Quantum - Corpus

quantum info ← extension from classical info

Descriptions of quantum info:

1 simplified description

- Quantum states represented by vectors
  - operations represented by unitary matrices
- enough for most quantum algorithms

* General description

- quantum states rep. by density matrices.
- includes both simplified + classical info (including pos states) as special case.

[classical info]

consider physical sys. of info "x"

Let x be in 1 of a finite n # of classical states at each moment.

Let Σ = {n ∈ Classical finite states}

if x - bit: $\Sigma = \{0, 1\}$

if x - 6/s/dice: $\Sigma = \{1, \ldots, 6\}$

Σ cannot be empty (because 2 pos. states)

E.g. if X is a bit:

Assume $0 \to P: \frac{3}{4}$ and $1 \to P: \frac{1}{4}$

Probabilistic state $\begin{cases} Pr(X=0) = \frac{3}{4} \ \& \\ Pr(X=1) = \frac{1}{4} \end{cases}$

column vector: $\begin{pmatrix} 3/4 \\ 1/4 \end{pmatrix} \begin{array}{l} \leftarrow 0 \\ \leftarrow 1 \end{array}$ } Probability vector

$\forall u \in \vec{P},\ u \in \mathbb{R}^+$ s.t u is a probability

Assumption 1 entry for each possible state.

Dirac Notation

Context: Classical info

Let Σ be any classical state set, assume

$\forall u \in \Sigma$, u is in corr. with $\mathbb{N}$: $1, \dots, |\Sigma|$

Denote $|a\rangle$ - column vector s.t $\begin{cases} 1 & \text{if } x = a \\ 0 & \text{if } x \in \Sigma \setminus \{a\} \end{cases}$ $x \in \Sigma$

If $\Sigma = \{0, 1\}$, then:

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}\begin{array}{l} -0 \\ -1 \end{array} \text{ and } |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}\begin{array}{l} -0 \\ -1 \end{array}$$

classical state set (finite, nonempty)

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vectors of this form are called standard basis vectors → can be exp. as unique linear comb.

$$\begin{pmatrix} 3/4 \\ 1/4 \end{pmatrix} = \frac{3}{4}|0\rangle + \frac{1}{4}|1\rangle$$

Measuring Probabalistic States

Measure while "X" in prob. state?

look and recognize its state

we see classical state acc. to probabilities.

Let cs. $a \in \Sigma \Rightarrow \Pr(X = a) = 1$

we ref. prob. state by $|a\rangle$

consider p.s. of a bit X s.t:

$$\Pr(X=0) = \frac{3}{4} \quad \Pr(X=1) = \frac{1}{4}$$

Measuring X selects (or reveals) a transition outcome.

$$\frac{3}{4}|0\rangle + \frac{1}{4}|1\rangle$$

$|0\rangle \qquad |1\rangle$

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Deterministic Operations

Perform op → state changes

Every function $f : \Sigma \to \Sigma$ desc. a deterministic op.

transforms $a \to f(a)$ $\forall a \in \Sigma$

Given any $f : \Sigma \to \Sigma$, $\exists M$ satisfying:

$$M|a\rangle = |f(a)\rangle \quad \forall a \in \Sigma$$

M will have exactly 1, 1 in each col, 0 all else : entry whose row corr. b, and col : a

$$M(b,a) = \begin{cases} 1 & b = f(a) \\ 0 & b \neq f(a) \end{cases}$$

This action of this operation is desc. by :

matrix-vector-mult :

$$\vec{v} \to M\vec{v}$$

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Example for $\Sigma = \{0,1\}$, there are four $f : \Sigma \to \Sigma$

a	$f_1(a)$
0	0
1	0

a	$f_2(a)$
0	0
1	1

a	$f_3(a)$
0	1
1	0

a	$f_4(a)$
0	1
1	1

↓ respect. Matrix

$$M(\overset{\text{row}}{b},\overset{\text{col}}{a}) = \begin{cases} 1 & b = f(a) \\ 0 & b \neq f(a) \end{cases}$$

$$M_1 = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} \quad M_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad M_3 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad M_4 = \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix}$$

$$M|a\rangle = |f(a)\rangle$$

Let Σ be a classical state set, assume $\forall a \in \Sigma$, a are in correspondence $1, \dots, |\Sigma|$

We denote $\langle a|$ the row vectors having a 1 in the entry corresponding to $a \in \Sigma$, with 0 for all else

eg: if $\Sigma = \{0,1\}$, then:

$$\langle 0| = (1\ 0) \quad \text{and} \quad \langle 1| = (0\ 1)$$

in general $(x\ x\ x \dots)\begin{pmatrix} x \\ x \\ ? \\ ? \end{pmatrix} = (x)$ scalar

arising it's

$$\langle a||b\rangle = \begin{cases} 1 & a = b \\ 0 & a \neq b \end{cases}$$

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$$|0\rangle\langle 0| = \begin{pmatrix}1\\0\end{pmatrix}\begin{pmatrix}1 & 0\end{pmatrix} = \begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}$$

$$|0\rangle\langle 0| = \begin{pmatrix}1\\0\end{pmatrix}\begin{pmatrix}\cancel{1\ 0}\\0\ 1\end{pmatrix} = \begin{pmatrix}0 & 0\\0 & 0\end{pmatrix}$$

vector $\begin{cases} |\text{elmt}\rangle & \text{– ket} \\ \langle\text{elmt}| & \text{– bra}\end{cases}$

The matrix $|a\rangle\langle b|$ has 1 in (a, b) – entry, 0 all else.

Unitary matrix: orthogonality (prop. s.t two things are independent of eachother) over $\mathbb{R}$ field no influence

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$x^2 = -1$ is a number i s.t $i^2 = -1$ or $i = \sqrt{-1}$

Assume to show like an ordinary number

e.g $\underbrace{3+5}\times i$ { neither real nor imaginary

complex #: $c = a + b \times i = a + bi$, $a, b \in \mathbb{R}$

real part, imaginary

e.g $c_1 = 3 - i$ $\quad c_1 + c_2$
$c_2 = 1 + 4i$ $\quad = 3 - 1 + 1 + 4i$
$= (3+1) + (+4 + -1)i$
$= 4 + 3i$

im Not "x", real $i^2 = -1$.

~~Fnd~~ Fundamental Thm Algebra

Every polynomial equation of one var. with complex coefficients has a complex solution.

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