Big Data Visual Analytics (CS 661)

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Study Materials for Lecture 14

- A Gentle Tutorial of the EM Algorithm and its Application to Parameter Estimation for Gaussian Mixture and Hidden Markov Models
- EM Algorithm: https://stephens999.github.io/fiveMinuteStats/intro_to_em.html
- SLIC Superpixels Compared to State-of-the-Art Superpixel Methods, Achanta et al.
- Homogeneity guided probabilistic data summaries for analysis and visualization of large-scale data sets, Dutta et al.
- Statistical visualization and analysis of large data using a value-based spatial distribution, Wang et al.

Mid Sem Exam

- Saturday 22nd Feb 8-10am
- Location: L18 and L19 (OROS)
- **Please bring your institute id card (mandatory)**
- No classes during Mid Sem week
- Syllabus:
  - Everything up to today's class

Final Project Group Formation

- Form your project team by March 2nd and update the google sheet with details of project members
  - https://docs.google.com/spreadsheets/d/1ZsWmnCRK4XEZV6YezgwM0CNk1nLMaZXSIPpv1d3WvAs/edit?usp=sharing
  - **Group size: 8**
  - Those who will not be part of a team, I will help them get assigned to groups

Parametric Distribution: GMM

- Gaussian Mixture Model (GMM): Represent a probability distribution function as a convex combination of multiple Gaussian functions

$$p(X) = \sum_{i=1}^{K} \omega_i * N(X \mid \mu_i, \sigma_i)$$

ω = Weights of the Gaussian components
K = Number of Gaussian components in the mixture model

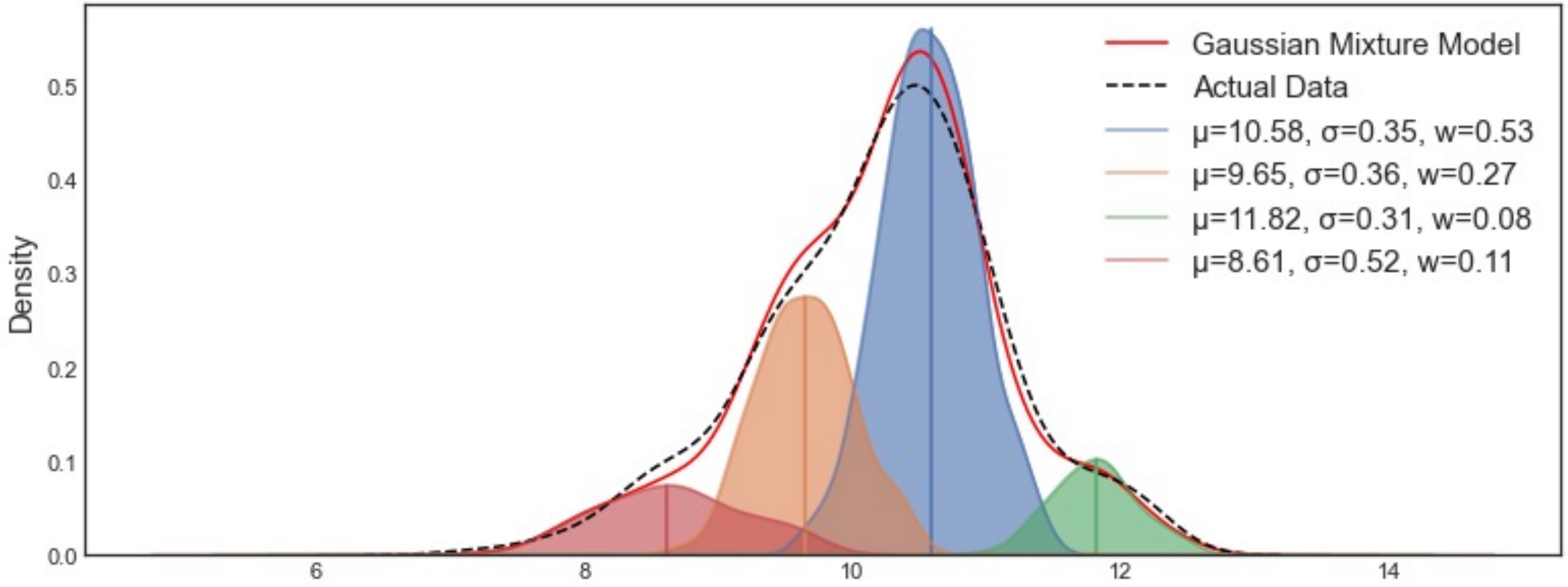


Fig. source: https://stats.stackexchange.com

Parameter Estimation Techniques

- Estimation of Gaussian distribution parameters are trivial
  - Maximum Likelihood Estimate (MLE)
  - Same as computing mean and variance

- Estimation of GMM parameters require Expectation Maximization (EM) algorithm
  - Iterative technique to fit GMM parameters

- Incremental schemes for GMM parameter estimation
  - Fast and approximate method to estimate GMM parameters
  - Can model streaming time-varying data

