Quantum - Corpus

quantum info ← extension from classical info

Descriptions of quantum info:

1 simplified description

- Quantum states represented by vectors
  - operations represented by unitary matrices
- enough for most quantum algorithms

* General description

- quantum states rep. by density matrices.
- includes both simplified + classical info (including prob states) as special case.

[classical info]

consider physical sys. of info "x"

Let x be in 1 of a finite n # of classical states at each moment.

Let $\Sigma = \{n \in \text{Classical finite states}\}$

if x - bit: $\Sigma = \{0, 1\}$

if x - 6/s/dice: $\Sigma = \{1, \ldots, 6\}$

Σ cannot be empty (release 2 pos. state)

E.g. if X is a bit:

Assume $0 \to p : \frac{3}{4}$ and $1 \to p : \frac{1}{4}$

Probabilistic state $\begin{cases} Pr(X=0) = \frac{3}{4} \ \& \\ Pr(X=1) = \frac{1}{4} \end{cases}$

column vector: $\begin{pmatrix} 3/4 \\ 1/4 \end{pmatrix} \begin{array}{l} \leftarrow 0 \\ \leftarrow 1 \end{array}$ } Probability vector

thus $\vec{P}$, $u \in \mathbb{R}^+$ s.t u is a probability

Assumption 1 entry for each possible state.

Dirac Notation

Context: Classical info

Let Σ be any classical state set, assume

thus Σ, u is in corr. with $\mathbb{N}$: $1, \dots, |\Sigma|$

Denote $|a\rangle$ - column vector s.t $\begin{cases} 1 & \text{if } x = a \\ 0 & \text{if } x \in \Sigma \setminus \{a\} \end{cases}$ $x \in \Sigma$

If $\Sigma = \{0, 1\}$, then:

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{array}{l} -0 \\ -1 \end{array} \text{ and } |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \begin{array}{l} -0 \\ -1 \end{array}$$

classical state set (finite, nonempty)

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vectors of this form are called standard basis vectors → can be exp. as unique linear comb.

$$\begin{pmatrix} 3/4 \\ 1/4 \end{pmatrix} = \frac{3}{4}|0\rangle + \frac{1}{4}|1\rangle$$

Measuring Probabilistic States

Measure while "X" in prob. state?

look and recognize its state

we see classical state acc. to probabilities.

Let cs. $a \in \Sigma \Rightarrow \Pr(X = a) = 1$

we ref. prob. state by $|a\rangle$

consider p.s. of a bit X s.t:

$$\Pr(X=0) = \frac{3}{4} \quad \Pr(X=1) = \frac{1}{4}$$

Measuring X selects (or reveals) a transition outcome.

$$\frac{3}{4}|0\rangle + \frac{1}{4}|1\rangle$$

$|0\rangle \qquad |1\rangle$

QC

Deterministic Operations

Perform op → state changes

Every function $f: \Sigma \to \Sigma$ desc. a deterministic op.

transforms $a \to f(a)$ $\forall a \in \Sigma$

Given any $f: \Sigma \to \Sigma$, $\exists M$ satisfying:

$$M|a\rangle = |f(a)\rangle \quad \forall a \in \Sigma$$

M will have exactly 1, 1 in each col, 0 all else: entry whose row corr. b, and col: a

$$M(b,a) = \begin{cases} 1 & b = f(a) \\ 0 & b \neq f(a) \end{cases}$$

This action of this operation is desc. by matrix-vector-mult:

$$\vec{v} \to M\vec{v}$$