Ejercicio 5

Demostrar en deducción natural que las siguientes fórmulas son teoremas sin usar principios de razonamiento clásicos salvo que se indique lo contrario. Recordemos que una fórmula σ es un teorema si y sólo si vale $\vdash \sigma$:

I. *Modus ponens* relativizado:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma, \rho \vdash \rho \Rightarrow \sigma \Rightarrow \tau}\text{ax} \qquad \dfrac{}{\Gamma, \sigma \Rightarrow \tau \vdash \rho}\text{ax}}{\Gamma \vdash \sigma \Rightarrow \tau}\Rightarrow_e \qquad \dfrac{\dfrac{}{\Gamma \vdash \rho \Rightarrow \sigma}\text{ax} \qquad \dfrac{}{\Gamma, \sigma \vdash \rho}\text{ax}}{\Gamma \vdash \sigma}\Rightarrow_e}{\Gamma = \rho \Rightarrow (\sigma \Rightarrow \tau), \rho \Rightarrow \sigma, \rho \vdash \tau}\Rightarrow_e}{(\rho \Rightarrow (\sigma \Rightarrow \tau)), (\rho \Rightarrow \sigma) \vdash (\rho \Rightarrow \tau)}\Rightarrow_i}{(\rho \Rightarrow (\sigma \Rightarrow \tau)) \vdash ((\rho \Rightarrow \sigma) \Rightarrow (\rho \Rightarrow \tau))}\Rightarrow_i}{\vdash (\rho \Rightarrow (\sigma \Rightarrow \tau)) \Rightarrow ((\rho \Rightarrow \sigma) \Rightarrow (\rho \Rightarrow \tau))}\Rightarrow_i$$

II. Reducción al absurdo

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma \vdash \rho \Rightarrow \bot}\text{ax} \qquad \dfrac{}{\Gamma \vdash \rho}\text{ax}}{\Gamma = (\rho \Rightarrow \bot), \rho \vdash \bot}\Rightarrow_e}{(\rho \Rightarrow \bot) \vdash \neg\rho}\neg_i}{\vdash (\rho \Rightarrow \bot) \Rightarrow \neg\rho}\Rightarrow_i$$

III. Introducción de la doble negación

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\rho, \neg\rho \vdash \rho}\text{ax} \qquad \dfrac{}{\rho, \neg\rho \vdash \neg\rho}\text{ax}}{\rho, \neg\rho \vdash \bot}\neg_e}{\rho \vdash \neg\neg\rho}\neg_i}{\vdash \rho \Rightarrow \neg\neg\rho}\Rightarrow_i$$

IV. Eliminación de la triple negación

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma \vdash \rho}\text{ax}}{\Gamma \vdash \neg\neg\rho}\neg\neg_i \qquad \dfrac{}{\Gamma \vdash \neg\neg\neg\rho}\text{ax}}{\Gamma = \neg\neg\neg\rho, \rho \vdash \bot}\neg_e}{\neg\neg\neg\rho \vdash \neg\rho}\neg_i}{\vdash \neg\neg\neg\rho \Rightarrow \neg\rho}\Rightarrow_i$$

V. Contraposición

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma \vdash \rho \Rightarrow \sigma}\text{ax} \qquad \dfrac{}{\Gamma \vdash \rho}\text{ax}}{\Gamma \vdash \sigma}\Rightarrow_e \qquad \dfrac{}{\Gamma \vdash \neg\sigma}\text{ax}}{\Gamma = (\rho \Rightarrow \sigma), \neg\sigma, \rho \vdash \bot}\neg_e}{(\rho \Rightarrow \sigma), \neg\sigma \vdash \neg\rho}\neg_i}{\rho \Rightarrow \sigma \vdash \neg\sigma \Rightarrow \neg\rho}\Rightarrow_i}{\vdash (\rho \Rightarrow \sigma) \Rightarrow (\neg\sigma \Rightarrow \neg\rho)}\Rightarrow_i$$

VI. Adjunción

Probar $((\rho \wedge \sigma) \Rightarrow \tau) \Leftrightarrow (\rho \Rightarrow \sigma \Rightarrow \tau)$ se reduce a probar

