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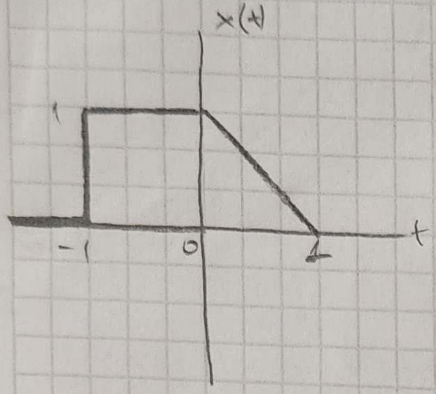
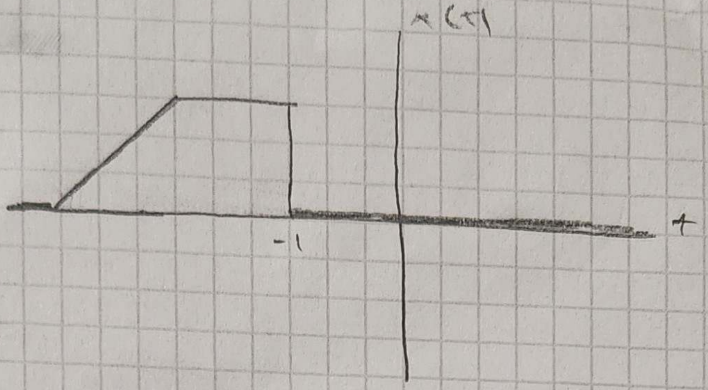
1)

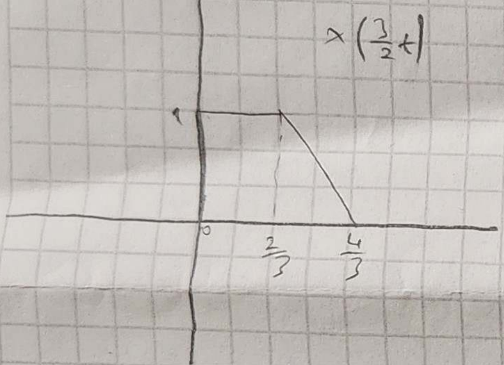
a) $x(t+1)$:



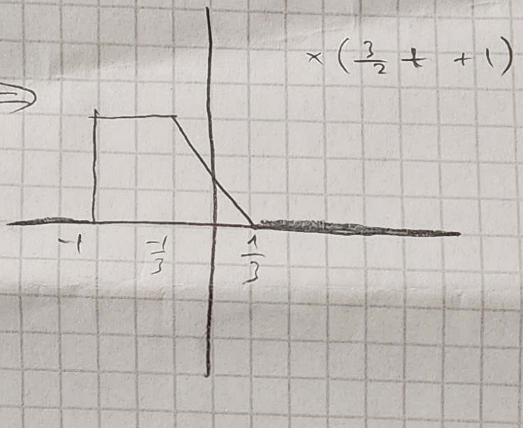
b) $x(-t+1)$



c) $x(\frac{3}{2}t+1)$



$\Rightarrow$



2) The given signal is a periodic triangular waveform. Since the signal is periodic, it cannot have finite energy, it would integrate to ∞, so it's not an energy signal.

The signal is periodic with period $T=2$, and the pattern between $t=-1$ to 1 repeats.

$$x(t) = \begin{cases} t+1, & -1 \leq t < 0 \\ 1-t, & 0 \leq t \leq 1 \end{cases}$$

so

$$P = \frac{1}{T}\int_0^T |x(t)|^2 dt = \frac{1}{2}\left[\int_{-1}^{0}(t+1)^2 dt + \int_0^1 (1-t)^2 dt\right]$$

$$\int_{-1}^{0}(t+1)^2 dt = \int_0^1 u^2 du = \frac{1}{3}$$

$$\int_0^1 (1-t)^2 dt = \int_0^1 u^2 du = \frac{1}{3}$$

$$P = \frac{1}{2}\left(\frac{1}{3}+\frac{1}{3}\right) = \frac{1}{3}$$

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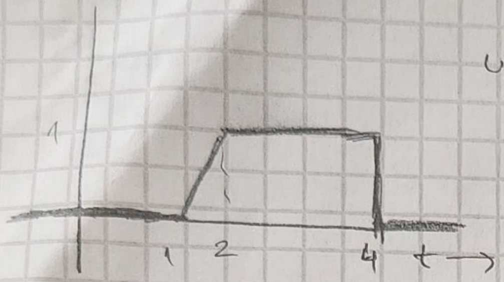
3) The signal rises linearly from 0 to 1 between $t=1$ and $t=2$, then stays at 1 until $t=4$, then drops to 0.



