we manage to develop a daily transaction determination mechanism with high generalization capability.

Nevertheless, considering about the drastic volatility pattern of Bitcoins, we also come up with the recommendation that while employing our trading algorithm to conduct daily determination of transaction volumes and time,[7] the trader should also focus more attention on the external forces and abnormal change in Bitcoin market to properly adjust the investment strategy frequently since the product has the property of high volatility, exposing the investors to both high spread opportunities and relatively high risks.

5 Transaction Cost Sensitivity Analysis

To analyze the effects of transaction costs on our strategy and result, we set different bitcoin commissions: 0.01, 0.02, and 0.04 respectively. For each commission level, we implement the strategy with different "K" selections and observe the comparative results of each.

α	$\mathbb{K} = \{1\}$	$\mathbb{K} = \{1, 3\}$	$\mathbb{K} = \{1, 3, 5\}$	$\mathbb{K} = \{1, 3, 5, 10\}$
$\alpha = 0.01$	$\$1,279,456$	$\$\mathbf{2,624,967}$	$\$2,376,976$	$\$1,771,741$
$\alpha = 0.02$	$\$466,017$	$\$\mathbf{533,323}$	$\$143,048$	$\$128,312$
$\alpha = 0.04$	$\$14,954$	$\$12,868$	$\$\mathbf{44,765}$	$\$\mathbf{44,765}$

Table 3: Sensitivity Analysis

Table 3 shows the outcomes of each condition. According to the results, the variation of commissions have considerable impacts on investment. As the commission rises, potential profits are greatly reduced. In addition, we discover that when the commission is low, a short term, or frequent, trading strategy will generate more potential returns; and when the commission is high, a long term trading strategy will generate more potential returns.

The strategy implementations with different commissions are presented in Figure 9 also supports our findings. When the commission is higher, less total returns and less frequent transactions are suggested; and vice versa.

This sensitivity analysis demonstrates that our method is adaptive to changing parameters. The trading strategy effectively and reasonably determines the optimal trading options and finely evaluates the impacts of transaction costs.

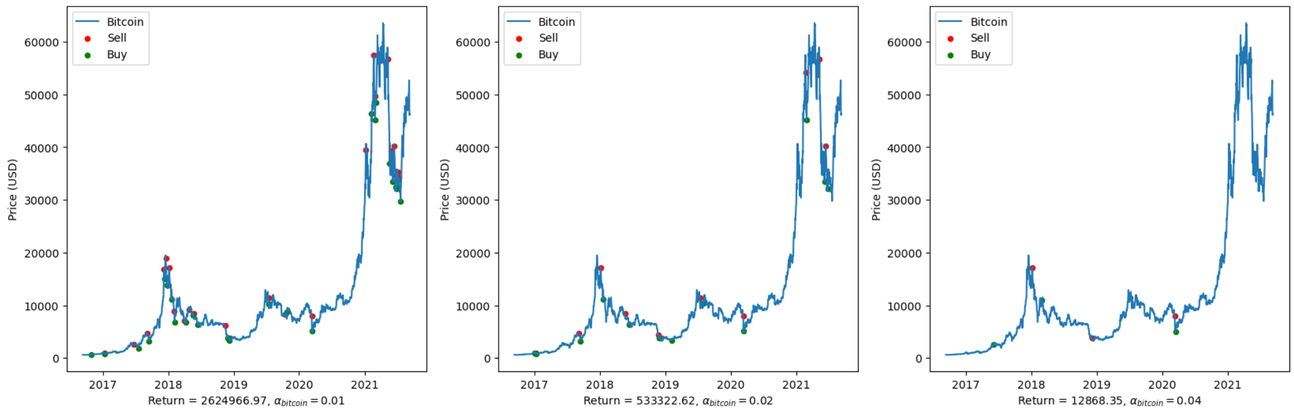

Figure 9: Sets of Buying and Selling Points Under different α_{bitcoin} level, $\mathbb{K} = \{1, 3\}$

6 Model Evaluation

6.1 Strengths

6.1.1 Price Forecasting Models

Improved comprehensiveness: As the GRU model captures time factor in asset price forecasting, the additional gradient boosted tree model also takes the impacts of other influencing factors into consideration and increases precision.

