MATH 350 — Numerical Analysis

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Taylor Series

The series expansion, Taylor Series, is a function of infinite sums whose terms are expressed as the derivatives of the function being approximated.

0 Taylor Series

We use polynomials of degree n in order to approximate existing functions that we know. These are functions like $sin(x)$, $cos(x)$, $e^{(x)}$, or $ln(x)$. It is a useful method of approximation for complex functions using polynomial expansion to an nth degree at some center $x = \alpha$.

Definition (Taylor polynomial of a function f that is n times differentiable at $x = \alpha$)**.** A Taylor polynomial of a n times differentiable function at some center $x = \alpha$ is the polynomial

$$T_\infty(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-\alpha)^n = f(a) + f'(a)(x-a) + \cdots + \frac{f^{(n)}(a)}{n!}(x-a)^n$$

Accuracy of approximation is ensured as the degree n is increased or the size of the interval is decreased.