unit ramp function from 1 to 2:

$$x_1(t) = r(t-1) - r(t-2)$$

unit step function from 2 to 4:

$$x_2(t) = u(t-2) - u(t-4)$$

final expression: $x(t) = \left[r(t-1) - r(t-2)\right] + \left[u(t-2) - u(t-4)\right]$

4) System $y(t) = \frac{d}{dt} x(t)$

a) Casual or non-casual?

A system is casual if the output at time t depends only on values of the input at time t or earlier. Differentiation only uses instantaneous and past values, so its casual.

b) Time-varying or invariant?

A system is time-invariant if a timeshift in the input causes an identical time shift in the output. Differentiation is a linear and time-invariant operation.

c) Memoryless or with memory?

A system has memory if the output at t depends on past or future values of the input. To compute a derivative, we need values of $x(t)$ just before and just after time t. It depends on nearby values of $x(t)$ and not just at t. Therefore, the system has memory.

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5) $(D^2+2D+1)\,y(t) = (D+2)\,x(t) \quad y_0(0)=1, \; \dot{y}_0(0)=2$

to find zero-input response, $x(t)=0$

$(D^2+2D+1)\,y_0(t) = 0 \Rightarrow \ddot{y}_0 + 2\dot{y}_0 + y_0 = 0$ (second order homogeneous diff equation) with repeated roots

$r^2+2r+1=0 \Rightarrow (r+1)^2 = 0 \quad r=-1$

general solution: $y_0(t) = (A+Bt)e^{-t}$

Apply initial conditions: $y_0(0) = \underline{A = 1}$

$\dot{y}_0(t) = [-A - Bt + B]e^{-t} \Rightarrow \dot{y}_0(0) = -A+B = 2 \Rightarrow \underline{B=3}$

final equation $\Rightarrow$ $\underline{y_0(t) = (1+3t)e^{-t}}$

6) $h(t) = (3e^{-3t} + 2e^{-t})u(t)$ for input $x(t) = 5e^{-2t}$

$y(t) = h(t) * x(t) = (3e^{-3t} + 2e^{-t}) * 5e^{-2t}$

$= 3\cdot(e^{-3t} * 5e^{-2t}) + 2\cdot(e^{-t} * 5e^{-2t})$

convolution table identity: $e^{-at} * e^{-bt} = \frac{1}{b-a}(e^{-at} - e^{-bt}), \; a \neq b$

1. $e^{-3t} * e^{-2t} = \frac{1}{-2+3}(e^{-2t} - e^{-3t}) = (e^{-2t} - e^{-3t})$

2. $e^{-t} * e^{-2t} = \frac{1}{-2+1}(e^{-2t} - e^{-t}) = -1(e^{-2t} - e^{-t})$

$y(t) = 3\cdot 5(e^{-2t} - e^{-3t}) + 2\cdot 5(-e^{-2t} + e^{-t}) = 15(e^{-2t} - e^{-3t}) - 10(e^{-2t} - e^{-t})$

$= (5e^{-2t} - 15e^{-3t} + 10e^{-t})$

$\underline{y(t) = (10e^{-t} + 5e^{-2t} - 15e^{-3t})\,u(t)}$

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7) Let $x[n]$ be the students enrolled in semester n.

Books from students enrolled in: reused books:

- semester $n-1 = \frac{1}{3} \cdot x[n-1]$
- semester $n-2 = \frac{1}{3} \cdot x[n-2]$
- semester $n-3 = \frac{1}{3} \cdot x[n-3]$

$$\Rightarrow \frac{1}{3} \cdot \left[x[n-1] + x[n-2] + x[n-3]\right]$$

So, since $x[n]$ students will buy books in total, the books sold by publisher:

$$y[n] = x[n] - \frac{1}{3}\left(x[n-1] + x[n-2] + x[n-3]\right)$$

8) $y[n+2] - y[n+1] + 0.24y[n] = x[n+2] - 2x[n+1]$ $\quad y[-2] = 1$ ← not given in question but it makes sense to solve correctly.