Expectation Maximization (EM) for GMM

- Initialize: means (μ), covariances (Σ), and weights (ω)
- Iterate until convergence:
  - **E-step:** Evaluate posterior probabilities given current parameters

$$\gamma_k^n = \frac{\omega_k \mathcal{N}(x^n | \mu_k, \Sigma_k)}{\sum_{j=1}^{k} \omega_k \mathcal{N}(x^n | \mu_j, \Sigma_j)}$$

  - **M-step:** Update the parameters to maximize the expected log-likelihood of the observed data

$$\omega_k = \frac{N_k}{N} \quad \text{and} \quad N_k = \sum_{n=1}^{N} \gamma_k^n$$

$$\mu_k = \frac{1}{N_k} \sum_{n=1}^{N} \gamma_k^n \; x^n$$

$$\Sigma_k = \frac{1}{N_k} \sum_{n=1}^{N} \gamma_k^n \, (x^n - \mu_k)\,(x^n - \mu_k)^{\mathrm{T}}$$

- Evaluate log likelihood at the end of each iteration and check for convergence

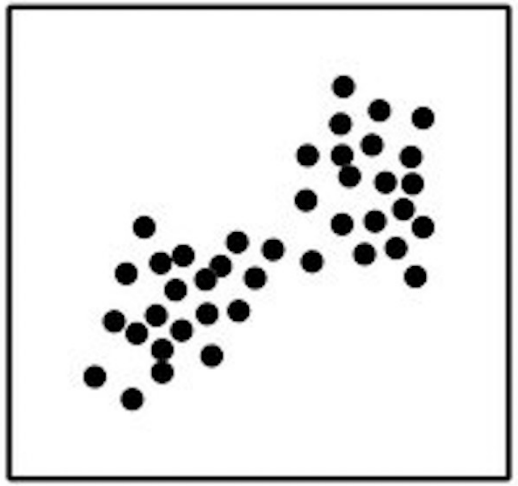
The theory of the method is much more involved!

<u>For a detailed derivation:</u>

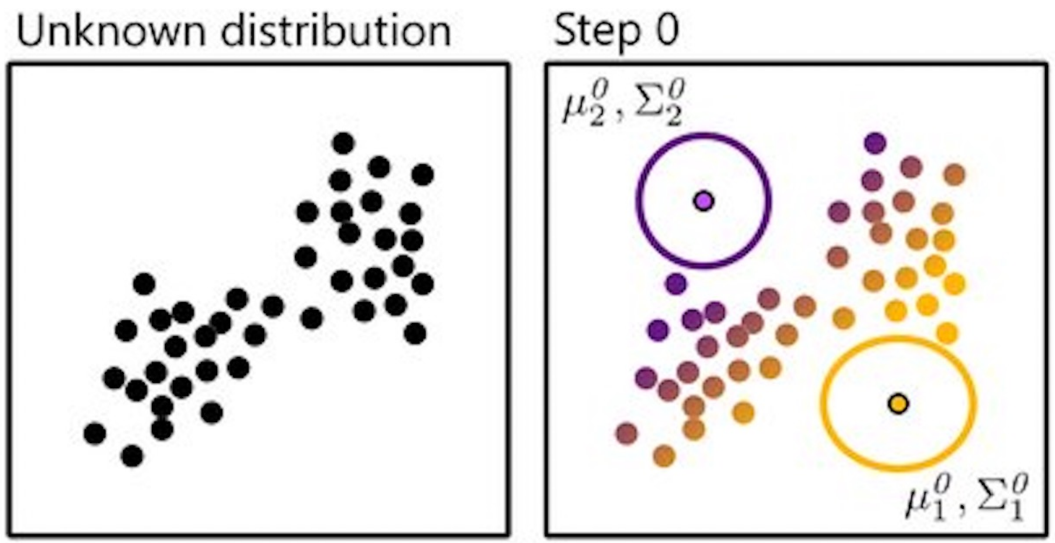
- A Gentle Tutorial of the EM Algorithm and its Application to Parameter Estimation for Gaussian Mixture and Hidden Markov Models
- https://stephens999.github.io/fiveMinuteStats/intro_to_em.html