$((\rho \wedge \sigma) \Rightarrow \tau) \Rightarrow (\rho \Rightarrow \sigma \Rightarrow \tau)$ y $(\rho \Rightarrow \sigma \Rightarrow \tau) \Rightarrow ((\rho \wedge \sigma) \Rightarrow \tau)$

Caso 1

$$
\begin{prooftree}
\AxiomC{}
\RightLabel{ax}
\UnaryInfC{$\Gamma \vdash (\rho \wedge \sigma) \Rightarrow \tau$}
\AxiomC{}
\RightLabel{ax}
\UnaryInfC{$\Gamma \vdash \rho$}
\AxiomC{}
\RightLabel{ax}
\UnaryInfC{$\Gamma \vdash \sigma$}
\RightLabel{$\wedge_i$}
\BinaryInfC{$\Gamma \vdash (\rho \wedge \sigma)$}
\RightLabel{$\Rightarrow_e$}
\BinaryInfC{$\Gamma = (\rho \wedge \sigma) \Rightarrow \tau, \rho, \sigma \vdash \tau$}
\RightLabel{$\Rightarrow_i$}
\UnaryInfC{$((\rho \wedge \sigma) \Rightarrow \tau), \rho \vdash (\sigma \Rightarrow \tau)$}
\RightLabel{$\Rightarrow_i$}
\UnaryInfC{$((\rho \wedge \sigma) \Rightarrow \tau) \vdash \rho \Rightarrow (\sigma \Rightarrow \tau)$}
\RightLabel{$\Rightarrow_i$}
\UnaryInfC{$\vdash ((\rho \wedge \sigma) \Rightarrow \tau) \Rightarrow (\rho \Rightarrow \sigma \Rightarrow \tau)$}
\end{prooftree}
$$

Caso 2

$$
\begin{prooftree}
\AxiomC{}
\RightLabel{ax}
\UnaryInfC{$\Gamma \vdash \rho \Rightarrow \sigma \Rightarrow \tau$}
\AxiomC{}
\RightLabel{ax}
\UnaryInfC{$\Gamma \vdash \rho \wedge \sigma$}
\RightLabel{$\wedge_{e_1}$}
\UnaryInfC{$\Gamma \vdash \rho$}
\RightLabel{$\Rightarrow_e$}
\BinaryInfC{$\Gamma \vdash \sigma \Rightarrow \tau$}
\AxiomC{}
\RightLabel{ax}
\UnaryInfC{$\Gamma \vdash \rho \wedge \sigma$}
\RightLabel{$\wedge_{e_2}$}
\UnaryInfC{$\Gamma \vdash \sigma$}
\RightLabel{$\Rightarrow_e$}
\BinaryInfC{$\Gamma = (\rho \Rightarrow \sigma \Rightarrow \tau), (\rho \wedge \sigma) \vdash \tau$}
\RightLabel{$\Rightarrow_i$}
\UnaryInfC{$(\rho \Rightarrow \sigma \Rightarrow \tau) \vdash ((\rho \wedge \sigma) \Rightarrow \tau)$}
\RightLabel{$\Rightarrow_i$}
\UnaryInfC{$\vdash (\rho \Rightarrow \sigma \Rightarrow \tau) \Rightarrow ((\rho \wedge \sigma) \Rightarrow \tau)$}
\end{prooftree}
$$

VII. de Morgan (I)

Preguntar

Queremos probar $\neg(\rho \vee \sigma) \Leftrightarrow (\neg\rho \wedge \neg\sigma)$:

