Bootcamp Assessment 2020

Name: _____

Please show your work for all problems, even if you do not arrive at a solution!

1. For which values of $p > 0$ does $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converge? _____

2. Let $X_1, \ldots, X_n$ be iid samples from $Pois(\lambda)$. Show that both $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$ and $S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2$ are unbiased estimators for λ.

3. Skewness is a measure of the asymmetry of a probability distribution of a random variable about its mean. We define the skewness of a r.v. X as $\mathsf{E}\left[\left(\frac{X-\mu}{\sigma}\right)^3\right]$. A random variable that is right-skewed will have positive skewness, and a variable that is left-skewed will have negative skewness.

   If $X \sim Exp(\lambda)$, what is the skewness of X? _____

4. Let $X_1, \ldots, X_n$ be iid $Beta(\theta, 1)$ random variables, where $\theta > 0$. Find the MLE of $\dfrac{1}{\theta}$.

MLE = _____

5. The joint pdf of X and Y is

$$f(x,y) = \frac{e^{-yx^2/2}}{\sqrt{2\pi/y}} \cdot ye^{-y}, \quad x \in \mathbb{R}, y > 0$$

(a) Find the conditional density $f_{X|Y}(x|y)$ of X given $Y = y$. _____
Hint: consider decomposing the joint into the product of conditional and marginal densities.

(b) What is $\mathsf{E}[X|Y]$? _____

(c) What is $\mathsf{Var}(X|Y)$? _____

(d) What is $\mathsf{Var}(X)$? _____