Reduced deviation: We involve cross-validation in gradient boosted tree model, avoiding over-fitting and increase accuracy and reliability of the model.

Accurate predictions: The price results predicted by our models is precise and consistent with the real data for the past five years.

6.1.2 Trading Strategy Analysis

Flexibility: The selected trading strategy compares the potential returns of relative short term and long term investments, and makes automated adjustments according to updated price predictions.

Outstanding performance: Our optimal strategy is expected to generate considerably high returns during the five years period.

High Sensitivity: The suggested trading strategies vary as the commission fees are changed.

6.2 Limitations

6.2.1 Price Forecasting Models

The forecasting models are effective in predicting price trends which are extended from historical market developments, but has difficulties in capturing the external changes and factors of the financial market.

6.2.2 Trading Strategy Analysis

The trading strategy is not able to hedge the volatility and potential risks of market randomness by managing the given two assets in the portfolio.

6.3 Improvement Strategy

6.3.1 Consideration Factor Variety

From the perspective of data scientists, we manage to ensure the general application capability of our current strategy with the use of cross-validation and boosting[13]. However, from the perspective of a trader and banker, the model can be further improved by including more diverse indicators and factors influencing each single market, ranging from macroeconomic inflation rate, global political news, financial regulations, and etc.

6.3.2 Hedging-used Penalty Term

Since the generally-employed portfolio trading strategy is proved to have low fitness with the market, the traditional risk-hedging method cannot work for the Bitcoin-Gold transactions. Hence, some further risk-hedging methods such as the introduction of an effective penalty term to the profit optimization function should be explored to alleviate the risks brought by over-optimistic or over-pessimistic predictions. However, considering about several historical unexpected changes of Bitcoin market beyond the normal volatility range, the selection of penalty term requires more comprehensive and detailed analysis and test to fit our model.

7 Conclusion

By evaluating the time-series property of the pricing data set, we observe the ineffectiveness of traditional time-series models such as ARIMA model on the current database, shifting focus on more self-adjusting machine learning algorithms. By further identifying the relative price fluctuations of gold and bitcoin and related technical indicators based on historical data, we first construct the GRU model and gradient boosted tree model to provide measures that fairly predict the future price changes of gold and bitcoin over time.

With the predicted price trends obtained, we attempt to construct both hedging portfolio trading strategy and return optimization trading strategy, and compare the the performance of the two. To design the hedging strategy, we build DCC-MGARCH model and find that the correlation between gold and bitcoin is continuously varying. With further conduction of the co-integration test, the relative stability of gold and high volatility of bitcoin indicate that it is not plausible to pair these two assets in our portfolio.

Then, we propose the profit optimization trading strategy to maximize the profit of the independent transaction returns, detecting the high fitness of single Bitcoin market "all-weight" trading. Through simulation, we construct a more detailed and comprehensive optimal transaction time and volume determination mechanism in terms of the given commissions. To avoid extra commission costs, we evaluate the potential profits from different time period trading strategies and select the most profitable transaction options.

By comparing the returns of conducting this trading strategy based on different commissions and time period selection, we conclude that our strategy effectively response to the changing parameters and generate high investment returns.

8 Memorandum

Date: 2022-02-21
To: Mr./Ms. Trader
From: Team #2218394 of 2021 MCM
Subject: Findings and Recommendation Trading Strategies on Bitcoin and Gold Market

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Dear Sir or Madam:

It is our great honor to be invited to propose trading strategies on Bitcoin and Gold market for you. Based on the 5-year historical pricing data in both markets, we worked to devise a cost-effective daily trading determination mechanism with the employment and design of various mathematical and machine learning models. In the rest part of this memorandum, we will elaborate more on our approaches, findings, and recommended strategies.