How EM Algorithm Works

Unknown distribution



How EM Algorithm Works



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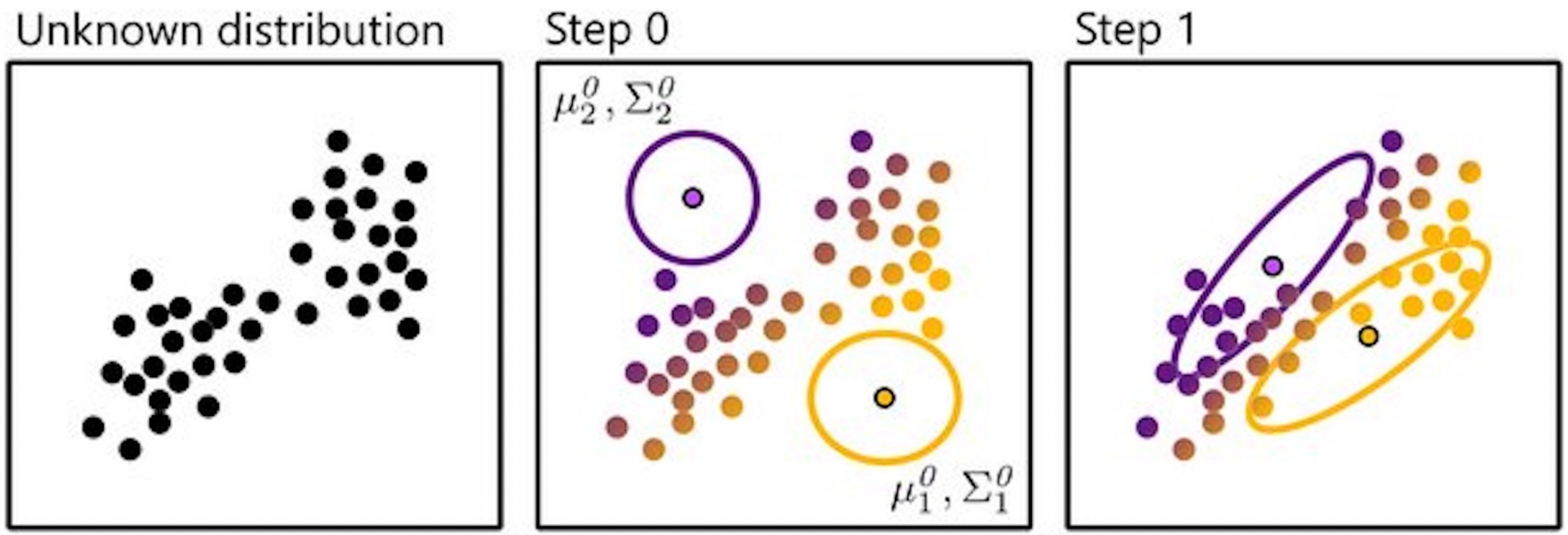

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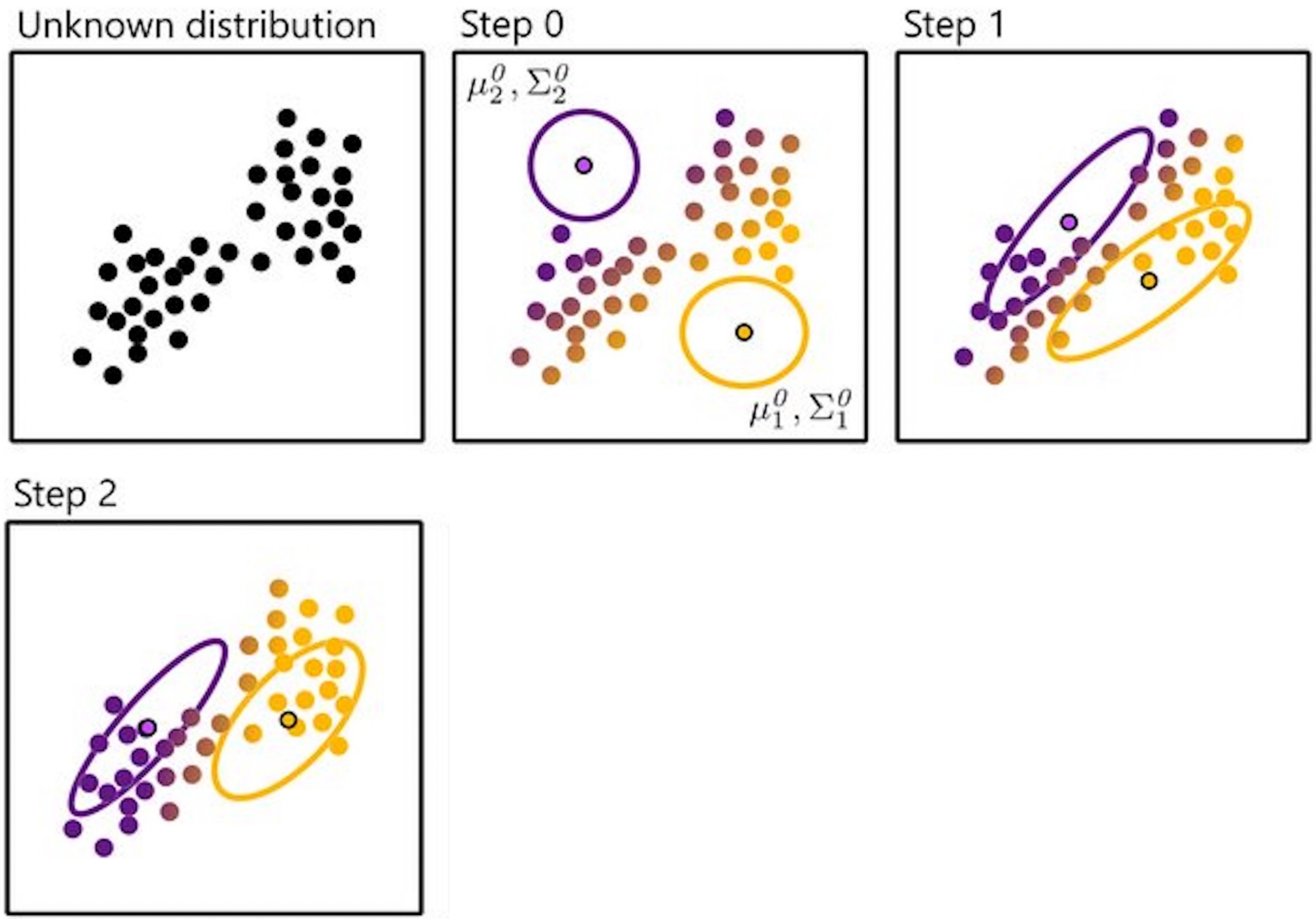

