1

a.

$x \leftarrow$ válido, término

b.

$x\ x \leftarrow$ válido, término

c.

$M \leftarrow$ inválido

d.

$M\ M \leftarrow$ inválido

e.

true false $\leftarrow$ válido, término

f.

true succ(false true) $\leftarrow$ válido, término

g.

$\lambda x.\text{isZero}(x) \leftarrow$ inválido

h.

$\lambda x : \sigma.\text{succ}(x) \leftarrow$ inválido

i.

$\lambda x : \text{Bool. succ}(x) \leftarrow$ válido, término

j.

$\lambda x : \text{if true then Bool else Nat}.x \leftarrow$ inválido

k.

$\sigma \leftarrow$ inválido

l.

Bool $\leftarrow$ válido, tipo

m.

Bool $\rightarrow$ Bool $\leftarrow$ válido, tipo

n.

Bool $\rightarrow$ Bool $\rightarrow$ Nat $\leftarrow$ válido, tipo

ñ.

(Bool $\rightarrow$ Bool) $\rightarrow$ Nat $\leftarrow$ válido, tipo

o.

succ true $\leftarrow$ inválido

p.

λx : Bool. if zero then true else zero succ(true) $\leftarrow$ válido, término

3

a.

Marcar las ocurrencias del término x como subtérmino en $\lambda x : \text{Nat. succ}((\lambda x : \text{Nat. } x)\ x)$.

Podemos plantear una α equivalencia tal que:

$\lambda x : \text{Nat. succ}((\lambda x : \text{Nat. } x)\ x) =_{\alpha} \lambda z : \text{Nat. succ}((\lambda y : \text{Nat. } y)\ z)$

Por lo que hay 0 ocurrencias.

b.

¿Ocurre x_1 como subtérmino en $\lambda x_1 : \text{Nat. succ}(x_2)$?

No

c.

¿Ocurre $x\ (y\ z)$ como subtérmino en $u\ x\ (y\ z)$?

$u\ x\ (y\ z) = (u\ x)\ (y\ z)$, entonces no.

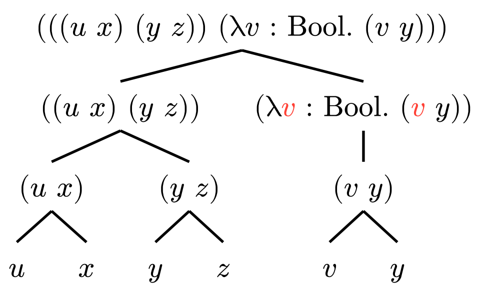
4

a.