$(\Longrightarrow)$

$$
\begin{prooftree}
\AxiomC{}
\RightLabel{ax}
\UnaryInfC{$\Gamma, \rho \vdash \rho$}
\AxiomC{}
\RightLabel{ax}
\UnaryInfC{$\Gamma, \rho \vdash \rho$}
\RightLabel{$\vee_{i_1}$}
\UnaryInfC{$\Gamma, \rho \vdash (\rho \vee \sigma)$}
\AxiomC{}
\RightLabel{ax}
\UnaryInfC{$\Gamma, \rho \vdash \rho$}
\AxiomC{}
\RightLabel{ax}
\UnaryInfC{$\Gamma, \rho, \sigma \vdash \rho$}
\RightLabel{$\vee_e$}
\TrinaryInfC{$\Gamma, \rho \vdash \neg\rho$}
\RightLabel{$\neg_e$}
\BinaryInfC{$\Gamma, \rho \vdash \bot$}
\RightLabel{$\neg_i$}
\UnaryInfC{$\Gamma \vdash \neg\rho$}
\AxiomC{}
\UnaryInfC{$\Gamma, \sigma \vdash \tau$}
\AxiomC{}
\UnaryInfC{$\Gamma, \sigma \vdash \neg\tau$}
\RightLabel{$\neg_e$}
\BinaryInfC{$\Gamma, \sigma \vdash \bot$}
\RightLabel{$\neg_i$}
\UnaryInfC{$\Gamma \vdash \neg\sigma$}
\RightLabel{$\wedge_i$}
\BinaryInfC{$\Gamma = \neg(\rho \vee \sigma) \vdash (\neg\rho \wedge \neg\sigma)$}
\RightLabel{$\Rightarrow_i$}
\UnaryInfC{$\vdash \neg(\rho \vee \sigma) \Rightarrow (\neg\rho \wedge \neg\sigma)$}
\end{prooftree}
$$

$(\Longleftarrow)$

$$
\begin{prooftree}
\AxiomC{}
\UnaryInfC{$\vdash (\neg\rho \wedge \neg\sigma) \Rightarrow \neg(\rho \vee \sigma)$}
\end{prooftree}
$$

VIII. de Morgan (II)

IX. Conmutatividad ($\land$)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\rho\land\sigma\vdash\rho\land\sigma}\,\text{ax}}{\rho\land\sigma\vdash\rho}\land_{e_1}\qquad\dfrac{\dfrac{}{\rho\land\sigma\vdash\rho\land\sigma}\,\text{ax}}{\rho\land\sigma\vdash\sigma}\land_{e_2}}{\rho\land\sigma\vdash\sigma\land\rho}\land_i}{\vdash(\rho\land\sigma)\Rightarrow(\sigma\land\rho)}\Rightarrow_i$$

X. Asociatividad ($\land$)

$((\rho\land\sigma)\land\tau)\Leftrightarrow(\rho\land(\sigma\land\tau))$

($\Longrightarrow$)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma\vdash(\rho\land\sigma)\land\tau}\,\text{ax}}{\Gamma\vdash\rho\land\sigma}\land_{e_1}}{\Gamma\vdash\rho}\land_{e_1}\qquad\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma\vdash(\rho\land\sigma)\land\tau}\,\text{ax}}{\Gamma\vdash\rho\land\sigma}\land_{e_1}}{\Gamma\vdash\sigma}\land_{e_2}\qquad\dfrac{\dfrac{}{\Gamma\vdash(\rho\land\sigma)\land\tau}\,\text{ax}}{\Gamma\vdash\tau}\land_{e_2}}{\Gamma\vdash\sigma\land\tau}\land_i}{\Gamma=((\rho\land\sigma)\land\tau)\vdash(\rho\land(\sigma\land\tau))}\land_i}{\vdash((\rho\land\sigma)\land\tau)\Rightarrow(\rho\land(\sigma\land\tau))}\Rightarrow_i$$

($\Longleftarrow$)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma\vdash\rho\land(\sigma\land\tau)}\,\text{ax}}{\Gamma\vdash\rho}\land_{e_1}\qquad\dfrac{\dfrac{\dfrac{}{\Gamma\vdash\rho\land(\sigma\land\tau)}\,\text{ax}}{\Gamma\vdash\sigma\land\tau}\land_{e_2}}{\Gamma\vdash\sigma}\land_{e_1}}{\Gamma\vdash\rho\land\sigma}\land_i\qquad\dfrac{\dfrac{\dfrac{}{\Gamma\vdash\rho\land(\sigma\land\tau)}\,\text{ax}}{\Gamma\vdash\sigma\land\tau}\land_{e_2}}{\Gamma\vdash\tau}\land_{e_2}}{\Gamma=(\rho\land(\sigma\land\tau))\vdash((\rho\land\sigma)\land\tau)}\land_i}{\vdash(\rho\land(\sigma\land\tau))\Rightarrow((\rho\land\sigma)\land\tau)}\Rightarrow_i$$