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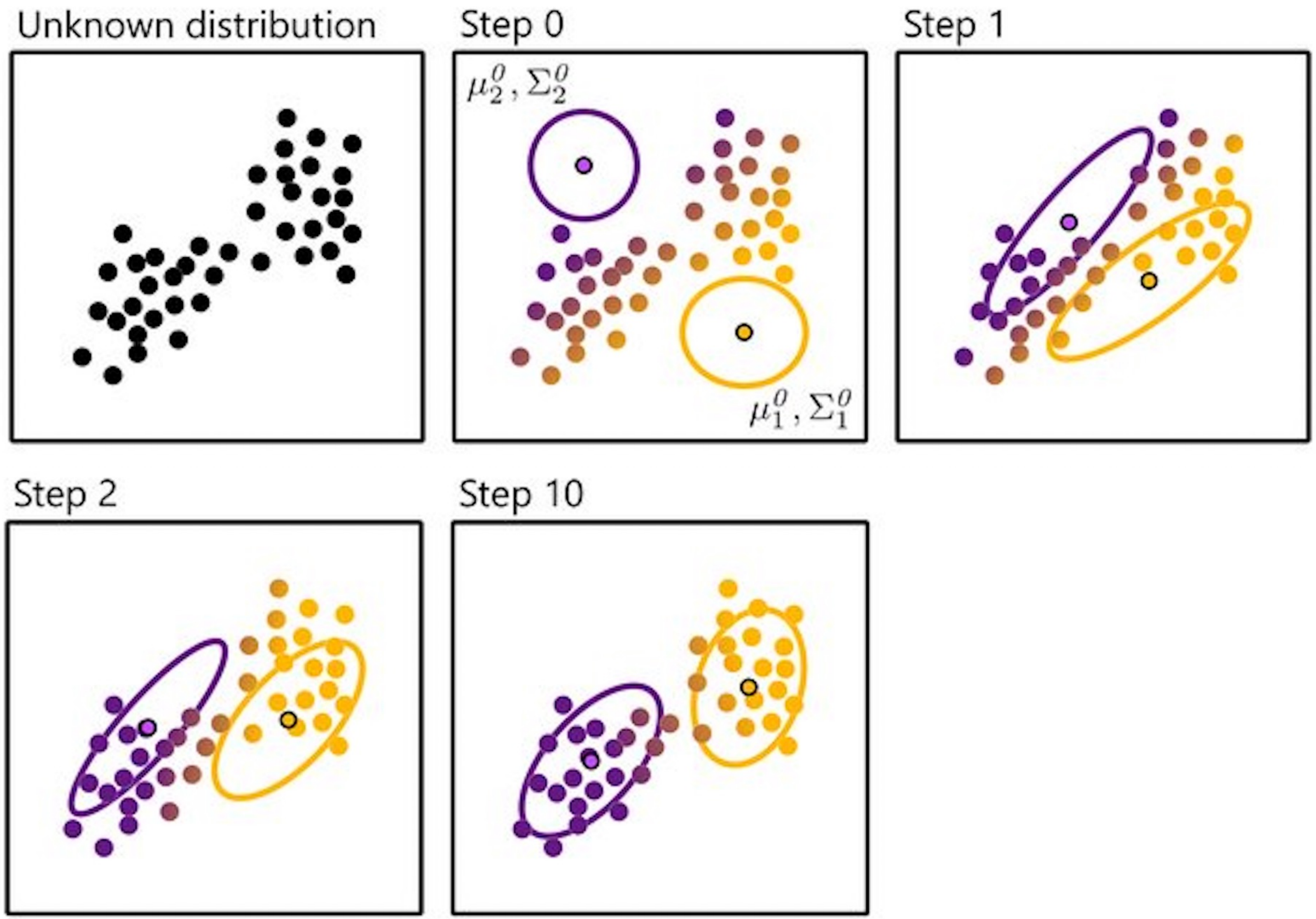