$u\ x\ (y\ z)\ (\lambda v : \text{Bool. } v\ y)$

$= (((u\ x)\ (y\ z))\ (\lambda v : \text{Bool. } (v\ y)))$

Árbol

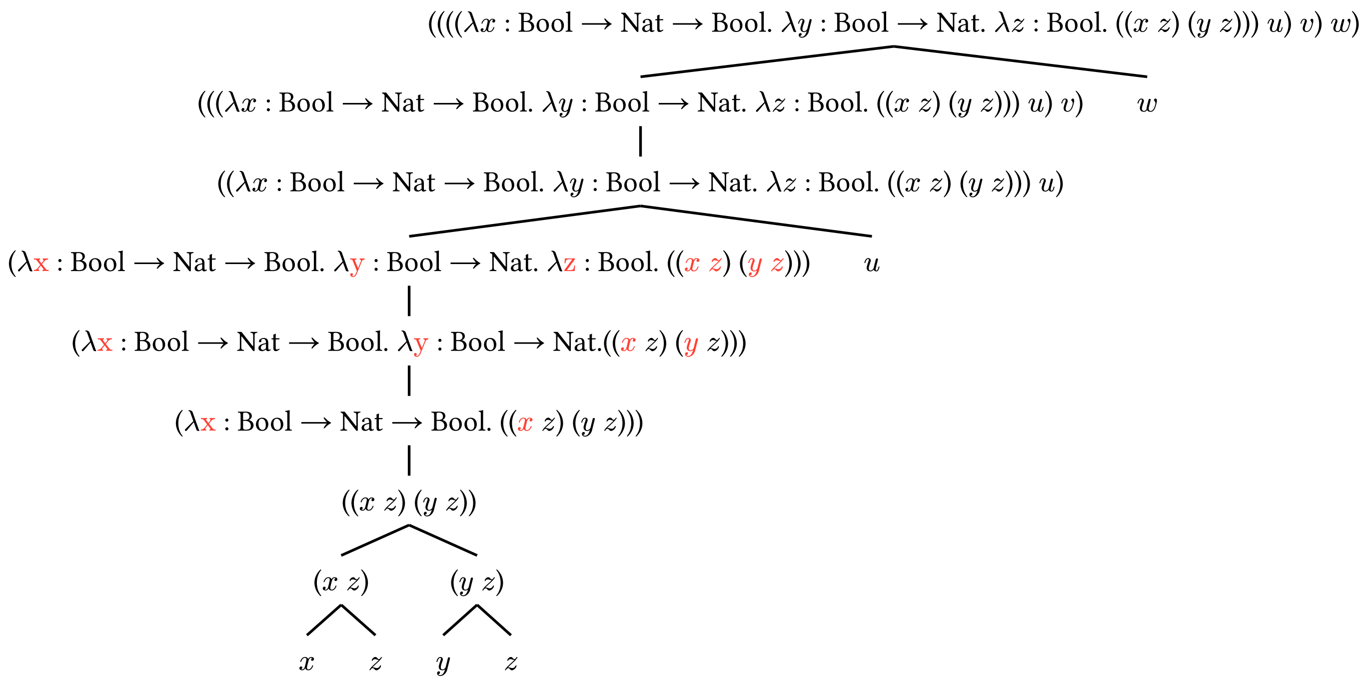


b.

$(\lambda x : \text{Bool} \to \text{Nat} \to \text{Bool. } \lambda y : \text{Bool} \to \text{Nat. } \lambda z : \text{Bool. } x\ z\ (y\ z))\ u\ v\ w$

$= ((((\lambda x : \text{Bool} \to \text{Nat} \to \text{Bool. } \lambda y : \text{Bool} \to \text{Nat. } \lambda z : \text{Bool. } ((x\ z)\ (y\ z)))\ u)\ v)\ w)$

Árbol

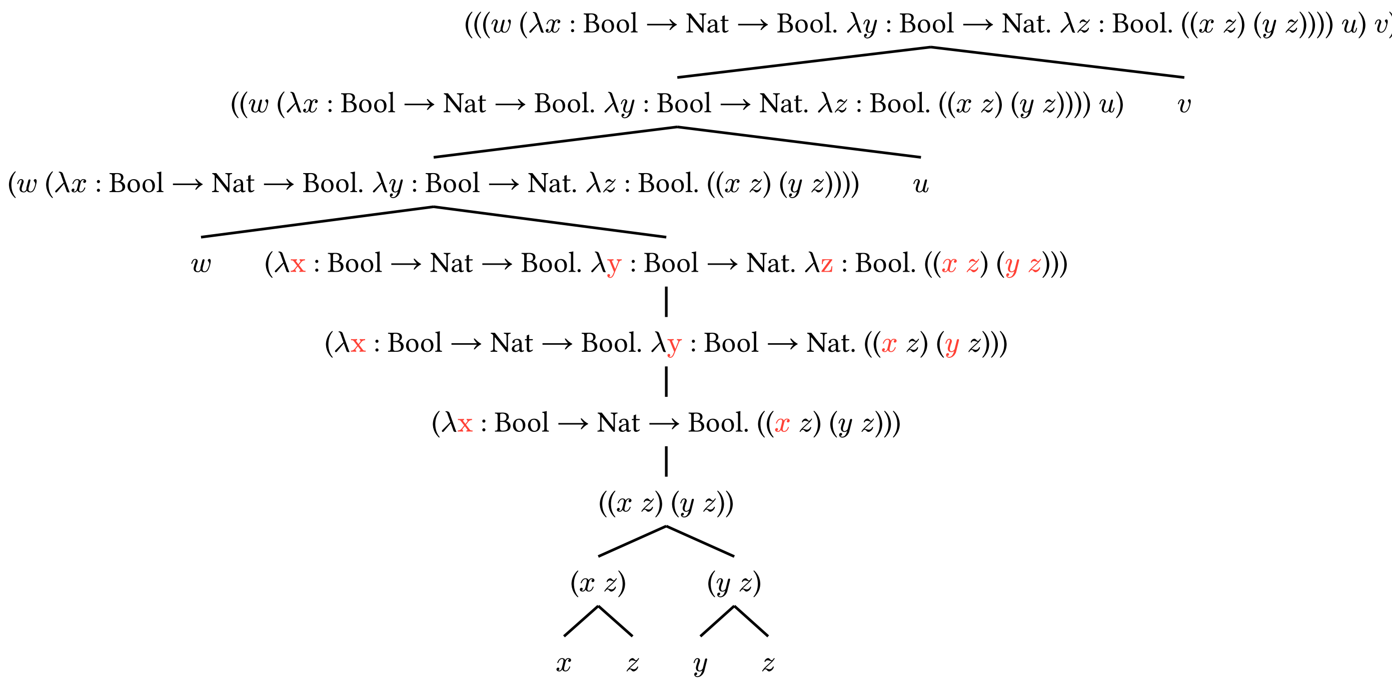


c.