XI. Conmutatividad ($\lor$)

$$\dfrac{\dfrac{\dfrac{}{(\rho\lor\sigma)\vdash\rho\lor\sigma}\,\text{ax}\qquad\dfrac{\dfrac{}{(\rho\lor\sigma),\rho\vdash\rho}\,\text{ax}}{(\rho\lor\sigma),\rho\vdash\sigma\lor\rho}\lor_{i_2}\qquad\dfrac{\dfrac{}{(\rho\lor\sigma),\sigma\vdash\sigma}\,\text{ax}}{(\rho\lor\sigma),\sigma\vdash\sigma\lor\rho}\lor_{i_1}}{(\rho\lor\sigma)\vdash\sigma\lor\rho}\lor_e}{\vdash(\rho\lor\sigma)\Rightarrow(\sigma\lor\rho)}\Rightarrow_i$$

XII. Asociatividad ($\lor$)

$((\rho\lor\sigma)\lor\tau)\Leftrightarrow(\rho\lor(\sigma\lor\tau))$

($\Longrightarrow$)

$$\dfrac{\dfrac{\dfrac{}{\Gamma\vdash(\rho\lor\sigma)\lor\tau}\,\text{ax}\qquad\dfrac{\dfrac{}{\Sigma\vdash\rho\lor\sigma}\,\text{ax}\qquad\dfrac{\dfrac{}{\Sigma,\rho\vdash\rho}\,\text{ax}}{\Sigma,\rho\vdash\rho\lor(\sigma\lor\tau)}\lor_{i_1}\qquad\dfrac{\dfrac{\dfrac{}{\Sigma,\sigma\vdash\sigma}\,\text{ax}}{\Sigma,\sigma\vdash\sigma\lor\tau}\lor_{i_1}}{\Sigma,\sigma\vdash\rho\lor(\sigma\lor\tau)}\lor_{i_2}}{\Sigma=\Gamma,(\rho\lor\sigma)\vdash\rho\lor(\sigma\lor\tau)}\lor_e\qquad\dfrac{\dfrac{\dfrac{}{\Gamma,\tau\vdash\tau}\,\text{ax}}{\Gamma,\tau\vdash\sigma\lor\tau}\lor_{i_2}}{\Gamma,\tau\vdash\rho\lor(\sigma\lor\tau)}\lor_{i_2}}{\Gamma=((\rho\lor\sigma)\lor\tau)\vdash(\rho\lor(\sigma\lor\tau))}\lor_e}{\vdash((\rho\lor\sigma)\lor\tau)\Rightarrow(\rho\lor(\sigma\lor\tau))}\Rightarrow_i$$

$(\Longleftarrow)$

$$
\begin{prooftree}
\AxiomC{}
\RightLabel{ax}
\UnaryInfC{$\Gamma \vdash \rho \vee (\sigma \vee \tau)$}
\AxiomC{}
\RightLabel{ax}
\UnaryInfC{$\Gamma, \rho \vdash \rho$}
\RightLabel{$\vee_{i_1}$}
\UnaryInfC{$\Gamma, \rho \vdash \rho \vee \sigma$}
\RightLabel{$\vee_{i_1}$}
\UnaryInfC{$\Gamma, \rho \vdash (\rho \vee \sigma) \vee \tau$}
\AxiomC{}
\RightLabel{ax}
\UnaryInfC{$\Sigma \vdash \sigma \vee \tau$}
\AxiomC{}
\RightLabel{ax}
\UnaryInfC{$\Sigma, \sigma \vdash \sigma$}
\RightLabel{$\vee_{i_2}$}
\UnaryInfC{$\Sigma, \sigma \vdash \rho \vee \sigma$}
\RightLabel{$\vee_{i_1}$}
\UnaryInfC{$\Sigma, \sigma \vdash (\rho \vee \sigma) \vee \tau$}
\AxiomC{}
\RightLabel{ax}
\UnaryInfC{$\Sigma, \tau \vdash \tau$}
\RightLabel{$\vee_{i_2}$}
\UnaryInfC{$\Sigma, \tau \vdash (\rho \vee \sigma) \vee \tau$}
\RightLabel{$\vee_e$}
\TrinaryInfC{$\Sigma = \Gamma, (\sigma \vee \tau) \vdash (\rho \vee \sigma) \vee \tau$}
\RightLabel{$\vee_e$}
\TrinaryInfC{$\Gamma = (\rho \vee (\sigma \vee \tau)) \vdash ((\rho \vee \sigma) \vee \tau)$}
\RightLabel{$\Rightarrow_i$}
\UnaryInfC{$\vdash (\rho \vee (\sigma \vee \tau)) \Rightarrow ((\rho \vee \sigma) \vee \tau)$}
\end{prooftree}
$$

