

# COMP318: RDFS entailment

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# Where were we

- SPARQL
  - RDF query language
    - Based on simple entailment
  - Syntax and exercises

# Semantics

- RDF(S) vocabulary has built-in “meaning”
- RDF(S) Semantics
  - Makes meaning explicit
  - Defines what follows from an RDF graph
- Goals
  - Evaluate the truth of a triple/graph
  - Characterise the state of the world that makes a triple/graph true.

# RDF(S) entailment

- An RDF(S) graph entails implicit triples
  - triples not explicitly contained in the graph, but that can be derived from an RDF(S) graph
    - **using the special semantics of the vocabulary of the graph**
      - **vocabulary of the graph:** set of names which occurs as the subject, predicate, object
  - **Interpretations** assign special meaning to the symbols in a particular vocabulary
  - **Interpretations** (Normative)
    - Mapping of RDF assertions into an abstract model, based on set-theory
    - With an “interpretation operator”  $I()$ , maps a RDF graph into a highly abstract set of high-cardinality sets
    - Highly theoretical model, useful to prove mathematical properties
  - **Entailments** (Informative)
    - Transformation rules to derive new assertions from existing ones
    - May be proven complete and consistent with the formal interpretation

# Entailment regimes

- Three entailment regimes
  - **simple entailment:** no particular extra conditions are posed on a vocabulary, including the RDF vocabulary itself;
    - it involved only graph transformations.
  - **RDF entailment:** based on the interpretation of the RDF vocabulary;
  - **RDFS entailment:**
    - based on the interpretation of the RDFS vocabulary
    - some extra conditions are posed by in the form of axiomatic triples and semantic conditions

# Inference rules

- An inference rule is a rule of the form:

$$\frac{\phi_1, \phi_2, \dots, \phi_n}{\psi}$$

- where  $\phi_1, \phi_2, \dots, \phi_n$  are sentences in the language (**assumptions**), whilst  $\psi$  is a new sentence derived from the assumptions (**conclusion**)
- Inference rules are a formal description of the process for constructing new expressions from existing ones.
  - In RDF, inferences corresponding to **entailments** are described as **correct** or **valid**.

# Proof theory

- Every formal logic has a set of inference rules that can be used to “prove” some formula  $\mu$  from a given set of formulas  $\Gamma$
- A **formal proof** is the sequential application of the inference rules that starts with  $\Gamma$  and ends with  $\mu$ 
  - $\Gamma \vdash \mu$ ,  $\mu$  can be proved from  $\Gamma$

# Soundness and completeness

- An inference mechanism is **sound** if it derives only sentences that are entailed.
  - If  $\Gamma \vdash \mu$  then  $\Gamma \models \mu$
- An inference mechanism is **complete** if derives all the sentences that are entailed.
  - If  $\Gamma \models \mu$  then  $\Gamma \vdash \mu$



# RDF inference rules

- The W3C recommendation “RDF Semantics” provides the inference rules that corresponds to the various form of entailments mentioned;
  - simple entailment
  - RDF entailment
  - RDFS entailment

# Notation

- a, b,
  - refer to any arbitrary URI
    - (i.e. anything that can appear in the predicate of a triple)
- u, v,
  - refer to any arbitrary URI or blank node ID
    - (i.e. anything that can appear in the subject of a triple)
- x, y,
  - refer to an arbitrary URI, blank node ID or literal
    - (i.e. anything that can appear in the object of a triple)
- \_:n,
  - refer to the ID of a blank node
    - (i.e. appearing as a subject or object)
- l,
  - refers to a literal
    - (i.e. a string that is sometimes found in the object)
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# RDF Entailment

- The RDF entailment has 4 inference rules:
- **(rdfax)** Infer the triple  $u \text{ a } x$ . for every RDF axiomatic triple  $u \text{ a } x$ .
- **(lg, literal generalisation)** If  $G$  contains  $u \text{ a } l$ . then infer the triple  $u \text{ a } \_ : n$ .
  - Specialised version of SE1 that allows generalisation of a literal by a blank node
    - Other properties of this literal can be inferred via this blank node: e.g. the literal is an instance of a class
    - Literals can only appear as objects in a triple

# RDF Entailment

- The RDF entailment has 4 inference rules:
- **(rdf1)** If G contains a triple  $u \text{ a } y$ . then we can infer
  - $a \text{ rdf:type rdf:Property}$ .
- **(rdf2)** If G contains a triple  $u \text{ a } l$ . where  $l$  is a well-formed XML literal then we can infer
  - $\_ : n \text{ rdf:type rdf:XMLLiteral}$ .

# RDF entailment

- **Theorem.** A graph  $G1$  RDF-entails a graph  $G2$  if and only if there is a graph  $G1'$  that can be derived from  $G1$  by using the rules `rdfax`, `lg`, `rdf1` and `rdf2` such that  $G1'$  simply entails  $G2$ .