$w\ (\lambda x : \text{Bool} \to \text{Nat} \to \text{Bool. } \lambda y : \text{Bool} \to \text{Nat. } \lambda z : \text{Bool. } x\ z\ (y\ z))\ u\ v$

$= (((w\ (\lambda x : \text{Bool} \to \text{Nat} \to \text{Bool. } \lambda y : \text{Bool} \to \text{Nat. } \lambda z : \text{Bool. } ((x\ z)\ (y\ z))))\ u)\ v)$

Árbol



LAS VARIABLES ROJAS SON LIGADAS

El término buscado aparece en el punto (b) al tercer nivel

6

a.

$$\dfrac{\dfrac{}{\vdash \text{true} : \text{Bool}}\,\text{T-True} \qquad \dfrac{}{\vdash \text{zero} : \text{Nat}}\,\text{T-Zero} \qquad \dfrac{\dfrac{}{\vdash \text{zero} : \text{Nat}}\,\text{T-Zero}}{\vdash \text{succ}(\text{zero}) : \text{Nat}}\,\text{T-Succ}}{\vdash \text{if true then zero else succ}(\text{zero}) : \text{Nat}}\,\text{T-If}$$

b.

$$\dfrac{\dfrac{}{\Gamma \vdash \text{true} : \text{Bool}}\,\text{T-True} \qquad \dfrac{}{\Gamma \vdash \text{false} : \text{Bool}}\,\text{T-False} \qquad \dfrac{\dfrac{\dfrac{}{\Gamma, z : \text{Bool} \vdash z : \text{Bool}}\,\text{T-Var}}{\Gamma \vdash (\lambda z : \text{Bool}.z) : \text{Bool} \to \text{Bool}}\,\text{T-Abs} \qquad \dfrac{}{\Gamma \vdash \text{true} : \text{Bool}}\,\text{T}}{\Gamma \vdash (\lambda z : \text{Bool}.z)\ \text{true} : \text{Bool}}\,\text{T}}{\Gamma = x : \text{Nat}, y : \text{Bool} \vdash \text{if true then false else}\ \ (\lambda z : \text{Bool}.z)\ \text{true} : \text{Bool}}\,\text{T-If}$$

c.

$$\dfrac{\dfrac{\color{red}{\textbf{INVALIDO}}}{\vdash (\lambda x : \text{Bool}.x) : \text{Bool}} \qquad \dfrac{}{\vdash \text{zero} : \text{Nat}}\,\text{T-Zero} \qquad \dfrac{\dfrac{}{\vdash \text{zero} : \text{Nat}}\,\text{T-Zero}}{\vdash \text{succ}(\text{zero}) : \text{Nat}}\,\text{T-Succ}}{\vdash \text{if}\ \ \lambda x : \text{Bool}.x\ \text{then zero else succ}(\text{zero}) : \text{Nat}}\,\text{T-If}$$

Es inválido porque $(\lambda x : \text{Bool}.x)$ es $\text{Bool} \to \text{Bool}$

d.

$$\dfrac{\dfrac{}{\Gamma \vdash x : \text{Bool} \to \text{Nat}}\,\text{T-Var} \qquad \dfrac{}{\Gamma \vdash y : \text{Bool}}\,\text{T-Var}}{\Gamma = x : \text{Bool} \to \text{Nat}, y : \text{Bool} \vdash (x\ y) : \text{Nat}}\,\text{T-App}$$

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Comparamos:

$$\frac{\Gamma \vdash M : \text{Bool} \leftarrow \text{INVALIDOOOO}}{\Gamma \vdash \lambda x : \text{Bool} \to \text{Bool}.M : \text{Bool} \to \text{Bool} \to \text{Bool}} \to_{i2}$$

y el original:

$$\frac{\Gamma, x : \text{Bool} \to \text{Bool} \vdash M : \text{Bool}}{\Gamma \vdash \lambda x : \text{Bool} \to \text{Bool}.M : \text{Bool} \to \text{Bool} \to \text{Bool}} \to_{i}$$

