Lecture 2 – Mathematical Preliminaries – Quiz Solutions

Slide 6 - Quiz

Find $x + y$

Just add the corresponding dimensions:

1+3=4   2+2=4   3+1=4

Find $x \otimes y$

To find the element-wise product of x and y just multiply them together:

1x3=3   2x2=4   3x1=3

Find $x^T y$

To find the inner product of x and y just multiply them together and then add them up:

1x3=3   2x2=4   3x1=3

3+4+3=10

Find xy^T

Find the outer product of x and y:

1x3=3   1x2=2   1x1=1

2x3=6   2x2=4   2x1=2

3x3=9   3x2=6   3x1=3

So now you have a 3 by 3 matrix for the outer product:

$$\begin{matrix} 3 & 2 & 1 \\ 6 & 4 & 2 \\ 9 & 6 & 3 \end{matrix}$$

Slide 8 - Quiz

Compute A+B:

Just add the corresponding elements together:

1+0=1   2+1=3   3+0=3   4+1=5   5+2=7   6+3=9   7+-1=6 8+0=8   9+1=10

$$\begin{matrix} 1 & 3 & 3 \\ 5 & 7 & 9 \\ 6 & 8 & 10 \end{matrix}$$

Compute B+A:

This should be the same result as A+B above.

$$\begin{matrix} 1 & 3 & 3 \\ 5 & 7 & 9 \\ 6 & 8 & 10 \end{matrix}$$

Compute AB:

Multiply the 1st row in matrix A by the 1st column in matrix B (inner product between the 2 vectors), then to get the 1st element you add them up:

1st element: 1x0=0   2x1=2   3x-1=-3   0+2+-3=-1

To get the 2nd element you multiply the 1st row in matrix A by the 2nd column in matrix B, then add:

2nd element: 1x1=1   2x2=4   3x0=0   1+4+0=5

3rd element: 1x0=0   2x3=6   3x1=3   0+6+3=9

Same again for the 2nd row in matrix A multiplied by the columns in matrix B:

1st element: 4x0=0   5x1=5   6x-1=-6   0+5+-6=-1

2nd element: 4x1=4   5x2=10 6x0=0   4+10+0=14

3rd element: 4x0=0   5x3=15 6x1=6   0+15+6=21

3rd row of A multiplied by the 1st 2nd and 3rd columns of B:

1st element: 7x0=0   8x1=8   9x-1=-9   0+8+-9=-1

2nd element: 7x1=7   8x2=16 9x0=0   7+16+0=23

3rd element: 7x0=0   8x3=24 9x1=9   0+24+9=33

So your matrix for AB should now be:

$$\begin{matrix} -1 & 5 & 9 \\ -1 & 14 & 21 \\ -1 & 23 & 33 \end{matrix}$$

Compute BA:

Multiply the 1st row in matrix B by the 1st column in matrix A and then add them up:

1st row 1st element:	0x1=0	1x4=4	0x7=0	0+4+0=4
1st row 2nd element:	0x2=0	1x5=5	0x8=0	0+5+0=5
1st row 3rd element:	0x3=0	1x6=6	0x9=0	0+6+0=6
2nd row 1st element:	1x1=1	2x4=8	3x7=21	1+8+21=30
2nd row 2nd element:	1x2=2	2x5=10	3x8=24	2+10+24=36
2nd row 3rd element:	1x3=3	2x6=12	3x9=27	3+12+27=42
3rd row 1st element:	-1x1=-1	0x4=0	1x7=7	-1+0+7=6
3rd row 2nd element:	-1x2=-2	0x5=0	1x8=8	-2+0+8=6
3rd row 3rd element:	-1x3=-3	0x6=0	1x9=9	-3+0+9=6

So, your matrix for BA should be:

$$\begin{matrix} 4 & 5 & 6 \\ 30 & 36 & 42 \\ 6 & 6 & 6 \end{matrix}$$

You can see the computations for AB and BA are not equal. So the matrix product is not commutative in general since they don't give you the same results.

Slide 10 - Computing the inverse of a 2x2 matrix and Slide 12 – Matrix Inversion

You basically multiply your 2 by 2 matrix by its inverse.

Matrix A

$$\begin{matrix} 1 & 2 \\ -2 & 1 \end{matrix}$$

Use this formula:

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{\det\begin{pmatrix} a & b \\ c & d \end{pmatrix}} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$$= \frac{1}{\det\begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix}} \begin{pmatrix} 1 & -2 \\ -(-2) & 1 \end{pmatrix}$$

Find the matrix determinant according to formula:

$det\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ = ad – bc

= $1 \cdot 1 - 2(-2) = 5$

Multiply $\frac{1}{5}$ by your inverse matrix:

$\frac{1}{5}\begin{pmatrix} 1 & -2 \\ -(-2) & 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{5} & \frac{-2}{5} \\ \frac{2}{5} & \frac{1}{5} \end{pmatrix}$

Note that you can check this answer is correct by performing matrix multiplication AA^{-1} (see slide 9 in lecture 2) and the result should be according to this:

Identity Matrix I = $\begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}$

Remember it says all diagonal elements are set to 1 and non-diagonal elements are set to 0. So to check this we can multiply the matrix A by its inverse matrix A^{-1} which is what we already did above:

A = $\begin{matrix} 1 & 2 \\ -2 & 1 \end{matrix}$

$A^{-1} = \begin{pmatrix} \frac{1}{5} & \frac{-2}{5} \\ \frac{2}{5} & \frac{1}{5} \end{pmatrix}$

So, then if you multiply rows by columns and then add them together:

1st row 1st element: 1x$\frac{1}{5}$ + 2x$\frac{2}{5}$

1st row 2nd element: 1x$\frac{-2}{5}$ + 2x$\frac{1}{5}$

2nd row 1st element: -2x$\frac{1}{5}$ + 1x$\frac{2}{5}$

2nd row 2nd element: -2x$\frac{-2}{5}$ + 1x$\frac{1}{5}$

$\frac{1}{5} + \frac{4}{5}$ $\quad$ $\frac{-2}{5} + \frac{2}{5}$

$\frac{-2}{5} + \frac{2}{5}$ $\quad$ $\frac{4}{5} + \frac{1}{5}$

So then you have proved the answer to be correct since it’s the same as the identity matrix:

$\begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}$