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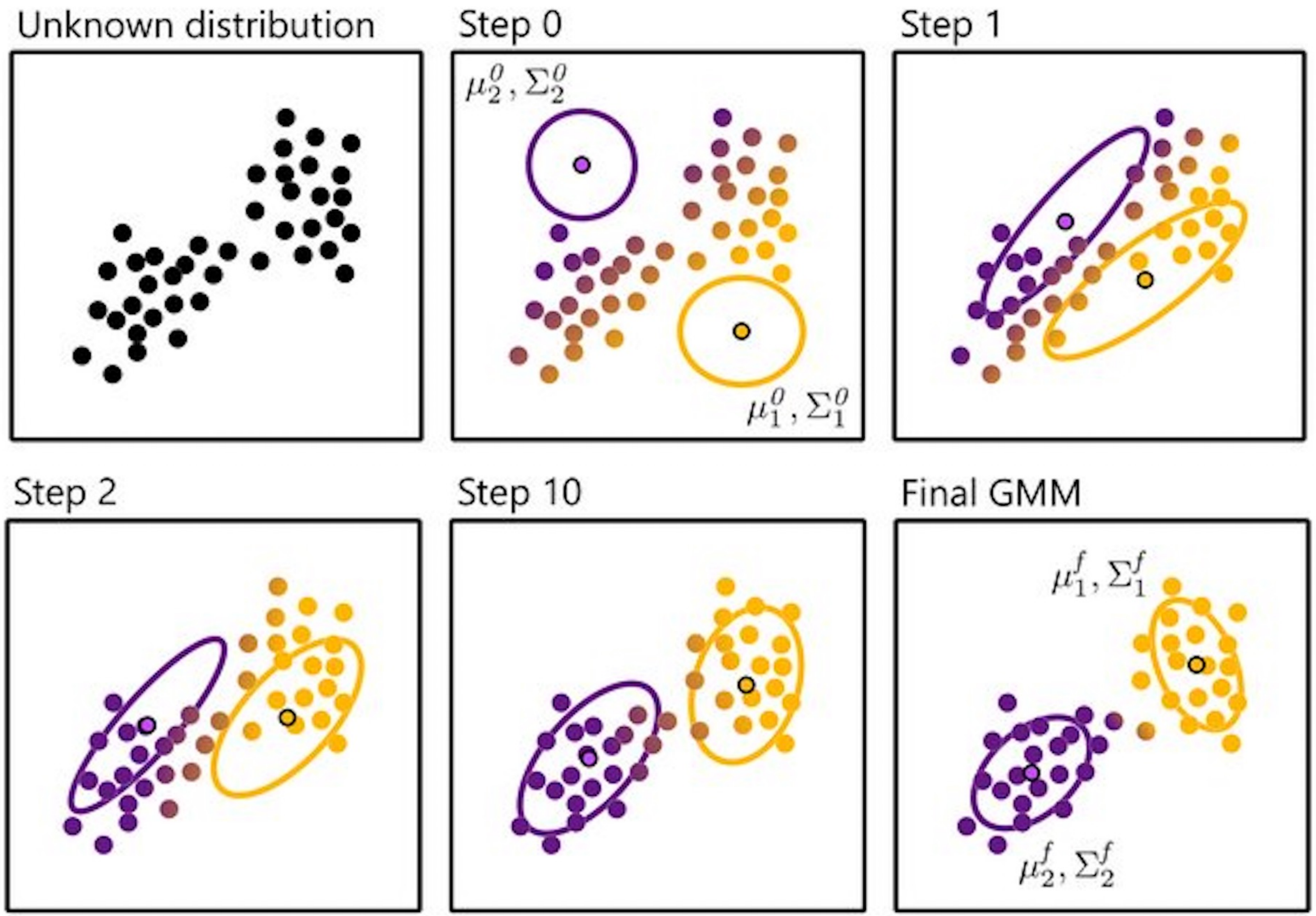
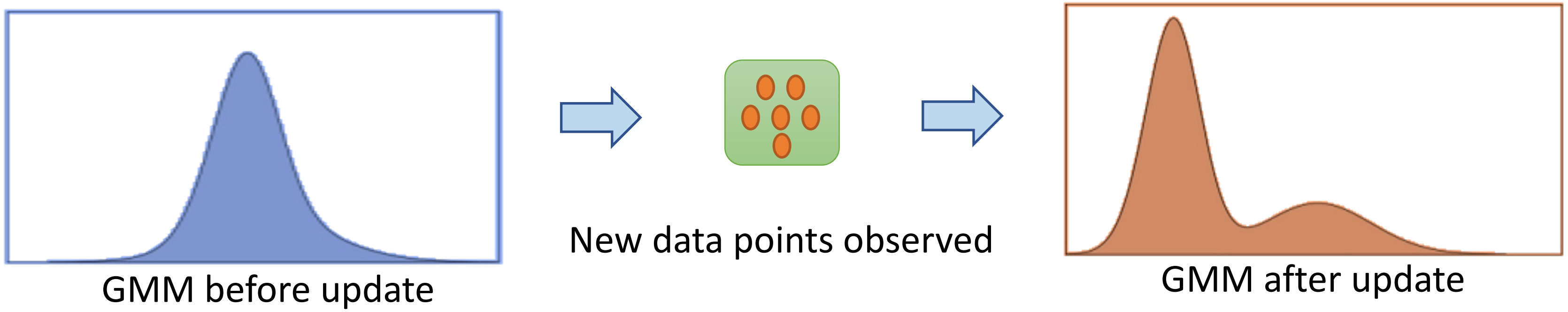


Incremental GMM Modeling for Time-varying Data



- Update weights as:

$$\omega_{k,t} = (1-\alpha)\omega_{k,t} + \alpha * M_{k,t}\,, \quad M_{k,t} = 1 \text{ for matched dist., 0 for others}$$