9

a. $\sigma \to \tau \to \sigma$

$$\dfrac{\dfrac{\dfrac{}{x : \sigma, y : \tau \vdash x : \sigma}\text{T-Var}}{x : \sigma \vdash \lambda y : \tau . x : \tau \to \sigma}\text{T-Abs}}{\vdash \lambda x : \sigma . \lambda y : \tau . x : \sigma \to \tau \to \sigma}\text{T-Abs}$$

b. $(\sigma \to \tau \to \rho) \to (\sigma \to \tau) \to \sigma \to \rho$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma \vdash x : \sigma \to \tau \to \rho}\text{T-Var} \quad \dfrac{}{\Gamma \vdash z : \sigma}\text{T-Var}}{\Gamma \vdash x\ z : \tau \to \rho}\text{T-App} \quad \dfrac{\dfrac{}{\Gamma \vdash y : \sigma \to \tau}\text{T-Var} \quad \dfrac{}{\Gamma \vdash z : \sigma}\text{T-Var}}{\Gamma \vdash y\ z : \tau}\text{T-App}}{\Gamma = \{x : \sigma \to \tau \to \rho, y : \sigma \to \tau, z : \sigma\} \vdash (x\ z)\ (y\ z) : \rho}\text{T-App}}{x : \sigma \to \tau \to \rho, y : \sigma \to \tau \vdash \lambda z : \sigma . x\ z\ (y\ z) : \sigma \to \rho}\text{T-Abs}}{x : \sigma \to \tau \to \rho \vdash \lambda y : \sigma \to \tau . \lambda z : \sigma . x\ z\ (y\ z) : (\sigma \to \tau) \to \sigma \to \rho}\text{T-Abs}}{\lambda x : \sigma \to \tau \to \rho . \lambda y : \sigma \to \tau . \lambda z : \sigma . x\ z\ (y\ z) : (\sigma \to \tau \to \rho) \to (\sigma \to \tau) \to \sigma \to \rho}\text{T-Abs}$$

c. $(\sigma \to \tau \to \rho) \to \tau \to \sigma \to \rho$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma \vdash x : \sigma \to \tau \to \rho}\text{T-Var} \quad \dfrac{}{\Gamma \vdash z : \sigma}\text{T-Var}}{\Gamma \vdash x\ z : \tau \to \rho}\text{T-App} \quad \dfrac{}{\Gamma \vdash y : \tau}\text{T-Var}}{\Gamma = \{x : (\sigma \to \tau \to \rho), y : \tau, z : \sigma\} \vdash x\ z\ y : \rho}\text{T-App}}{x : (\sigma \to \tau \to \rho), y : \tau \vdash \lambda z : \sigma . x\ z\ y : \sigma \to \rho}\text{T-Abs}}{x : (\sigma \to \tau \to \rho) \vdash \lambda y : \tau . \lambda z : \sigma . x\ z\ y : \tau \to \sigma \to \rho}\text{T-Abs}}{\vdash \lambda x : \sigma \to \tau \to \rho . \lambda y : \tau . \lambda z : \sigma . x\ z\ y : (\sigma \to \tau \to \rho) \to \tau \to \sigma \to \rho}\text{T-Abs}$$

d. $(\tau \to \rho) \to (\sigma \to \tau) \to \sigma \to \rho$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma \vdash x : \tau \to \rho}\text{T-Var} \quad \dfrac{\dfrac{}{\Gamma \vdash y : \sigma \to \tau}\text{T-Var} \quad \dfrac{}{\Gamma z : \sigma}\text{T-Var}}{\Gamma \vdash y\ z : \tau}\text{T-App}}{\Gamma = \{x : (\tau \to \rho), y : (\sigma \to \tau), z : \sigma\} \vdash x\ (y\ z) : \rho}\text{T-App}}{x : (\tau \to \rho), y : (\sigma \to \tau) \vdash \lambda z : \sigma . x\ (y\ z) : \sigma \to \rho}\text{T-Abs}}{x : \tau \to \rho \vdash \lambda y : \sigma \to \tau . \lambda z : \sigma . x\ (y\ z) : (\sigma \to \tau) \to \sigma \to \rho}\text{T-Abs}}{\vdash \lambda x : \tau \to \rho . \lambda y : \sigma \to \tau . \lambda z : \sigma . x\ (y\ z) : (\tau \to \rho) \to (\sigma \to \tau) \to \sigma \to \rho}\text{T-Abs}$$

10

a.

