1. Find the sequence generation rule of 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610

Ans. 這是一個 Fibonacci 數列，其生成的公式為

- $F_0 = 0$
- $F_1 = 1$
- $F_n = F_{n-1} + F_{n-2}\ (n \geqq 2)$

2. Extra credit

Use Berlekamp–Massey algorithm to find out the sequence rule of 0, 1, 1, 2, 3, 5, 8, 13, 21, 34

Ans.

$S(x) = x^8 + x^7 + 2x^6 + 3x^5 + 5x^4 + 8x^3 + 13x^2 + 21x + 34$

$r(x) = x^9$

$1\ 1\ 2\ 3\ 5\ 8\ 13\ 21\ 34 \overline{)\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0}\ \ 1-1$

$1\ 1\ 2\ 3\ 5\ 8\ 13\ 21\ 34$

$-1\ -2\ -3\ -5\ -8\ -13\ -21\ -34\ 0$

$-1\ -1\ -2\ -3\ -5\ -8\ -13\ -21\ -34$

$-1\ -1\ -2\ -3\ -5\ -8\ -13\ 34$

$\Rightarrow r(x) = S(x)(x-1) + (-x^5 - x^6 - 2x^5 - 3x^4 - 5x^3 - 8x^2 - 13x + 34)$

$-1\ -1\ -2\ -3\ -5\ -8\ -13\ 34 \overline{)\ 1\ 1\ 2\ 3\ 5\ 8\ 13\ 21\ 34}\ \ -1$

$1\ 1\ 2\ 3\ 5\ 8\ 13\ -34$

$55\ 34$

$\Rightarrow S(x) = (-x^5 - x^6 - 2x^5 - 3x^4 - 5x^3 - 8x^2 - 13x + 34)\cdot(-x) + 55x + 34$

$\Rightarrow \dfrac{S(x) - (55x+34)}{-x} = (-x^5 - x^6 - 2x^5 - 3x^4 - 5x^3 - 8x^2 - 13x + 34)$

$\Rightarrow r(x) = S(x)(x-1) - x^{-1}S(x) + 55 + 34x^{-1}$

$\times x \Rightarrow r(x)\cdot x = S(x)(x^2 - x - 1) + 55x + 34$

$\Rightarrow r(x)\cdot x + (-x^2 + x + 1)S(x) = 55x + 34$

$\Rightarrow (-x)\cdot r(x) + (x^2 - x - 1)S(x) = -55x - 34$

$\Rightarrow x^2 - x - 1 = 0 \Rightarrow x^2 = x + 1$

結論: 根據 Berlekamp–Massey algorithm，也能夠推導出生成公式 x^2 = x + 1