$$\begin{aligned} length2 &= length1 \\ length2 &\underset{\{L2\}}{=} = foldr \ (\backslash_ \ res \rightarrow 1 + res) \end{aligned}$$

Por extensionalidad, basta con probar \forall x
s $\ldots[a]$

Por inducción en xs

$$P(xs): (foldr (_res \rightarrow 1+res) 0) xs = length1 xs$$

Caso base:

$$\begin{array}{c} \text{foldr } (\setminus _\text{res} \to 1 + \text{res}) \ 0 \ [] \ \underset{\{F0\}}{\overset{=}{=}} \\ \\ \text{length1} \ [] \ \underset{\{L10\}}{\overset{=}{=}} \ 0 \end{array}$$

Paso inductivo xs = y:ys:

$$\begin{split} \mathrm{HI} = P(\mathrm{ys}) \ \mathrm{foldr}(\backslash_{-} \ \mathrm{res} \ \rightarrow 1 + \mathrm{res} \ 0) \ \mathrm{ys} = \mathrm{length1} \ \mathrm{ys} \\ \mathrm{TI} : P(\mathrm{y} : \mathrm{ys}) \\ \mathrm{foldr}(\backslash_{-} \ \mathrm{res} \rightarrow 1 + \mathrm{res}) \ 0 \ (\mathrm{y} : \mathrm{ys}) = \mathrm{length1} \ (\mathrm{y} : \mathrm{ys}) \\ &\stackrel{=}{\underset{\{\mathrm{F1}\}}{=}} (\backslash_{-} \ \mathrm{res} \rightarrow 1 + \mathrm{res}) \ y \ (\mathrm{foldr} \ (\backslash_{-} \ \mathrm{res} \ -> 1 + \mathrm{res}) \ 0 \ \mathrm{ys}) \\ &\stackrel{=}{\underset{2\beta}{=}} 1 + \mathrm{foldr} \ (\backslash_{-} \ \mathrm{res} \ -> 1 + \mathrm{res}) \ 0 \ \mathrm{ys} \\ &\stackrel{=}{\underset{\{\mathrm{HI}\}}{=}} 1 + \mathrm{length1} \ \mathrm{ys} \\ &\stackrel{=}{\underset{\{\mathrm{LI1}\}}{=}} \ \mathrm{length1} \ (\mathrm{y} : \mathrm{ys}) \Box \end{split}$$