

# MATH 350 — Numerical Analysis

Based on lectures by Dr. Kevin Murphy

Notes taken by Pablo C. Bedolla Ortiz

Spring 2025

These notes are my own and are not endorsed by the lecturers. I have often made significant modifications to them after the lectures, so they may not accurately reflect the content presented in class. Any errors are almost certainly my own.

## **Taylor Series**

The series expansion, Taylor Series, is a function of infinite sums whose terms are expressed as the derivatives of the function being approximated.

## 0 Taylor Series

We use polynomials of degree  $n$  in order to approximate existing functions that we know. These are functions like  $\sin(x)$ ,  $\cos(x)$ ,  $e^{(x)}$ , or  $\ln(x)$ . It is a useful method of approximation for complex functions using polynomial expansion to an  $n$ th degree at some center  $x = \alpha$ .

**Definition** (Taylor polynomial of a function  $f$  that is  $n$  times differentiable at  $x = \alpha$ ). A Taylor polynomial of a  $n$  times differentiable function at some center  $x = \alpha$  is the polynomial

$$T_{\infty}(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x - \alpha)^n = f(a) + f'(a)(x - a) + \cdots + \frac{f^{(n)}(a)}{n!} (x - a)^n$$

Accuracy of approximation is ensured as the degree  $n$  is increased or the size of the interval is decreased.