$$\dfrac{\dfrac{\dfrac{}{x : \sigma \vdash x : \sigma = \text{Nat}}\ \text{T-Var}}{x : \sigma \vdash \text{succ}(x) : \text{Nat}}\ \text{T-Succ}}{x : \sigma \vdash \text{isZero}(\text{succ}(x)) : \tau = \text{Bool}}\ \text{T-IsZero}$$

$\sigma = \text{Nat}, \tau = \text{Bool}$

b.

$$\dfrac{\dfrac{\dfrac{}{x : \sigma \vdash x : \sigma}\ \text{T-Var}}{\vdash (\lambda x : \sigma . x) : \sigma \to \sigma}\ \text{T-Abs} \qquad \dfrac{\dfrac{}{y : \text{Bool} \vdash \text{zero} : \tau = \text{Nat}}\ \text{T-Zero}}{\vdash (\lambda y : \text{Bool}.\text{zero}) : \sigma = \text{Bool} \to \tau}\ \text{T-Abs}}{\vdash (\lambda x : \sigma . x)(\lambda y : \text{Bool}.\text{zero}) : \sigma}\ \text{T-App}$$

$\sigma = \text{Bool} \to \text{Nat}$

c.

$y : \tau \vdash \text{if } (\lambda x : \sigma . x) \text{ then } y \text{ else succ(zero)} : \sigma$

Se nota a simple vista que no tipa, $(\lambda x : \sigma . x)$ es de $\sigma \to \sigma$

d.

$$\dfrac{\dfrac{}{x : \sigma \vdash x : \rho \to \tau = \sigma}\ \text{T-Var} \qquad \dfrac{}{x : \sigma \vdash y : \rho}}{x : \sigma \vdash x\ y : \tau}\ \text{T-App}$$

Queda trabado ahí, $y : \rho$ no está en el contexto.

e.

$$\dfrac{\dfrac{}{x : \sigma, y : \tau \vdash x : \tau \to \tau = \sigma}\ \text{T-Var} \qquad \dfrac{}{x : \sigma, y : \tau \vdash y : \tau}\ \text{T-Var}}{x : \sigma, y : \tau \vdash x\ y : \tau}\ \text{T-App}$$

$\sigma = \tau \to \tau$

f.

$$\dfrac{\dfrac{}{x : \sigma \vdash x : \text{Bool} \to \tau}\ \text{T-Var} \qquad \dfrac{}{x : \sigma \vdash \text{true} : \text{Bool}}\ \text{T-True}}{x : \sigma \vdash x \text{ true} : \tau}\ \text{T-App}$$

$\sigma = \text{Bool} \to \tau$

g.

$$\dfrac{\dfrac{}{x : \sigma \vdash x : \text{Bool} \to \sigma} \qquad \dfrac{}{x : \sigma \vdash \text{true} : \text{Bool}}\ \text{T-True}}{x : \sigma \vdash x \text{ true} : \sigma}\ \text{T-App}$$

Pero $\sigma \neq \text{Bool} \to \sigma$, no tipa.

h.

$$\dfrac{\dfrac{}{x : \sigma \vdash x : \rho \to \tau} \qquad \dfrac{}{x : \sigma \vdash x : \rho}}{x : \sigma \vdash x\ x : \tau}\ \text{T-App}$$

Pero $\tau \neq \rho \to \tau$, no tipa.

13

a.

$$(\lambda y:\sigma.x\ (\lambda x:\tau.x))\{x := (\lambda y:\rho.x\ y)\}$$

$$\underset{\alpha}{=} (\lambda z:\sigma.x\ (\lambda w:\tau.w))\{x := (\lambda y:\rho.x\ y)\}$$

$$\stackrel{\text{def}}{=} (\lambda z:\sigma.(\lambda y:\rho.x\ y)\ (\lambda w:\tau.w))$$

b.

$$(y\ (\lambda v:\sigma.\ x\ v))\{x := (\lambda y:\tau.\ v\ y)\}$$

$$\underset{\alpha}{=} (y\ (\lambda w:\sigma.\ x\ w))\{x := (\lambda z:\tau.\ v\ z)\}$$

$$\stackrel{\text{def}}{=} (y\ (\lambda w:\sigma.\ (\lambda z:\tau.\ v\ z)\ w))$$

15

a.

$(\lambda x : \text{Bool}.x)\ \text{true} \underset{\beta}{\rightarrow} \text{true}$

Es valor.

b.