- Update means and covariances for the matched distribution as:

$$\mu_t = (1-\rho)\,\mu_{t\text{-}1} + \rho x_t$$

$$\sigma_t^2 = (1-\rho)\,\sigma_{t-1}^2 + \rho(x_t - \mu_t)^T(x_t - \mu_t)\,, \quad \rho = \alpha * N(x_t | \mu_{\mathrm{k}}, \sigma_{\mathrm{k}})$$

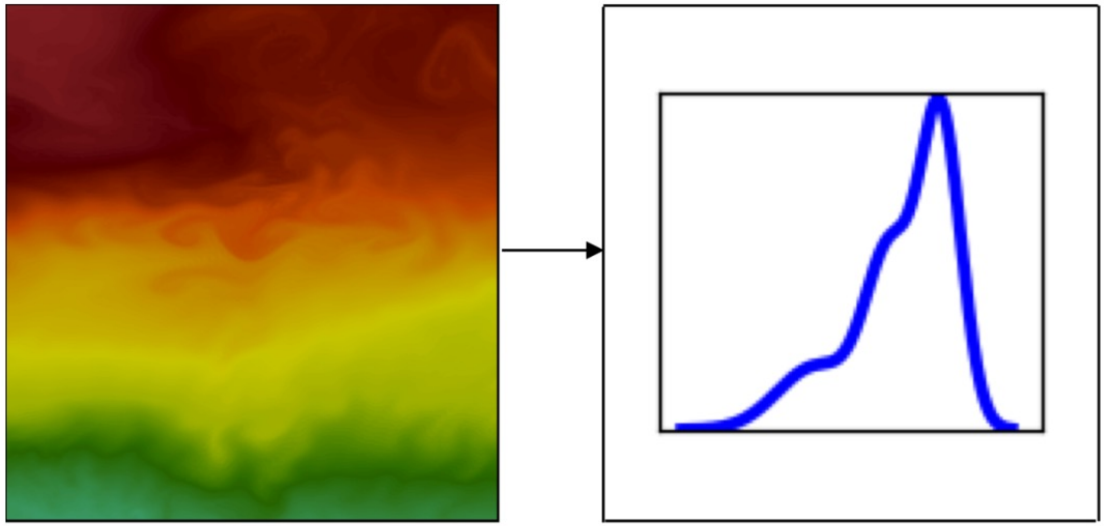
$\Sigma_{k,t} = \sigma_k^2 \mathrm{I}$, where I = Identity matrix, α = learning rate

Distribution-based Large Data Summarization and Visualization

Distribution Models for Big Data Summarization

- Distribution models that can be estimated efficiently and has a compact memory footprint is preferred over other models
- For Univariate Data:
  - Histograms: Fast but takes more space
  - KDE: Computationally expensive and takes more space
  - Gaussian: Fast but often the model is too simple
  - Gaussian mixture model: Parameter estimation can be a little expensive, but representation is compact
- For Multivariate Data:
  - Many of the standard multivariate models become either slow or space inefficient
  - Statistical Copula functions can be used effectively

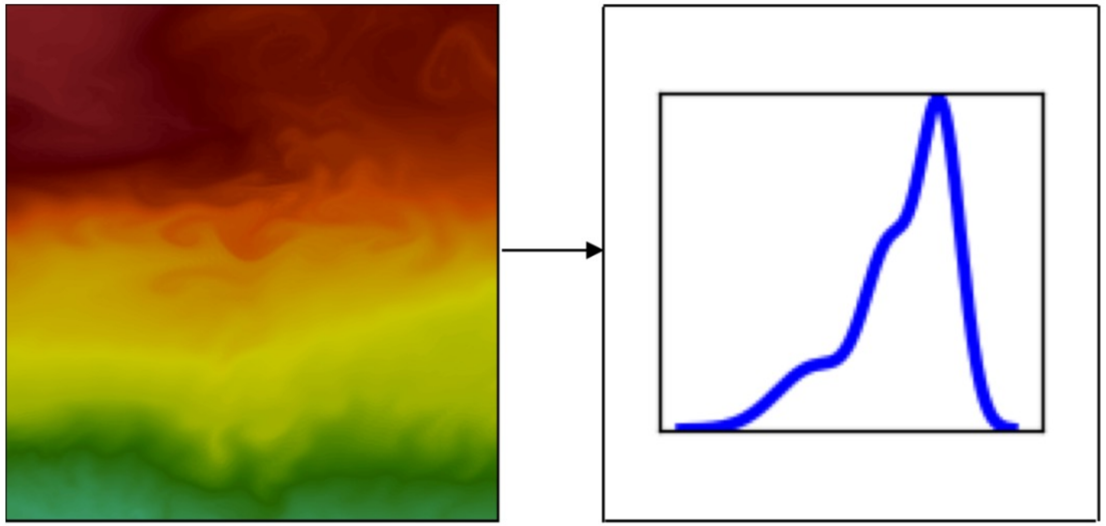
Distribution-based Data Summarization Strategies

