Slide 14 - Quiz: Linear independence

You need to prove that **v** cannot be expressed as **a** and **b** and show the contradiction.

Using coefficients λ and μ, when you multiply these coefficients into **a** and **b** respectively, you should get **v**.

v = $(1, 2, -3, 4)^T$

a = $(1, 1, 0, 2)^T$

b = $(-1, -2, 1, 1)^T$

Use the coefficients and take the sum:

v = **a** + **b**

$1 = (1 \times \lambda) + (-1 \times \mu) = \lambda - \mu$

$2 = (1 \times \lambda) + (-2 \times \mu) = \lambda - 2\mu$

$-3 = (0 \times \lambda) + (1 \times \mu) = \mu$

$4 = (2 \times \lambda) + (1 \times \mu) = 2\lambda + \mu$

So, $\mu = -3$

Now find λ, plug $\mu = -3$ into the above equations:

$4 = 2\lambda + \mu$

$4 = 2\lambda + -3$

$3 + 4 = 2\lambda$

$7 = 2\lambda$

$\lambda = 7/2 = 3.5$

$\lambda = 3.5$

Plug in values for λ and μ to check for contradictions:

$2 = \lambda - 2\mu$

$2 = 3.5 - (2 \times -3) = 9.5$

This is a contradiction since 9.5 does not equal 2. So **v** cannot be expressed as a linear combination of **a** and **b**.

Slide 16 - Quiz

To find the ranks for these matrices **A** and **B**. Choose any row or column and try to make any elements 0.

Matrix A

1	0	1
0	1	1
0	0	0

3rd row are all 0's, so rank should be 2 since you can completely omit this row from matrix. So the number of linearly independent rows for matrix A is 2, so the rank is 2.

Matrix B

1	2	1
-2	-3	1
3	5	0

Eliminate elements in 1st column under 1st element.

Add 2 to all elements in 2nd row to get 0 for 1st element in row 2 and multiply by elements in row 1 r2 + (2r1):

-2 + (2x1) = 0

-3 + (2x2) = 1

1 + (2x1) = 3

Subtract 3 from all elements in 3rd row to get 0 for 1st element in row 3 and multiply by elements in row 1

r3 – (3r1):

3 - (3x1) = 0

5 - (3x2) = -1

0 - (3x1) = -3

1	2	1
0	1	3
0	-1	-3

Eliminate elements in 2nd column under 2nd element.

Add 1 to get 0 for 2nd column under 2nd element and multiply by elements in row 2 (leave column 1 alone)

r3 + (1 x r2):

-1 + (1x1) = 0

-3 + (1x3) = 0

1	2	1
0	1	3
0	0	0

So row 3 has 0s, so rank of matrix is therefore 2.

<u>Slide 19 - Quiz</u>

Find the eigenvalues and the corresponding eigenvectors of A.

4	2
1	3

Eigenvalues of A are the roots of the characteristic equation det $(A - \lambda I) = 0$

$(4 - \lambda)$	2	
1	$(3-\lambda)$	= 0

$\therefore (4-\lambda)\times(3-\lambda)-2\times1 = 0$

$\therefore (12-7\lambda+\lambda^2)-2 = 0$

$\therefore (\lambda^2-7\lambda+10) = 0$

$\therefore (\lambda-2)(\lambda-5) = 0$

$\therefore (\lambda-2) = 0$ or $(\lambda-5) = 0$

$\therefore$ The eigenvalues of the matrix A are given by $\lambda=2, 5$

Using these values you can get the eigenvectors which should be:

$[-1, 1]^T$ where $\lambda=2$

$[2, 1]^T$ where $\lambda=5$