# Inference Rules for RDFS-entailment

- Assign “meaning” to the RDFS vocabulary
- **(rdfsx)** Infer the triple  $u \text{ a } x.$  for every RDFS axiomatic triple  $u \text{ a } x.$
- **(RDFS1, literal)** If G contains  $u \text{ a } l.$  where  $l$  is a plain literal (with or without language information), then infer the triple:
  - $\_ : n \text{ rdf:type rdfs:literal.}$

# Domain and range restrictions

- **(rdfs2)** If G contains a triples `a rdfs:domain x`.  
`u a y`. then we can infer
  - `u rdf:type x`.
- **(rdf3)** If G contains a triples `a rdfs:range x`.  
`u a v`. then we can infer
  - `v rdf:type x`.

# Everything is a resource

- **(rdfs4a)** If G contains a triple  $u \text{ a } x$ . then we can infer
  - $u \text{ rdf:type rdfs:Resource}$ .
- **(rdfs4b)** If G contains a triple  $u \text{ a } v$ . then we can infer
  - $v \text{ rdf:type rdfs:Resource}$ .
- We do not need an inference rule for predicates:
  - the relevant triple can be derived using **rdf1** and **rdfs4**



# Important property of binary relations

- binary relation  $R$  on a set  $A$  is said to be:
  - Reflexive: if  $x R x$ , for all  $x \in A$ 
    - A number **is equal to** itself
  - Irreflexive: if not  $x R x$ , for all  $x \in A$ 
    - A number  $x$  **is not equal to**  $(x+1)$
  - Symmetric: if  $x R y$  implies  $y R x$ , for all  $x, y \in A$ 
    - **marriedTo**: if Ross **marriedTo** Rachel then Rachel **marriedTo** Ross
  - Asymmetric: if  $x R y$  not  $A$  implies  $y R x$ , for all  $x, y \in A$ 
    - **parentOf**: if Rachel **parentOf** Emma then it does not imply Emma **parentOf** Rachel
  - Transitive: if  $x R y$  and  $y R z$  implies  $x R z$ , for all  $x, y, z \in A$ 
    - **friendOf**: if Monica **friendOf** Joey and Joey **friendOf** Phoebe then Monica **friendOf** Phoebe

# Reflexivity and Transitivity of `rdfs:subPropertyOf`

- **(rdfs5)** If G contains the triples  
`u rdfs:subPropertyOf v.` and `v rdfs:subPropertyOf x.` we can infer
  - `u rdfs:subPropertyOf x.`
- **(rdfs6)** If G contains the triple  
`u rdf:type rdf:Property.` we can infer
  - `u rdfs:subPropertyOf u.`

# More on Subproperties

- **(rdfs7)** If G contains the triples  
`a rdfs:subPropertyOf b.` and `u a y.` we can infer
  - `u b y.`

# Classes and instances

- **(rdfs8)** If G contains the triple  
`u rdf:type rdfs:Class.` we can infer
  - `u rdfs:subClassOf rdfs:Resource.`
- **(rdfs9)** If G contains the triples  
`u rdfs:subClassOf x.` and `v rdf:type u.` we can infer
  - `v rdf:type x.`

# Reflexivity and Transitivity of `rdfs:subClassOf`

- **(rdfs10)** If G contains the triple  
`u rdf:type rdfs:Class.` we can infer
  - `u rdfs:subClassOf u.`
- **(rdfs11)** If G contains the triples  
`u rdfs:subClassOf v.` and `v rdfs:subClassOf x.` we can infer
  - `u rdfs:subClassOf x.`

# Containers

- (rdfs12) If G contains the triple  
u rdf:type  
rdfs:ContainerMembershipProperty. we can  
infer
  - u rdfs:subPropertyOf rdfs:member.

# Datatypes

- **(rdfs13)** If G contains the triple  
`u rdf:type rdfs:Datatype.` we can infer
  - `u rdfs:subClassOf rdfs:Literal.`

# Bijection between literals and surrogate bonds

- **(gl inverse of lg)** If G contains the triple  $u \text{ a } \_ : n .$  we can infer
  - $u \text{ a } 1 .$  However:
    - this rule can be applied only when  $\_ : n$  identifies a bnode that was introduced earlier by weakening the literal 1 via the rule **lg**
    - This inference rule is necessary to bring back a literal that has been substituted by a blank node using rule **lg**, then some other inference rule produced a triple with this blank node ( $\_ : n$ ) in the object position, and rule **gl** can now be used to bring this literal back!
      - E.g. `:Murray atp:name "Andy Murray".`  
`:atp:name rdfs:range atp:PlayerName.`
      - Would entail `"Andy Murray" a atp:PlayerName.` which is problematic. Why?



# Bijection between literals and surrogate bonds

- The latter triple is not a valid RDF triple
  - a literal should not appear in the subject position!
- thus it will not be inferred (the domain of the ?v variable in the rdfs3 rule would prevent the inference). And so, to achieve the valid inference:

- `_:AndyMurray a atp:PlayerName .`

Requires the surrogate blank node (`_:AndyMurray`) to be used through the rule lg.

- The inverse rule gl then allows surrogates to “travel” back as literals into the object position, though examples of such behaviour are not necessarily intuitive.

# Conclusion

- RDF entailment rules
- RDFS entailment rules