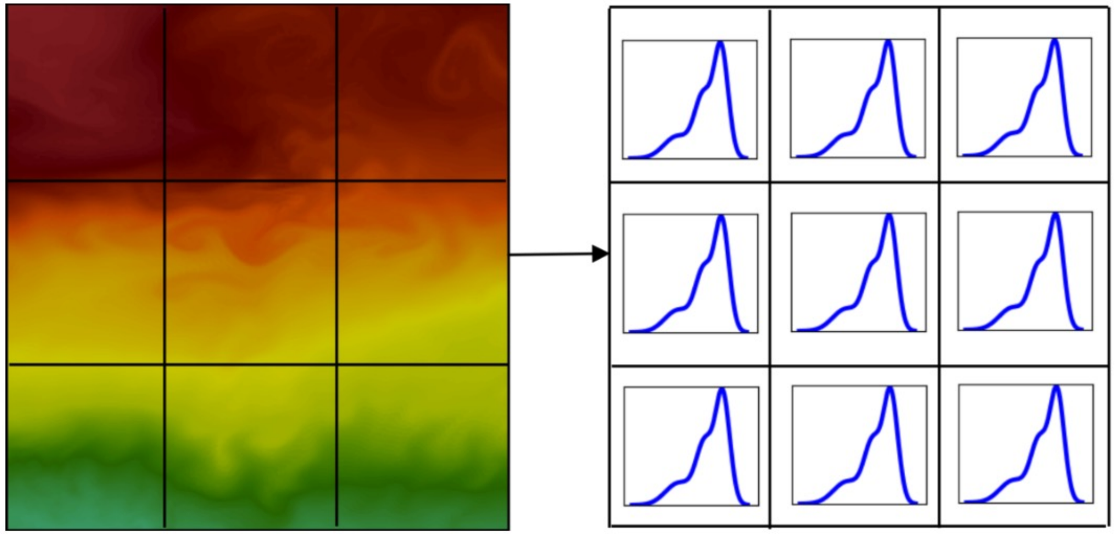


<u>Global distribution model:</u> A single distribution model to represent the entire data set

- Significant data reduction is possible
- Coarse representation of the data
- Not suitable for fine grained visual analysis

Distribution-based Data Summarization Strategies





Global distribution model: A single distribution model to represent the entire data set

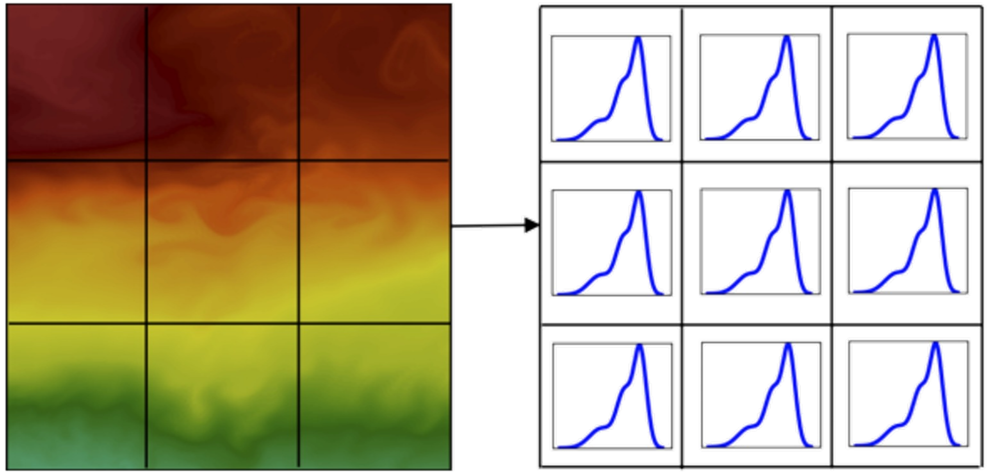
- Significant data reduction is possible
- Coarse representation of the data
- Not suitable for fine grained visual analysis

Local distribution model: Data is divided into small blocks and then each block is summarized using a separate distribution model

- Data reduction at an acceptable range is possible
- Fine details of the data and statistical properties are preserved
- Preferred over global model for scientific data summarization

Local/Region-wise Distribution-based Data Modeling

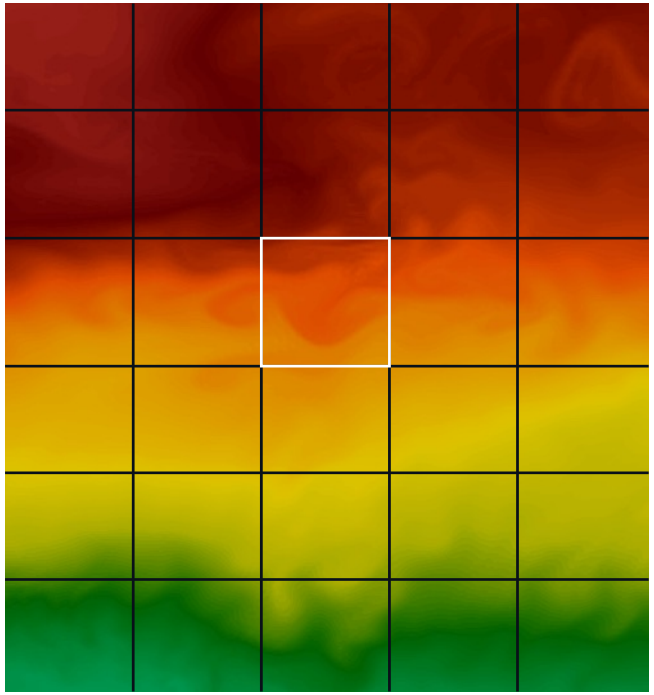
- Local Statistical distribution-based data modeling
  - Partition data into local regions
  - Summarize each region with a statistical distribution model
  - Benefits:
    - Distributions preserve local statistical data properties
    - Reduce data size significantly
    - Enables sampling-based analysis and reconstruction
    - Allows uncertainty quantification