$\lambda x : \text{Bool}.2 \overset{\text{def}}{=} \lambda x : \text{Bool}.\text{succ}^2(\text{zero}) = \lambda x : \text{Bool}.\ \text{succ}(\text{succ}(\text{zero})) \underset{\beta}{\rightarrow} \text{succ}(\text{succ}(\text{zero}))$

Es valor.

c.

$\lambda x : \text{Bool}\ .\text{pred}(2) = \lambda x : \text{Bool}\ .\text{pred}(\text{succ}(\text{succ}(\text{zero})))$

$\underset{\text{E-PredSucc}}{\rightarrow} \lambda x : \text{Bool}.\ \text{succ}(\text{zero}) \underset{\beta}{\rightarrow} \text{succ}(\text{zero})$

Es valor

d.

$\lambda y : \text{Nat}.(\lambda x : \text{Bool}.\text{pred}(2))\ \text{true}$

$= \lambda y : \text{Nat}.(\lambda x : \text{Bool}.\text{pred}(\text{succ}(\text{succ}(\text{zero}))))\ \text{true}$

$\underset{\text{E-Pred\_Succ}}{\rightarrow} \lambda y : \text{Nat}.(\lambda x : \text{Bool}.\text{succ}(\text{zero}))\ \text{true}$

$\underset{\beta}{\rightarrow} \lambda y : \text{Nat}.\ \text{succ}(\text{zero})$

$\underset{\beta}{\rightarrow} \text{succ}(\text{zero})$

Es valor

e.

x no es un valor

f.

succ(succ(zero)) = $\underline{2}$

Es valor

16

I.

$(\lambda x : \text{Bool}.x)\ \text{true} \underset{\beta}{\to} x\{x := \text{true}\} = \text{true}$

Es un programa, forma normal, valor

II.

$\lambda x : \text{Nat}.\text{pred}(\text{succ}(x))$

Es un programa, forma normal, valor

III.

$\lambda x : \text{Nat}.\text{pred}(\text{succ}(y))$

No es un programa

IV.

$(\lambda x : \text{Bool}.\ \text{pred}(\text{isZero}((x)))\ \text{true} \underset{\beta}{\to} \text{pred}(\text{isZero}((x)))\{x := \text{true}\} = \text{pred}(\text{isZero}((\text{true})))$

No es un programa

V.

$(\lambda f : \text{Nat} \to \text{Bool}.f\ \text{zero})\ (\lambda x : \text{Nat}.\ \text{isZero}(x))$

Es un programa, no hay variables libres, el 2do lambda suelta algo de tipo Nat, y la 1era recibe algo de tipo Nat, por lo que tipa

Forma normal, valor

VI.

$(\lambda f : \text{Nat} \to \text{Bool}.x)\ (\lambda x : \text{Nat}.\ \text{isZero}(x))$

No es un programa

VII.

$(\lambda f : \text{Nat} \to \text{Bool}.f\ \ \text{pred}(\text{zero}))\ (\lambda x : \text{Nat}.\ \text{isZero}(x))$

Es un programa, mismo argumento que en el **V**, forma normal, error

VIII.

$\text{fix}\ \lambda y : \text{Nat}.\ \text{succ}(y)$

Es un programa, forma normal, pero nunca termina… ¿runtime error?

20

a.

$$\dfrac{\Gamma \vdash M : \sigma \qquad \Gamma \vdash N : \tau}{\Gamma \vdash \langle M, N\rangle : \sigma \times \tau}\text{T-Pares}$$

$$\dfrac{\Gamma \vdash M : \sigma \times \tau}{\Gamma \vdash \pi_1(M) : \sigma}\text{T-}\pi_1$$

$$\dfrac{\Gamma \vdash M : \sigma \times \tau}{\Gamma \vdash \pi_2(M) : \tau}\text{T-}\pi_2$$

b.

I. $\sigma \to \tau \to (\sigma \times \tau)$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma \vdash x : \sigma}\text{T-Var} \qquad \dfrac{}{\Gamma \vdash y : \tau}\text{T-Var}}{\Gamma = \{x : \sigma, y : \tau\} \vdash \langle x, y\rangle : (\sigma \times \tau)}\text{T-Pares}}{x : \sigma \vdash \lambda y : \tau.\langle x, y\rangle : \tau \to (\sigma \times \tau)}\text{T-Abs}}{\vdash \lambda x : \sigma.\lambda y : \tau.\langle x, y\rangle : \sigma \to \tau \to (\sigma \times \tau)}\text{T-Abs}$$

