<u>COMP310 Tutorial 9</u>

<u>Nash equilibria and Dominant Strategies</u>

For each of the payoff matrices, state whether each agent has a dominant strategy, and identify any outcomes that are in Pure Strategy Nash Equilibrium.

Solutions:

First, we have to remind ourselves of what is meant by a "strategy", a "dominant strategy" and by "Pure Strategy Nash Equilibrium".

When we talk about a strategy that an agent plays, we are generally referring to which action it chooses and how it chooses that particular action. The combination of actions played by agents in the game (…. Or mutli agent interaction) will then have an influence on the outcome.

As we know, agents have preferences over outcomes and are motivated to seek the outcome that they prefer the most. We represent the agents' preferences using **utility**, and this utility is often displayed as a payoff matrix.

We now discuss dominant strategies and nash equilibrium. It is important to note that when we are talking about an agent deviating from their current strategy (ie: changing from playing "D" to "C" and vice versa), then we assume that **all other agents moves stay the same**.

An agent's strategy is **dominant** if by playing this particular strategy, the agent is guaranteed the best payoff (as in, it can do no better by changing to another strategy). This must be true for every possible strategy that can be played by the opponents. To put it differently, the agent will always come off better by playing their dominant strategy, and will not regret playing by this strategy.

A particular outcome in a game is in **nash equilibrium** if no single agent has incentive to deviate from their current strategy, as they would not achieve payoff greater than what they already achieve. By that, we mean that no agent to change their mind, because there is no benefit to doing so.

Game 1

	Agent i Defects	Agent i Cooperates
Agent j Defects	2 2	0 5
Agent j Cooperates	5 0	3 3

Dominant strategy:

To check if a strategy is dominant for an agent, we must see that for each outcome reached by playing this strategy, the agent can't do any better by playing a different strategy, therefore we have to check the payoff values.

Let's check to see if "defect" is a dominant strategy for Agent i: there are 2 things we have to check here:

Ui(D,D) >= Ui(C,D) AND Ui(D,C) >= Ui(C,C)

If both of these hold, we know that Agent i is **always better off** by playing "Defect".

By looking at the payoff matrix for Game 1, we see that 2>0 and 5>3. Therefore, both of our conditions hold and we know that **"Defect" is a dominant strategy for Agent i.**

Let's now consider if "cooperate" is is dominant strategy for Agent j.

For this we have to check Agent J's payoff values when agent I plays both defect and cooperate.

There are 2 things we have to check here:

Uj(D,C) >= Uj(D,D) AND Uj(C,C) >= Uj(C,D)

If both of these hold, then "cooperate" is a dominant strategy for agent j.

Looking at the payoff matrix, we see that : 0>2 does **not hold**, and 3>5 **does not hold**, meaning that "cooperate" is **not a dominant strategy** for agent j.

By following the above method again for the other strategies of the agents, we will see **that both agents have a dominant strategy of "defect".**

Nash Equilibrium

To find out if an outcome is in nash equilibrium, we have to do the following:

- Fix all agent's strategies
- For each agent in turn, see if they get a better (strictly greater) payoff if they were to deviate from their current strategy

If **even just one** agent gets a better payoff by deviating, then this is **not** in nash equilibrium.

If **all** agents stick to their current strategy (ie: they can't do any better), then this is indeed **in nash equilibrium**.

This is important, as it means that by deviating, the agent knows it can actually perform better and gain a greater utility.

Lets now consider outcome (D,D).

To see if this is in nash equilibrium, we can follow the steps outlined above:

- Fix all strategies (agent I plays D, Agent J plays D)
- Does agent I get more payoff by playing C?
  - Ui(C,D) = 0; Ui(D,D) = 2; **No – Agent I does not deviate**
- Does agent J get more payoff by playing C?
  - Uj(D,C) = 0; Uj(D,D) 2; **No – Agent J does not deviate**
- **Therefore, (D,D) is in Nash Equilibrium.**

By following the above method for the other outcomes, we can see that (D,D) is the **only outcome** in this game that is in nash equilibrium.

We now give the answers for the remaining games, using the methods we have described.

Game 2

	Agent i Defects	Agent i Cooperates
Agent j Defects	1 1	0 2
Agent j Cooperates	2 0	3 3

Dominant strategies: None

Nash Equilibrium: (D,D), (C,C)

Game 3

	Agent i Defects	Agent i Cooperates
Agent j Defects	0 0	1 3
Agent j Cooperates	3 1	2 2

Dominant strategies: None

Nash Equilibrium: (D,C), (C,D)