Ejercicio 6

Demostrar en deducción natural que vale $\vdash \sigma$ para cada una de las siguientes fórmulas. Para estas fórmulas es imprescindible usar lógica clásica:

I. Absurdo clásico

$$
\begin{prooftree}
\AxiomC{}
\RightLabel{ax}
\UnaryInfC{$\Gamma \vdash \neg\tau \Rightarrow \bot$}
\AxiomC{}
\RightLabel{ax}
\UnaryInfC{$\Gamma \vdash \neg\tau$}
\RightLabel{$\Rightarrow_e$}
\BinaryInfC{$\Gamma = \{\neg\tau \Rightarrow \bot, \neg\tau\} \vdash \bot$}
\RightLabel{PBC}
\UnaryInfC{$\neg\tau \Rightarrow \bot \vdash \tau$}
\RightLabel{$\Rightarrow_i$}
\UnaryInfC{$\vdash (\neg\tau \Rightarrow \bot) \Rightarrow \tau$}
\end{prooftree}
$$

II. Ley de Peirce

$$
\begin{prooftree}
\AxiomC{}
\RightLabel{ax}
\UnaryInfC{$\Gamma, \neg\tau \vdash (\tau \Rightarrow \rho) \Rightarrow \tau$}
\AxiomC{}
\RightLabel{ax}
\UnaryInfC{$\Gamma, \neg\tau, \tau \vdash \tau$}
\AxiomC{}
\RightLabel{ax}
\UnaryInfC{$\Gamma, \neg\tau, \tau \vdash \neg\tau$}
\RightLabel{$\neg_e$}
\BinaryInfC{$\Gamma, \neg\tau, \tau \vdash \bot$}
\RightLabel{$\bot_e$}
\UnaryInfC{$\Gamma, \neg\tau, \tau \vdash \rho$}
\RightLabel{$\Rightarrow_i$}
\UnaryInfC{$\Gamma, \neg\tau \vdash (\tau \Rightarrow \rho)$}
\RightLabel{$\Rightarrow_e$}
\BinaryInfC{$\Gamma, \neg\tau \vdash \tau$}
\AxiomC{}
\RightLabel{ax}
\UnaryInfC{$\Gamma, \neg\tau \vdash \neg\tau$}
\RightLabel{$\neg_e$}
\BinaryInfC{$\Gamma, \neg\tau \vdash \bot$}
\RightLabel{PBC}
\UnaryInfC{$\Gamma = ((\tau \Rightarrow \rho) \Rightarrow \tau) \vdash \tau$}
\RightLabel{$\Rightarrow_i$}
\UnaryInfC{$\vdash ((\tau \Rightarrow \rho) \Rightarrow \tau) \Rightarrow \tau$}
\end{prooftree}
$$

III. Tercero excluido

Esto se puede probar con PBC pero ya tenemos dado LEM.

$$
\begin{prooftree}
\AxiomC{}
\RightLabel{LEM}
\UnaryInfC{$\vdash \tau \vee \neg\tau$}
\end{prooftree}
$$