II. $(\sigma \times \tau) \to \sigma$ **y** $(\sigma \times \tau) \to \tau$

Caso 1

$$\dfrac{\dfrac{\dfrac{}{x : (\sigma \times \tau) \vdash x : (\sigma \times \tau)}\text{T-Var}}{x : (\sigma \times \tau) \vdash \pi_1(x) : \sigma}\text{T-}\pi_1}{\vdash \lambda x : (\sigma \times \tau).\pi_1(x) : (\sigma \times \tau) \to \sigma}\text{T-Abs}$$

Caso 2

$$\dfrac{\dfrac{\dfrac{}{y : (\sigma \times \tau) \vdash y : (\sigma \times \tau)}\text{T-Var}}{y : (\sigma \times \tau) \vdash \pi_2(y) : \tau}\text{T-}\pi_2}{\vdash \lambda y : (\sigma \times \tau).\pi_2(y) : (\sigma \times \tau) \to \tau}\text{T-Abs}$$

III. $(\sigma \times \tau) \to (\tau \times \sigma)$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{x : (\sigma \times \tau) \vdash x : (\sigma \times \tau)}\text{T-Var}}{x : (\sigma \times \tau) \vdash \pi_2(x) : \tau}\text{T-}\pi_2 \qquad \dfrac{\dfrac{}{x : (\sigma \times \tau) \vdash x : (\sigma \times \tau)}\text{T-Var}}{x : (\sigma \times \tau) \vdash \pi_1(x) : \sigma}\text{T-}\pi_1}{x : (\sigma \times \tau) \vdash \langle \pi_2(x), \pi_1(x)\rangle : (\tau \times \sigma)}\text{T-Pares}}{\vdash \lambda x : (\sigma \times \tau).\langle \pi_2(x), \pi_1(x)\rangle : (\sigma \times \tau) \to (\tau \times \sigma)}\text{T-Abs}$$

IV. $((\sigma \times \tau) \times \rho) \to (\sigma \times (\tau \times \rho))$ **y** $(\sigma \times (\tau \times \rho)) \to ((\sigma \times \tau) \times \rho)$

Caso 1

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma \vdash x : (\sigma \times \tau) \times \rho}\text{T-Var}}{\Gamma \vdash \pi_1(x) : (\sigma \times \tau)}\text{T-}\pi_1}{\Gamma \vdash \pi_1(\pi_1(x)) : \sigma}\text{T-}\pi_1 \qquad \dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma \vdash x : (\sigma \times \tau) \times \rho}\text{T-Var}}{\Gamma \vdash \pi_1(x) : (\sigma \times \tau)}\text{T-}\pi_1}{\Gamma \vdash \pi_2(\pi_1(x)) : \tau}\text{T-}\pi_2 \qquad \dfrac{\dfrac{}{\Gamma \vdash x : (\sigma \times \tau) \times \rho}\text{T-Var}}{\Gamma \vdash \pi_2(x) : \rho}\text{T-}\pi_2}{\Gamma \vdash \langle \pi_2(\pi_1(x)), \pi_2(x)\rangle : (\tau \times \rho)}\text{T-Pares}}{\Gamma = x : ((\sigma \times \tau) \times \rho) \vdash \langle \pi_1(\pi_1(x)), \langle \pi_2(\pi_1(x)), \pi_2(x)\rangle\rangle : (\sigma \times (\tau \times \rho))}\text{T-Pares}}{\vdash \lambda x : ((\sigma \times \tau) \times \rho).\langle \pi_1(\pi_1(x)), \langle \pi_2(\pi_1(x)), \pi_2(x)\rangle\rangle : ((\sigma \times \tau) \times \rho) \to (\sigma \times (\tau \times \rho))}\text{T-Abs}$$

Caso 2

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma \vdash x : (\sigma \times (\tau \times \rho))}\text{T-Var}}{\Gamma \vdash \pi_1(x) : \sigma}\text{T-}\pi_1 \qquad \dfrac{\dfrac{\dfrac{}{\Gamma \vdash x : \sigma \times (\tau \times \rho)}\text{T-Var}}{\Gamma \pi_2(x) : (\tau \times \rho)}\text{T-}\pi_2}{\Gamma \vdash \pi_1(\pi_2(x))) : \tau}\text{T-}\pi_1}{\Gamma \vdash \langle \pi_1(x), \pi_1(\pi_2(x))\rangle : (\sigma \times \tau)}\text{T-Pares} \qquad \dfrac{\dfrac{\dfrac{}{\Gamma \vdash x : (\sigma \times (\tau \times \rho))}\text{T-Var}}{\Gamma \vdash \pi_2(x) : (\tau \times \rho)}\text{T-}\pi_1}{\Gamma \vdash \pi_2(\pi_2(x)) : \rho}\text{T-}\pi_2}{\Gamma = x : (\sigma \times (\tau \times \rho)) \vdash \langle \langle \pi_1(x), \pi_1(\pi_2(x))\rangle, \pi_2(\pi_2(x))\rangle : ((\sigma \times \tau) \times \rho)}\text{T-Pares}}{\vdash \lambda x : (\sigma \times (\tau \times \rho)).\langle \langle \pi_1(x), \pi_1(\pi_2(x))\rangle, \pi_2(\pi_2(x))\rangle : (\sigma \times (\tau \times \rho)) \to ((\sigma \times \tau) \times \rho)}\text{T-Abs}$$