$y[-1] = 2$

$x[n] = n u[n] = n$ for $n \geq 0$, 0 otherwise

$\boxed{n = -2}$

$y[0] - 2 + 0.24(1) = 0$, $y[0] = 2 - 0.24 = 1.76$

$\boxed{n = -1}$

$y[1] - 1.76 + 0.24(2) = 1$, $y[1] = 1 + 1.76 - 0.48 = 2.28$

$\boxed{n = 0}$

$y[2] - 2.28 + 0.24 \cdot (1.76) = 0$, $y[2] = 2.28 - 0.4224 = 1.8576$

$\boxed{y[0] = 1.76}$ $\boxed{y[1] = 2.28}$ $\boxed{y[2] = 1.8576}$

[redacted] [redacted]

9) $h[n] = \{1,1,1\}$ for $n = 0,1,2$ we need to find the

$x[n] = \{0.5, 2\}$ for $n = 0, 1$ Convolution $\underline{y[n] = x[n] * h[n]}$

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k].$$

$y[0] = x[0]\,h[0] = 0.5$

$y[1] = x[0]\,h[1] + x[1]\,h[0] = 0.5(1) + 2(1) = 2.5$

$y[2] = x[0]\,h[2] + x[1]\,h[1] = 0.5(1) + 2(1) = 2.5$

$y[3] = x[1]\,h[2] = 2 \cdot 1 = 2$

$y[n] = \{0.5, 2.5, 2.5, 2\}$

10) $\dfrac{8s+10}{(s+1)(s+2)^3}$ find the unilateral transform.

Partial fraction decomposition:

$$\frac{8s+10}{(s+1)(s+2)^3} = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{(s+2)^2} + \frac{D}{(s+2)^3}$$

equalize the denominators:

$$8s+10 = A(s+2)^3 + B(s+1)(s+2)^2 + C(s+1)(s+2) + D(s+1)$$

$\underline{\text{for } s=-2}$ $\quad -6 = -D$, $\underline{D=6}$

$\underline{\text{for } s=-1}$ $\quad 2 = A$, $\underline{A=2}$

now equation becomes $\Rightarrow$ $8s+10 = 2(s+2)^3 + B(s+1)(s+2)^2 + C(s+1)(s+2) + D(s+1)$

try two more values and eliminate one of B or C to find it.

$\underline{\text{for } s=0}$

$10 = 2 \cdot 2^3 + 4B + 2C + 6 \Rightarrow \overset{-3}{4B + 2C = -12}$

for $s=1$

$18 = 54 + 18B + 6C + 12 \Rightarrow + \underline{18B + 6C = -48}$

$6B = -12$

$\underline{B=-2}$ $\quad \underline{C=-2}$

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so, the equation becomes:

$$\frac{8s+10}{(s+1)(s+2)^3} = \frac{2}{s+1} - \frac{2}{s+2} - \frac{2}{(s+2)^2} + \frac{6}{(s+2)^3}$$

now take the inverse laplace transform term by term:

$$\mathcal{L}^{-1}\left\{\frac{1}{s+a}\right\} = e^{-at}$$

$$\mathcal{L}\left\{\frac{1}{(s+a)^n}\right\} = \frac{t^{n-1}}{(n-1)!} \cdot e^{-at}$$

$$\mathcal{L}^{-1}\left\{\frac{2}{s+1}\right\} = 2e^{-t}$$

$$\mathcal{L}^{-1}\left\{\frac{-2}{s+2}\right\} = -2e^{-2t}$$

$$\mathcal{L}^{-1}\left\{\frac{-2}{(s+2)^2}\right\} = -2te^{-2t}$$

$$\mathcal{L}^{-1}\left\{\frac{6}{(s+2)^3}\right\} = 6\frac{t^2}{2!}e^{-2t} = 3t^2e^{-2t}$$

$$\boxed{= 2e^{-t} - 2e^{-2t} - 2te^{-2t} + 3t^2e^{-2t}}$$