Local distribution-based data model

Goals of a Region-wise Statistical Summarization

- Produce coherent partitions
  - Similar data values are grouped together
  - Partitions are spatially contiguous
- Preserve the statistical properties of the data accurately
  - Minimize sampling errors
  - Efficient feature analysis
- Use appropriate distribution models for summarization
  - A compact storage representation



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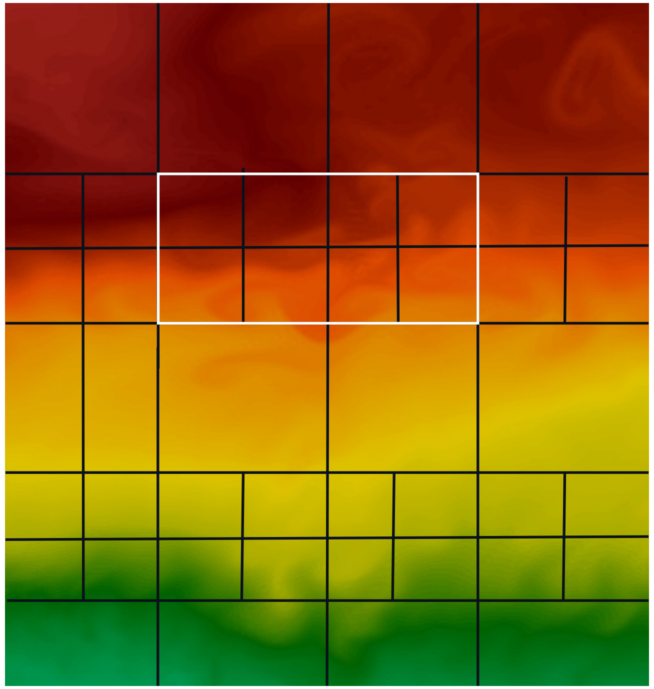
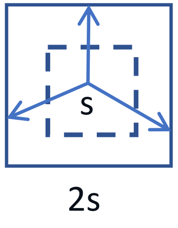
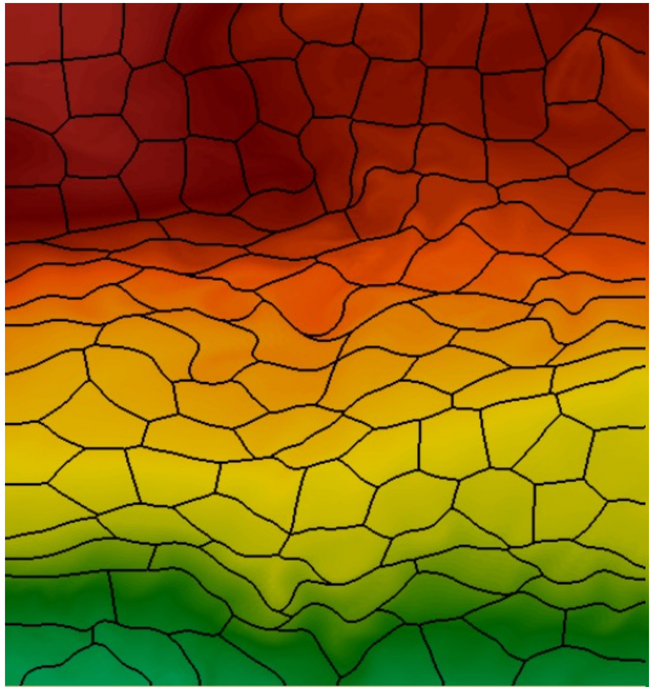


A Superior Solution for Region-wise Statistical Summarization

- Generate partitions based on data homogeneity
- Simple Linear Iterative Clustering (SLIC)
  - Produces irregular shaped partitions/clusters
  - Value variation inside partitions is minimized
  - Reduced sampling error



SLIC concept



$$dist(i,j) = \alpha . \| C_i - P_j \|_2 + (1-\alpha) . | val_i - val_j |$$

SLIC Algorithm Steps

Algorithm 1 Efficient superpixel segmentation

```
Initialize cluster centers C_k = [l_k, a_k, b_k, x_k, y_k]^T by sampling pixels at regular grid steps S.
Perturb cluster centers in an n × n neighborhood, to the lowest gradient position.
repeat
    for each cluster center C_k do
        Assign the best matching pixels from a 2S × 2S square neighborhood around the cluster center according to the distance measure (Eq. 1).
    end for
    Compute new cluster centers and residual error E {L1 distance between previous centers and recomputed centers}
until E ≤ threshold
Enforce connectivity.
```