Methodologies:

In terms of our methodologies, we firstly evaluate the quality of, and conduct the exploratory data analysis of the market data. Based on the observed time-series data properties such as the stability, auto-correlation of the series, we properly select multiple machine-learning models such as LSTM and GRU models instead of traditional time-series models to forecast the future prices of both products, among which the best-performed one was finally selected. To improve our model adaptability to the real market, we also include some popular financial technical indicators in a new model, improving the forecasting accuracy. Furthermore, based on the predicted prices, we then try to evaluate and improve the fitness of popular statistical arbitrage strategies in the two markets through the conduction of DCC-MGARCH and co-integration test; In addition, we also design a entirely new daily trading time and quantity determination system by taking both optimization factors, transaction period, and daily fluctuations into account.

Findings:

Through our analysis process, some findings are concluded below:

- The time-series property of auto-correlation for both Bitcoin and Gold pricing data is weak with near-zero values, indicating the ineffectiveness of traditional time-series model employment for the analysis in this market.
- The time-varying conditional correlation coefficient between the price change of Bitcoin and Gold is highly floating across time, indicating the difficulty to include them into a hedging paired portfolio.

- The price change volatility of the Bitcoin is extremely high, with drastic change in almost every day, whereas the volatility of gold price change is approximately fixed, signalling the higher profitability nature of Bitcoin market and the lower risk nature of Gold market.

Recommendation Strategies:

Based on the findings above, analysis of mathematical models, and simulation results obtained from machine-learn algorithms, we propose the following strategies:

- Considering about the special time-series property of Bitcoin and Gold price, to forecast the future price of both financial products, self-learning and adjusting machine-learning algorithms is preferred over the traditional time-series model.
- Because of the highly volatile time-varying conditional correlation coefficient between the price change of Bitcoin and Gold, the industry prevalent statistical arbitrage strategy does not work well in this market as intuitively assume.
- To achieve the goal of profit maximization, our return-optimization algorithm can be applied since it will automatically help you to determine the trading time and volume on a daily basis with the transaction period length and daily fluctuations into account. (The implementation code is attached below)
- The drastic pattern difference between the price change volatility of the Bitcoin and Gold market suggests that the Bitcoin market offers higher profitable opportunities with the sacrifice of risk levels. Hence, while employing the profit-optimization model we have designed, extra attention on relevant global events and external forces news is also recommended.

We hope these findings and proposed strategies are helpful for your investment decisions. If you have any further questions about the strategies and models, please feel free to contact us.

Sincerely,
Consultant Team #2218394

9 Appendices

9.1 Profit Maximization Code (Excerpt)

```
def profit_max(C,B,G,q_b,q_g,alpha_b,alpha_g):

    positions = []

    # set the objects of the optimization function
    obj=[alpha_b - q_b, alpha_g - q_g]

    A_ub = [[alpha_b + 1, alpha_g + 1]]
    b_ub = [C + B * (1 + alpha_b) + G * (1 +alpha_g)]

    # set the bound for our variable
    bnd = [(B, float("inf")),  # Bounds of \hat{B_{d+x}}
           (G, float("inf"))]  # Bounds of \hat{G_{d+x}}

    # invoke the function
    strategy = linprog(c=obj, A_ub=A_ub, b_ub=b_ub, bounds=bnd,
                       method="revised simplex")
    positions.append(strategy.x)
```

9.2 XGBoost Traning Code (Excerpt)

```
for forward_step in FORWARD_STEPS:

    X = df_clean[["Bitcoin % Change", "Gold % Change",
                  f"Bitcoin % Naive Prediction ({forward_step} Days)",
                  "30-day Bitcoin rolling standard deviation",
                  "Bitcoin PSY",
                  "Bitcoin RSI"]].iloc[:-forward_step, :].copy()
    y = df_clean["Value"][forward_step:].values / df_clean["Value"]
    [:-forward_step].values - 1

    X_train, X_val, y_train, y_val = train_test_split(X, y, test_size=0.25)

    model = get_model()

    model.fit(X_train, y_train,
              eval_metric="rmse",
              eval_set=[(X_val, y_val)],
              early_stopping_rounds=300,
              verbose=50)

    df[f"Bitcoin % Final Prediction ({forward_step} Days)"] = pd.Series(model.
    predict(df_clean[X.columns]), index=df_clean.index)
```

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