V. $((\sigma \times \tau) \to \rho) \to (\sigma \to \tau \to \rho)$ **y** $(\sigma \to \tau \to \rho) \to ((\sigma \times \tau) \to \rho)$

Caso 1

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma \vdash f : (\sigma \times \tau) \to \rho}\text{T-Var} \qquad \dfrac{\dfrac{}{\Gamma \vdash x : \sigma}\text{T-Var} \qquad \dfrac{}{\Gamma \vdash y : \tau}\text{T-Var}}{\Gamma \vdash \langle x, y\rangle : (\sigma \times \tau)}\text{T-Pares}}{\Gamma = f : (\sigma \times \tau) \to \rho, x : \sigma, y : \tau \vdash f\langle x, y\rangle : \rho}\text{T-App}}{f : (\sigma \times \tau), x : \sigma \to \rho \vdash \lambda y : \tau.f\langle x, y\rangle : \tau \to \rho}\text{T-Abs}}{f : (\sigma \times \tau) \to \rho \vdash \lambda x : \sigma.\lambda y : \tau.f\langle x, y\rangle : \sigma \to \tau \to \rho}\text{T-Abs}}{\vdash \lambda f : (\sigma \times \tau) \to \rho.\lambda x : \sigma.\lambda y : \tau.f\langle x, y\rangle : ((\sigma \times \tau) \to \rho) \to (\sigma \to \tau \to \rho)}\text{T-Abs}$$

Caso 2

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma \vdash f : \sigma \to \tau \to \rho}\text{T-Var} \qquad \dfrac{\dfrac{}{\Gamma \vdash p : (\sigma \times \tau)}\text{T-Var}}{\Gamma \vdash \pi_1(p) : \sigma}\text{T-}\pi_1}{\Gamma \vdash f\ \pi_1(p) : \tau \to \rho}\text{T-App} \qquad \dfrac{\dfrac{}{\Gamma \vdash p : (\sigma \times \tau)}\text{T-Var}}{\Gamma \vdash \pi_2(p) : \tau}\text{T-}\pi_2}{\Gamma = f : (\sigma \to \tau \to \rho), p : (\sigma \times \tau) \vdash f\ \pi_1(p)\ \pi_2(p) : \rho}\text{T-App}}{f : (\sigma \to \tau \to \rho) \vdash \lambda p : (\sigma \times \tau).f\ \pi_1(p)\ \pi_2(p) : ((\sigma \times \tau) \to \rho)}\text{T-Abs}}{\lambda f : (\sigma \to \tau \to \rho).\lambda p : (\sigma \times \tau).f\ \pi_1(p)\ \pi_2(p) : (\sigma \to \tau \to \rho) \to ((\sigma \times \tau) \to \rho)}\text{T-Abs}$$

c.

$V ::= \ldots \mid \langle V, V\rangle$

d.

Si $M \to M'$ **entonces** $\pi_1(M) \to \pi_1(M')$

Si $M \to M'$ **entonces** $\pi_2(M) \to \pi_2(M')$

$\pi_1(\langle V, W\rangle) \to V$

$\pi_2(\langle V, W\rangle) \to W$

Si $M \to M'$ **entonces** $\langle M, N\rangle \to \langle M', N\rangle$

Si $N \to N'$ **entonces** $\langle V, N\rangle \to \langle V, N'\rangle$

e.

La demostración queda de ejercicio al lector

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a.

Primer ejemplo

$$\text{case zero} :: \text{succ(zero)}\ []_{\text{Nat}}\ \text{of}\ \{[] \rightsquigarrow \text{false} \mid x :: xs \rightsquigarrow \text{izZero}(x)\} \twoheadrightarrow \text{true}$$

Segundo ejemplo