IV. Consecuencia milagrosa

$$
\begin{prooftree}
\AxiomC{}
\RightLabel{ax}
\UnaryInfC{$\Gamma \vdash \neg\tau \Rightarrow \tau$}
\AxiomC{}
\RightLabel{ax}
\UnaryInfC{$\Gamma \vdash \neg\tau$}
\RightLabel{$\Rightarrow_e$}
\BinaryInfC{$\Gamma \vdash \tau$}
\AxiomC{}
\RightLabel{ax}
\UnaryInfC{$\Gamma \vdash \neg\tau$}
\RightLabel{$\neg_e$}
\BinaryInfC{$\Gamma = \{(\neg\tau \Rightarrow \tau), \neg\tau\} \vdash \bot$}
\RightLabel{PBC}
\UnaryInfC{$(\neg\tau \Rightarrow \tau) \vdash \tau$}
\RightLabel{$\Rightarrow_i$}
\UnaryInfC{$\vdash (\neg\tau \Rightarrow \tau) \Rightarrow \tau$}
\end{prooftree}
$$

V. Contraposición clásica

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma \vdash \tau}\text{ax} \qquad \dfrac{\dfrac{}{\Gamma \vdash \neg\rho \Rightarrow \neg\tau}\text{ax} \qquad \dfrac{}{\Gamma \vdash \neg\rho}\text{ax}}{\Gamma \vdash \neg\tau}\Rightarrow_e}{\Gamma = \{(\neg\rho \Rightarrow \neg\tau), \tau, \neg\rho\} \vdash \bot}\neg_e}{(\neg\rho \Rightarrow \neg\tau), \tau \vdash \rho}\text{PBC}}{(\neg\rho \Rightarrow \neg\tau) \vdash (\tau \Rightarrow \rho)}\Rightarrow_i}{\vdash (\neg\rho \Rightarrow \neg\tau) \Rightarrow (\tau \Rightarrow \rho)}\Rightarrow_i$$

VI. Análisis de casos

No compila completo, habría que cambiar **esta línea**, tuve que separarlo.

$$\dfrac{\dfrac{}{\Gamma, \neg\rho \vdash \tau \Rightarrow \rho}\text{ax} \qquad \dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma, \neg\rho, \neg\tau \vdash \neg\tau \Rightarrow \rho}\text{ax} \qquad \dfrac{}{\Gamma, \neg\rho, \neg\tau \vdash \neg\tau}\text{ax}}{\Gamma, \neg\rho, \neg\tau \vdash \rho}\Rightarrow_e \qquad \dfrac{}{\Gamma, \neg\rho, \neg\tau \vdash \neg\rho}\text{ax}}{\Gamma, \neg\rho, \neg\tau \vdash \bot}\neg_e}{\Gamma, \neg\rho \vdash \tau}\text{PBC}}{\Gamma, \neg\rho \vdash \rho}\Rightarrow_e$$

$(*)$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\uparrow}{\Gamma, \neg\rho \vdash \rho}\Rightarrow_e \qquad \dfrac{}{\Gamma, \neg\rho \vdash \neg\rho}\text{ax}}{\Gamma, \neg\rho \vdash \bot}\neg_e}{\Gamma = \{(\tau \Rightarrow \rho), (\neg\tau \Rightarrow \rho)\} \vdash \rho}\text{PBC}}{(\tau \Rightarrow \rho) \vdash (\neg\tau \Rightarrow \rho) \Rightarrow \rho}\Rightarrow_i}{\vdash (\tau \Rightarrow \rho) \Rightarrow ((\neg\tau \Rightarrow \rho) \Rightarrow \rho)}\Rightarrow_i$$

VII. Implicación vs disyunción

$(\tau \Rightarrow \rho) \Leftrightarrow (\neg\tau \vee \rho)$

$(\Longrightarrow)$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{(\tau \Rightarrow \rho), \neg(\neg\tau \vee \rho) \vdash} \qquad \dfrac{}{(\tau \Rightarrow \rho), \neg(\neg\tau \vee \rho) \vdash}}{(\tau \Rightarrow \rho), \neg(\neg\tau \vee \rho) \vdash \bot}\neg_e}{(\tau \Rightarrow \rho) \vdash (\neg\tau \vee \rho)}\text{PBC}}{\vdash (\tau \Rightarrow \rho) \Rightarrow (\neg\tau \vee \rho)}\Rightarrow_i$$

$(\Longleftarrow)$

$$\dfrac{}{(\neg\tau \vee \rho) \Rightarrow (\tau \Rightarrow \rho)}$$